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Understanding the Riskiness of a GLWB Rider for FIAs

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ABSTRACT

GLWB guarantees have different risks when attached to an FIA vis-a-vis a VA. In this article we will assess the risks associated with this rider and analyze how different modeling choices can affect these risks. In particular, the impacts of improving the estimate of future caps will be explored.

INTRODUCTION

Insurance guarantees are exotic in nature because they have to take into consideration not only actuarial parameters (e.g., mortality) but have to address financial market guarantees and be tailored to more detail.

Given that exotic derivatives can be, in general, very sensitive to all kinds of modeling assumptions we immediately see that their appropriate modeling is a key for a company dealing with more and more narrow profit margins and lower returns on investments.

Cliquets/Monthly Sum Cap is a sequence of forward starting options—such a simple feature surprisingly results in quite a lot of pricing difficulties. When one puts a guarantee on such an index (FIA GLWB) one sees that it may inherit such sensitivities. One can encounter even a bit more exotic modification which would be GLWB on a Monthly Sum Cap on a Vol Controlled index.

SETUP

In the next few sections we will lay out assumptions underlying our analysis as well as, for the sake of completeness, recall some standard definitions.

We will be focusing on exploring how assumptions about the cap value for a Point to Point construct affects the price and Greeks of a policy with a GLWB rider.

Renewal cap setting for Point to Point.

In order to investigate different approaches to modeling the cap setting we need to make a few assumptions which are the most relevant for the analysis. Those will include Index Modeling, General Account/Budget assumption and Hedging assumptions.

Index Modeling

While it may be the simplest case, we are going to focus on a Point to Point indexing to illustrate the concept of cap renewal.

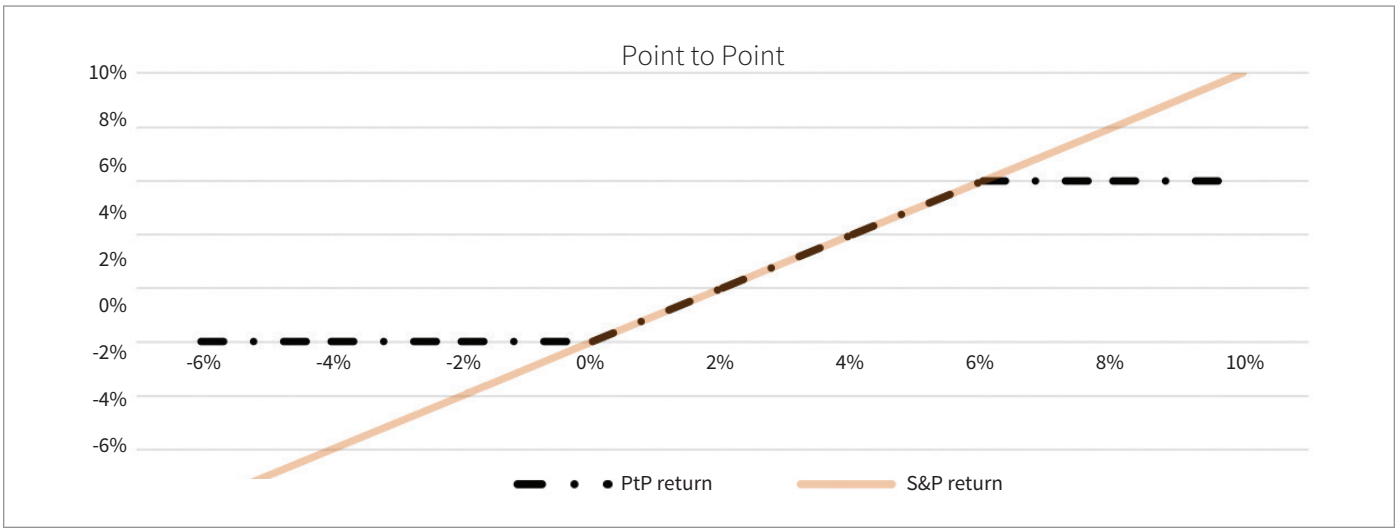
The Point to Point structure

The cap is a limit on what an FIA policy can return. FIAs have caps and floors. The caps help control costs and make it cheaper to have a floor. For the policyholder, the floor is the index guarantee they are buying and in order to keep the costs lower they give up some upside potential (set by the cap).

