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BENCHMARK SURPLUS FORMULAS

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- o How can risk be measured and quantified?
  --Practical application of cash-flow analysis.
- o Cash-flow-based surplus (CFS)--A new concept of financial strength.
- o Factors that affect benchmark surplus--Illustrations using C-3 risk single premium deferred annuity (SPDA) and guaranteed investment contract (GIC) products.
- o How should multiple risks be combined for benchmark surplus?
- o Benchmark surplus and the valuation actuary.
- o Uses of benchmark surplus formulas.

MR. SIDNEY A. LEBLANC: No matter how sophisticated the calculation of a benchmark surplus formula, the input assumptions are subjective. If you knew what was going to cause you to go broke, you could do something about it. Using that logic about 12 years ago, I suggested that we measure our company's surplus versus other mutual companies our size using a surplus formula which we borrowed. That way, if everybody else went broke, we wanted to go broke because drastic action is possible then, but we didn't want to be the first one to go broke. Unfortunately, while the logic has validity, this simplified approach isn't good enough anymore.

The motivations for looking at benchmark surplus include computations of profits as a return on surplus and the mutual company surplus tax. A primary reason is the role of the valuation actuary. The opinion of the valuation actuary that the reserves make good and sufficient provision under all plausible fluctuations adds considerable impetus to considerations of required surplus levels. However, to my mind, the most important reason for benchmark surplus review is in order to understand risk and thereby learn how to manage and reduce risk.

A great deal of work on this topic has been done in the Society of Actuaries' Committee on Valuation and Related Problems, of which Mr.

Cody is chairman, and the Combination of Risks Task Force, of which Mr. Mateja is chairman, and Mr. Geyer has been a very active member.

MR. MICHAEL E. MATEJA: My goal today is to present the work of the Combination of Risks Task Force (CORTF) and relate that work to the general subject of benchmark surplus formulas. It seems appropriate to begin with a few remarks about the origin of the Task Force and its charge. The real origin of the Task Force probably dates back exactly six years ago this month when the Trowbridge Committee issued its landmark report. I doubt that that committee fully realized what a Pandora's box it was opening.

The Combination of Risks Task Force was formally organized about two years ago. Some of you may recall that at that time the C-3 Risk Task Force under the chairmanship of Mr. Carl Ohman was concluding its work on the analysis of mismatch risk. I was a member of the C-3 Risk Task Force, and Mr. Geyer and I did the analysis of mismatch risk for nonparticipating individual life insurance.

As a result of the work of the C-3 Risk Task Force, it became clear that the traditional approaches to valuation and solvency did not make adequate provision for mismatch risk in the financial statements of insurers. Furthermore, there was increased concern about asset default and pricing risks, spawned by an uncertain economic climate and a rush of new interest-sensitive products to the marketplace.

For many years, risk management in an insurance company had been taken for granted. In the context of the early 1980s, there was a painful realization that risk management, which fundamentally was at the heart of the insurance business, was not well developed or understood. Mr. Cody asked me to be Chairman of the CORTF anticipating a broad-based assault on these specific problems and other related concerns.

The charge to the Task Force is presented in the Society Year Book and is briefly summarized as follows:

CORTF Goals:

- understand and quantify combination of risks
- implications on
  - statutory reserves
  - surplus
  - valuation statutes
  - early warning tests
  - corporate planning

Our charge can be summarized as an effort to respond to all questions related to the management of risk in an insurance enterprise. It was an ambitious undertaking, but the Task Force, in fact, has developed results responsive to many of these original goals. The problem of understanding and quantifying risk is not an easy one, so we don't have a neat package of simple answers. We have learned a great deal, but our results are more a beginning rather than an end.

BENCHMARK SURPLUS FORMULAS

How does the work of the Combination of Risks Task Force relate to benchmark surplus formulas? As our business has become financially more sophisticated, there has been a general realization that we need to better understand the surplus required to drive our business. While surplus was once in abundance, it has become a scarce resource. There is increasing pressure to use it wisely. Quite simply, good benchmark surplus formulas can help us to use our surplus wisely. Developing good surplus formulas, however, requires a good understanding of risk, since surplus is fundamentally needed to manage risk in an insurance company. If our own work is representative, then it is probable that the industry is struggling to understand the problem of risk management. We have learned a good deal about risk in the work of the CORTF which we feel will be helpful in addressing the general problem of risk management and control and the development of benchmark surplus formulas.

Let's talk about risk. We all have an intuitive understanding of it. I suspect though that any of you who have tried to quantify the risk exposure of your companies didn't have an easy time of it. There certainly isn't much help in the literature. Even if you were successful or thought you were, I suspect there was at least a lingering doubt as to whether you had the right answer.

Risk by its very nature is not easy to understand and not easy to quantify. But insurance is a risk-taking business, and fundamentally we, the actuaries, should know something about the problem of quantifying risk. I suspect we will all be forced to learn more about this subject in the years ahead.

I worked in the area of risk quantification at the Aetna before my work with the CORTF began. I was responsible for developing benchmark surplus formulas that were used in our planning process to allocate the surplus that we held among our various lines of business. The allocation was used to determine return on equity and understand something about the ability of each line to finance its own growth. Our original efforts to quantify risk followed what I would describe as a statistical-based methodology. I outlined some of our thinking in this regard in a panel presentation at the Society meeting in Anaheim in 1979. Mr. Cody outlined some of the ideas he used in quantifying risk at the New England Mutual in a paper which he presented in 1982 at the Society meeting in Orlando. His approach also was statistically oriented.

Mr. Geyer and I had been actively involved in the work of the C-3 Risk Task Force, and many of you may have reviewed our presentations on nonparticipating individual life insurance which were published in the Record. In retrospect, the work on the C-3 risk represented something of a breakthrough in our own understanding of risk. We came away from this effort with a clear understanding that insurance is fundamentally a cash-flow business.

This background had a profound impact on our early thinking as we began work on the problem of combination of risks. We knew that it was possible to understand mismatch risk by an analysis of cash flow, and it was intuitively obvious that a focus on cash flow could also reveal something about pricing and asset default risk as well.

For example, this next figure shows what could be considered the manifestation of pricing risk related to mortality. In this case, the risk is a greater outflow of cash relative to expectations. Pricing risk in terms of cash flows is intuitively clear for many of our products and follows this simple example:

PRICING RISK

Cash Outflows

$100	$100	$100	$100	$100	$100	Estimated
$ 80	$ 80	$ 80	$ 70	$150	$ 80	Actual
1	2	3	4	5	6	

Duration

In the following figure, which illustrates asset default risk, the cash inflows from our assets fail to materialize and then the repayment schedule is extended. While the relationship of default to cash flows is obvious, I had not thought of asset default risk in these terms until the CORTF work began.

ASSET DEFAULT RISK

Cash Inflows

$100	$100	$100	$100	$1,100			Anticipated
$100	$ 0	$ 0	$ 80	$ 80	$ 80	$880	Actual
1	2	3	4	5	6	7	

Duration

This line of thinking, at any rate, led to the conclusion that risk fundamentally represents a deviation from expected cash flow. It is a simple notion, but it has endured throughout the work of the Task Force.

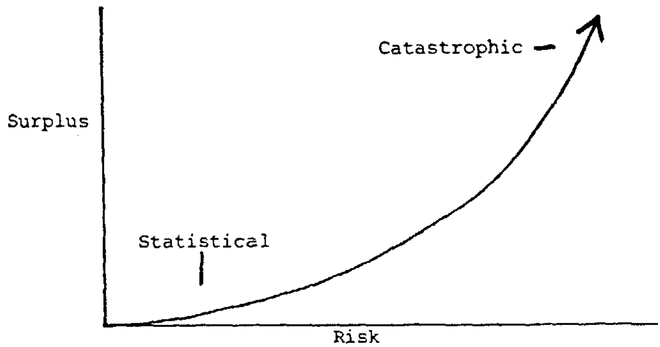
We concluded, therefore, that if we could develop a model that focused on cash flows and understand how cash flows are affected by the various risks, we would have a basis for better understanding and quantifying combination of risks as anticipated in our charge. In a relatively short time we developed a simple model of an insurance company that projected cash flows which could be varied to reflect different levels of risk. It was easy to see in the model the relationship of cash flow deviations to risk. Practical analysis of the cash-flow streams, however, proved to be quite a challenge.

Much of our original thinking on the general subject of risk is presented in our Preliminary Report (published in this Record at the end of Mr. Geyer's presentation).

Our approach to quantifying risk was scenario-oriented. That is, if we specified a given level of mismatch, asset default and pricing risk, our model could develop the surplus required to manage that particular combination of risk. This was the same approach used by the C-3 Risk Task Force. The C-3 Risk Task Force basically showed how to quantify a given mismatch risk situation, but it stopped considerably short of specifying what level of surplus would manage all C-3 risk. The CORTF, in effect, was preparing to produce similar results.

This became a key point as we have tried to bring our work to a conclusion. Specifying the surplus required to manage a specified level of various risks does not necessarily yield the amount of surplus required to manage the full range of combination of those risks. The important point is that the more we focused on the problem of quantifying risk, the less confident we became that it was possible to specify a surplus level that would manage the full range of a specific risk or some combination of risks. Our concern was based on a rather simplistic view of risk as illustrated in figure 1.

FIGURE 1



All risks (except perhaps the C-4 risk) exhibit statistical characteristics with random deviations about a mean or expected value. Relatively modest amounts of surplus are required to manage this type of risk. In fact, as our subsequent work has revealed, most of this type of risk can be managed by pricing margins, i.e. profit, and conservatism in valuation reserves. It is conceptually possible to think about managing risk at this level, and we have envisioned that this approach would be suitable for setting valuation reserve levels.

The problem comes at the extreme of risk which I associate with catastrophes. Surplus requirements increase dramatically as illustrated in the figure, and some risks literally are "off the scale" depending on how the real world behaves in a catastrophe. The problem is simply that it's impossible to draw a line. How high can interest rates go? Five years ago, they reached levels that were "off the scale" of at least one actuary who was concerned about interest rates in pricing GIC products in the early 1970s. How many assets will default if a 1930s type depression materializes? My discussions with our investment area leave me a bit nervous when they characterize today's typical investment as many times riskier than the typical 1930s investment. Then there are epidemics, earthquakes, and nuclear accidents to round out the variety of catastrophes. Throw in something for the extremes of C-4 risk, and you have the makings of real uncertainty.

There are no firm answers to the question of surplus levels required to manage these extremes of risk. The oversimplistic answer to how much surplus you need is "as much as you're willing to hold." The marketplace, of course, controls the final management decision regarding surplus levels.

While there necessarily must be considerable doubt about any effort to define surplus levels to assure solvency, it is possible to reasonably quantify the surplus required for a catastrophe of a given level of severity. For instance, it should be possible to develop the surplus required to survive an epidemic producing a 50 percent increase in mortality. This is the approach taken by the CORTF; we have defined a catastrophe for the various risks at roughly a 1 in 100 level of probability. The choice of severity levels is admittedly arbitrary, but they are in the right ball park. Remember, we are more concerned about the problem of combination of risks rather than quantifying specific risks. Quantifying individual risks on this basis proved relatively straightforward, and the problem was how to combine them.

This view of risk and surplus suggests that any approach to benchmark surplus formulas has inherent limitations. Do we need benchmark surplus formulas? I believe the answer is yes! Even imperfect formulas can help us to better understand risk management and control throughout the company, and they can provide a reasonable measure of risk relativity. Developing a reasonable understanding of risk relativity among various business segments is the most useful end product of benchmark surplus formulas. With such an understanding, it is possible to allocate surplus and develop a consistent approach to measuring profitability for our various lines.

This view of risk and surplus has also shaped our attitude toward the Joint Committee on the Role of the Valuation Actuary in the United States proposal regarding the valuation actuary. I have little hope of quantifying the extremes of risk that I associate with catastrophes on any disciplined basis. Therefore, I have difficulty supporting the idea that the valuation actuary should be responsible to regulators for determining "internally designated surplus" required for "plausible" fluctuations from expected assumptions. This simply extends actuarial science into the realm of actuarial guesswork. I am content though in advising my management on my views regarding surplus.

In regard to cash flows, anyone who has worked with cash-flow analysis undoubtedly has found that developing the basic cash flows, while somewhat tedious, is not impossible. The real problem is to compare one cash-flow stream to another. We started working on this problem when we addressed the C-3 risk for nonparticipating individual life insurance. We had problems at that time dealing with federal income tax (FIT) and shareholder dividend cash flows, and we struggled with before-tax and after-tax discount rates in the computation of present values. It was no surprise, therefore, when we encountered the same problems on a larger scale with the CORTF model.

We finally made a breakthrough when Mr. Geyer developed a disciplined analysis of all of the cash flows associated with a simple insurance arrangement. This work produced a discipline for the concept of cash-flow based surplus or CFS.

CFS can be thought of as the economic value of a book of business. If you can accept that the economic value of a book of business can vary depending upon what level of risk is realized over the lifetime of that business, you can see where CFS can be used to get a handle on the problem of quantifying risk. Remember that risk is manifest as a deviation from expected cash flow. It is worthy to note that CFS can also quantify gains.

