SOCIETY OF ACTUARIES

Article from

ARCH 2017.1 Proceedings

The optimal decumulation strategy during retirement with the purchase of deferred annuities

ANRAN CHEN

CASS BUSINESS SCHOOL, CITY UNIVERSITY LONDON

JULY 2016

Motivation

- Traditional DB pension scheme → DC pension scheme
- Low demand of annuities - "Annuity Puzzle"
- A decumulation strategy comprising
  - A deferred annuity purchased at retirement
  - Optimal consumptions before the commencement of the annuity

Previous Research – Deferred annuities

Major advantages of deferred annuities:

- It requires a smaller initial investment, which overcomes a potential psychological barrier to annuitisation
- It provides almost as much longevity insurance as an immediate annuity product
- It suffers much less actuarial unfairness
- It improves and simplifies the process of retirement wealth decumulation while retaining the chance to leave a bequest

Supporting literature:

Milevsky (2005); Gong and Webb (2010); Scott (2008); Sexauer and Siegel (2013).

Previous Research – Decumulation strategies

- A first year withdrawal of 4%, followed by inflation-adjusted withdrawals (Bengen, 1994)
- DCDB: a laddered Treasury Inflation Protected Securities (TIPS) for 20 years and a 20-year deferred annuity (Sexauer et al., 2012)
- A annually recalculated virtual annuity (Waring and Siegel, 2015)

Objectives

- The Utility-maximisation decumulation strategy
  - The optimal fraction of wealth to be invested in a deferred annuity
  - The optimal consumption path that investors should follow
- The impact of following additions to the model: smooth consumption requirement, the state benefit, the target replacement ratio, the bequest motive.
- The optimal decumulation strategy for irrational retirees with:
  - Time-inconsistent preferences
  - Subjective mortality rates

Optimal decumulation strategies

Optimal decumulation strategies

A decumulation strategy comprising of

(a) A deferred life annuity purchased at the point of retirement

(b) Optimal consumptions before the annuity commences

$$V = \max_{C_{65}, C_{66}, \ldots} \sum_{i=0}^{54} \delta(i) \ {}_{i}P_{65} \, u(C_{65+i})$$

Subject to $$\sum_{i=0}^{d-1} C_{65+i} \, v^{i} + \sum_{i=d}^{54} C_{65+i} v^{i} \ {}_{i}P_{65} (1+L) = 1$$

where: C_x: consumption at age x, for $65 \leq x \leq 119$

α: percentage of wealth to be invested in the deferred annuity

r: deterministic risk free interest rate, $r = 3\%$

v: one-period exponential discount rate, $v = 1/(1+r)$

$u(C_x)$: CRRA utilities, $u(C_x) = {C_x}^{1-\gamma}/(1-\gamma)$, $\gamma = 2$

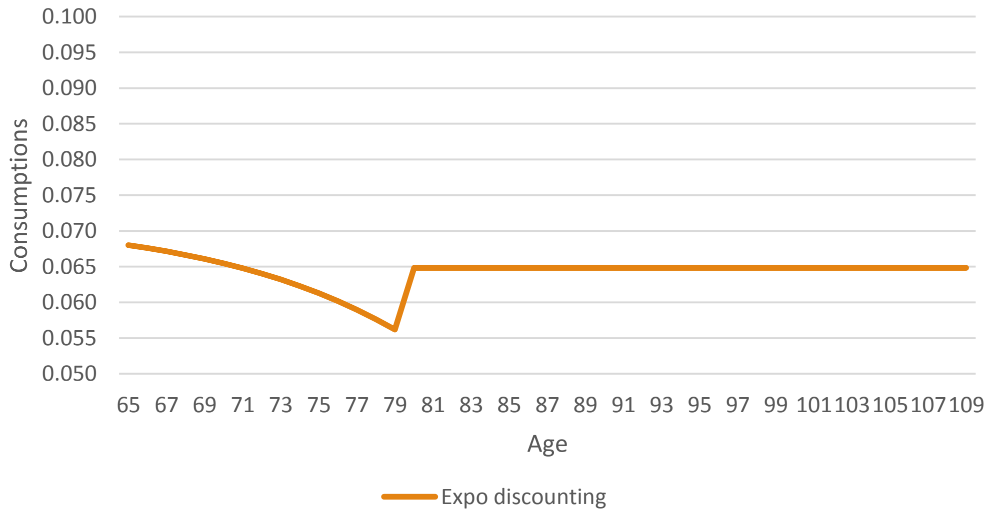
L: profit loading factor, $L = 10\%$

Optimal decumulation strategies – Fixed deferred annuity

M01: the available deferred annuity has a fixed deferred period *d*.

