QUANTIFYING INTER-GENERATIONAL EQUITY UNDER DIFFERENT TARGET BENEFIT PLAN DESIGNS

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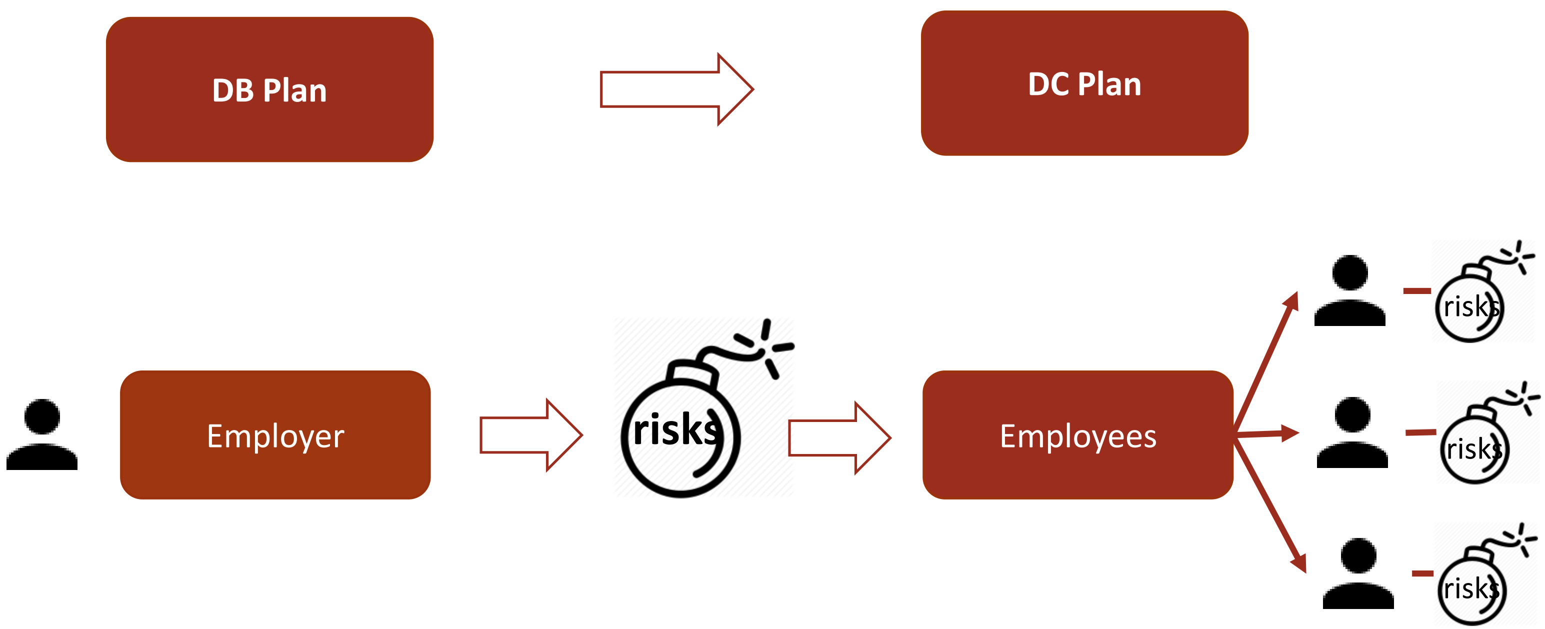
BACKGROUND – TREND