CFS is defined as the present value of asset cash flows less the present value of liability cash flows. Recently there has been much discussion of this general approach to quantifying a cash-flow stream, but there has never been a clear articulation of how to calculate it. Our analysis indicates that great care must be exercised in the definition of the cash flows and in the choice of interest rates used for discounting purposes. Otherwise, results can be very misleading.

The problems are best illustrated with a simple example:

BASIC CASH FLOWS

Duration	Inflows	Outflows
1	$ 140	-
2	140	-
3	140	-
4	1,140	$1,630
PV-B[efore] FIT	1,000	965
PV-A[fter] FIT	1,167	1,161

Consider these cash flows associated with a simple insurance arrangement where the liability or outflow is a 4-year compound GIC with an interest guarantee of 13 percent. Expenses are ignored. Thus, there is one cash outflow at the maturity of the contract for the amount indicated. Assume that the premium for this GIC was invested in a 14 percent bond with annual coupons which matures when the liability matures. The cash inflows are then as shown.

At the bottom of this example, I have shown the present value of the inflows and outflows on both a before- and after-tax basis. Before-tax interest rates are 14 percent, and after-tax rates are 8.8 percent which reflect a 36.8 percent tax rate. If these were the only cash flows associated with this insurance arrangement, then clearly the difference between the present value of inflows and outflows would be positive on either a before- or after-tax basis. But which approach is correct? It is clear that the value of CFS would vary materially depending on whether a before- or after-tax approach is used.

Unfortunately, there are other cash flows associated with an insurance arrangement that complicate the problem. Even ignoring expenses, we have FIT and shareholder dividends to deal with. In the normal operations of an insurance company, these are important cash flows, and under conditions of adversity, the treatment of these cash flows becomes very material in the overall assessment of risk.

This next example shows what I have called operations cash flow for our simple insurance arrangement. Gross income is equal to interest earned minus interest credited. The income column is simply presented for reference--it is important to understand that this is not a cash flow. FIT is equal to 36.8 percent of gross income.

OPERATIONS CASH FLOWS

	Annual Dividend			Final Dividend		
Dur.	Gross Income	FIT	Div.	Gross Income	FIT	Div.
1	10.0	3.7	6.3	10.0	3.7	0
2	11.3	4.2	7.1	12.2	4.5	0
3	12.8	4.7	8.1	14.7	5.4	0
4	14.4	5.3	9.1	17.7	6.5	34.5
PV-BFIT	xx	12.7	21.9	xx	14.2	20.4
PV-AFIT	xx	14.3	24.6	xx	16.0	24.6

Operations cash flows have been developed for two dividend policies. The first assumes that all statutory earnings are paid out as earned, and this appears under the annual shareholder dividend section. The second assumes a final shareholder dividend at the time that the liabilities are paid out. In both cases, it is assumed that any cash is invested to mature at the time the liability matures. Yield curve considerations are ignored, but it should be clear that this would affect the analysis in practice.

Because of the retained earnings under the final shareholder dividend assumption, there are additional interest earnings which increase gross income and FIT accordingly.

Several interesting observations can be made from the present values of these cash flows. On a before-tax basis, the present value of the dividends is different under the two dividend assumptions, whereas it is the same on an after-tax basis. This is because when earnings are retained, they grow in the surplus accountant at an after-tax basis. When they are discounted on an after-tax basis, the timing of payment of the dividends becomes immaterial. On the other hand, when retained earn- ings are accumulated on an after-tax basis and then discounted on a pretax basis, the present value of the dividends changes as illustrated. This fact quickly established in our minds that a before-tax approach to calculating present values wasn't going to provide satisfactory answers.

The present value of all of the cash flows associated with the insurance arrangement are summarized in table 1.

TABLE 1

	Annual Shareholder Dividend		Final Shareholder Dividend	
Assets (Inflows)	$1,000	$1,167	$1,000	$1,167
Liabilities (Outflows)				
- Benefits	$ 965	$1,161	$ 965	$1,161
- FIT	13	14	15	16
A - L	22	(8)	20	(10)
Shareholder Dividend	22	25	20	25

We are looking for a difference between the present value of inflows and outflows, and it is apparent that there is a wide choice. The before-tax results have the advantage that the difference between the present value of asset and liability cash flows is equal to the present value of the shareholder dividends. This makes sense intuitively. The disadvantage with the before-tax basis is that the answer varies depending upon the dividend policy. It does not make sense that a measure of economic value should depend on a company's dividend policy.

The after-tax results simply do not make any sense. The asset and liability present values appear meaningless, and it is unclear what the difference means. The fact that the present value of shareholder dividends is the same regardless of the dividend policy on an after-tax basis has prompted us to do some additional research on how to compute the present values of cash flows on an after-tax basis. The end result has been the methodology for the calculation of CFS.

The following illustration shows the CFS-basis cash flows for our simple insurance arrangement. The calculation of the cash flows is fairly simple and straightforward, but it is not intuitively obvious. Basically, it is necessary to tax-effect the transactions that affect the tax liability on a current basis. For example, the coupon inflow of $140 each year is multiplied by the complement of the tax rate of 36.8 percent to yield a net after-tax inflow of $88. Note that the present value of the inflows of $88 annually at an after-tax interest rate of 8.8 percent yields $1,000 which is the same as the statutory statement value of the asset.

CFS BASIS

BASIC CASH FLOWS

Duration	Inflows	Outflows
1	$ 88	$ (48)
2	88	(54)
3	88	(61)
4	1,088	(69) + 1630
PV - AFIT	$1,000	$ 975

Note: (130) x (1 - .368) = (82)
(130) - (82) = (48)

The cash outflows are a little trickier. For instance, when the first year interest credit of $130 is tax-effected, the result is $82. The interest credit, however, is simply an accounting entry, not a cash flow. But the difference between the before- and after-tax credit of $48, which is developed in the note at the bottom of the illustration, effectively becomes a cash inflow paid by the federal government. That explains the negative values in the outflow column.

Tax-effecting the cash flows automatically accounts for the actual FIT cash flow. On the inflow side, tax-effecting procedures a $52 outflow, $140 less $88. On the outflow side, there is a $48 inflow. The difference represents a net outflow of $4 which is the actual FIT paid in the first year. The present value of the various cash outflows, including the tax credits, yields $975.

The present values of all the cash flows are summarized in table 2. Note that the difference between asset and liability present values is the same regardless of the shareholder dividend assumption. Moreover, in both cases the difference between the asset and liability present values equals the present value of shareholder dividends. In our paper on CFS, we have a rather simple proof that CFS will always equal the present value of shareholder dividends if it is computer properly.

TABLE 2

	Annual Shareholder Dividend	Final Shareholder Dividend
Assets (Inflows)	$1,000	$1,000
Liabilities (Outflows)		
- Benefits	$1,161	$1,161
- FIT	(186)	(186)
CFS = A-L	25	25
Shareholder Dividend	25	25

There is one other important point regarding the methodology for calculating CFS--discounting. In this example we have used only level interest rates which greatly simplify the process since common discount factors can be used. When the future interest rate varies, it is necessary to reflect both the pattern of the new-money rates and the reinvestment assumptions in the discounting process. The approach is to separately accumulate inflows and outflows forward to the end of the modeling period reflecting the new-money rate and reinvestment assumptions. It is also necessary to accumulate $1 invested at time zero to the end of the modeling period, again properly reflecting new-money rates, rollover rates, and reinvestment assumptions. When the accumulated inflow and outflow values are divided by the accumulated value of $1, present values are obtained that appropriately reflect the assumed interest scenario and reinvestment assumptions.

While we developed the concept of CFS with the benefit of these oversimplified insurance arrangements, we subsequently went on to incorporate the CFS methodology into our Task Force model and proved the relationship holds under a variety of complicated scenarios.

After extensive analysis and testing, we concluded that CFS had two unique properties which, we think, will make it of great value in addressing the general problem of analyzing cash flows and quantifying risk. As developed in figure 2, CFS is equal to the present value of shareholder dividends. This also follows logically from the intuitive notion that the shareholder interest is what is left over after payments of benefits to policyholders and payments of taxes to the federal government. For participating business, CFS can be thought of as the permanent contribution to surplus.

CFS

- PV of Shareholder Dividend
- CASH REMOVED--Still Mature Benefits

CFS

- Cash cost of risk

The more interesting property of CFS for quantifying risk is that it is equal to the amount of cash that could be removed from the beginning assets so that the remaining funds are just sufficient to mature benefits. Conceptually, this is the shareholder dividend that could be paid at the beginning of the insurance arrangement if there was certainty about the future cash flows. Of course, there is no certainty about the future cash flows, and it should be apparent that if the cash flows change, CFS will change accordingly. The change in CFS may be viewed as the cash cost of risk.

Suppose in the following example that interest rates, instead of remaining level at 14 percent, increase to 14.4 percent and that the liability matures at the end of one year by virtue of a policyholder election to exercise a discretionary withdrawal right. This is a classic example of mismatch risk.

	CFS
Level 14%	$ 24.59
Increase to 14.4%	(.06)
Cash Cost of Mismatch	$ 24.65

When we computed CFS based on these revised cash flows, the result was a negative $.06 versus the positive $24.59 in the level interest case. In effect, the shareholders would realize a loss of $24.65 relative to their expectations. The difference of $24.65 can also be thought of as the cash cost of the mismatch risk.

It should be apparent that CFS can be used to quantify other cash flow deviations, for example, those associated with pricing or asset default risks. CFS, of course, also works for any combination of cash flows associated with different risks.

USES OF CFS

- quantify risk
- actuarial opinion
- benchmark surplus

The good news, therefore, is that CFS can be used to understand and quantify risk. The bad news is that we're not completely sure how to use it. CFS addresses the economics of the business without regard to statutory solvency requirements. Thus, it is possible that CFS may be positive, while statutory surplus over the life of a book of business may become negative. The relationship of CFS to statutory surplus can provide insight into the real financial strength of an insurer. For instance, if statutory surplus is positive, while CFS is negative, this is a clear indication of future insolvency. The valuation actuary will be in the best position to use CFS in interpreting the results of the analysis of cash flows which will likely become an essential requirement of the actuarial opinion. More work is necessary to understand the relationship of CFS to statutory surplus.

CFS can also be used to develop benchmark surplus formulas that will be fairly easy for management to understand. We have found that surplus requirements based on cash-flow deviations associated with a given level of asset default, mortality, or mismatch risk are easily understood. The problem is to introduce some discipline on the process of combining cash flows for the various risks. We are currently in the middle of an effort to update benchmark surplus formulas at the Aetna using the cash-flow methodology, and we are encouraged by the results thus far. Given the overall subjectivity associated with quantification of risk, we are finding that the cash-flow-analysis technique produces reasonable results in a relatively simple and straightforward manner. The major difficulty is developing appropriate relativity in the choice of cash flows. Relativity, of course, has always been a problem even when we used a statistical approach; but now the relativity problem is easier for everyone to comprehend because the focus is on cash flows rather than means, standard deviations, and statistical formulas. We will eventually develop viable solutions to these problems.

MR. JAMES A. GEYER: Over the last several years, the CORTF has worked hard to impose discipline on the process of assessing benchmark surplus, and has lately delved deeply into statistical theory. In the end though, we have concluded that the determination of benchmark surplus formulas is indeed quite subjective. By being more sophisticated, one merely changes the areas where the subjective judgments enter in.

Nevertheless, our research provides valuable insight into the problem. My goal this afternoon is to review recent research that Mr. Mateja and I have conducted at the Aetna for the CORTF and then to discuss the implications of this research on efforts to determine benchmark surplus. I also intend to discuss briefly our own efforts to develop benchmark surplus formulas for our various lines at the Aetna.

I will begin by reviewing our research with respect to the C-3 risk with regard to SPDA products. We conducted the C-3 studies largely to test the computer model we had developed for the CORTF project and to establish some groundwork for our combination of risks research. Some of these results confirm findings from the work of the C-3 Risk Task Force, but some are new. All of it has a material bearing on the problem of determining benchmark surplus and of combining risks.

The major findings of the C-3 risk study are as follows*.

- The strategy for determining the level of interest credited to policyholders has a significant impact on levels of required surplus and the emergence of the statutory losses.

- There are several critical variables that affect the required surplus result. These include:

  - Critical variables
    - Investment strategy,
    - Assumed future interest rates, and
    - Assumed future lapse rates.

  - Less critical variables
    - Owner dividend policy,
    - Reserve conservatism and earnings margin, and
    - Federal income tax assumptions.

- One can determine a formula for estimating required surplus that accounts for the critical variables.

This study also reinforced our views concerning the interrelationships between reserves and surplus.