α	Deferred period				
	5	10	15	20	25
Exponential discounting	67.62%	41.19%	21.60%	9.13%	2.83%

The optimal percentage of wealth to be invested in the deferred annuities



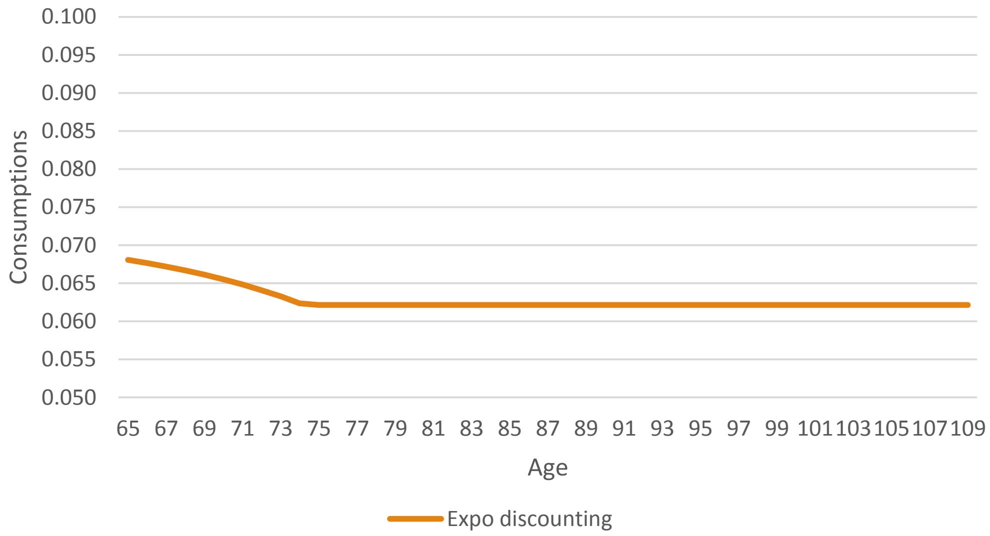
The optimal consumption path at retirement (*d*=15)

Optimal decumulation strategies – Flexible deferred annuity

M02: the deferred period, *d*, is at the choice of investors.

α	Profit Loading				
	5%	10%	15%	20%	25%
	Optimal deferred period				
	4	6	8	10	11
Exponential discounting	73.2%	61.83%	51.54	42.25%	38.2%

The optimal percentage of wealth to be invested in the deferred annuities



The optimal consumption path at retirement (*L*=10%)

Model extensions

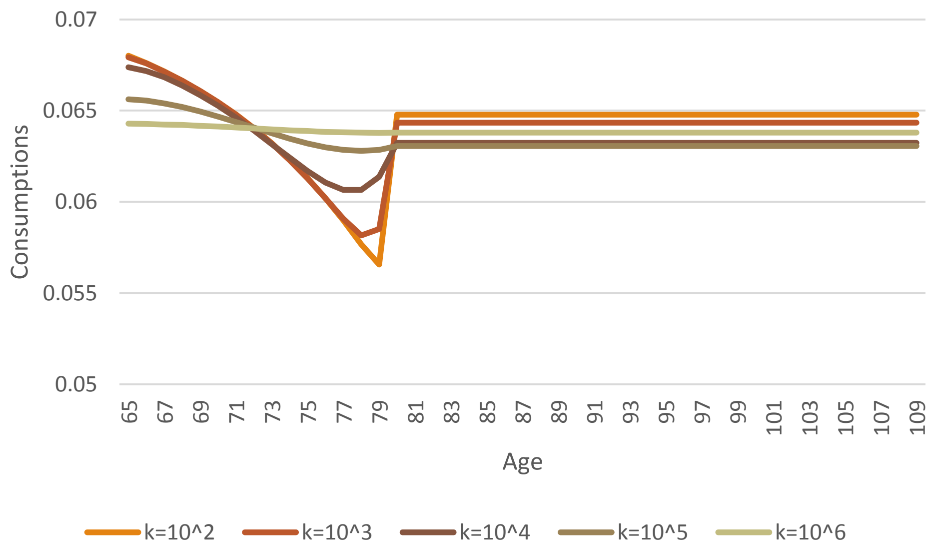
Optimal decumulation strategies – Smooth consumptions

$$V = \max_{C_{65}, C_{66}, \ldots} \sum_{i=0}^{54} \delta(i) \ {}_iP_{65}\, u(C_{65+i}) - \kappa \sum_{i=0}^{53} (C_{65+i+1} - C_{65+i})^2$$

