



SOCIETY OF ACTUARIES

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# Implementing the Longstaff-Schwartz Model

by Pete Smith

## Background

The HJM (Heath-Jarrow-Morton) class of stochastic interest models has been important in finance because the use of forward interest rates enables the development of arbitrage-free periods over expected forward rates, calibration to important markets such as swaptions and reproducing stylized facts well. However, the HJM models typically are not representable as recombining trees, which may lead to computational complexities and inability to express model solutions as closed-form formulas. For example, see Rebonato 1998.

There is another significant problem with the HJM models as they are typically developed as four- or five-factor forward rate models. This leads to high dimensionality, especially when the HJM models are combined with other models to model more complex situations as  $f/x$  quanto models. Defining the model dimensionality as the sum of the independent model factors and using the quick-and-dirty rule of thumb that the number of requisite scenarios for statistical credibility is about  $10^s$  (within perhaps one half an order of magnitude) and  $s$  is dimensionality, we see that the number of requisite scenarios may readily exceed a facility's computational capacity, especially with liability models containing numerous cells.

The Longstaff-Schwartz string model is very similar to the BGM (Brace-Gatarek-Musiela) implementation of HJM but with much lower dimensionality. The Longstaff-Schwartz string model has a dimensionality equal to the number of factors less the sum of the correlations of adjacent forward rates. As these correlations tend to be quite high, the Longstaff-Schwartz string model should have a dimensionality not significantly greater than 1.

The Longstaff-Schwartz model produces significantly lower dimensionality than BGM, so considerably fewer scenarios are required at a given level of statistical credibility. However, actual scenario generation computational complexity is about the same as BGM because there are still as many forward factors created as for BGM. Also the statistical calibration of the string model is computationally intense.

## Implementation Note

The following overview of the Longstaff string model is obtained from the paper "Throwing Away a Billion Dollars."

## Equation 12

Equation 12 on page 12 states,  
 $(12) dD = r D dt + J^{-1} \sigma F dZ,$   
 where:

$D$  is the vector of the discount bond prices obtained from forward rates.

$dD$  is the derivative of  $D$ .

$r$  is the risk-free rate.

$J^{-1}$  is the inverse of the Jacobian matrix, i.e. partial derivatives of discount bond prices,  $D$ , with respect to the forward rates  $f_1, \dots, f_n$ .  $J$  is a simple banded diagonal matrix because the partial derivatives are approximated by the finite difference approximation to the derivative by successive  $[D(t+1) - D(t)] / [f(t+1) - f(t)]$ .

$F$  is the vector of associated fixed coupons for swap contracts of maturities up to 15 years, such that the expected initial swap contracts have a value of zero.

$\sigma$  is the vector of volatilities of the  $F(i)$ .

$dZ$  is a vector of Brownian shocks.

Equation 12 provides the definition of changes to the discount bond values from time  $t$  to  $t + 1$ .

Equation 10 states:

$$dF_i = \alpha_i F_i dt + \sigma_{i1} F_i dZ_1 + \dots + \sigma_{iN} F_i dZ_N$$

## Parameterization

Let  $H$  be the historical correlation matrix of percentage changes in forward rates. The forward rates are obtained from cubic interpolation of estimated bond prices.

$H = U \Lambda U'$  where,

$U$  = matrix of eigenvectors, the first four principal eigenvectors are used.

$\Lambda$  = diagonal matrix of eigenvalues

Assume,

$$\Sigma = U\Psi U'$$

$\Sigma$  is the implied covariance matrix.

where  $\Psi$  is a matrix of non-negative elements that best fits  $\Sigma$ .

$\Psi$  is solved for stochastically by generating the usual set of random shocks and solving equations '10' and '12' for the best fit over the stochastic set. ☒

## References

R.Rebonato, *Interest Rate Option Models*, 1998, Wiley

Brace, Gatarek and Musiela, "The Market Rate Model of Interest Rate Dynamics", *Mathematical Finance* 7, p. 127-155

F.Longstaff, P. Santa-Clara, E. Schwartz, "Throwing Away A Billion Dollars", to be published, available at Longstaff's Web page.

F.Longstaff, P. Santa-Clara, E. Schwartz, "The Relative Valuation of Caps and Swptions: Theory and Empirical Evidence", to be published, available at Longstaff's web page.

F.Longstaff, E. Schwartz, "Valuing American Options by Simulation: A Simple Least Squares Approach", to be published, available at Longstaff's Web page.

## Publication Schedule for the Next Three Issues of *Risks and Rewards*

In an effort to even out work flow and assure timely production of section newsletters, the Society of Actuaries has scheduled publication dates for next year's issues of *Risks and Rewards* as follows:

Issue	Editor	Deadline	E-mail
February 2003	Boezio	December 2	<a href="mailto:NBoezio@sympatico.ca">NBoezio@sympatico.ca</a>
July 2003	Koltisko	May 5	<a href="mailto:joseph_koltisko@agfg.com">joseph_koltisko@agfg.com</a>
October 2003	Wendt	August 4	<a href="mailto:Wendtd@towers.com">Wendtd@towers.com</a>

Unlike past practice, these deadlines are firm. Articles received after the deadline will be included in the next issue. If you have an article or an idea for article, please contact the editor of the next issue. Reports from seminars or meetings, summaries of interesting books or papers in other publications of interest to the members are welcome. ☒

## Results of Investment Section Council Election:

The following three members have been elected to the Investment Section Council for three-year terms beginning October, 2002: Bryan E. Boudreau, Michael J. O'Connor and Steven W. Easson. Bryan Boudreau was elected to the reserved pension seat.

The following officer nominations have been received for the Investment Section Council for a one-year term beginning October 2002:

Chairperson Douglas A. George  
Vice Chairperson Mark W. Bursinger  
Treasurer Craig Fowler ☒