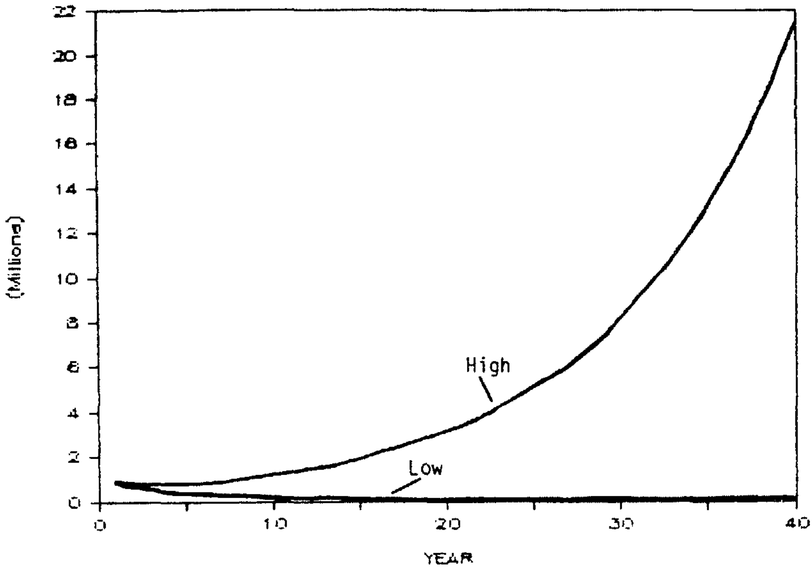
* The C-3 Risk Study for SPDA products appears as an addendum to this presentation.

I have a series of graphs and charts to illustrate these results for you. Because I want to devote most of my time to the combination of risks research and benchmark surplus, I will run through these quickly.

Graph 1 shows the effect on the liabilities in force for different crediting strategies.

GRAPH 1

LIABILITY BALANCE



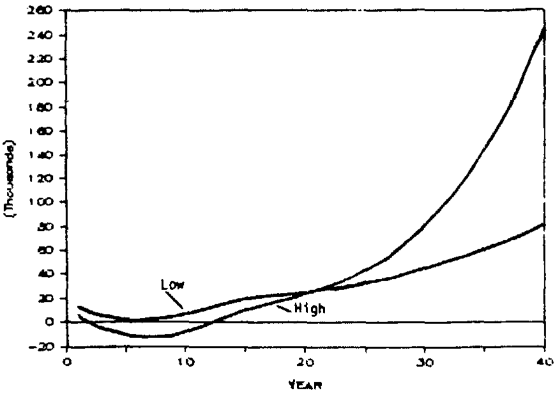
The "low" strategy involves keeping the credited rate fixed at the initial rate, despite rising new-money rates. In this scenario, lapse rates are high, and the liability balance falls off quickly. In the "high" strategy, we set the credited rate between the earned rate and the new-money rate; this keeps lapses at a fairly low level so that the liability balance grows with credited interest.

Graph 2 shows statutory gain from operations (GFOs) for the same two crediting strategies.

The pattern of GFOs by year are also quite different for the two crediting strategies. Losses develop immediately under the high strategy, as we credit more interest than is earned. Ultimately though, after the initial assets have rolled over, strong positive earnings emerge on the large liability balance. Under the low strategy, losses from high lapses are deferred. In fact, the interest on the surplus is sufficient to absorb the losses from the withdrawals. This is because, to be consistent, the amount of initial surplus used for both runs is the same and was determined from the high crediting scenario; this amount of surplus is actually much higher than is needed for the low scenario. In later years, the earnings under the low strategy derive solely from the interest on surplus, as the contracts have all lapsed.

GRAPH 2

GFO AFIT

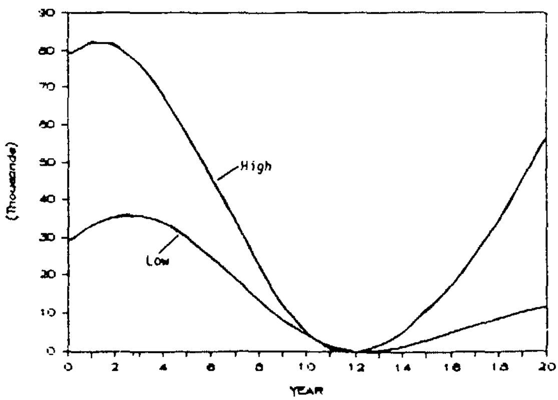


The required surplus for the high strategy was 7.9 percent versus 2.9 percent with the low strategy. Thus, crediting strategy has a large impact on required surplus.

Graph 3 illustrates what we mean by required surplus.

GRAPH 3

SURPLUS



The required surplus is the least amount of initial statutory surplus required to prevent statutory surplus from going negative. In each crediting strategy, the initial surplus balance is reduced by the series of negative GFOs, until the surplus reaches zero at approximately year 12. Thereafter, the credit rate is less than the earned rate, so that we have positive GFOs, and the surplus balance begins to grow again.

For our remaining tests, we assumed the insurer would follow the low credit strategy, i.e., keep the credited rates at the initial rate.

Variables Affecting Required Surplus

Assumptions relative to investments, future interest rates, and future lapse rates are key determinants of required surplus. Following are some results which illustrate how critical these factors are.

The first critical factor is asset length. To test this we fixed the interest rate level and lapse rate assumptions and varied asset length as indicated in this example:

INVESTMENT STRATEGY

Asset	Asset Duration (Years)	Required Surplus
10-year mortgage	3.9	0.0%
12-year mortgage	4.6	0.9
15-year mortgage	5.5	3.7
20-year mortgage	7.0	9.1
30-year mortgage	10.4	23.5

Required surplus takes off with increasingly long assets. For an intermediate length asset such as a 10-year mortgage, no statutory losses will develop, and hence, no statutory surplus is required. For longer assets, the amount of surplus required can be quite severe.

The next critical factor is the future interest rate level. In this case, we kept the asset (15-year mortgage) and lapse rates fixed:

INTEREST RATES

Future Interest Rate	Required Surplus
17%	1.2%
20	3.7
25	7.8
30	11.9

As the assumed future interest rate rises, the amount of required surplus increases. Thus, the amount of required surplus is a function of how bad you think interest rate conditions can get.

Finally, surplus also increases for higher assumed lapse rates as shown in the following:

LAPSE RATES

Lapse	Liability Duration (Years)	Required Surplus
20%	6.6	0.4%
25	4.5	3.7
30	3.7	6.6
40	2.7	10.4

Higher lapse rates effectively reduce the liability duration, thereby increasing the mismatch for a given asset.

There are several other factors or assumptions affecting the required surplus result to a lesser extent than the first three.

1. Owner dividend policy--We generally have assumed that there existed a minimum dividend that would have to be paid even in bad years. One can view this requirement as funding owner dividends, fixed overhead expenses, and/or requirements for growth. These required cash flows are important factors in a period of adversity.

2. Reserve conservatism and earnings margin--It should be intuitively clear that the amount of required surplus is dependent on the assumed earnings margin and the reserve conservatism.

3. Federal income tax--In periods of negative GFO, you need to assume whether or not the company will get negative FIT credit for its losses. This depends on your view as to whether there will be other lines in the company with positive GFO and on whether the losses would follow a period of years with tax gains so that the FIT carryback provisions could be utilized.

Formula for Estimating Required Surplus

We were able to develop the following formula for required surplus which accounts for the previous critical variables.

$$\text{Required Surplus} = (i - i_o)\ (D^A - D^L) - C.$$

The three critical variables are reflected by i (the assumed future new-money rate), the duration of the assets and the duration of the liabilities, which reflects the assumed lapse rates. The initial interest rate is i_o. Both i and i_o are after-tax. The constant C appears to be a function of the earnings margin, owner dividend policy, and FIT assumptions.

We derived the formula and the value of C using regression analysis on our results for a range of assets and lapse rates. What we found fascinating was how well our derived formulas matched actual statutory required surplus amounts. This is illustrated in the following.

FORMULA VERSUS ACTUAL SURPLUS

Asset	Formula	Actual
10-year bond	1.6	2.3%
15-year mortgage	4.2	3.7
20-year mortgage	9.9	9.1
15-year bond	10.1	10.4
20-year bond	16.0	16.6
30-year mortgage	18.5	17.9
30-year bond	23.2	23.5

For a given interest rate and lapse assumption, the formula and actual surplus results are quite close for a large range of assets. However, this formula should be viewed as a useful rule of thumb. It provides approximate, not exact, results. For example, two asset portfolios with identical durations can produce different required surplus results due to different cash-flow patterns. Also, if yield curves shift dramatically, and the asset and liability cash flows are badly mismatched, the formula may not produce good results.

The following table shows the relationship of earnings margin to the C-factor in our formula.

EFFECT OF EARNINGS MARGIN

Earnings Margin (Basis Points)	Required Surplus
50	3.8%
100	2.9
150	2.1

As the earnings margin increases, the required surplus decreases. Hence, higher earnings margins imply higher values of C. This decrease in required surplus reflects the fact that earnings provide the first line of defense for managing risk.

Reserves versus Surplus

The next table shows the results on required surplus of holding different levels of reserves.

BENCHMARK SURPLUS FORMULAS

RESERVE CONSERVATISM

Reserve Margin	Required Surplus
0% of CSV	2.9%
2	1.7
4	0.7

Holding an extra 2 percent of cash-surrender value as a reserve only reduces the required surplus by roughly 1 percent. The problem is that, as we encounter the period of adversity, the reserve conservatism is not available to cover the losses. Instead, we must continue to fund reserves at x percent over the cash-surrender value. The extra reserve conservatism does help through somewhat higher earnings on the extra assets and the reserve conservatism that is released on lapsing policyholders.

Note that we would not get this result if the conservative reserves were set by the actuary through the Valuation Actuary Opinion process and if the amount of conservatism remaining was allowed to decrease in a period of adversity. The problem is that the extra conservatism must be maintained in good times and bad times.

So what is the significance of these results to today's discussion? Several things:

1. There is no unique answer to the question of what level of benchmark surplus is required for something like C-3 risk. For example, companies selling in different markets or with different distribution systems will have considerably different lapse experience in a given interest scenario. Furthermore, there are so many factors that effect the surplus results, with each factor requiring an actuary's judgment, that different actuaries will inevitably arrive at entirely different answers.

2. On the other hand, these studies are valuable for understanding risk and understanding what the critical variables are.

3. The sensitivity of the required surplus results to these critical variables has implications to combining risks.

Combination of Risks

The CORTF was formed to investigate how surplus required for each of the C-1, C-2, and C-3 risks should be combined.

A basic question is, can the required surplus results for each of the risks be combined through some formula approach, or does one need to model the underlying cash flows and develop surplus standards using the deterministic methods we have been using for C-3 risk? I will examine the formula approach first, since it would certainly be much easier to apply.

Mr. Cody presented the following formula in his paper, "The Future Outlook for Stock Company Profitability and Mutual Company Surplus Position," RSA 8:2 (1982), pages 689-723.

$$S_c = \sqrt{S_1^2 + S_2^2 + (2)(r)(s_1)(s_2)}, \text{ where}$$

- S_1 is the surplus required for one risk for a given probability level,
- S_2 is the required surplus at the same probability level for another risk,
- r = correlation coefficient between risk 1 and risk 2, and
- S_c = surplus required for the combination of risks at that same probability level.

Our committee's final paper will include the derivation of this formula. I will now just review the underlying assumptions needed to derive this formula:

- Risks additive--If X is the surplus required for risk 1 at a given probability level, and Y is the surplus required for risk 2 at the same probability level, then the required surplus at the same probability level for risk 1 combined with risk 2 must be X+Y.
- Probability distributions for required surplus--(1) Distributions are assumed to be normal. (2) Distribution must be symmetrical around the mean, so that both gains and losses are possible for the given risk. For example, for the risk of extra mortality, there is the potential for gain from lower than expected mortality as well as loss from higher than expected mortality. Also, in Mr. Cody's formula, the mean is assumed to be zero.

Unfortunately, these assumptions do not hold for all risks, especially for C-3 risk. As a result, this formula approach might not be expected to work well where C-3 risk is involved. I will review with you where and why the assumptions do not hold.

This table presents some data from our model which demonstrates that the risks are not additive.

C-1 Risk		C-3 Risk		Combined		
				Required Surplus		
Default Rate	Required Surplus	Interest Rate	Required Surplus	Sum	Actual	Percent Error
1%	0.6	16%	0.0%	0.6	0.6	nil
2	2.6	18	0.4	3.0	3.6	17%
3	6.0	20	3.2	9.2	10.3	11

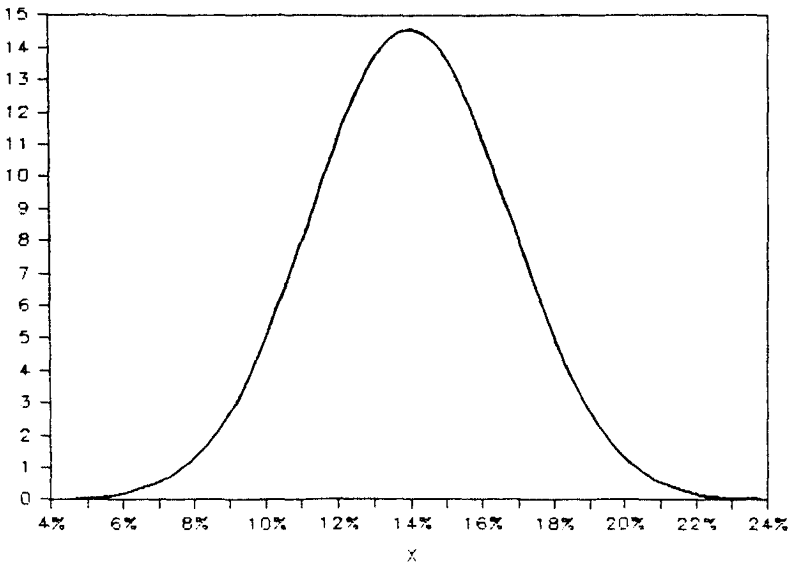
These data demonstrate that the required surplus for a given combination of C-1 and C-3 risk is different from the sum of the surplus required for the two risks taken one at a time. This results primarily from two factors: (1) With increasing interest rates under the C-3 risk, the assets are worth less, so that manifestation of another risk is more painful than when the C-3 Risk is not present. (2) The "credit" for the earnings margin (recall our regression formula) is reflected in each of the individual results but can only be used once in the combined results.