BACKGROUND – RISK SHARING

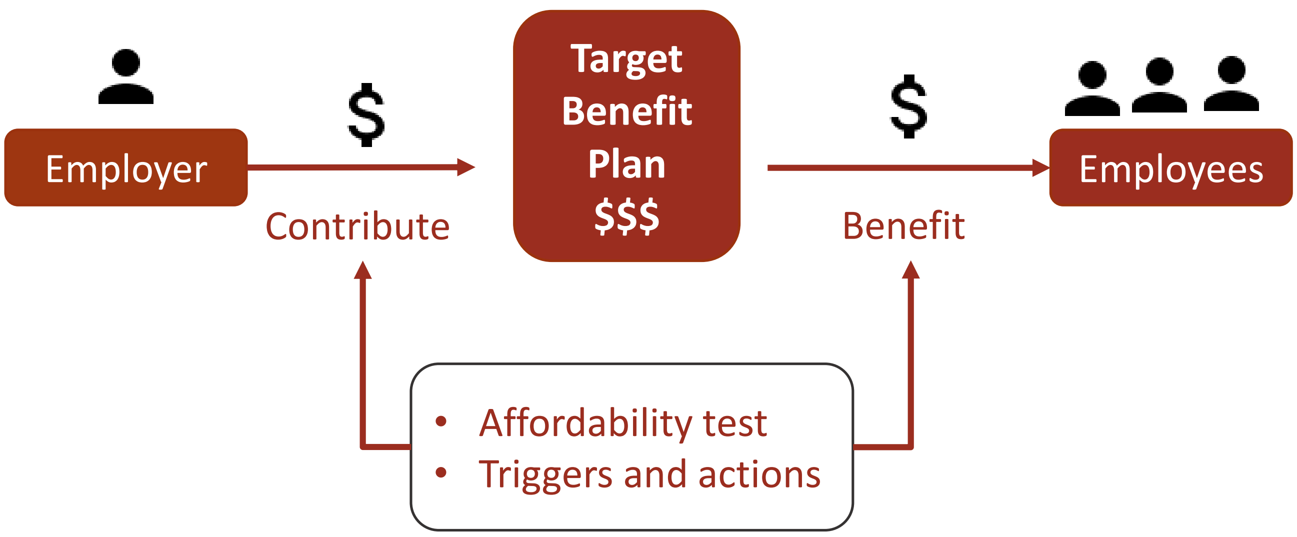
- Between employers and employees
- Across different generations:
  - Different age groups have different risk profiles

- Benefits of risk sharing discussed in Bovenberg et al. (2007), Gollier (2008), Blommestein et al. (2009), Cui et al. (2011)

BACKGROUND – TARGET BENEFIT PLAN

Target Benefit: e.g. 1% of Final Average Earnings * Service

Contribution: e.g. 15% of salary



BACKGROUND – TARGET BENEFIT PLAN

At each valuation point:

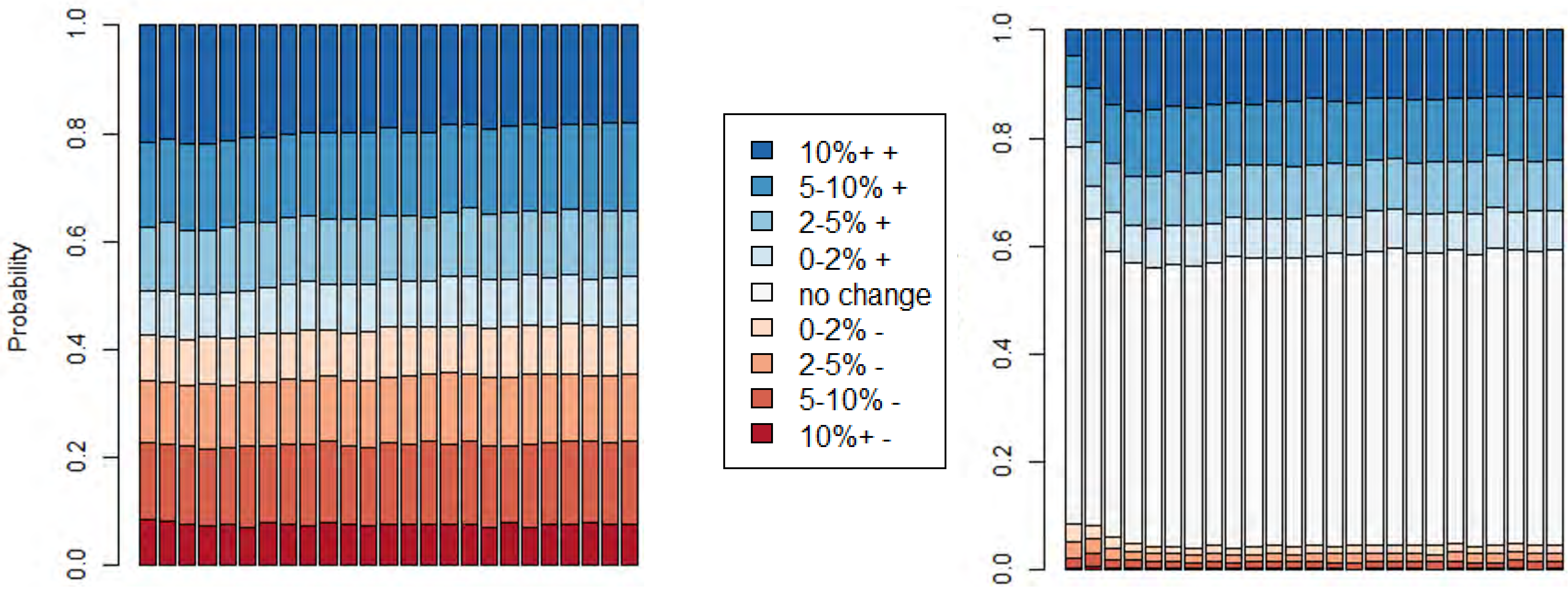
- **Affordability test: whether the target benefit is affordable**
  - Funded ratio = $\frac{Assets}{Liabilities}$