Insurers have the right to reset these caps on every renewal year. In simulations, however, often times that cap setting does not change. It is reasonable though to model the dynamic nature of that feature in the simulation.

Figure 1

Point to Point with a floor at 0 percent and cap of 6 percent and participation rate of 100%



The cap determined based on the available budget coming from returns on the General Account as well as market environment (cost of options in case of a static hedge).

We will discuss the General Account setup next, but for the completeness of the discussion we recall some relevant structures met in VAs and FIAs for a side by side comparison.

The following are formulas for index growth for Variable Annuities and for Fixed Index Annuities (two of the three most commonly used: Monthly Sum Caps and Point to Point (PTP)):

- VA Fund Performance

$$Fund\ return\ = \frac{S_T}{S_0} - 1$$

- FIA Crediting Mechanics

$$PTP\ = \min\left(cap, p \cdot max\left(0, \frac{S_T}{S_0} - 1\right)\right)$$

$$Monthly\ Sum\ Caps\ = \max\left(0, \sum \min\left(cap, \frac{S_i}{S_{i-1}} - 1\right)\right)$$

where p in the PTP formula is the participation rate.

Looking through the prism of put/call payoff formulas, we immediately see that PTP as well as *Monthly Sum Caps* exhibit that option-like structure.

For VA, capital is invested directly into funds and no optionality is involved. On the other hand, index credits are awarded for the FIAs depending on performance. Market risk is transferred from policyholder to insurer.

Option Budget

The General Account modeling is a common problem for many areas of insurance, including VAs and FIAs. The General Account is mainly composed of a diversified fixed income portfolio invested in Treasuries, corporates and mortgages. It enjoys yield from rates, credit spreads and structure premia.

A Fixed Annuity credits policyholders with the income of this portfolio, less a spread; an Indexed Annuity swaps this credit for an equity option.

In this study we accept a 10y swap rate as a proxy for the crediting rate. It is a reasonable assumption and, most of all, fairly straightforward to simulate.

The 10y swap rate will evolve through time and on each path independently. So will option indices which are used to find the right cap, given the budget.

Hedging Assumptions

In general, insurance companies should consider static or dynamic hedging. In this article, we focus on the static hedge.

Static Hedging is going to be a perfect hedge for the underlying index. However, due to mismatch of lapse assumptions and experience, an overhedge may appear.

Dynamic Hedging on the other hand will require hedge strategy replicated in the coding and requires pricing of assets throughout the projection.

One may also perform hedging of the base policy and GLWB together.

RENEWAL CAP SETTING. ANALYSIS.

The main theme of our article is focused on this problem. We consider two approaches:

- A static cap of 4 percent
- A dynamic cap, which is determined as follows:
  - Option budget estimated as 10y swap rate
  - Obtain European call prices using American Monte Carlo framework
  - Solve for cap/call spread using the option budget and option prices
  - Cap is reset annually

RESULTS OF THE STUDY

Details of the policy contract being modeled:

Base Policy

- Point to Point
- Age 60
- Account Value of $100 at start of projection

GLWB Rider

- Rollup Rate of 5 percent
- Rider Fee of 0.85 percent
- Withdrawals begin at 72 at 6 percent of Benefit Base

The results of the simulation look as follows:

	Static Cap	Resetting Cap
PV (Fees – Claims)	$(0.26)	$2.23
Fair GLWB Fee	0.88%	0.61%
Delta ($)	$0.043	$0.040
Rho ($)	$0.335	$0.605

We immediately see that for this particular setup (in this particular market environment), Resetting Cap case shows higher PV and in consequence a lower fee could be offered to the client. We also observe higher sensitivity to interest rates for Resetting Cap. This is to be expected because the budget for the cap (and hence the cap level) now depends on the level of interest rates (10y swap rate).

We emphasize the dependence of the results on the market environment as it is a crucial component which may swing Static Cap vs Resetting Cap results. It depends on the relationship between levels of interest rates and the cap assumption in the static case.