It was exactly this interaction that initially led us to doubt the validity of statistical approaches that work by combining surplus results for the individual risks.

Graph 4 illustrates the probability distribution curve that we have developed for the C-3 risk. I will show how this distribution compares to the normal assumption of the formula approach.

GRAPH 4

INTEREST RATES

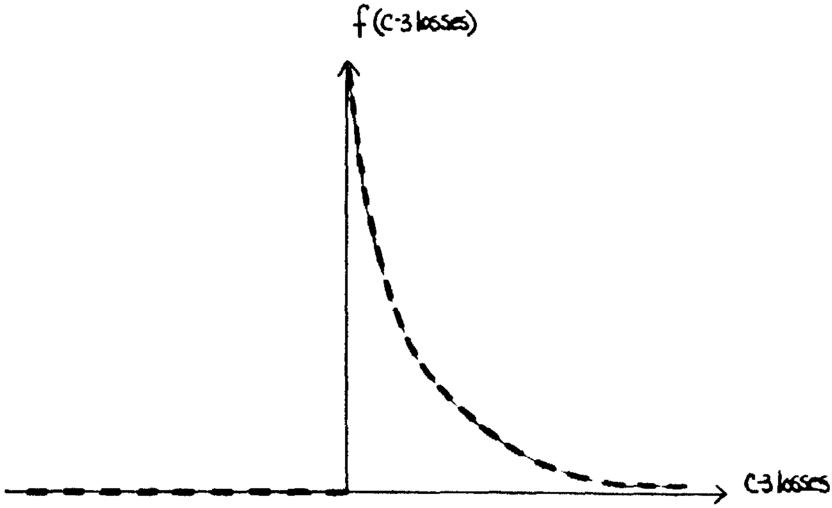


For interest rates, we have assumed that the distribution for the future interest rate is normally distributed. Keep in mind that the interest rate from this distribution applies to all future years. Thus a 20 percent result is taken to be representative of future scenarios where rates will go up and down, but the average for all future years is 20 percent. We developed the distribution shown here by assuming that the 20 percent future interest rate scenario was a 1-in-100 type of catastrophe. By assuming a mean of 14 percent and a normal distribution we defined the curve for the other points. We also assumed interest rates are as likely to go down as to go up.

To say that interest rates are normally distributed is quite different though from saying that required surplus for C-3 risk is normally distributed. Required surplus for C-3 risk mushrooms as interest rates increase. Furthermore, we assume that the company will lower its credited rates should interest rates fall so that there is no gain potential symmetrical with the loss potential.

Consequently, the probability density function for required surplus is quite different from the normal, as shown in Graph 5.

GRAPH 5



In particular, there is no probability associated with negative surplus values (gains). There is very high probability associated with zero surplus since zero surplus results from the half of scenarios with decreasing rates, and with moderate increase in rates. At the right, the tail of the distribution falls much more slowly than the normal which reflects the property that small increases in interest produces large increases in required surplus.

Thus for C-3 risk, we have neither the additive property nor the normal distribution that the formula requires.

In summary, there is reason to be skeptical of the formula approach. Our skepticism has led us to the original commitment to build the CORTF computer model and to develop an understanding of how cash-flow deviations from various risks interact.

On the other hand, merely trying different risk combination scenarios in our model to develop surplus standards deterministically does not have much appeal. We need some discipline for deciding how we select those combinations of risk that would be consistent with our worst-case assumptions for the individual risks. For example, if we are assuming that 20 percent future interest rates are a 1-in-100 scenario, how do we determine those combinations of C-1, C-2, and C-3 risk that would provide consistent 1-in-100 disaster scenarios?

For C-3 risk, we merely chose our worst-case scenarios and did not worry about any underlying probability distribution for interest rates or interest rate scenarios. For combining risks, we need the underlying probability distributions, in order to determine what combinations to test.

Combining Risks--Basic Theory

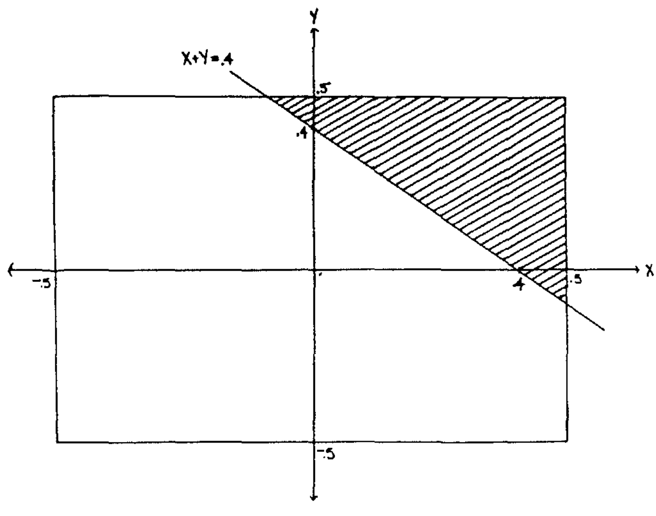
Suppose we want to combine two very simple risks. Assume that,

X = Random variable of the loss for Risk 1, $-.5 \le X \le .5$
Y = Random variable of the loss for Risk 2, $-.5 \le Y \le .5$

For both risks, the maximum loss is .5 and there are symmetrical gains possible (gains are represented by negative value of X and Y). Furthermore, assume that required surplus for a given level of risk is equal to the loss from that risk. Given these assumptions, it is clear that the total loss (and, hence, required surplus) for a combination of a given level of each risk is the simple sum of the two loss amounts (X+Y).

Graph 6 shows all possible combinations of X and Y.

GRAPH 6



Also shown are all combinations that require exactly .4 of surplus; these combinations form the line X+Y=.4. Thus, .4 of surplus would be exactly enough to cover combinations of X and Y that lie on this line. For example, X and Y can each be .2, or X can be negative .1 (a gain) if Y is +.5, and so on. Furthermore, .4 would be sufficient to cover all combinations of X and Y then lie to the lower left of the .4 line. For example, when X=.1, and Y=.2, the total loss of .3 is less than .4 of surplus, so we are covered. On the other hand, .4 of surplus would be insufficient to cover all combinations of X and Y that lie in the shaded area above the X+Y=.4 line.

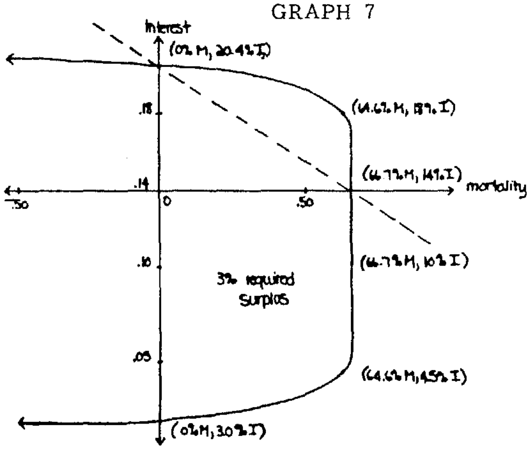
We have named such lines "lines of constant surplus," since all points on the line require the same amount of surplus.

Our ultimate goal is to find how much surplus is required to assure adequacy at some high probability level, say 99 percent. Thus, in this example, we wish to find a value, say K, such that the total probability associated with all combinations of points in the area to the upper right of the X+Y=K line is 1 percent; the total probability associated with all combinations of points to the lower left of the line is, thus, 99 percent.

It is not enough to measure the relative areas above and below the line to determine K. Instead, we need to assume a probability frequency function for the random variables, so that we can sum up the relative probabilities of the combinations of points on either side. It is here that we need probability distribution functions for the two random variables. Furthermore, if these were two normal distributions, we should be able to duplicate the results of the statistical formula presented earlier.

Consider Graph 7 showing a line of constant surplus for combinations of interest rate and excess mortality risk. In dealing with mortality risk here, we have essentially shifted from our SPDA contract to a universal life contract. Also, the extra mortality is expressed as a percentage over expected. Thus, if we expected 2 deaths per 1,000, a 50 percent increase would be 3 deaths per 1,000.

GRAPH 7



Note how different the line is from the simple X+Y line seen previously. This line corresponds to 3 percent surplus. Thus, 3 percent surplus is exactly enough to cover 0 percent excess mortality and 20.4 percent interest, 66.7 percent excess mortality and 14 percent interest, or 66.7 percent excess mortality and 10 percent interest, and so on. Sample points that were used to draw this line were developed through testing with our computer model. As with the simple example, you can see that the 3 percent surplus will be more than adequate for all combinations of interest rate and mortality risk that lie to the left (or "inside") of the 3 percent line. This is because all such combinations are less severe than a given point on the line. On the other hand, 3 percent surplus would be inadequate for any point to the right (or "outside") of the 3 percent line.

The shape of the 3 percent line is interesting in that it is quite different from the straight X+Y lines we saw in the simple example. The shape of the curve reflects the C-3 risk characteristics we reviewed earlier. In particular, when interest rates rise from 14 percent to about 17.5 percent, there are no statutory losses and, in fact, very little if there is any reduction in statutory gains. The same is true as interest rates drop below 14 percent, until about 5 percent. As a result, the 3 percent surplus can cover 67 percent extra mortality for any interest rate from 5 to 17 percent. This explains why the line is a straight vertical line for this interest rate range.

Once interest rates exceed 18 percent, required surplus mushrooms. As interest rates rise slightly, the 3 percent surplus is adequate only if the excess mortality drops substantially. At very high interest rates, we have practically a horizontal line, since a slight increase in interest rate requires big improvement in mortality. A similar result occurs for very low interest rates, since we assume the guaranteed minimum credited rate is 4 percent.

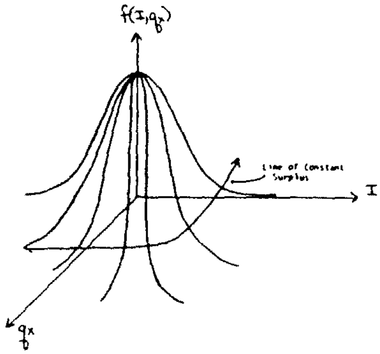
The dashed straight line reflects what one might expect if the two risks were directly additive. You can easily see how far away from this assumption we are.

This line of constant surplus is totally independent of the probability distributions assumed for interest rates and excess mortality. Instead, it depends solely on our computer modeling. To determine the probability adequacy level of, say 3 percent surplus, we must assume a probability distribution function for each of the risks. See graph 8.

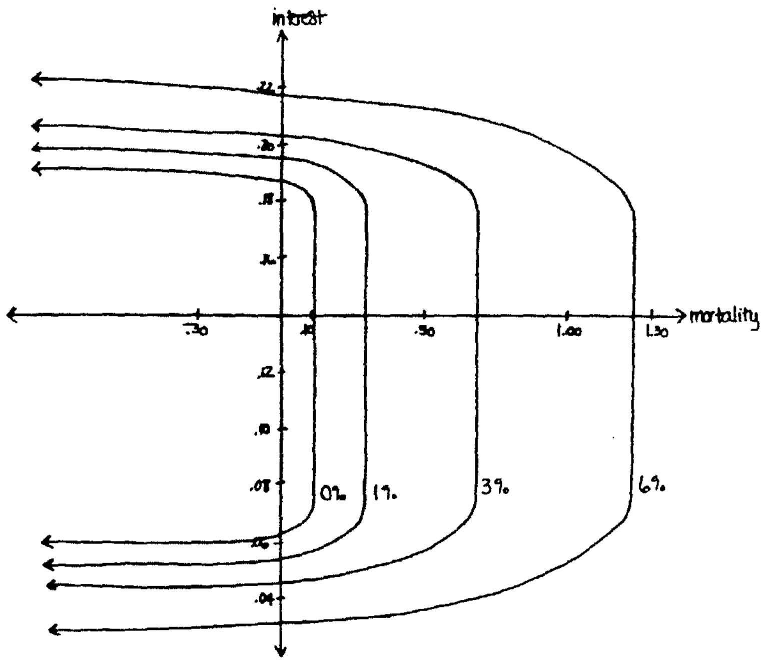
To determine the probability adequacy of a given surplus amount, we must measure the probability volume that lies above the reason the I, q_x plane that is within the line of constant surplus. If, for example, 90% of the total volume lines inside, then this surplus level provides 90% adequacy.

If we want to know how much surplus is required for the 99 percent level, we essentially have to work backwards. In other words, we would find the probability level for various lines of constant surplus, until we could "close in" on the 99% level line. See graph 9.