- **Triggers: whether we should take action**
  - Immediate action: e.g. funded ratio ≠ 100%
  - Delayed action: e.g. funded ratio corridor { Upper bound: 110%; Lower bound: 90% }

- **Actions: what adjustment to make**
  - Benefits (past and/or future accruals)
  - Contributions
  - Investment strategy

SANDERS (2016)

Distribution of benefit adjustments by size



Without "Corridor"

With "Corridor" (90%-110%)

OUR OBJECTIVE

- Use value-based ALM approach
  - Hoevenaars and Ponds (2007)
  - Soer (2012)
  - Lekniute, Beetsma, and Ponds (2014)
- Quantify value transfer when moving between designs

SHINNY APP

- https://retirement.shinyapps.io/new_app/

DEMOGRAPHIC MODEL

- **Entry age**: 30
- **Retirement age**: 65
- **Age at death**: 85

- **Stationary population**: 100 people at each age at all times
- Past service is recognized at plan inception

The pension fund is assumed to be liquidated after 25 years

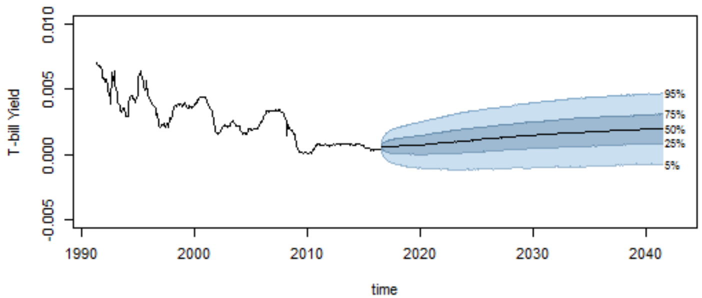
ASSET MODEL: VAR(1)

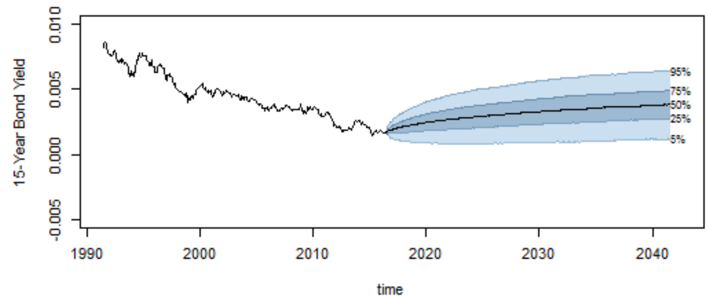
$$\boldsymbol{z_{t+1}} = \boldsymbol{v} + \boldsymbol{B z_t} + \boldsymbol{\Sigma \varepsilon_{t+1}} \quad \text{where } \varepsilon_{t+1} \sim N(0, I)$$