Another illustration worth looking at is the average claims and fees level:

Figure 2
Dynamic Cap Setting

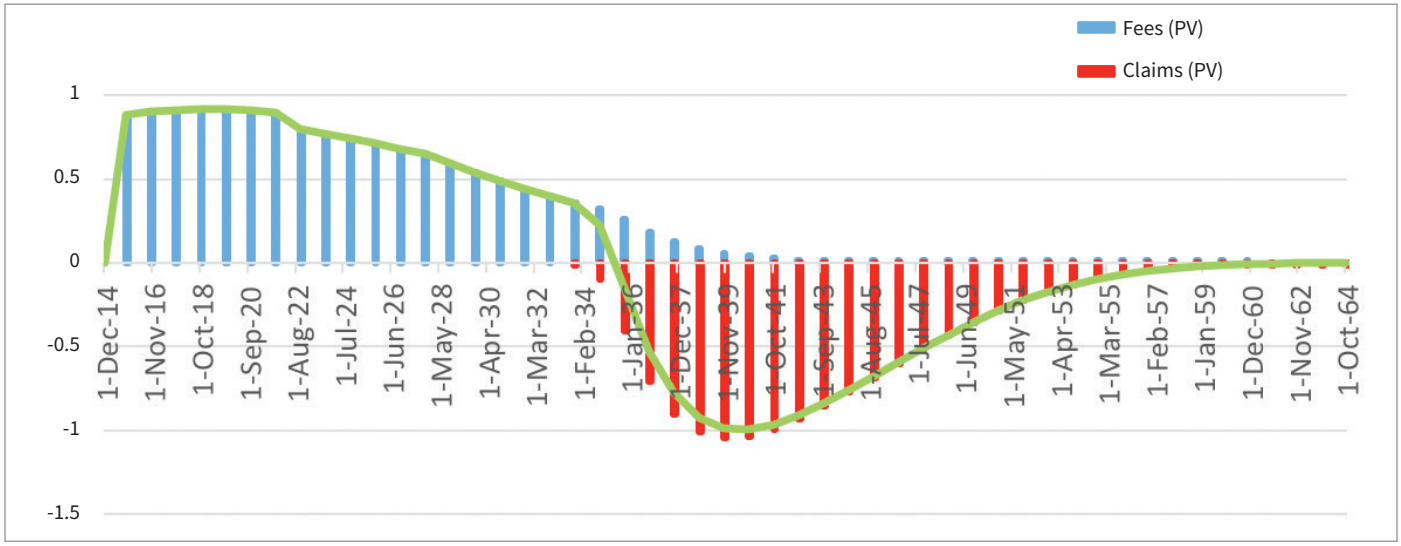
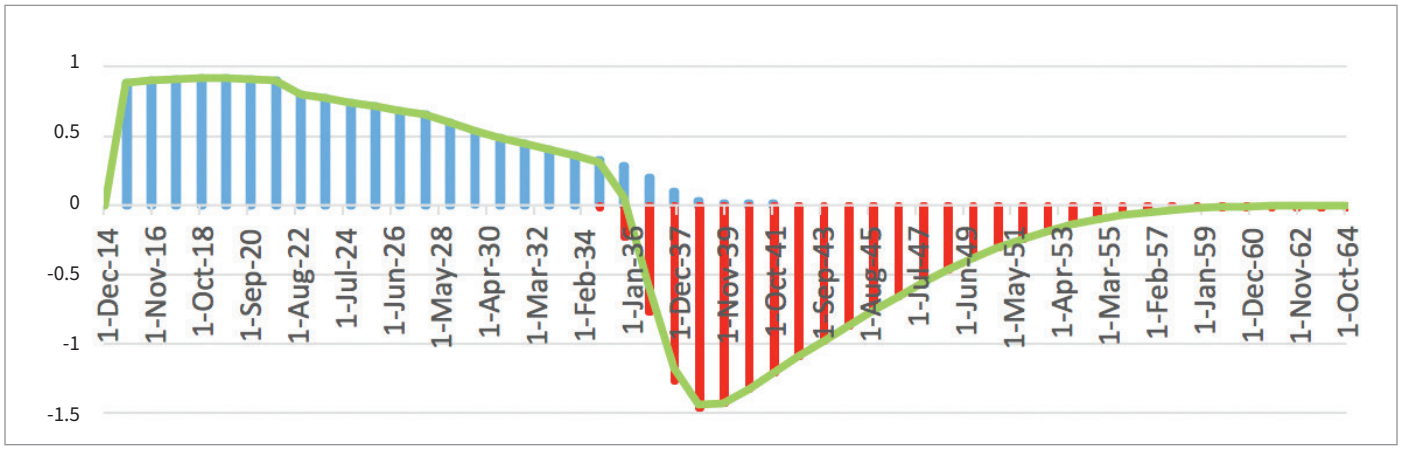


Figure 3
Static Cap



We see that claims (on average) begin sooner with Dynamic Cap Setting. That is to be expected because there will be paths where the cap remains below static 4 percent resulting in lower AV and claims occurring sooner. At the same time, the magnitude of those is smaller due to the term structure allowing one to have a higher than 4 percent cap.

