Ruin Probabilities in Multivariate Risk Models with Periodic Common Shock

Ionica Groparu-Cojocaru\(^1\) and Jose Garrido

\(^1\) Concordia University, Canada; ionica.groparu-cojocaru@concordia.ca

In an insurance company, for a given portfolio of insurance policies of different types such as health, automobile, or house insurance, it is advantageous to have an accurate forecast of the expected liability of these policies. In practice, there are situations in which the assumption of independent policies is not verified. For example, in the case of a catastrophe such as an earthquake, the damages covered by homeowners and private passenger automobile insurance cannot be considered independent. Therefore, it is desirable to develop models which assume that different policies are dependent in order to increase the accuracy of the estimation of the costs associated to different policies.

These reasons motivate us to consider investigating multivariate risk processes which may be useful in studying ruin problems for insurance companies handling dependent classes of business. As pointed out in Chan et al. (2003), ruin theory under multidimensional risk models is very complex. Even in a two-dimensional case, the problem is challenging. In ruin theory under multidimensional risk models, it is usually difficult to derive explicit results for the probability of ruin.

We assume that an insurance company has \(m\) different classes of insurance business allowing for dependence between claim sizes and dependence among the numbers of claims across classes. As a method to model the dependence between the number of claims, we propose a more general model with common shock, in the sense that, for each class of business, the individual shocks arrive according to homogeneous Poisson processes, while the common shock which affects all classes of business is arriving according to a non-homogeneous periodic Poisson process. The reason of considering the common shock as a non-homogeneous periodic Poisson process is due to the fact that extreme events such as hurricanes, tornados appear with some periodicity, specific to some seasons during the year term, which may cause claims on all types of insurance classes.

In this multivariate setting, we derive upper bounds of Lundberg-type for the probability that ruin occurs in all classes simultaneously, which are numerically illustrated in a bivariate risk model considering special cases for the periodic claim intensity function. These cases include a trigonometric form and the cases of single- and double-beta periodic intensity functions introduced by Lu and Garrido (2005). An asymptotic upper estimate of the finite-time ruin probability of this type in the case of dependent heavy-tailed claims, as the initial surplus for each class of business increases, is also obtained.