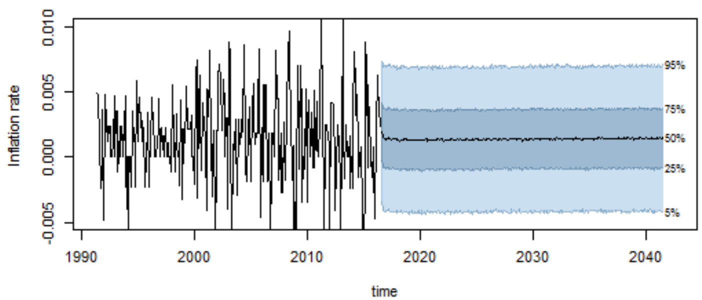
State variables:

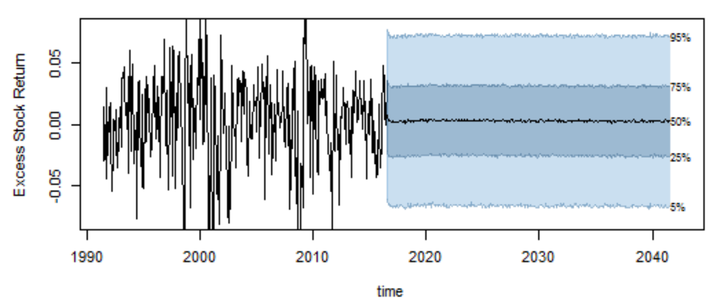
- 1-month T-bill yield
- 15-year zero coupon bond yield
- Inflation rate
- TSX stock return in excess of the 1-month T-bill rate
- Dividend yield

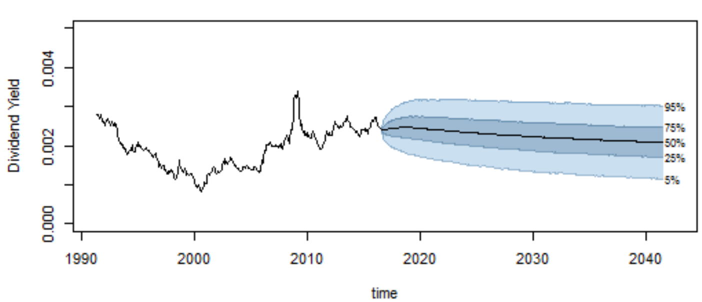
MODEL AND ASSUMPTIONS











SIMULATION STUDY

Age\t	0	1	…	20	21	…	23	24	25
6	0	0	…	0	0	…	0	$-C_{24,30}$	R_6
7	0	0	…	0	0	…	$-C_{23,30}$	$-C_{24,31}$	R_7
…	…	…	…	…	…	…	…	…	…
30	$-C_{0,30}$	$-C_{1,31}$	…	$-C_{20,50}$	$-C_{21,51}$	…	$-C_{23,53}$	$-C_{24,54}$	R_{30}
…	…	…	…	…	…	…	…	…	…
64	$-C_{0,64}$	$B_{1,65}$	…	$B_{20,84}$	$B_{21,85}$	…	0	0	0
65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

SIMULATION STUDY

Age\t	0	1	…	20	21	…	23	24	25
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…	…	…	…	…	…	…	…	…	…
30	$-C_{0,30}$	$-C_{1,31}$	…	$-C_{20,50}$	$-C_{21,51}$	…	$-C_{23,53}$	$-C_{24,54}$	R_{30}
…	…	…	…	…	…	…	…	…	…
64	$-C_{0,64}$	$B_{1,65}$	…	$B_{20,84}$	$B_{21,85}$	…	0	0	0
65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

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…	…	…	…	…	…	…	…	…	…
64	$-C_{0,64}$	$B_{1,65}$	…	$B_{20,84}$	$B_{21,85}$	…	0	0	0
65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

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30	$-C_{0,30}$	$-C_{1,31}$	…	$-C_{20,50}$	$-C_{21,51}$	…	$-C_{23,53}$	$-C_{24,54}$	R_{30}
…	…	…	…	…	…	…	…	…	…
64	$-C_{0,64}$	$B_{1,65}$	…	$B_{20,84}$	$B_{21,85}$	…	0	0	0
65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