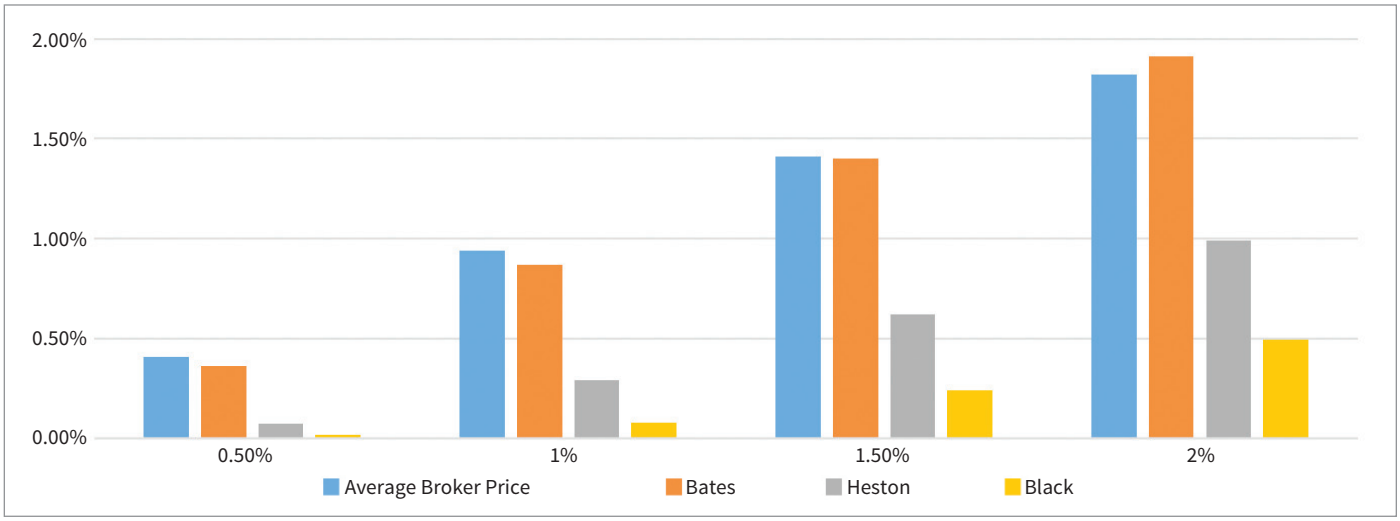
Forward starting optionality may be sensitive to modeling assumptions. We would like to present another example of how drastic of a difference market model choice can make.

MONTHLY SUM CAP: ILLUSTRATION OF IMPACT OF THE MARKET MODEL CHOICE

Monthly Sum Caps (Cliquets) are a popular FIA index choice. Their modeling can be quite challenging and we will illustrate its impact on the prices.

In the case of a base contract being statically hedged by purchasing OTC options, one needs to model the price of those in the future. The following graph illustrates how much of an impact different choices of market models can have on the price:

