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GMDB Pricing:

Comparing a Lognormal Model to a Regime-Switching Lognormal Model

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A paper entitled "A Regime-Switching Model of Long-Term Stock Returns" appears in the April 2001 issue of the *North American Actuarial Journal*. In it, Dr. Mary Hardy describes a model for future equity returns and some reasons why that model might better reflect real equity price movements than the often-used lognormal model. Among her reasons are that the regime-switching model exhibits volatility bunching and generates the occasional extreme return like that seen in October 1987. Several applications of the model are described by Dr. Hardy, including measuring risks associated with Canadian segregated fund contracts. The purpose of this article is to apply a two-regime model to variable annuity guaranteed minimum death benefit (GMDB) pricing and to compare the results to those from a lognormal model.

In short, the two-regime model assumes equity returns arise from two regimes or states. Each state's returns are assumed to be lognormal, with each having different mean and volatility assumptions. A Markov process is assumed to determine which regime the equity price model is in at any given time.

Dr. Hardy fit her model to monthly S&P 500 total return data for the period 1956-1999. Her maximum likelihood estimates (MLEs) for mean monthly log return (0.9 percent) and annual volatility (14.38 percent) are used in this paper for the lognormal model. Parameters for the two-regime model are the ones derived via maximum likelihood estimation by Dr. Hardy (again fitted to the S&P data). Consequently, regime one has a mean monthly log return of 1.26 percent with 12.1 percent annual volatility and regime two has respective parameters –1.85 percent and 25.9 percent. Price processes in regime one are assumed to move to regime two with probability .04 while those in regime two move to regime one with probability .38.

Each model was used to project equity prices, account values, and expected death benefits for a male, age 65, issued a variable annuity having one of four death benefit types: return of premium, 3 percent roll-up, 2-year ratchet and a combination of the ratchet and roll-up. All deposits are assumed to be invested in equity funds, and expected mortality is set equal to the Annuity 2000 Basic table. For simplicity, the effect on GMDB costs of Actuarial Guideline 34 was not included. The present value of calculated costs was converted to an additional mortality and expense charge (account values were projected using a roughly 100 bp MAE charge). Mean results for each benefit as well as 90th percentile results are shown on the following page for each model.

Although the models are fit to the same data, it is evident that the effect of the two-regime approach is significant. The 90th percentile values, often used by companies for setting GMDB costs, are 33-50 percent higher for the two-regime model than for the lognormal model.

TABLE 1

	Cost as Additional Basis Points of MAE			
	Lognormal Model		Two-Regime Model	
Benefit	Mean	90th Percentile	Mean	90th Percentile
Return of Premium	1.2	3.2	1.8	4.8
3% Rollup	2.4	6.5	3.3	9.4
2-Year Ratchet	5.5	10.9	7.0	14.5
Max (Rollup, Ratchet)	6.2	12.5	7.9	16.7

TABLE 2

Lognormal Model: Cost of Return of Premium Benefit				
Annual Volatility	14.4% (MLE)	15.0%	17.0%	20.0%
90th Percentile (bps)	3.2%	3.8%	6.0%	10.4%

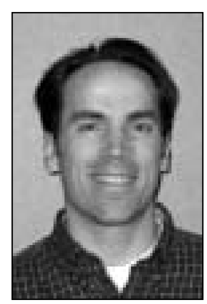
The actual basis point costs shown above will vary depending on items such as the level of mortality and expense, the data to which the model parameters are fit, and the age of the insured. What is significant is the comparative difference between the models, given that they were fit to the same data and use otherwise identical assumptions. Also, although not the focus of this article, some companies find it more appropriate to use a current market volatility assumption instead of one based on historical data. Any one of the illustrated values could change markedly depending on the assumptions used.

To illustrate the sensitivity of the lognormal model to a change in volatility assumptions, note the different costs of the return of premium benefit using the following volatilities (assumed mean log returns are held constant at the MLE used in this article).

For this rather common and seemingly inexpensive benefit, cost varies markedly using the lognormal model, depending on assumed volatility.

What approach does your company use? It is appropriate to choose a future equity model that is consistent with history (both the lognormal and two-regime models are fitted to historical data) and which brackets the historical highs and lows, including returns like October 1987. For uses like determining GMDB cost, it is imperative to include and extend beyond such extremes, since it is the rare (and maybe never-before-seen) values that determine whether you have significant cost (or risk) at all. This is where the two-regime model has an advantage over the lognormal model: it captures the extreme values that history has proven are possible. And by including such possibilities, benefits like GMDBs might be viewed as more costly to an insurance company than previously assumed.

Hardy, M. R. 2001. "A Regime-Switching Model of Long-Term Stock Returns." North American Actuarial Journal 5(2): 41-53.



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