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Risk Aggregation and Diversification

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OVERVIEW

This report reviews the academic literature on risk aggregation and diversification as well as the regulatory approaches. We will point out the advantages and disadvantages of the different approaches with a focus on model risk issues.

We first discuss, in section 1, the basic fundamentals of measuring aggregated risk. Specifically, we review the concept of a risk measure as a suitable way to measure the aggregate risk. We discuss desirable properties of risk measures and illustrate our discussion with the study of value-at-risk (VaR) and tail value-at-risk (TVaR).

Section 2 explores the question of diversification benefits associated with risk aggregation and the potential limitations of correlations as the only statistic to measure dependence. We go beyond correlations and explain that a full multivariate model is needed to obtain a correct description of the aggregate risk position.

We then explore the regulators approach to risk aggregation and diversification in section 3, and provide some observations on the implicit assumption made by international regulators and different approaches that can be taken.

We end our review by highlighting that model risk becomes a key issue in measuring risk aggregation and diversification. In section 4, we explore a framework that allows practical quantification of model risk and which has been recently developed in Bernard and Vanduffel [2015a][1] (building further on ideas of Embrechts et al. [2013][2]). Details are provided in appendices A and B. Appendix C presents the definitions of the mathematical notations used throughout the research paper.

INTRODUCTION

The risk assessment of high-dimensional portfolios $(X_1, X_2, \ldots, X_d)$ is a core issue in risk management of financial institutions. In particular, this problem appears naturally for an insurance company. An insurer is typically exposed to different risk factors (e.g., non-life risk, longevity risk, credit risk, market risk, operational risk), has different business lines or has an exposure to several portfolios of clients. In this regard, one typically attempts to measure the risk of a random sum, $S = \sum_{i=1}^{d} X_i$, in which the individual risks X_i depict losses (claims of the different customers, changes in the different market risk factors, etc.) using a risk measure such as the variance, the VaR or the TVaR[3]. It is clear that solving this problem is mainly a numerical issue once the joint distribution of $(X_1, X_2, \ldots, X_d)$ is completely specified. Unfortunately, estimating a multivariate distribution is a difficult task. In many cases, the actuary will be able to use mathematical and statistical techniques to describe the marginal risks X_i fruitfully but the dependence among the risks is not specified, or only partially specified. In other words, the assessment of portfolio risk is prone to model misspecification (model risk).

From a mathematical point of view, it is then often convenient to assume that the random variables X_i are mutually independent, because powerful and accurate computation methods such as Panjer's recursion and the technique of convolution can then be applied. In this case, one can also take advantage of the central limit theorem, which states that the sum of risks, S, is approximately normally distributed if the number of risks is sufficiently high. In fact, the mere existence of insurance is based on the assumption of mutual independence among the insured risks, and sometimes this complies, approximately, with reality. In the majority of cases, however, the different risks will be interrelated to a certain extent. For example, a sum S of dependent risks occurs when considering the aggregate claims amount of a non-life insurance portfolio because the insured risks are subject to some common factors such as geography, climate or economic environment. The cumulative distribution function of S can no longer be easily specified.

Standard approaches to estimating a multivariate distribution among dependent risks consist in using a multivariate Gaussian distribution or a multivariate Student t distribution, but there is ample evidence that these models are not always adequate. More precisely, while the multivariate Gaussian distribution can be suitable as a fit to a data set "on the whole", it is usually a poor choice if one wants to use it to obtain accurate estimates of the probability of simultaneous extreme (tail) events, or, equivalently, if one wants to estimate the VaR of the aggregate portfolio $S = \sum_{i=1}^{d} X_i$ at a given high confidence interval; see McNeil et al. [2010][4]. The use of the multivariate Gaussian model is also based on the (wrong) intuition that correlations[5] are enough to model dependence (Embrechts et al. [1999][6], Embrechts et al. [2002][7]). This fallacy also underpins the variance-covariance standard approach that is used for capital aggregation in Basel III and Solvency II, and which also appears in many risk management frameworks in the industry. Furthermore, in practice, there are not enough observations that can be considered as tail events. In fact, there is always a level beyond which there is no observation. Therefore if one makes a choice for modelling tail dependence, it has to be somewhat arbitrary, at least not based on observed data.

There is recent literature on the development of flexible multivariate models that allow a much better fit to the data using, for example, pair-copula constructions and vines (see e.g., Aas et al. [2009][8] or Czado [2010][9] for an overview). While these models have theoretical and intuitive appeal, their successful use in practice requires a data set that is sufficiently rich. However, no model is perfect, and while such developments are clearly needed for an accurate assessment of portfolio risk, they are only useful to regulators and risk managers if they are able to significantly reduce the model risk that is inherent in risk assessments.

In this review, we provide a framework that allows practical quantification of model risk and which has been recently developed in Bernard and Vanduffel [2015a][10] (building further on ideas of Embrechts et al. [2013][11] and references herein). Technically, consider N observed vectors $\{(X_{1i}, \ldots, X_{di})\}_{i=1,\ldots,N}$ and assume that a multivariate model has been fitted to this data set. However, one does not want to trust the fitted multivariate model in areas of the support that do not contain enough data points (e.g., tail areas). The idea is thus to split $\mathbb{R}^d$ into two subsets, the first subset $\mathcal{F}$ is referred to as the "fixed part" and the second subset $\mathcal{U}$ is the "unfixed part," which will incorporate all the areas for the fitted model is not giving an appropriate fit. This incorporates the two directions discussed above for risk aggregation. If one has a perfect trust in the model, then all observations are in the "fixed" part ($\mathcal{U} = \emptyset$) and there is no model risk. If one has no trust at all in the fit of the dependence, then $\mathcal{F} = \emptyset$ and we are in the setting of Embrechts et al. [2013][12] who derive risk bounds for portfolios when the marginal distributions of the risky components are known but no dependence information is available. The approach of Bernard and Vanduffel [2015a][13] makes it possible to consider dependence information in a natural way and may lead to more narrow risk bounds. This framework is also supplemented with an algorithm allowing actuaries to deal with model risk in a very practical way, as we will show in full detail. ■

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ENDNOTES

1 This paper received the 2014 PRMIA Award for New Frontiers in Risk Management. Carole Bernard and Steven Vanduffel. A New Approach to Assessing Model Risk in High Dimensions. *Journal of Banking and Finance*, 58:166–178, 2015a

2 Paul Embrechts, Giovanni Puccetti, and Ludger Rüschendorf. Model uncertainty and VaR aggregation. *Journal of Banking & Finance*, 37(8):2750–2764, 2013.

3 In the literature it is also called the expected shortfall, the conditional value at risk and the tail value-at-risk, among others.

4 Alexander J. McNeil, Rüdiger Frey, and Paul Embrechts. *Quantitative Risk Management: Concepts, Techniques, and Tools*. Princeton University Press, 2010.

5 It should be clear that using correlations is not enough to model dependence, as a single number (i.e., the correlation) cannot be sufficient to describe the interaction between variables unless additional assumptions are made (e.g., a Gaussian dependence structure).

6 Paul Embrechts, Alexander McNeil, and Daniel Straumann. Correlation: Pitfalls and Alternatives. *Risk London, Risk Magazine*, 12:69–71, 1999.

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8 Kjersti Aas, Claudia Czado, Arnoldo Frigessi, and Henrik Bakken. Pair-copula constructions of multiple dependence. *Insurance: Mathematics and Economics*, 44(2):182–198, 2009.

9 Claudia Czado. Pair-copula constructions of multivariate copulas. In *Copula Theory and Its Applications*, pages 93–109. Springer, 2010.

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