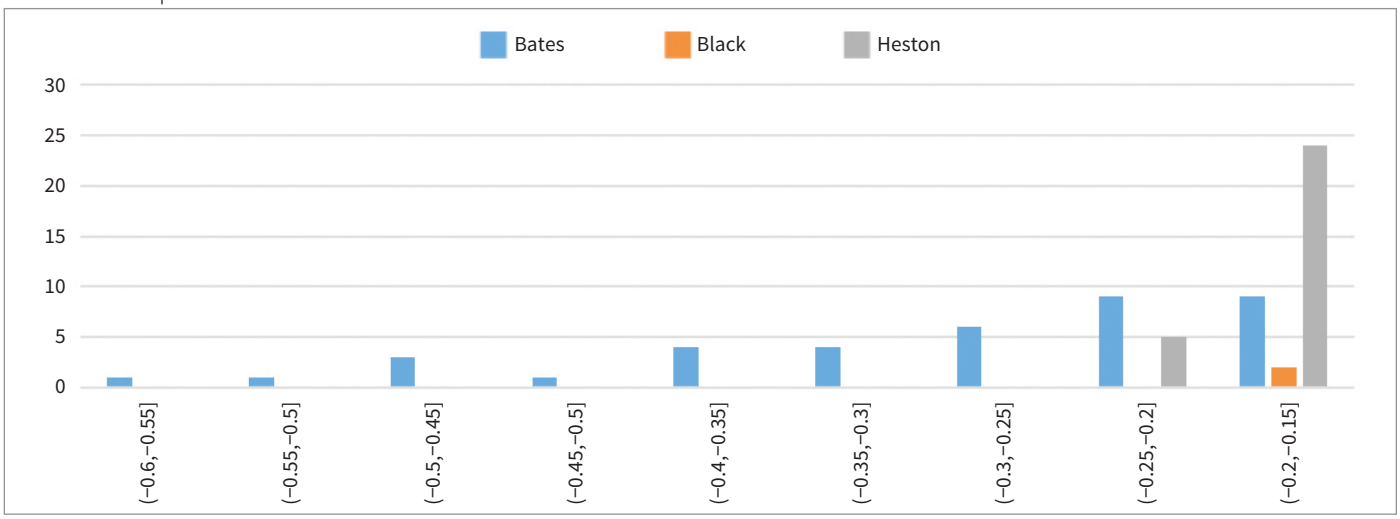
Figure 4
Monthly Sum Cap prices compared with counterparty quotes as of 2/24/15



We see that the choice of using the Bates equity model to price those deals gives one the closest price to the market quotes, but what is more important is that other choices (Heston and Black) result in drastically lower values.

The reason behind such drastic differences lies behind the distribution of returns for those different models (calibrated to the same market data). To further illustrate this observation, the graph below shows a left tail distribution of returns for Bates, Heston and Black models calibrated to the same market data.