Subject to: $\sum_{i=0}^{d-1} C_{65+i}\, v^i + \sum_{i=d}^{54} C_{65+i}\, v^i \ {}_iP_{65}(1+L) = 1$

κ	10^2	10^3	10^4	10^5	10^6
α	21.58%	21.43%	21.06%	21.00%	21.25%
max C_i	6.8%	6.8%	6.7%	6.6%	6.4%
min C_i	5.7%	5.8%	6.1%	6.3%	6.4%
max-min	1.1%	1.0%	0.7%	0.3%	0.0%

The optimal percentage of wealth to be invested in a 15-year deferred annuity and a conclusion of consumption levels



The optimal consumption path at retirement (d=15)

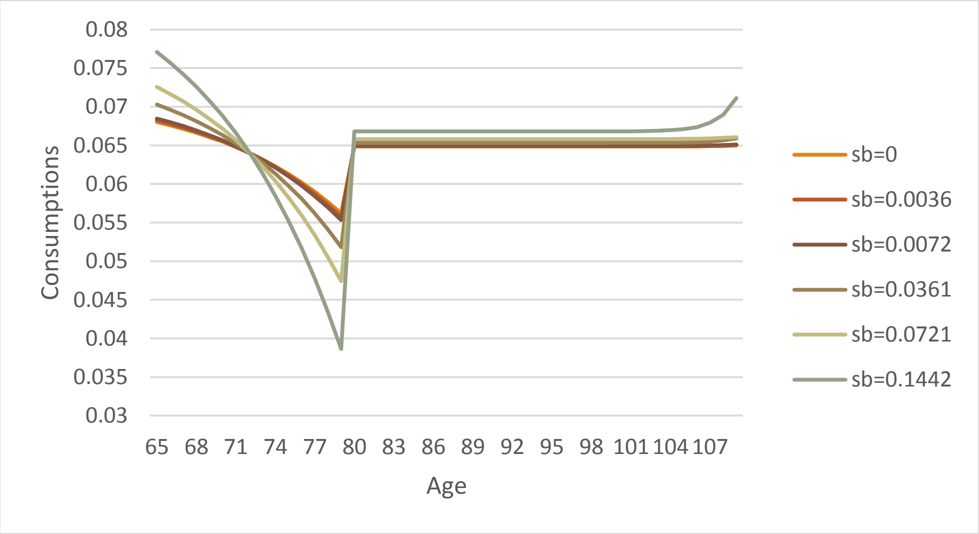
Optimal decumulation strategies – State benefits

$$V = \max_{c_{65}, c_{66}, \dots} \sum_{i=0}^{54} \delta(i) \; {}_iP_{65} \, u(C_{65+i} + sb)$$

Subject to: $\sum_{i=0}^{d-1} C_{65+i} \, v^i + \sum_{i=d}^{54} C_{65+i} \, v^i \; {}_iP_{65}(1+L) = 1$

sb	0.0036	0.0072	0.0361	0.0721	0.1442
α	21.60%	21.62%	21.76%	21.92%	22.25%

The optimal percentage of wealth to be invested In a 15-year deferred annuity



The optimal consumption path at retirement (*d*=15)

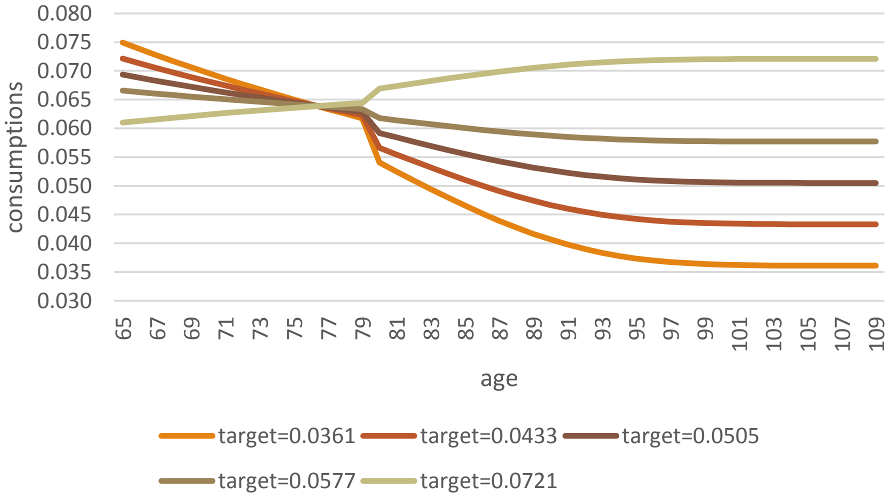
Optimal decumulation strategies –Target replacement ratio