SIMULATION STUDY

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…	…	…	…	…	…	…	…	…	…
30	$-C_{0,30}$	$-C_{1,31}$	…	$-C_{20,50}$	$-C_{21,51}$	…	$-C_{23,53}$	$-C_{24,54}$	R_{30}
…	…	…	…	…	…	…	…	…	…
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65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
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…	…	…	…	…	…	…	…	…	…
30	$-C_{0,30}$	$-C_{1,31}$	…	$-C_{20,50}$	$-C_{21,51}$	…	$-C_{23,53}$	$-C_{24,54}$	R_{30}
…	…	…	…	…	…	…	…	…	…
64	$-C_{0,64}$	$B_{1,65}$	…	$B_{20,84}$	$B_{21,85}$	…	0	0	0
65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

VALUE-BASED ALM ANALYSIS

V_6
V_7
…
V_{30}
…
V_{64}
V_{65}
…
V_{85}

Age\t	**0**	**1**	**…**	**20**	**21**	**…**	**23**	**24**	**25**
6	0	0	…	0	0	…	0	$-C_{24,30}$	R_6
7	0	0	…	0	0	…	$-C_{23,30}$	$-C_{24,31}$	R_7
…	…	…	…	…	…	…	…	…	…
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…	…	…	…	…	…	…	…	…	…
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65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

STOCHASTIC DISCOUNT FACTOR

Pricing kernel based on Ang and Piazzesi (2003) as in Hoevenaars and Ponds (2007)

$$\boldsymbol{M_{t+1} = e^{-(\delta_0+\delta_1 z_t+\frac{1}{2}\lambda_t'\Sigma'\Sigma\lambda_t+\lambda_t'\Sigma\,\varepsilon_{t+1})}}$$

One period stochastic discount factor

STOCHASTIC DISCOUNT FACTOR

Pricing kernel based on Ang and Piazzesi (2003) as in Hoevenaars and Ponds (2007)

$$\boldsymbol{M_{t+1} = e^{-\left(\underbrace{\delta_0 + \delta_1 z_t}_{\text{Short rate}} + \frac{1}{2}\lambda_t' \Sigma' \Sigma \lambda_t + \lambda_t' \Sigma\, \varepsilon_{t+1}\right)}}$$

Short rate

STOCHASTIC DISCOUNT FACTOR

Pricing kernel based on Ang and Piazzesi (2003) as in Hoevenaars and Ponds (2007)

$$\boldsymbol{M_{t+1} = e^{-(\delta_0 + \delta_1 z_t + \frac{1}{2}\lambda_t' \Sigma' \Sigma \lambda_t + \lambda_t' \Sigma \ \varepsilon_{t+1})}}$$

Time varying market risk premium

STOCHASTIC DISCOUNT FACTOR

Pricing kernel based on Ang and Piazzesi (2003) as in Hoevenaars and Ponds (2007)

$$\boldsymbol{M_{t+1} = e^{-(\delta_0+\delta_1 z_t+\frac{1}{2}\lambda_t'\Sigma'\Sigma\lambda_t+\lambda_t'\Sigma\,\varepsilon_{t+1})}}$$

Time varying market risk premium

$$\lambda_t = \lambda_0 + \Lambda_1 z_t$$

STOCHASTIC DISCOUNT FACTOR

Pricing kernel based on Ang and Piazzesi (2003) as in Hoevenaars and Ponds (2007)

$$\boldsymbol{M_{t+1} = e^{-(\delta_0+\delta_1 z_t+\frac{1}{2}\lambda_t'\Sigma'\Sigma\lambda_t+\lambda_t'\Sigma\,\varepsilon_{t+1})}}$$

Economic Value of $\boldsymbol{P_{t+k}}$

$$V_t[P_{t+k}] = E_t[M_{t+k}^* P_{t+k}], \text{ where } M_{t+k}^* = M_{t+1}M_{t+2} \dots M_{t+k}$$