Figure 5
Left tail of simple return distribution

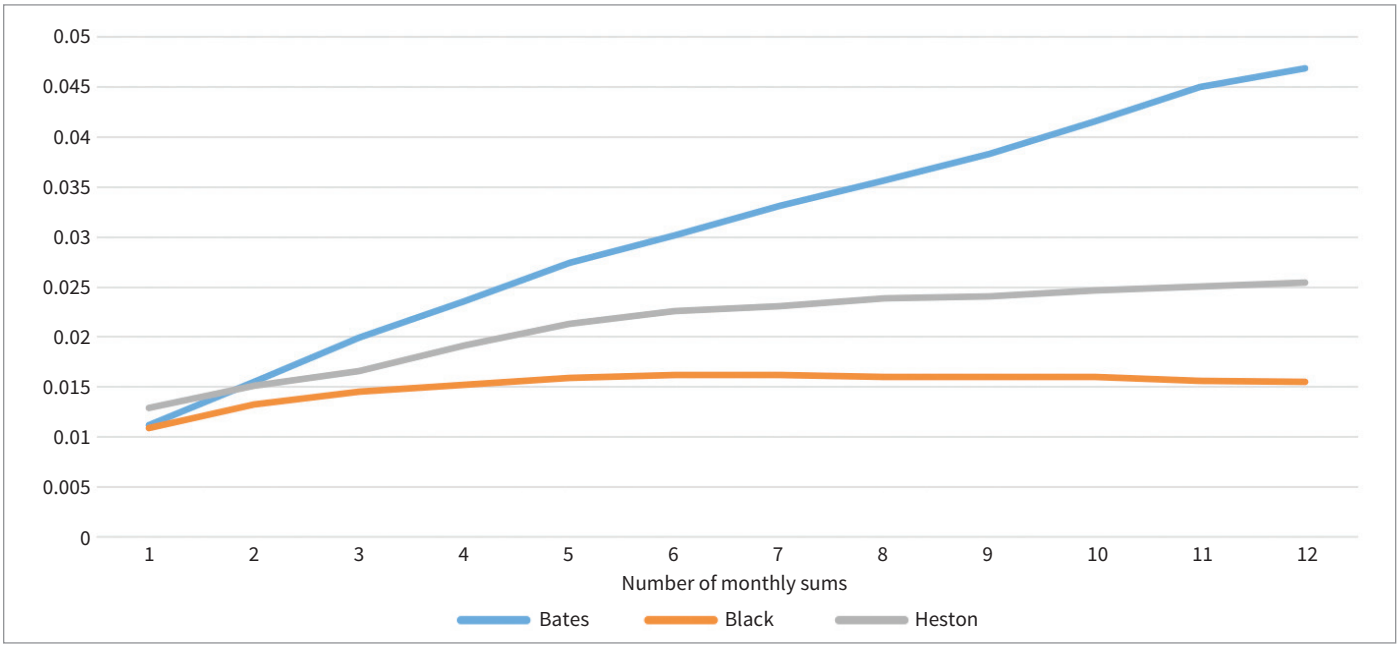


For the sake of readability we omit the full distribution range.

We see, as expected, that Bates and Heston models are heavier in tails than Black. Bates model is much heavier in negative returns than Heston. All of that is relevant to the monthly sum cap. Drawing from a qualitatively same distribution and capping the returns (monthly sum cap) will make those differences even more prominent.

In order to back up that observation with real numbers, we provide the following graph of price of a sum cap under different models and depending on how many sum-periods are considered.

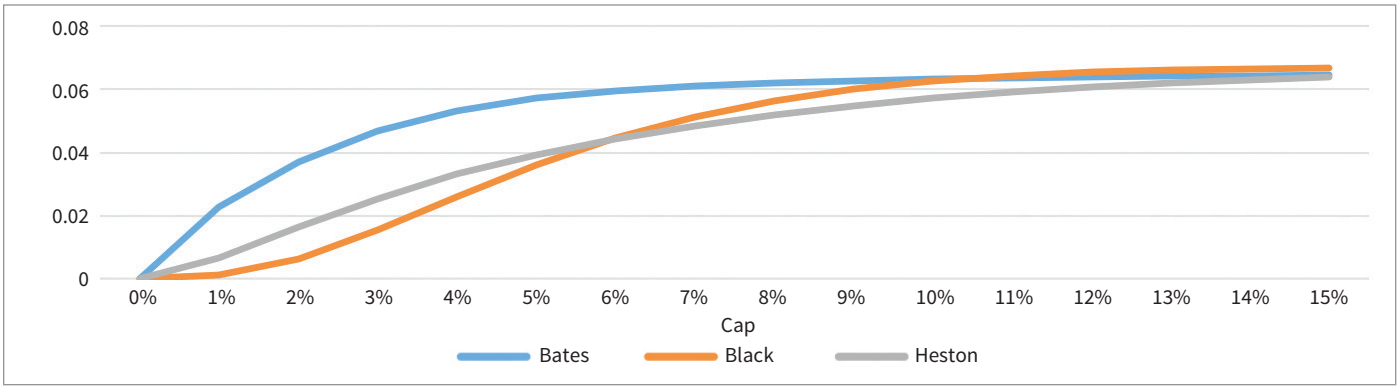
Figure 6
Price of Monthly Sum Cap – 3% cap



Moreover, it is expected that the value of the cap will have significant influence on the results. To see that more easily, one should think of similarities between applying the cap and the shape of the Cumulative Distribution Function of a given distribution. When the cap is (artificially) high, it will have limited effect on the value of the monthly sum cap and one ends up with just valuing the floor. Hence one expects all models to value it similarly. On the other hand, setting the cap equal to zero (see the formula for the monthly sum cap) results in 0 value of the monthly sum cap, regardless of the model.

The shape of the curve (x axis being the cap value, y axis being the value of the sum cap) will depend on the distribution of returns. For the completeness of this part of the study, we present a graph depicting these results:

Figure 7
Value of the Monthly Sum Cap



SUMMARY

In this article, we focus on some problems insurance companies may be facing when modeling Fixed Index Annuities. We illustrate that those challenges are quite impactful and should be taken into consideration in the modeling process. Forward starting optionality, as well as modeling choices (effectively the choice of distribution of returns)—which is seen in both the resetting cap feature as well as pricing of a MSC—should be treated with care and will have significant impact. ■

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