SOCIETY OF ACTUARIES

Article from:

Risks and Rewards Newsletter

October 2002 – Issue No. 40

How Many Scenarios?

by David Ingram

As the Risk Management Task Force was forming, we found that we had many more topics that we were curious about than we had time to pursue. One of those topics related to the appropriate number of scenarios. This is how the questions were framed by one of the RMTF members:

"How many scenarios are necessary for various uses of Monte Carlo models? Stochastic simulation models are used to determine values for many nonlinear risk factors. Practical considerations on computer run times have sometimes limited the number of random scenarios that are used. With newer models and more powerful computers, run time is now a smaller constraint. At the same time, work on extreme value theory and fat-tailed distributions has heightened awareness of the importance of looking carefully at low-frequency situations rather than screening them out of consideration as unreliable outliers. Research into the criteria needs to be applied to determine the number of stochastic scenarios that are adequate for different actuarial problems such as ALM, credit loss, mortality, morbidity, operational risk and equity market models when used in pricing, valuation and risk management situations. Does the number of scenarios needed to obtain credible results vary based on the underlying random process that is being modeled and/or the purpose of the model? What is needed to obtain credible results if multivariate models are used where several high correlation and low correlation random variables are used?"

Instead of forming a study group, we decided to try polling *Risk & Rewards* readers to see if the collective wisdom offered any answers. The poll asked the following questions:

1. How many scenarios do you run?
2. How did you determine the number?
3. What confidence interval does your result have?

The model can generate random numbers on the fly (by using a positive seed number) or generate adjusted random numbers to reduce the dependence of the results on the particular seed chosen (by using a negative seed number). The DI Working Group used the latter. For a specific set of assumptions, seven seed numbers were tested under the number of scenarios shown in Table 3.

TABLE 3

	Standard Deviation of Results from Seven Seed Numbers	
Number of Scenarios	On the Fly Generation	Adjusted
1,000	3.17%	2.79%
5,000	1.66%	0.95%
10,000	0.79%	0.49%
20,000	0.82%	0.50%
30,000	0.89%	0.38%
40,000	0.47%	0.22%
50,000	0.53%	0.13%

The seed - 100,000 was close to the average result at all the scenario levels. Our recommendations are based on 50,000 scenarios and a seed of -100,000. A complete description of the aggregate model is in Appendix C. The model is an Excel file, which is available from the Academy.

We only got a few responses (probably not credible to estimate the answer for the entire class of readers), but they were interesting answers.

Max Rudolph, United of Omaha, forwarded some work from a couple of years ago that concluded that "The hybrid Sobol sequence with antithetic variates outperforms other methods of Monte Carlo integration. Even with this method, one still needs in excess of 100 scenarios for an accurate estimate. But this compares with many thousands for pure Monte Carlo. The accuracy of the estimate seems to depend only on the number of scenarios and not the random numbers used in the interpolation." This was regarding modeling of interest rates for evaluation of an SPDA business.

Dennis Lauzon, NY Insurance Department, forwarded the following from the DI RBC Working Group of the JointDI/LTC/SL/LB Task Force Final Report March 22, 2001. Dennis says, "It addresses the two questions of the importance of number of scenarios and the importance of seed number. The power of using variance reduction techniques (the adjusted results) is evident."

Jason Alleyenne said, "I work for a small insurer in a developing country. We use the Canadian Regulatory, so the Canadian approach of seven scenarios that work to identify exposure to understandable duration and convexity mismatch is a starting point.

But to take this further, the use of scenarios should always be used to convey understanding to senior management (non-actuaries) of the potential risk facing the current business model of the enterprise. If the management appetite and knowledge base allows one to present results from 10,000 scenarios and percentile results, then so certainly do so. But my management certainly don't want to see a 100-page report that only tells them their assets are too short."

Fred Travan, Canada Life, responded, "Our company uses 1000 scenarios for products linked to stock market performance. The theory behind this is the same as outlined in your e-mail, so it represents a 95 percent confidence interval. We have rounded the 983 theoretical figure to 1000 for practical reasons."

The e-mail theory that Fred was referring to said that the number of scenarios, n, for a 99 percent confidence interval should be:

$n >= 38{,}416\ s^2/x^2$, where s is the sample variance and x is the sample mean

For example, if you are modeling bond prices, the standard deviation of bond prices was just under 8 percent of the price in 1999. Substituting into the formula above, we get n > = 246. In 1999, stock prices had a standard deviation about 16% of the price. That would lead to n > = 983.

Pete Smith, of AIG sent the following explanation:
"A rule-of-thumb for the number necessary scenarios is that approximately 10^s pseudo-random scenarios are typically necessary for statistical credibility, where s is the dimensionality of the model. When constructing actual models, the statistical credibility of the number of scenarios should be computed based on an estimate of the error term or computationally estimated. However, the 10^s rule-of-thumb is very useful in planning and conceptualizing the complexity and likely feasibility of the model. Quasi-random numbers may significantly reduce the number requisite scenarios. A rough rule-of-thumb is that use of quasi-random sequences, such as Sobol or Faure, reduces the number of requisite scenarios by approximately a factor of 10. Other variance reduction techniques, such as a Brownian Bridge, may potentially reduce the number of requisite scenarios by an additional factor of 10."

In a separate survey of company stress testing procedures, I asked six companies how many scenarios they used for stress testing. The answers from companies ranged from 10 to 10,000. Keep in mind that stress testing does not require a confidence interval and may be a small set of subjectively determined "disaster" scenarios.

As I said at the outset, this is not a large enough response to be able to say what the most common practices are. However, these comments do raise a number of questions that you may want to consider the next time you perform a stochastic simulation.

In physics, Heisenberg postulated that the observer has an impact on the results of any observation. With financial market models, it is most likely true that a good and accurate model will stop being accurate as soon as it is widely used, no matter how many scenarios are used.

David Ingram, FSA, MAAA, is a consulting actuary at Milliman USA in NY and a member of the Investment Section Council. Dave is a FRM (2001) and a PRMIA CRM (2002). He can be reached at david.ingram @milliman.com.