VALUE-BASED ALM ANALYSIS

V_6
V_7
…
V_{30}
…
V_{64}
V_{65}
…
V_{85}

M ⇐

Age\t	0	1	…	20	21	…	23	24	25
6	0	0	…	0	0	…	0	$-C_{24,30}$	R_6
7	0	0	…	0	0	…	$-C_{23,30}$	$-C_{24,31}$	R_7
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30	$-C_{0,30}$	$-C_{1,31}$	…	$-C_{20,50}$	$-C_{21,51}$	…	$-C_{23,53}$	$-C_{24,54}$	R_{30}
…	…	…	…	…	…	…	…	…	…
64	$-C_{0,64}$	$B_{1,65}$	…	$B_{20,84}$	$B_{21,85}$	…	0	0	0
65	$B_{0,65}$	$B_{1,66}$	…	$B_{20,85}$	0	…	0	0	0
…	…	…	…	…	…	…	…	…	…
85	$B_{0,85}$	0	…	0	0	…	0	0	0

$$V_x = E(CF_{x,0} + \sum_{t=1}^{r+l-e-1} M_t^* * CF_{x,t})$$

SHINY APP

- https://retirement.shinyapps.io/new_app/

CONCLUSION

- Calibrated simple asset model based on Canadian market data
- Modeled operation of Canadian target benefit plan designs
- Applied value-based ALM method developed by Hoevenaars and Ponds (2007)
- Created Shiny app to demonstrate value shift between generations of plan members

  → App can help plan actuaries to visualize and understand the impact of each design element

FUTURE WORK

- Asset model (Lekniute et al. (2014))
- More options for affordability test, triggers and actions (Sanders (2006))
- Separate surplus and deficit options (Kocken (2008) and Soer (2012))

REFERENCES

- Ang, A., & Piazzesi, M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics,* 50(4), 745-787. doi: 10.1016/S0304-3932(03)00032-1.
- Blommestein, H. J., Janssen, P., Kortleve, & N., Yermo, J. (2009). Evaluating the Design of Private Pension Plans: Costs and Benefits of Risk-Sharing. *OECD Working Papers on Insurance and Private Pensions*, No. 34, OECD Publishing. doi: 10.1787/225162646207.
- Bovenberg, L., Koijen, R., Nijman, T., & Teulings, C. (2007). Saving and Investing Over The Life Cycle and The Role of Collective Pension Funds. *De Economist,* 155, No. 4. doi: 10.1007/s10645-007-9070-1.
- Cui, J., Jong F., & Ponds, E. (2011). Intergenerational Risk Sharing within Funded Pension Schemes. *Journal of pension economics and finance,* 10(01), 1-29. doi: 10.2139/ssrn.989127.
- Cochrane, J.H., & Piazzesi, M. (2005). Bond Risk Premia. *The American Economic Review*, 95(1), 138-160. doi: 10.1257/0002828053828581.
- Gollier, C. (2008). Intergenerational Risk-sharing and Risk-taking of A Pension Fund. *Journal of Public Economics,* 92(5), 1463-1485. doi: 10.1016/j.jpubeco.2007.07.008.

REFERENCES

- Hoevenaars, R.P. (2008). *Strategic asset allocation & asset liability management.* PhD Dissertation. Universiteit Maastricht. Retrieved from http://digitalarchive.maastrichtuniversity.nl/fedora/get/guid:8e6f53dc-9a2d-4200-b103-4fd65302f516/ASSET1.
- Hoevenaars, R.P. and Ponds, E. Valuation of Intergenerational Transfers in Funded Collective Pension Schemes (2007). Available at SSRN: https://ssrn.com/abstract=942266.
- Kocken, T. Curious Contracts: Pension Fund Redesign for the Future. Uitgeverij Tutein Nolthenius in ‘s-Hertogenbosch, The Netherlands (2006). ISBN 90-72194-78-0, 242 pages
- Lekniute, Z., Beetsma, R.M., & Ponds, E.H. (2014). A Value-based Approach to the Redesign of US State Pension Plans. doi: 10.2139/ssrn.2438637.
- Sanders, B. (2016). Analysis of Target Benefit Plan Design Options. Retrieved from https://www.soa.org/Files/Research/research-2016-analysis-tbp-plan-design.pdf.
- Slagmolen, C. (2010). Economic Scenarios for an Asset and Liability Management Study of a Pension Fund. University of Groningen. Retrieved from http://arno.uvt.nl/show.cgi?fid=113711.
- Soer, M. (2012). Fairness between generations in the new Dutch pension agreement. University of Groningen. Retrieved from http://www.ag-ai.nl/download/13655-thesis.

QUESTIONS?

Thank you!