GRAPH 8



GRAPH 9



The following table presents the probability results for sample lines of constant surplus.

BENCHMARK SURPLUS FORMULAS

C-2, C-3 RISK COMBINED

Surplus	Probability Adequacy
0%	82%
1	94.6
2	97.3
3	98.4
4.6	99
6	99.7

Note that at 0 percent surplus, the bare reserves provide adequacy 82 percent of the time. This reflects the probability distribution functions that we have adopted for C-2 and C-3 risks; assumes the assets were invested in 10-year bonds (which have a duration of about 4.5 years); and assumes lapses are tied to the difference between earned and credited interest rates using the same formula that we used for SPDAs. This formula may be too drastic for universal life, but I do not think it is too far off. It is also assumed that the combination of interest crediting, mortality margin, and expenses provide a 100 basis point pretax profit margin.

As surplus increases even slightly from 0 percent, the probability adequacy improves dramatically at first, but then the improvement tails off as surplus is raised further. Thus, in going from 0 to 1 percent, the adequacy level goes up 12.6 percent; from 1 to 2 percent surplus, we gain only 2.7 percent adequacy, and from 2 to 3 percent, we pick up only another 1.1 percent adequacy level. To get the next .6 percent, i.e., from 98.4 to 99 percent, we must increase surplus, from 3 to 4.6 percent.

When we were dealing with the C-2 and C-3 risks individually, our position was that we wanted surplus to survive 20 percent interest rates and 50 percent extra mortality. We then defined these to be 1-in-100 type scenarios. The combination of risks results consistent with these 1-in-100 scenarios are the 4.6 percent surplus for the 99 percent probability.

Consider next how the combination results compare to the individual risk results.

REQUIRED SURPLUS AT 99% LEVEL

Individual Risks

C-2:

99% = 50% excess mortality = 2.2% Required Surplus

C-3:

99% = 20.4% interest rates = 3.0% Required Surplus

Combination of Risks

A. Simple Sum: 5.2%

B. 100% correlation (20.4%, 50% Q_x) : 5.6% Required Surplus

C. Formula: $(2.2)^2 + (3.0)^2 = 3.7\%$ Required Surplus

D. Actual: 4.6% Required Surplus

For C-2 risk alone, the 99 percent standard would imply 2.2 percent required surplus. Similarly, the 99 percent level for C-3 risk alone would imply required surplus at 3.0 percent. If we simply add these two together, we would get 5.2 percent required surplus. On the other hand, due to the interactions of various risks, if we actually experience 50 percent excess mortality at the same time as 20.4 percent interest, the required surplus would be 5.6 percent. This is the nonadditive property. The statistical formula suggests an answer of 3.7 percent. The actual result is roughly halfway between the statistical formula results and the full 100 percent correlation result (note that we assume 0 percent correlation).

The next table shows corresponding results for other probability levels.

REQUIRED SURPLUS

Probability Level	C-2	C-3	Sum	100% Correlation	Actual Combined	Formula
95%	.6	0	.6	.9	1.2	.6
97.5	1.1	.9	2.0	2.6	2.2	1.4
99	2.2	3.0	5.2	5.6	4.6	3.7

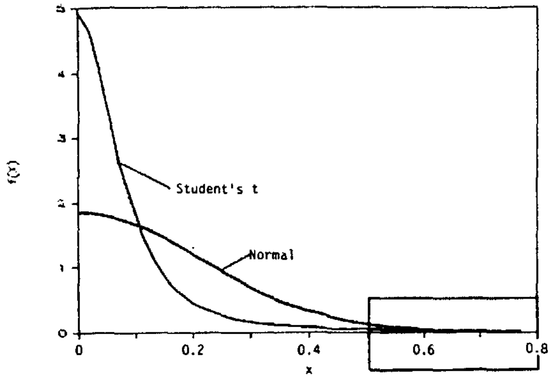
Again, the actual results exceed the formula results. At the 95 percent level, the actual result actually exceeds the 100 percent correlation results. Interestingly, the actual combined required surplus is somewhat close to the simple sum of the individual C-2, C-3 required surplus results.

We have also done some testing of different probability distribution assumptions to determine how critical such assumptions are to the final results. For the excess mortality assumption, we tried both a normal curve, and a Student's t curve, for the right-hand side, or excess mortality side, of the distribution. See graph 10.

Both of these curves were developed to give a 99 percent probability that excess mortality would be below 50 percent. Thus, the area under the curves up to 50 percent extra mortality is the same, and hence, the area under the curves in the lower box is the same for each curve.

GRAPH 10

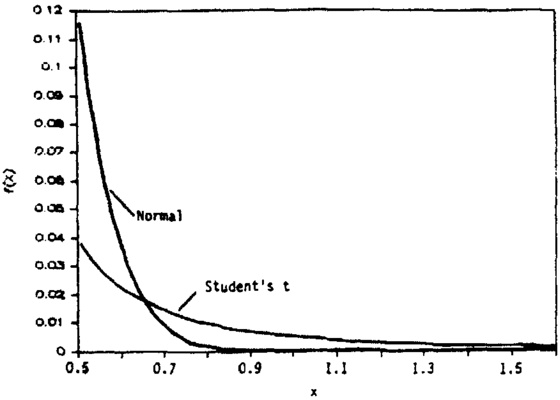
NORMAL VS STUDENTS T



The results thus far used the Student's t curve, which we felt provided a more realistic picture of this risk. (See graph 11). The Student's t has the desirable characteristic so that most of the time, we do not get more than 15 or 20 percent extra mortality (90 percent of the total probability lies to the left of 14 percent; with the normal, 90 percent is to the left of 28 percent). At higher excess mortality levels, the distribution does not fall off nearly as fast as the normal. Thus getting 100 percent extra mortality (that is, 4 deaths per 1,000 instead of 2 deaths per 1,000), is not that much more unlikely than 70 or 80 percent extra mortality. With the normal, as you can see in graph 11, there is still material probability associated with 70 percent extra mortality, but extremely small probability associated with 100 percent excess mortality.

GRAPH 11

BLOWUP OF BOXED AREA



The results for the two distributions are presented below.

EFFECT OF DISTRIBUTION FUNCTION

Adequacy Level	I: Normal Q_x: Student's t	I: Normal Q_x: Normal
66%	0%	0%
82	0	.6
90	.6	1.0
95	1.2	1.6
97.5	2.2	2.1
99	4.6	3.2

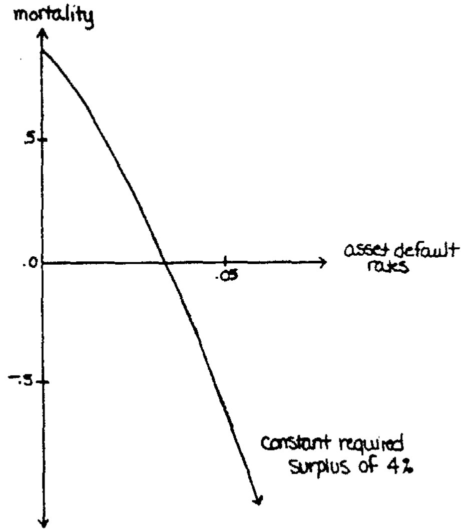
It is interesting how different the results are for the two different distributions and how the relationships between the two change with different probability levels. First, notice that whereas 0 percent surplus was good 82 percent of the time with the Student's t, it is good only 66 percent of the time with the normal. The explanation for this is that with the Student's t, the great bulk of the mortality probability was to the left of 10 to 15 percent extra mortality. The normal does not fall off so quickly, so that there is still substantial probability between 15 and 50 percent. At higher probability levels, we get the opposite result that less surplus is needed with the normal assumption than with the Student's t. The reason for this is that the normal falls off quickly after the 1-in-100 or 50 percent mortality point, whereas the Student's t has a considerably longer tail.

I feel that the Student's t curve provides a better fit for most of these catastrophic risks. For example, with something like asset defaults or interest rates, the great bulk of the time the experience will not be far from expected. However, at the extremes, or catastrophes, it seems unlikely that the probabilities could fall off as quickly as they do with the normal. For example, with the interest rate scenario, if 20 percent is 1-in-100, the normal would tell us that it is virtually impossible to get to 22 or 23 percent. This does not make a whole lot of sense for catastrophic risk. If interest rates can get as bad as 20 percent, why not 21 or 22 percent. If asset default rates can be 3 percent per year, why not 4 or 5 percent? If the 3 percent asset default rate is 1-in-100, the Student's t implies that 4 percent is 1-in-174. With the normal, if 3 percent is 1-in-100, 4 percent would be 1-in-1,110, or 11 times less likely.

In any case, given that we used a normal for interest rates instead of a Student's t, I suspect that our results for combination of risks at the 99 percent level are slightly too low (graph 10). I suspect that the true answer is closer to 5.2 percent, the simple sum of the two individual results.

We have just begun testing combinations of C-1 and C-2 risk. The initial results show that the lines of constant of surplus are essentially straight lines, which reflect the more additive nature of the risks than we had with C-3 risk.

GRAPH 12



Furthermore, our very initial results suggest that the statistical formula provides a reasonable approach for combining the individual surplus results. We still have a bit of work to finish up in this area though.

Acknowledgments

I would like to give credit to those who have helped Mr. Mateja and me in this work. Mr. Tim Corbett, an actuarial student in our area, was responsible for developing the CORTF model. Next, Ms. Linda Dinius, another Aetna employee, was instrumental in developing the statistical theory we used in the COR. Finally, Mr. Cody, and several other Task Force members (Mr. Don Sondergeld, Mr. Steven Meyers), provided valuable advice along the way.

Conclusions

I would now like to review what we have learned from our research.

1. Cash-flow analysis can be very revealing. It facilitates an understanding of:

    a. exactly how a given risk manifests itself,
    b. the effects of such things as the earnings margin, reserve conservatism, owner dividends, and
    c. how various risks interact.

    In short, it is a useful and worthwhile exercise.

2. We have learned that C-3 risk behaves differently from other risks. This applies to:

a. The distribution function of required surplus amounts, and
b. the fact that manifestation of a little C-3 risk added to another risk magnifies the effect of the other.

These two factors create problems for statistical formulas that operate by combining individual surplus results.

3. We are further convinced that we will never be able to determine surplus requirements precisely. There are simply too many unknowns:

   a. Is 20 percent interest the right 1-in-100 level, or should it be 18 or 25 percent? Historical data are of little use in measuring this.

   b. If dealing with C-3 risk alone, we can sidestep the question in (a) and simply declare that 20 percent is a sufficiently bad scenario that surplus should cover. However, when combining risks, we need to address the question and define a probability distribution curve. We then have a problem of determining the shapes of distributions at the extremes. We have found that the shape of the distribution function assumed does indeed matter.

   c. Furthermore, we also need to look at C-1 and C-2 risks and look for a "bad" scenario for those risks that is consistent with our choice of interest scenario. Getting the relativities right for each of these risks is unlikely.

   In the end, determining benchmark surplus is subjective and dependent both on one's view of how bad things can get and how bad a scenario you feel you ought to be able to survive.

4. The theoretical approach that we have developed for combining risks is appropriate for understanding the problem and understanding the interactions of various risks. We recognize that it is not very useful in practice, as it is exceedingly difficult to carry out. One needs to do a lot of work in developing the probability distributions, a great deal of computer modeling must be done to develop the lines of surplus, and calculating the volume under the surface in three dimensions is a complicated and painstaking job. Also, we cannot extend the methodology to combinations of three risks, as we would then have to deal in four dimensions, which is beyond our capabilities.

5. We have developed some rules of thumb. For risks other than C-3, we are finding that the formula approach provides generally reasonable results. However, C-3 risk operates quite differently such that we feel surplus for C-3 should be added to the others directly.

6. So how do we determine surplus? I think it is misguided optimism to think that we will even be able to specify surplus for a product

or line or company to provide 95 or 99 confidence. That ought not to be the purpose of these studies.

Actual surplus for a company is limited by such forces as competition, shareholder dividend paying needs, regulatory restrictions, and management's philosophy and views. From an assuring solvency standpoint, the only firm rule is that more surplus is better than less.

7. The purpose then of developing benchmark surplus is as follows:

   a. to understand risk,

   b. to measure risk relativities among your company's various lines, which can be used to allocate total surplus to the different lines, and

   c. To provide insight into management options to reduce risk, such as product changes, investment changes, crediting strategies, and so on.

8. At the Aetna, we use benchmark surplus standards in pricing and in allocating the company's total surplus to each of the lines. One of the profit measures used to measure profitability for each of the lines is return on total capital, which reflects the allocated surplus.

9. We are currently in the process of redetermining our benchmark surplus formulas. Our approach is essentially deterministic. We have agreed with the various lines to several "bad" scenarios which we speak of as being roughly 1-in-100 types of scenarios. Whether they really are 1-in-100 or 1-in-50 or 1-in-1,000, we cannot know and have decided that it does not matter that much. Our goal is to develop risk relativities among lines.

   Once we have defined the scenarios, we can use our computer models to develop a required surplus result for each line for those scenarios. Based on the results thus far from the CORTF research, we intend to combine surplus for the C-1 and C-2 risks using the statistical formula, but then add the required surplus for C-3 risk directly.