$$V = \max_{c_{65}, c_{66}, \ldots} \sum_{i=0}^{54} \delta(i) \ {}_iP_{65}\, u(C_{65+i}) - \kappa \sum_{i=0}^{54} (C_{65+i} - target)^2$$

Subject to: $$\sum_{i=0}^{d-1} C_{65+i}\, v^i + \sum_{i=d}^{54} C_{65+i}\, v^i \ {}_iP_{65}(1+L) = 1$$

		Replacement Ratio				
α		0.5	0.6	0.7	0.8	1
		target				
		0.0361	0.0433	0.0505	0.0577	0.0721
κ	0	21.60%	21.60%	21.60%	21.60%	21.60%
	10^2	20.88%	21.05%	21.22%	21.40%	21.78%
	10^3	18.72%	19.38%	20.07%	20.79%	22.32%
	10^4	16.43%	17.66%	18.92%	20.20%	22.81%
	10^5	15.83%	17.24%	18.65%	20.07%	22.92%
	10^6	15.75%	17.19%	18.62%	20.06%	22.93%

The optimal percentage of wealth to be invested in a 15-year deferred annuity



The optimal consumption path at retirement (*d*=15)

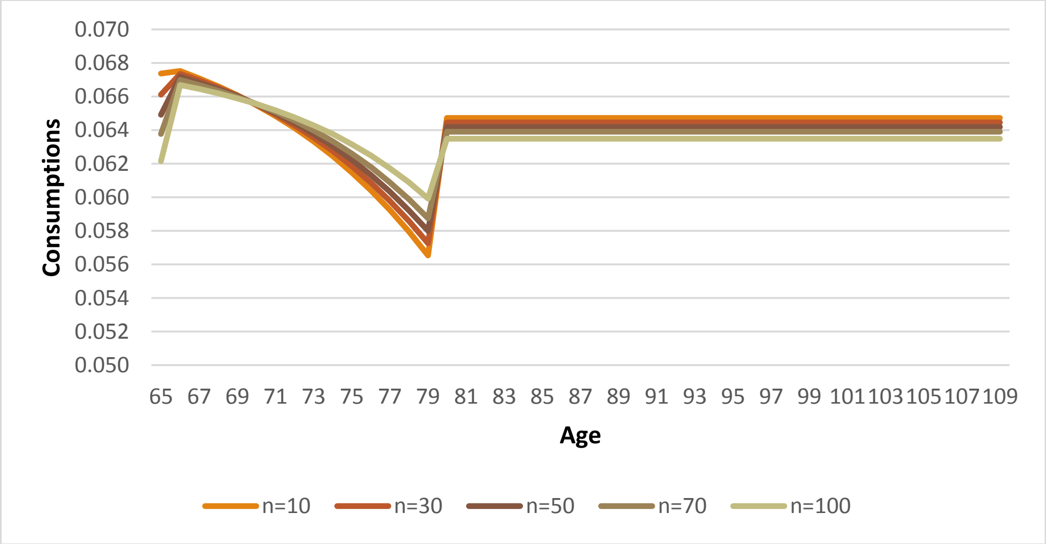
Optimal decumulation strategies –Bequest motives

$$V = \max_{c_{65},c_{66},\dots} \sum_{i=0}^{54}\{\delta(i)\ {}_iP_{65}u(C_i) + \delta(i+1)\ {}_iP_{65}q_{65+i}\ nW_{i+1}\}$$