SUMMARY STATISTICS

	x_t	y_t^{180}	inf_t	s_t	div_t
μ	0.0025	0.0044	0.0015	0.0021	0.0019
σ	0.0017	0.0018	0.0034	0.0421	0.0005

VAR ESTIMATE ($\boldsymbol{\beta}$)

	y_t^1	y_t^{180}	inf_t	s_t	div_t	R^2
y_{t+1}^1	0.9517 (0)	0.0215 (0.1982)	-0.0019 (0.6870)	-0.0003 (0.4903)	-0.0789 (0.0122)	0.9728
y_{t+1}^{180}	0.0185 (0.116)	0.9761 (0)	0.0005 (0.868)	-0.0003 (0.243)	-0.0050 (0.809)	0.9892
inf_{t+1}	0.0976 (0.6387)	-0.1509 (0.4422)	0.1659 (0.0037)	0.0106 (0.0195)	-0.5451 (0.1395)	0.0413
s_{t+1}	-2.2977 (0.3838)	1.9292 (0.4383)	-0.3387 (0.6379)	0.1617 (0.0052)	1.0039 (0.8299)	0.015
div_{t+1}	0.0029 (0.2779)	-0.0086 (0.0011)	-0.0010 (0.1842)	-0.0017 (0)	0.9782 (0)	0.9934

VAR ESTIMATE (υ)

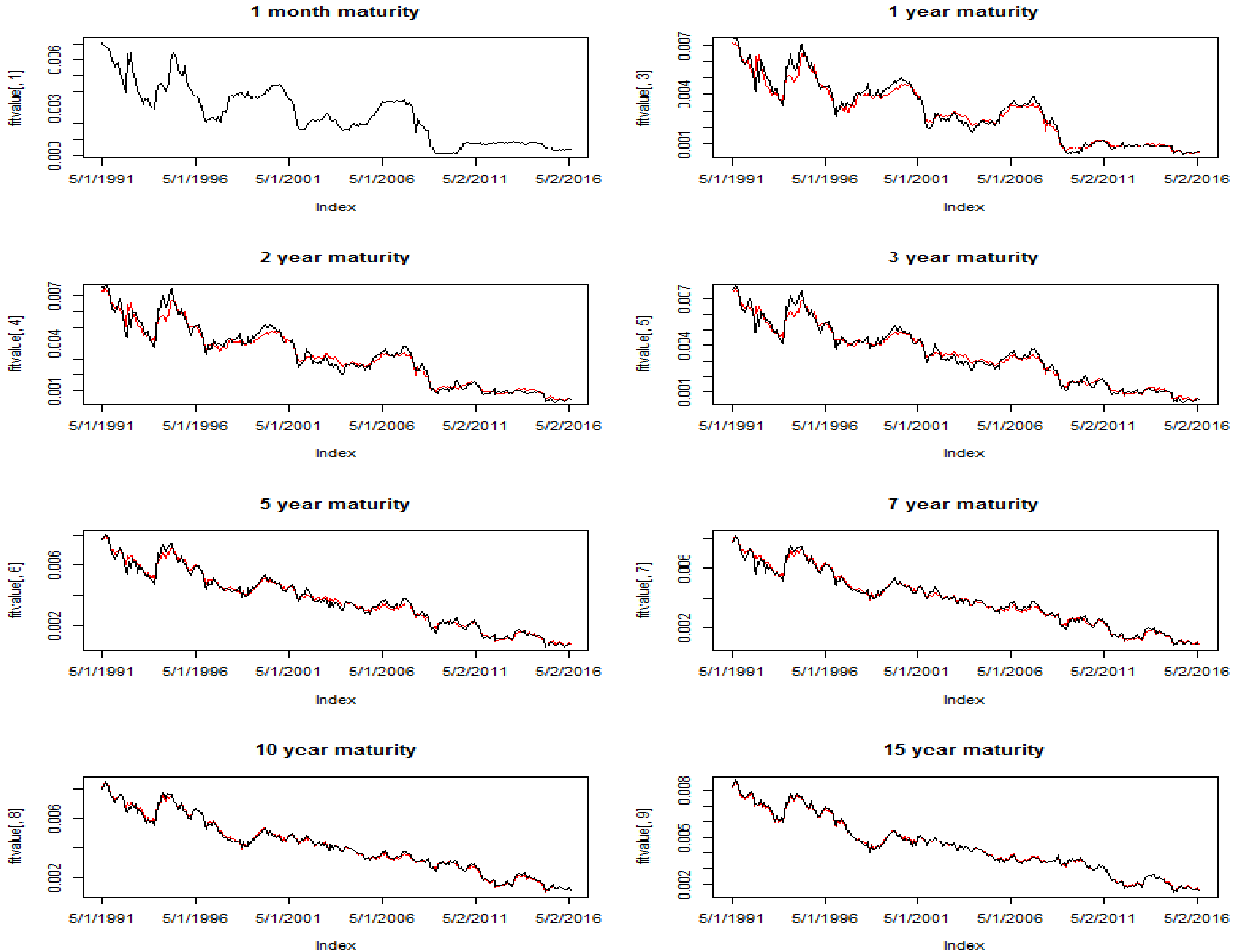
y_t^1	y_t^{180}	inf_t	s_t	div_t
0.0018	0.00007	0.00269	-0.00230	0.00007