   This first step will provide a measure in dollars, of surplus for each line. We will then use a ratio of actual total company surplus to total formula surplus to adjust the factors in the benchmark surplus formulas, so that the final formulas do reproduce total actual surplus.

ADDENDUM

C-3 RISK ASSOCIATED WITH THE SPDA PRODUCT

by
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The purpose of this report is to present the results of an analysis of C-3 risk associated with a typical SPDA product and an assessment of these results.

Background

When the CORTF decided to use cash-flow analysis in its effort to understand and quantify combinations of risks, it became necessary to develop a computer model to support the necessary analysis. The process of constructing such a model quickly established that while modeling future cash flows associated with an insurance product subject to different risks was relatively straightforward, understanding and interpreting the resultant cash flows was not nearly as simple and straightforward. A great deal of effort was eventually devoted to the subject of discounting cash flows and the end result was the concept of CFS.

Once the model took shape, it became obvious that the model offered the opportunity to more thoroughly understand mismatch risk. When a relationship between "required surplus" and "duration" was developed, it was concluded that it would be worthwhile to complete a more disciplined analysis of C-3 risk that would serve as a point of departure when this risk was combined with other risks in subsequent phases of the research effort.

Appendix I contains a detailed description of the assumptions used to model the SPDA product. The mechanics of the computer model will be described in the final report of the CORTF.

Major Findings

1. Management of the interest crediting strategy can influence the timing of reported losses associated with the C-3 risk but, perhaps, not the ultimate economic cost.

2. The degree of mismatch or C-3 risk as measured by required surplus is extremely sensitive to the investment strategy, the level to which future interest rates rise, and withdrawal rates. Other factors not commonly associated with C-3 risk have a significant bearing on required surplus including the treatment of negative FIT, the shareholder dividend policy, and the margin for adverse deviation contained in the reserves.

3. Assuming a level future interest rate environment, there appears to be a relationship between the difference between asset and liability durations, the basic earnings margin, and required surplus. Specifically, required surplus can be approximated by the following formula:

$$\text{Required Surplus} = (i_{actual} - i_{base}) \quad (D^A - D^L) - C$$

where

i_{actual} = the assumed level future new-money rate

i_{base} = the interest rate at which the initial assets are invested

D^A = the Macaulay duration of the assets

D^L = the Macaulay duration of the liabilities

C = a constant that is a function of the earnings margin present prior to an adverse interest scenario

This approximation formula was derived from regression analysis performed on the model results. It appears to hold on both a pre-tax and post-tax basis, where on a post-tax basis, i_{actual} and i_{base} are after-tax rates, and the durations are calculated using after-tax interest rates.

I. MANAGEMENT OF INTEREST CREDITING STRATEGY

The nature of the C-3 risk for a typical SPDA product as measured by required surplus is dependent on the interest crediting strategy adopted by management when interest rates rise dramatically. Testing has concentrated on the effects of two basic crediting strategies:

1. Low Crediting Strategy

   Keep credited rates low and risk a great outflow of cash which, if it produces negative cash flow, requires liquidation of assets at a loss or borrowing at high rates which produces an equivalent loss.

2. High Crediting Strategy

   Increase credited rates to prevent the cash outflow, thus producing a certain immediate loss due to crediting more interest than the assets are actually generating.

Results for a CORTF computer model run when the low crediting strategy was followed are summarized in table 1. The major assumptions are as follows: initial assets earn 14 percent with a duration of 5.5; interest rates immediately rise to and remain at 20 percent; the 13 percent credited rate is maintained indefinitely; policyholders lapse at the rate of 25 percent each year; and initial liabilities equal $1,000,000.

Assumed high lapses due to low credited rates immediately cause net cash flow to turn negative. In the CORTF computer model, negative net cash flow is covered by borrowing at the then current interest rate, i.e., 20 percent. If assets are liquidated, the losses would show up immediately. By borrowing, the losses are deferred, but not avoided.

The average earned rate on the net assets (invested assets less borrowing) falls steadily as more money is borrowed at 20 percent. Since the assumed asset is a 15-year mortgage, the last of the initial assets rolls over at the end of year 15, and the average earned rate stabilizes at -20 percent. (A negative sign is used to denote the situation where both interest and assets are negative.) In year 12, the asset balance becomes negative, indicating a net borrowed position.

TABLE 1

CORTF Model Results
Low Crediting Strategy

Year	New-Money Interest Rate	Credited Rate	Lapse Rate	Liability Balance EOY (000)	Net Cash Flow (000)	Average Earned Rate	GFO AFIT (000)
1	20.0%	13.0%	25.0%	$847.5	$-107.9	14.0%	$ 6.3
2	20.0	13.0	25.0	718.3	-92.1	13.2	1.5
3	20.0	13.0	25.0	608.7	-78.0	12.4	- 2.3
4	20.0	13.0	25.0	515.9	-64.6	11.6	- 5.4
5	20.0	13.0	25.0	437.2	-50.8	10.7	- 7.8
10	20.0	13.0	25.0	191.2	- 9.2	6.2	-11.8
15	20.0	13.0	25.0	83.6	-13.0	-33.5	-10.8
20	20.0	13.0	25.0	36.5	-46.1	-20.0	-21.2

Since lapses are assumed to occur at the end of the year, first-year earnings are unaffected by the interest rate increase. Earnings quickly become negative, however, as the average earned rate drops below the credited rate.

Consider next how the C-3 risk is manifested when the high crediting strategy is chosen, i.e., management increases the credited rate ("chases" the new-money rate) in an attempt to minimize cash outflow. Assume it is decided to credit what the average earned rate would have been if there were no lapses. Assume further that lapse rates are still high initially, but considerably reduced from the prior case. Table 2 presents the specific credited rate and lapse rate assumptions and the results.

TABLE 2

CORTF Model Results
High Crediting Strategy

Year	Interest Rate	Credited Rate	Lapse Rate	Liability Balance EOY (000)	Net Cash Flow (000)	Average Earned Rate	GFO AFIT (000)
1	20.0%	14.0%	22.0%	$ 888.9	$ -62.9	14.0%	$ 7.0
2	20.0	15.0	19.2	825.9	-19.1	13.6	- 0.7
3	20.0	15.8	16.9	794.6	13.5	13.5	- 5.0
4	20.0	16.6	14.6	791.2	46.5	13.6	- 8.3
5	20.0	17.3	12.6	811.6	78.1	14.0	-10.4
10	20.0	19.0	7.5	1,212.8	203.1	17.7	- 7.5
15	20.0	19.0	7.5	1,959.9	277.8	19.9	10.7
20	20.0	19.0	7.5	3,167.2	474.9	20.0	23.9

Note how different the manifestation of the risk now appears. Except for the first two years, the net cash flows are all positive. Lapses still create loss in that they reduce the cash flow otherwise available for investment at the high interest rates. But little borrowing at high rates is required because of lapse which produced the losses in the low crediting strategy. Losses in this second example can be directly related to the decision to credit at a rate greater than supported by the assets. What makes the high crediting strategy potentially attractive is that ultimately the normal profit margin is achieved, and there is a substantial book of business in force.

To absorb these statutory losses, a certain amount of initial statutory surplus would be required. We have defined "required surplus" to denote the least amount of statutory surplus that is required to maintain statutory solvency in each future year. Furthermore, we assume such required surplus is backed by assets having the same characteristics as those assets backing the liabilities.

Required surplus for these two crediting strategies is as follows:

Crediting Strategy	Required Surplus
Low	2.9% of initial liabilities
High	7.9

The higher required surplus for the high crediting strategy implies that this is a far riskier strategy. As developed, this conclusion is not necessarily accurate. Another means to compare these two crediting strategies is to determine the cash-flow based surplus, which equals the net present value of assets less liability cash flows. CFS is useful here to demonstrate the "economic" as opposed to the statutory impact of different crediting strategies.

Under the low crediting strategy, required surplus is 2.9 percent, and CFS is -1.5 percent, where both are percentages of initial liabilities. Under the high "chase the rate" crediting strategy, required surplus is 7.9 percent and CFS is -2.3 percent. The CFS results are sufficiently close for us to conclude that the two strategies are current economic equivalents. The material difference between the two strategies is that with the high "chase the rate" strategy a large, profitable (assuming no further C-3 risk manifestation) block of in-force business remains after the losses due to the C-3 risk disappear. However, the high crediting strategy may be difficult to implement since management must be prepared to accept immediate losses and surplus requirements to fund the losses are considerably higher.

A more moderate "chase the rate" strategy was investigated to see what effect variations in the lapse rate and credited rate would have on required surplus and CFS. Consider the following "intermediate" crediting strategy, which is assumed to produce lapses somewhere in between the two already presented. Table 3 summarizes the high and intermediate "chase the rate" assumptions. Remember that in the low crediting strategy, the credited rate was held at 13 percent with lapses at 25 percent.

TABLE 3

"Chase the Rate" Crediting Strategies

Year	High		Intermediate	
	Credited Rate	Assumed Lapse Rate	Credited Rate	Assumed Lapse Rate
1	14.0%	22.0%	13.5%	23.4%
2	15.0	19.2	14.0	22.0
3	15.8	16.9	14.5	20.6
4	16.6	14.6	14.8	19.8
5	17.3	12.6	15.2	18.6
6	17.9	10.8	15.5	17.8
7	18.4	9.3	15.7	17.2
8	18.8	8.1	15.9	16.6
9	19.0	7.5	16.0	16.3
10	19.0	7.5	16.2	15.8
11	19.0	7.5	16.6	14.6
12	19.0	7.5	17.2	12.9
13	19.0	7.5	17.9	10.8
14	19.0	7.5	18.4	9.3
15	19.0	7.5	18.8	8.1
16+	19.0	7.5	19.0	7.5

Under the intermediate strategy, the credited rates are pegged at a lower level relative to those assumed in the high strategy, so that we would expect greater lapse rates. With the assumptions of the intermediate strategy, required surplus is 4.1 percent, and CFS is -1.8 percent. These results fall within the range established by the high and low crediting strategy results. The CFS results suggest that, for a given mismatch situation, management of the interest credits will not appreciably alter the underlying economic loss.

Mismatch risk is dependent on asset cash flows as well as liability cash flows, i.e., lapse rates. Thus, the effects of the crediting strategy could be influenced by the assets backing the liabilities. Table 4 illustrates the effects on required surplus and CFS of the previous three strategies for four asset types. Required surplus and CFS are expressed as a percentage of initial liabilities.

These results clearly indicate increasing risk with increasing asset length, but interestingly, in all cases, required surplus materially overstates the economic loss as measured by CFS. It is also interesting to note that, for a particular investment, the economic loss as measured by CFS is remarkably stable for the various crediting strategies. The low crediting strategy produces the lowest level of loss on a present value basis primarily because the losses are deferred into the future whereas, under the intermediate and high crediting strategies, most of the loss is recognized up front.

TABLE 4

Effect of Interest Crediting Strategies
on Required Surplus and CFS
for Various Assets

CFS and Required Surplus as a Percentage of Initial Liabilities

	CFS			Required Surplus		
	Low	Intermediate	High	Low	Intermediate	High
10-year Bond	0.0%	- .1%	- .6%	1.5%	3.0%	6.3%
15-year Mortgage	- 1.5	- 1.8	- 2.3	2.9	4.1	7.9
20-year Bond	- 9.8	-10.1	-10.6	16.0	16.7	19.6
30-year Mortgage	-10.4	-10.7	-11.2	17.5	17.5	19.5

Through testing of various crediting strategies, it was found that these types of evaluations are highly dependent on the lapse assumptions. Thus, the "best" strategy is highly dependent on one's perceptions of how policyholders will react to the different levels of credited rates.

One additional interest crediting strategy was tested where the credited rate was immediately reduced to 4 percent. It was assumed that policyholders would lapse in droves: 35 percent in year 1, then 50 percent in later years. Using the 15-year mortgages, required surplus was 6.4 percent, and CFS was -2.6 percent. These results suggest that there is a lower limit to losses associated with low crediting strategies. Results, however, are extremely sensitive to lapse assumptions. For instance, if lapses remain at 30 percent in this example, no additional surplus would be required.

II. RISK FACTORS

The effect on required surplus of combinations of the various risks assumed by an insurance company is the focus of the CORTF research effort. As development of the CORTF model progressed, it became clear that required surplus is dependent on many other factors not immediately associated with the basic insurance risks. The Task Force felt it was necessary to develop a better understanding of these factors. The purpose of this section is to examine those factors which have been found to affect required surplus in a C-3 risk environment. Some of these factors have been previously addressed by the C-3 Risk Task Force. When a relationship between required surplus for C-3 Risk and duration was discovered, it was decided that it was necessary to reexamine some of the work of the C-3 Risk Task Force.

The following factors in particular were found to have a material effect on the degree of risk as measured by required surplus:

1. investment strategy
2. future interest rate assumption
3. withdrawal assumptions
4. owner dividend policy

5. treatment of negative FIT
6. reserve conservatism/earnings margin
7. interest crediting strategy

All tests presented in this report assume interest rates increase immediately to some high level and remain there. Thus, the future reinvestment strategy has no effect on required surplus; with variable future interest rates, the reinvestment strategy would have a significant effect.