Subject to: $\sum_{i=0}^{d-1} C_{65+i}\, v^i + (1+L)\sum_{i=d}^{54} C_{65+i} v^i\ {}_iP_{65} = 1$

n	10	30	50	70	100
α	21.56%	21.47%	21.38%	21.28%	21.14%

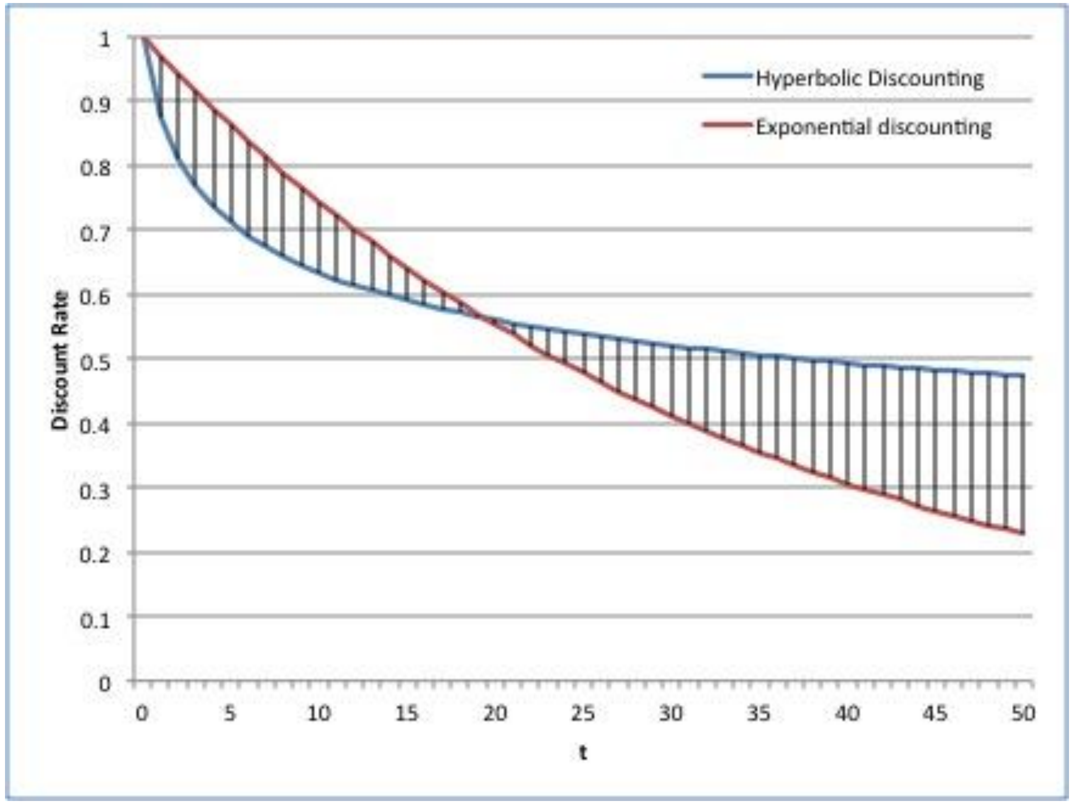
The optimal percentage of wealth to be invested in a 15-year deferred annuity



The optimal consumption path at retirement (*d*=15)

The decumulation strategy for retirees with time-inconsistent preferences

Hyperbolic discount model vs Exponential discount model

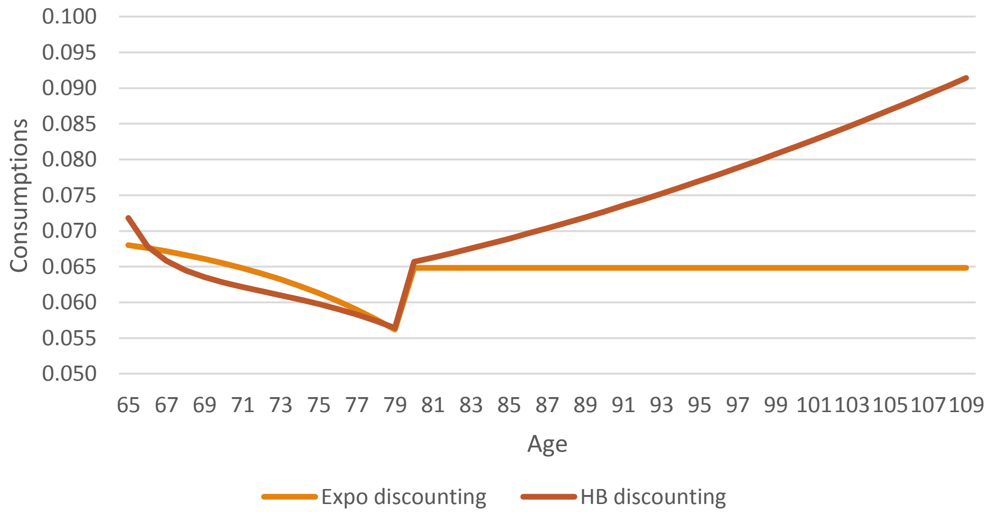


Optimal decumulation strategies – Fixed deferred annuity

M01: the available deferred annuity has a fixed deferred period *d*.