VAR ESTIMATE ($\boldsymbol{\Sigma}$)

	y_t^1	y_t^{180}	inf_t	s_t	div_t
y_t^1	0.00028				
y_t^{180}	0.00002	0.00018			
inf_t	0.00031	0.00037	0.00326		
s_t	-0.00006	0.00389	-0.00089	0.0415	
div_t	0	0	0	0	0.00004

RISK PREMIA ($\boldsymbol{\Sigma\lambda_0}$, $\boldsymbol{\Sigma\Lambda_1}$)

	$\boldsymbol{y^1}$	$\boldsymbol{y^{180}}$	$\boldsymbol{inf}$	$\boldsymbol{s}$	$\boldsymbol{div}$	$\boldsymbol{\Sigma\lambda_0}$
$\boldsymbol{y^1}$	-0.0010	-0.0299	-0.0021	0.0002	-0.00421	0.00015
$\boldsymbol{y^{180}}$	0.0222	-0.0250	0.00042	0	-0.0043	0.00004
$\boldsymbol{inf}$	0	0	0	0	0	0
$\boldsymbol{s}$	-2.2977	1.9291	-0.3387	-0.1616	1.0039	-0.00229
$\boldsymbol{div}$	0	0	0	0	0	0


1 month maturity
fitvalue[, 1]
0.000
0.003
0.006
1 year maturity
fitvalue[, 3]
0.001
0.004
0.007
2 year maturity
fitvalue[, 4]
0.001
0.004
0.007
3 year maturity
fitvalue[, 5]
0.001
0.004
0.007
5 year maturity
fitvalue[, 6]
0.002
0.006
7 year maturity
fitvalue[, 7]
0.002
0.006
10 year maturity
fitvalue[, 8]
0.002
0.006
15 year maturity
fitvalue[, 9]
0.002
0.005
0.008
5/1/1991
5/1/1996
5/1/2001
5/1/2006
5/2/2011
5/2/2016
Index

SIMULATION STUDY

Simple plan design to study

t=0 t=1

F0 F1

F_0: Initial Fund
B_0: Initial Benefit
C_0: Initial Contribution

$$F_1: \left(F_0 + (C_0 - B_0)\right) * (1 + Asset\ Return)$$

Target Accrued Liability

- Target Accrual rate
- Valuation rate
- Valuation method

$$FR = \frac{F_1}{TAL_1}$$

Triggers and Actions

C_1 , B_1

HOW DOES M WORK

Risk-Neutral distribution: $\pi^*(s) = \frac{M(s)}{E(M)}\pi(s)$

Risk-Neutral pricing: $P(x) = \frac{1}{R^f}\Sigma\pi^*(s)x(s)$

If M(s) = E(M) in all scenario => risk free

$$\frac{1}{R^f}\Sigma\pi^*(s)x(s) = \frac{1}{R^f}\Sigma\pi(s)x(s)$$

If m(s) ≠ E(m)

$$\frac{1}{R^f}\Sigma\pi^*(s)x(s) < \frac{1}{R^f}\Sigma\pi(s)x(s)$$