These risk factors were examined one at a time by holding all other risk factors constant and appropriately varying the relevant risk factor. Except when "chasing the rate," the credited risk is set equal to the average earned rate of the investments less the earnings margin but never less than the initial credited rate.

1. Investment Strategy. In the CORTF computer model, the characteristics of the initial assets are defined by two variables: the type of asset invested in and the distribution of the insurance cash flows invested over the prior ten-year period. Aggregate rollover rates of the initial block of assets can then be developed.

   Table 5 summarizes the asset and liability durations and required surplus for the various assets tested. (The duration index used is Macaulay Duration. It is equal to the weighted average number of years to each future cash flow where the weights are the present value of each cash flow. In symbols:

$$\frac{\sum t \quad v^t \quad CF_t}{\sum \quad v^t \quad CF_t}$$

   where $v = 1/(1+i)$. For the liability duration, the cash flows equal the amount of liabilities lapsed each year, using the projected lapse rates. For the asset duration, the cash flows equal the sum of investment income and principal rollover on the initial assets.) It is assumed that interest rates immediately rise to and remain at 20 percent and that lapse rates are 25 percent each year.

   The durations shown in this and subsequent tables are post-tax durations, calculated by using an after-tax interest rate. Although it is still not clear whether pre-tax or post-tax durations are "better," the latter produces regression formulas for required surplus as a function of duration that appear to have a better "fit."

   Note that as the asset length increases, and the spread between D^A and D^L increases, the required surplus increases dramatically. It is interesting to note though that there are exceptions (the 7-year bond), and some surplus is required even when the asset duration is less then the liability duration.

TABLE 5

Effect of Investment Strategy on Required Surplus
Durations as of Beginning of Projection Period
Required Surplus as a Percentage of Initial Liabilities

Asset Type	Duration: Asset	Duration: Liability*	Duration: D^A-D^L	Required Surplus
5-year Bond	2.8	5.3	(2.5)	0.0%
7-year Bond	3.9	5.1	(1.3)	0.3
10-year Mortgage	3.9	5.0	(1.1)	0.0
12-year Mortgage	4.6	4.6	0.0	0.9
10-year Bond	4.9	4.5	0.3	2.3
15-year Mortgage	5.5	4.5	1.0	3.7
20-year Mortgage	7.0	4.5	2.5	9.1
15-year Bond	7.0	4.5	2.5	10.4
20-year Bond	8.5	4.5	4.0	16.6
30-year Mortgage	9.2	4.5	4.7	17.9
30-year Bond	10.4	4.5	5.8	23.5

* The liability duration is influenced by the asset length through the effect on credited rates. Since the shorter assets roll over quicker, losses are not as large, and the average earned rate ultimately rises to the new money rate. These led to higher credited rates, and lower lapse amounts in later years.

The reason for these exceptions is that duration is not a perfect indicator of risk potential. Losses develop when negative net cash flows lower the average earned rate below the credited rate. Generally, assets with shorter durations produce greater cash flows, thereby reducing, and sometimes eliminating, the negative net cash flows. If all net cash flows are positive (where the credited rate is kept below the earned rate), statutory losses do not develop.

However, assets with similar durations can have very different cash-flow patterns. Generally, the mortgages produce more uniform cash flows by year than the bonds. With the 7-year bond, for example, although the duration is quite short, there is relatively little cash flow generated in the first few years, so that negative cash flows and statutory losses do develop.

2. Future Interest Rate Assumptions. Table 6 illustrates the effect on required surplus of different future interest levels for various assets. The scenario is the same as that described for table 5, where initial assets are invested at 14 percent, and lapse rates are 25 percent each year.

For a given asset type, required surplus increases dramatically with higher interest rate levels.

TABLE 6

Effect of Future Interest Assumptions
Requires Surplus as a Percentage of Initial Liabilities

Future Interest Rate	7-Year Bond	10-Year Bond	15-Year Mortgage	20-Year Mortgage	30-Year Mortgage	30-Year Bond
17%	0.0%	0.5%	1.2%	3.9%	8.3%	11.1%
20	0.3	2.3	3.7	9.1	17.9	23.5
25	1.3	5.2	7.8	17.5	33.2	43.3
30	2.3	8.0	11.9	25.6	47.4	61.6

In reality, the sensitivity of required surplus to different interest rate levels would be even greater, since withdrawal rates could be expected to increase as the spread between new-money rates and credited rates increases.

For example, consider the results for a 15-year mortgage where lapse rates are determined using a formula relating lapse rates to this spread (see Appendix 3) for the four preceding interest scenarios.

TABLE 7

Required Surplus as a Percentage of Initial Liabilities

Future Interest Rate	Flat 25% Lapse		Intermediate Lapse Formula		High Lapse Formula	
	Lapse Rate	Required Surplus	Lapse Rate	Required Surplus	Lapse Rate	Required Surplus
17%	25%	1.2%	16.3%	0.0%	20.4%	0.0%
20	25	3.7	24.8	3.6	39.4	10.2
25	25	7.8	37.6	18.3	75.0	29.6
30	25	11.8	48.4	33.8	75.0	43.5

The lapse rates shown for the two formulas are the highest lapse rates experienced in the given scenario according to the formula followed. Clearly, withdrawal assumptions together with the future interest rate assumptions can have a very material effect on required surplus.

3. Withdrawal Assumptions. For a given asset assumption, interest environment, and interest crediting strategy, the surplus required is very much a function of lapse rates. Table 8 illustrates the surplus required under a variety of withdrawal assumptions. A 15-year mortgage is assumed to be the underlying asset, and interest rates are assumed to go to 20 percent. A low crediting strategy is assumed where the credited rate is set equal to the average earned rate less 1.00 percent subject to a minimum of 13 percent which is the initial credited rate. (Low crediting strategy will be used in this context hereafter.)

TABLE 8

Effect of Withdrawal Assumptions
Required Surplus as a Percentage of Initial Liabilities

Year	A(mod)	B(ext)	C	D	E	F	G	H	I
	Lapse Rates								
1	24.8%	39.4%	20%	25%	15%	20%	30%	25%	40%
2	24.8	39.4	20	25	20	25	30	30	40
3	24.8	39.4	20	25	25	30	30	35	40
4	24.8	39.4	20	25	30	35	30	40	40
5	24.8	39.4	20	25	35	40	30	40	40
6+	24.8*	39.4	20	25	40	40	30	40	40
	Durations (Post-Tax)								
Liability	4.6	2.7	6.6	4.5	4.2	3.7	3.7	3.3	2.7
Asset-Liability	1.0	2.8	-1.0	1.0	1.3	1.8	1.8	2.2	2.9
	Required Surplus								
	3.6%	10.2%	0.4%	3.7%	4.0%	6.0%	6.6%	7.7%	10.4%

* In year 9, cash flow turns positive (thanks to scheduled rollover from initial assets relative to a smaller in-force), and earned rates begin to rise; in year 14, credited rates increase, and lapses decrease, reaching an ultimate level of 7.5 percent in year 16.

Tests A and B have lapse rates developed by the lapse formulas relating lapse rates to the spread between new-money and credited rates. A has "intermediate" rates, and B has "high" rates. Appendix 2 discusses the formulas and other considerations due to the withdrawal assumptions used throughout these tests. The intermediate lapse formula has generally been used unless otherwise indicated.

From table 8, it is evident that the surplus required increases substantially as the liability duration shortens or as the spread between asset and liability duration increases.

4. Owner Dividend Policy. In the testing to this point, it has been assumed, somewhat arbitrarily, that 50 percent of GFO after FIT would be paid out as owner dividends, and the remaining 50 percent would be retained in surplus. (The owner dividend requirement can also be viewed partially or fully as the requirement to fund new business.) Thus, a portion of earnings in early years accumulates in surplus, which is available to cover losses in later years.

In reality, even the retained portion of GFO after FIT might not be available for risk management. For an ongoing company, earnings from in-force blocks are used to fund new business. Amounts left over, if not needed to maintain surplus at a target level, are paid out in the form of dividends to owners.

Tests to this point have also assumed that a "minimum owner dividend" is paid even when there are statutory losses, or depressed earnings. The minimum owner dividend is equal to 32 basis points of the beginning of year liability balance. (The minimum dividend of 32 basis points was derived by multiplying the earnings margin of 100 basis points by 50% x (1 - FIT rate of 36.8%.) This assumption recognizes that an ongoing company may wish to maintain a certain level of dividends (or support new business), even if it is necessary to borrow money.

Table 9 illustrates the effect of these dividend assumptions. It is assumed that interest rates rise to and remain at 20 percent; withdrawal rates are at 25 percent; and the underlying asset is a 15-year mortgage invested at 14 percent. Further, the low crediting strategy is assumed.

TABLE 9

Effect of Dividend Policy
Required Surplus as a Percentage of Initial Liabilities

Scenario	Minimum Owner Dividends	% of Positive GFO as Dividends	Required Surplus
A	No	0%	2.2%
B	No	50	2.9
C	No	100	3.7
D*	Yes**	50	3.7
E	Yes**	100	4.5

* This is the assumed dividend policy used in earlier tests.
** 32 basis points times beginning of year liability balance.

The first three scenarios illustrate the effect of the assumption as to how much of the statutory earnings will be paid as dividends. Note that this assumption only matters in years of positive GFO. Under this particular scenario, GFO is positive in the first two years. The assumption that all earnings in this period are retained versus all earnings are paid out is worth 150 basis points of required surplus. If the assumptions were changed to defer the emergence of the loss to later years, it should be apparent that earnings accumulated during this period would provide substantial risk-management capacity. Clearly, the ability to retain earnings materially reduces the amount of initial surplus required to manage risk.

Scenarios B versus D and C versus E illustrate the effect that the necessity for management to pay dividends (or fund new business), even during periods of statutory losses, may have on required surplus. In these examples, the decision to borrow funds at high interest rates to pay dividends is worth 80 basis points of required surplus.

Unless otherwise noted, the minimum dividend requirement of 32 basis points is used in all tests. The only exception is with the "chasing the rate" scenarios. Minimum dividend requirements when "chasing the rate" are large since minimum dividends are defined as a function of the liability balance. The liabilities grow large quickly in the "chasing the rate" scenarios since credited rates are high and lapse rates are low. Consequently, the results presented earlier assumed no minimum dividends for all "chasing the rate" strategies presented nor for the other strategies that these were compared to.

Table 10 shows the effect of minimum dividend strategies on required surplus for various interest crediting strategies. The assumptions are consistent with those of table 9.

TABLE 10

Effect of Dividend Policy
Required Surplus as a Percentage of Initial Liabilities

Interest Crediting Strategy	Minimum Owner Dividends* No	Minimum Owner Dividends* Yes
Low	2.9%	3.7%
Intermediate	1.0	2.3
High	5.6	8.0

* It is also assumed that 50 percent of positive GFO is paid as dividend.

Again, these results are influenced by the assumptions as to the timing and duration of the loss that produces negative earnings, i.e., beginning of interest rate increase and assumed level of rates. If the loss extends over a longer period of time, required surplus would be higher and vice versa.

These results are sufficient to establish that the overall dividend policy is a material determinant of required surplus levels. In interpreting C-3 risk surplus requirements, it is essential to understand what dividend policy applies.

5. Treatment of Negative FIT. All of the preceding results assume negative FIT is available as a current year tax credit when GFO is negative. In other words, it is assumed there are other lines of business generating positive FIT. The practical effect is to reduce negative cash flows and temper losses that otherwise would materialize. In periods of adversity, this may not be a valid assumption. There also may not be positive FIT available from other lines if the company is growing rapidly.

Table 11 illustrates the effect that the negative FIT credit assumption has on required surplus. Interest rates are assumed to rise to and remain at 20 percent, withdrawal rates are 25 percent, the assets are 15-year mortgages, and there is no minimum dividend requirement. Further, a low crediting strategy is assumed.

TABLE 11

Effect of Negative FIT Credit
Required Surplus as a Percentage of Initial Liabilities

% of Positive GFO as Dividends	Required Surplus: FIT Credit	Required Surplus: No FIT Credit
0%	2.2%	3.0%
50	2.9	3.9
100	3.7	4.9

As would be expected, the availability of the FIT credit reduces required surplus, since the credit effectively reduces the size of the losses that would otherwise be covered by borrowing in the model.

Note that the required surplus associated with the "no FIT credit" scenario is somewhat overstated, in that the possibility of loss carryforwards and carrybacks are ignored.

Again, in assessing C-3 surplus requirements, it is essential to understand the effect of the tax credit assumption on the results.

6. Reserve Conservatism/Earnings Margin. The preceding results all assume that the statutory reserve equals the cash-surrender value (CSV). We have tested more conservative reserves to examine the effect on required surplus. These results are presented in table 12 for various assets. Interest rates are assumed to immediately rise to and remain at 20 percent, lapse rates are 25 percent, and there is no minimum dividend requirement. Further, a low crediting strategy is assumed.