α	Deferred period				
	5	10	15	20	25
Exponential discounting	67.62%	41.19%	21.60%	9.13%	2.83%
Hyperbolic discounting	67.78%	42.34%	22.97%	10.09%	3.26%

The optimal percentage of wealth to be invested in the deferred annuities



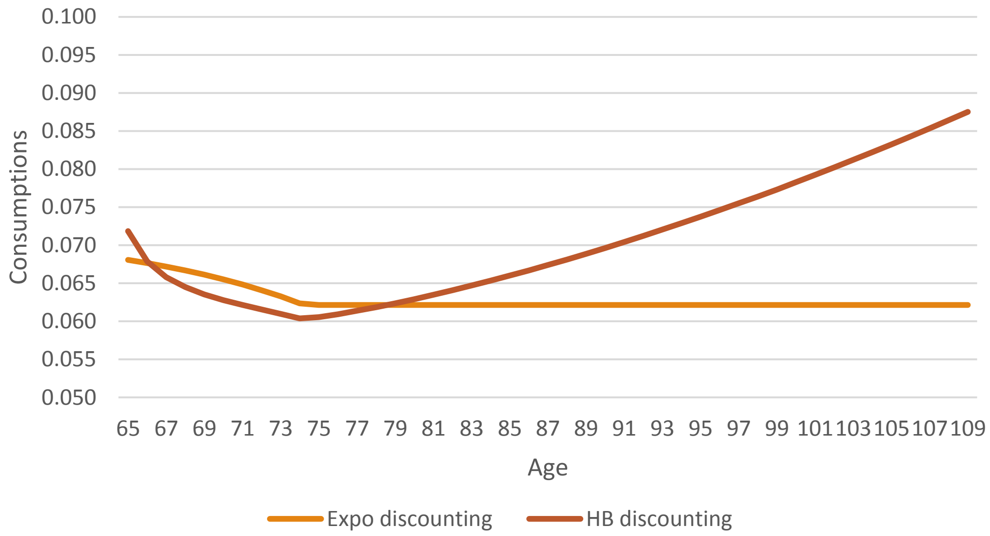
The optimal consumption path at retirement (*d*=15)

Optimal decumulation strategies – Flexible deferred annuity

M02: the deferred period, *d*, is at the choice of investors.

α	Profit Loading				
	5%	10%	15%	20%	25%
	Optimal deferred period				
	4	6	8	10	11
Exponential discounting	73.2%	61.83%	51.54	42.25%	38.2%
Hyperbolic discounting	71%	60.41%	50.84%	42.14%	38.33%

The optimal percentage of wealth to be invested in the deferred annuities



The optimal consumption path at retirement (L=10%)

Conclusions

Strategies	Asset allocation
M01	6.9% in a 20-year deferred annuity
M02	56.95% in a 6-year deferred annuity
DCDB	93.12% in 20-year TIPS + 6.88% in a 20-year deferred annuity

	M01	M02	DCDB
Reduce longevity risk	*	*	*
Reduce liquidity risk	*		*
Reduce inflation risk			*
Achieve higher utility	*	*	

Conclusions

- A target replacement ratio is an influential factor on wealth allocation.
- Smooth consumption requirement, state benefits and bequest motives do not show significant impacts on wealth allocation.
- Retirees with time-inconsistent preferences tend to prefer inflation protected deferred annuities.

References

Bengen, W. P., 1994. Determining withdrawal rates using historical data. Journal of Financial Planning 7 (4), 64-73.

Gong, G., Webb, A., 2010. Evaluating the advanced life deferred annuity - an annuity people might actually buy. Insurance: Mathematics and Economics 46 (210-221).

Milevsky, M. A., 2005. Real longevity insurance with a deductible: Introduction to advanced-life delayed annuities (ALDA). North American Actuarial Journal 9 (109-122).

Scott, J. S., 2008. The longevity annuity: An annuity for everyone? Financial Analysts Journal 64 (1).

Sexauer, S. C., Peskin, M. W., Cassidy, D., 2012. Making retirement income last a lifetime. Financial Analysts Journal 68 (1), 74–84.

Waring, M. B., Siegel, L. B., 2015. The only spending rule article you will ever need. Financial Analysts Journal 71 (1), 91–107.