TABLE 12

Effect of Liability Valuation on Required Surplus
Required Surplus as a Percentage of Initial Liabilities

Asset	Statutory Reserve Equals CSV Plus: 0% of CSV	2% of CSV	4% of CSV
10-year Bond	1.5%	0.4%	0.0%
15-year Mortgage	2.9	1.7	0.7
20-year Bond	16.1	14.8	13.6
30-year Mortgage	17.5	16.3	15.2

Note that the surplus is expressed as a percentage of initial liabilities, which includes the extra x percent of CSV. Also, note that an increase in the statutory reserve of x percent does not translate into a corresponding decrease in required surplus. In these examples, an extra 2 percent of reserves decreases required surplus by only 1.0-1.3 percent. This relationship arises because the additional reserve "requirement" must be maintained even when adversity strikes. In contrast, required surplus is available in full to cover losses as required.

Table 13 shoes how much of a margin must be introduced into the reserve so that no initial surplus is required. The low crediting strategy and the high "chasing the rate" strategy discussed in part I are used as examples here.

TABLE 13

Relationship between Reserve Conservatism and Required Surplus

Required Surplus as a Percentage of Initial Liabilities

Interest Crediting Strategy	Statutory Reserve Increased By x% of CSV	Required Surplus	Total Reserve Plus Surplus**
Low			
	0.0%*	2.9%*	2.9%
	2.9	1.2	4.1
	5.3	0.0	5.3
High			
	0.0*	7.9*	7.9
	7.9	3.8	11.7
	17.9	0.0	17.9

* These results relate to tables 1 and 2 respectively.

** Note that the increase in statutory reserve applies in all policies years, whereas the required surplus is expressed as a percentage of initial liabilities. Thus this total must be used with caution.

In both cases, reserves must be increased well in excess of the required surplus when based on reserves equal to cash surrender values. The necessary increase in reserves to get required surplus to zero is especially dramatic with the high "chase the rate" crediting strategy.

Under both strategies, the conservatism in the reserves is released upon lapse and then is available to cover losses. In low crediting strategy, the high level of lapses in the early years results in a high level of reserves conservatism released. In the high crediting strategy, which produces a lower level of lapse, relatively *more reserve conservatism is necessary to cover the* same level of loss.

Another interesting difference between the two strategies is the relationship between interest earned on the additional reserve less interest credited to support the increase in the additional reserve. Under the low crediting strategy, the extra reserve provides additional interest margin, which helps cover the losses and reduces the level of additional reserve required. However, under the high crediting strategy, the interest margin on the incremental reserve piece is actually negative, i.e., the extra reserve is increased by the credited rate which is greater than the interest earned on the assets. Thus, required reserve conservatism is further increased.

These results suggest that reserve conservatism, at least in the form of a fixed percentage increase, is not necessarily an effective means to assure solvency. Clearly, management would opt for the lowest combination of surplus and reserve conservatism that would assure that obligations could be matured, and these results indicate that this is achieved when all risk-management capacity is held in surplus.

Another form of reserve conservatism that is relied on for statutory purposes is to use a low discount rate in the present value calculations. Once a reserve with a conservative discount rate is established, the conservatism in later years emerges as a larger statutory earnings margin. In the model, this form of reserve conservatism was simulated by varying the margin between earned and credited interest.

For most of the runs, a pretax earnings margin of 100 basis points, net of expenses, has been assumed. Table 14 illustrates how various earnings margins impact required surplus. Interest rates are assumed to go to 20 percent; withdrawal rates are 25 percent; and there is no minimum dividend requirement.

TABLE 14

Effect of Earnings Margin on Required Surplus
Required Surplus as a Percentage of Initial Liabilities

Asset	Earnings Margin		
	50 Basis Points	100 Basis Points	150 Basis Points
10-year Bond	2.3%	1.5%	0.7%
15-year Mortgage	3.8	2.9	2.1
20-year Bond	16.9	16.1	15.2
30-year Mortgage	18.3	17.5	16.7

Since earnings are the first line of defense against risk, it would be expected that higher margins would decrease required surplus. This expectation is confirmed by the results in table 14 though the magnitude of the effect is smaller than was anticipated. The reasons for this are twofold: (1) FIT reduces the 50 basis point differentials to 32 basis points. (2) Lapses are so high that the liability balance, to which the 32 basis points apply, shrinks rapidly.

This approach of just varying the earnings margin does not necessarily simulate the reserve conservatism present when low discount rates are used, depending on the reserve/CSV relationship. Where low discount rates are used for both, such that reserves and CSVs are approximately equal, this should be an adequate representation. However, where lower discount rates are used for reserves, such that reserves exceed cash values, the impact on required surplus would be a combination of tables 12 and 14. This combination has not been investigated.

7. Interest Crediting Strategy. The effect of the interest crediting strategy on required surplus was discussed in detail in part I.

III. RELATIONSHIP BETWEEN DURATION AND REQUIRED SURPLUS

In the testing, an interesting relationship was discovered between required surplus and the difference between the asset and liability durations, where the liability duration reflects the projected lapse rates. Regression formulas for required surplus, based on the bulk of the data presented in the preceding tables, exhibit the following general relationship:

$$\text{Required Surplus} = (i_{actual} - i_{base})\,(D^A - D^L) - C$$

where

i_{actual} = the assumed level future new-money rate

i_{base} = the interest rate at which the initial assets are invested

D^A = the Macaulay duration of the assets

D^L = the Macaulay duration of the liabilities

C = a constant that is a function of the earnings margin present prior to an adverse interest scenario

We came upon this formula by applying regression analysis to our data, whereby we defined the independent variable to be the asset/liability duration difference (denoted by x), and the dependent variable to be the required surplus percentage (denoted by y).

Illustrations of our analysis follow. Only those results where the asset duration exceeds the liability duration and initial surplus is required were included in the calculation of the regression formulas unless otherwise noted.

Table 15 shows the first set of regression formulas derived from the data in table 6, wherein the effect of different future interest assumptions was examined. (Appendix 3 contains all the raw data used to develop the regression formulas that follow.) Both pretax and post-tax data were tested. (For post-tax, durations are computed as the post-tax investment rate, and $(i_a - i_b)$ is post-tax.)

TABLE 15

Regression Analysis for Various Future Interest Rate Levels

Future Interest Rate	$i_a - i_b$	Regression Formula	Correlation Coefficient
Pretax Interest Rates			
17%	3%	y = 3.02%x - 1.93%	.9855
20	6	y = 5.85%x - 1.89	.9880
25	11	y = 10.69%x - 2.56	.9910
30	16	y = 15.18%x - 2.80	.9925
Post-Tax Interest Rates			
17%	1.9%	y = 1.92%x - .43%	.9965
20	3.8	y = 3.91%x - .29	.9968
25	7.0	y = 7.01%x - 1.93	.9965
30	10.1	y = 9.92%x - 3.66	.9948

Note that the coefficient of x, the duration difference, determined by the regression analysis, is close to $(i_a - i_b)$ under both the pretax and post-tax basis. Note also that the high correlation coefficients indicate a very good fit. However, this may be somewhat misleading as relatively few points, 5 to 11, were used.

The post-tax results appear slightly "better," in that the correlation coefficients are closer to 1, and the coefficient of x remains closer to $(i_a - i_b)$ over the interest rate range.

Table 16 compares actual surplus vs. regression formula surplus for the 20 percent post-tax regression formula.

TABLE 16

Actual Surplus Versus Regression Surplus
Surplus as a Percentage of Initial Liabilities

	10-yr. Bond	15-yr. Mortg.	20-yr. Mortg.	15-yr. Bond	20-yr. Bond	30-yr. Mortg.	30-yr. Bond
Duration Difference (x)	.33%	1.01	2.45	2.51	4.01	4.66	5.85
3.91%x + .29%	1.6%	4.2%	9.9%	10.1%	16.0%	18.5%	23.2%
Actual Required Surplus	2.3%	3.7%	9.1%	10.4%	16.6%	17.9%	23.5%

Thus, the formula does remarkably well at matching the actual required surplus determined by the model.

The regression analysis was repeated, but using the data in table 14 wherein the effect of different earnings margins was examined. (See table 17.)

TABLE 17

Regression Formulas for Various Earnings Margin Levels

Earnings Margin	$i_a - i_b$	Regression Surplus Formula	Correlation Coefficient
Pretax Interest Rates			
0 bp	6%	y = 6.02x - .55	.9887
50	6	y = 5.98x - 1.62	.9873
100	6	y = 5.87x - 2.44	.9848
150	6	y = 6.15x - 4.38	.9837
Post-Tax Interest Rates			
0 bp	3.8%	y = 3.99x + 1.77	.9969
50	3.8	y = 4.01x + .49	.9969
100	3.8	y = 3.99x - .50	.9963
150	3.8	y = 3.91x - 1.27	.9946

Interestingly, the coefficient of the duration difference, x, remains approximately equal to the difference between the interest rates. But, the constant C seems to vary at some constant rate depending on the earnings margin. On a pretax basis, it appears to be an offset to required surplus, an offset that becomes larger as the earnings margin grows. This makes sense since the earnings margin is the first line of defense in the management of risk. Thus, the larger the earnings margin, the less surplus that is necessary to manage risk.

The same general reasoning applies to the post-tax formulas, even though the constant C is positive in the first two cases. Thus, the constant C is more negative (or less positive) as the initial earnings margin increases.

The fact that this constant is sometimes positive for the particular formula is somewhat troubling, however. For example, it suggest that surplus of 1.77 percent is required when asset and liability durations are equal, and no earnings margin is present. This may well be true in some situations, if cash flows are not well matched, but the pretax formula implies required surplus of negative 0.55 percent for the same case. All of this reinforces the observation that duration is not a perfect measure of risk, and these formulas are not necessarily precise.

Note that the correlation coefficients in table 17 suggest the post-tax formulas consistently provide the better fit. There is also less variation of ($i_{actual} - i_{base}$) on a post-tax basis than on a pretax one. These results led to the use of durations computed at post-tax interest rates in earlier sections of this report.

The next area of interest was the effect of various lapse assumptions, which influences the duration of the liabilities. The regression formulas shown in table 18 were derived from the following data:

1. Table 5--various assets, interest to 20 percent, 25 percent lapse.

2. Table 8--15-year mortgage, interest to 20 percent, various lapses.

3. Various assets, interest to 20 percent, intermediate lapse formula (results not shown in prior tables).

TABLE 18

Regression Formulas for Various Lapse Assumptions

Pretax Durations	Required Surplus Formula	Correlation Coefficient
1) Table 5	y = 5.85x - 1.89	.9880
2) Table 8	y = 4.60x - 1.05	.9981
3) Intermediate Lapse Formula	y = 6.60x - 4.37	.9909
4) Data Set Combined	y = 6.06x - 2.78	.9863
Post-Tax Durations		
1) Table 5	y = 3.91x + .29	.9968
2) Table 8	y = 3.62x - .23	.9912
3) Intermediate Lapse Formula	y = 4.03x - .25	.9970
4) Data Sets Combined	y = 4.06x - .52	.9946

Interestingly, there is remarkable consistency in the formulas derived from the different data sources, especially with the post-tax formulas. We expected some variation in the magnitude of the "C" factor, since the rate of lapse influences the distribution and amount of earnings over the period during which losses due to the C-3 risk are occurring. Although C does vary, the variation is not severe, which we find encouraging.

The preceding regression formulas were based only on tests where the asset duration exceeds the liability duration and initial surplus is required. Now consider the case where D^A is less than D^L which is encountered when we "chase the rate." The following set of regression formulas shown in table 19 is based on the different interest crediting strategies found in table 4.

Interestingly, the coefficient of x is reasonably stable, yet x in these high crediting runs is generally negative. However, the correlation coefficient and, thus, the fit of the regression formula for the intermediate and high scenarios are not nearly as good as in previous regressions. But again, there seems to be slightly better results among the post-tax formulas.

The large positive C factor with the intermediate and high crediting strategies again appears to be related to the earnings margin. In the intermediate and high strategies, more interest is credited than is earned, thus producing negative earnings margins. This logically leads to the need for an addition to required surplus, which would not be a direct function of the duration difference. Instead, the large losses occur in the first few years and are roughly of the same magnitude with different assets.

TABLE 19

Regression Formulas for Different Interest Crediting Scenarios

Scenario	Required Surplus Formula	Correlation Coefficient
	Pretax Durations	
Low	y = 5.87x - 2.44	.9848
Intermediate	y = 4.77x + 15.38	.9545
High	y = 4.73x + 38.17	.9792
	Post-Tax Durations	
Low	y = 3.99x - .50	.9963
Intermediate	y = 3.29x + 24.64	.9836
High	y = 3.41x + 51.46	.9834

The relationship between duration difference and CFS was also examined. The relationships found were not as strong as those associated with required surplus, although correlations are still very high. This was surprising since CFS is a present value of cash-flow measure, and certainly appears more closely related to the duration difference in the regression formula. Results of regression analysis with CFS are presented in Appendix 3.

In summary, these formulas are at best approximations. However, the closeness of fit is intriguing, and the formulas have been quite useful for gaining appreciation for the magnitude and sensitivity of risk to the various factors that affect C-3 risk.

IV. CONCLUSIONS

As demonstrated by the tests presented herein, initial surplus required to manage manifestations of C-3 risk is dependent upon a variety of variables. Some variables, such as interest rate, lapse, and investment assumptions, have a strong effect on required surplus, and they have commonly been associated with C-3 risk.

Other variables, which are to a large extent controlled by management of an insurance company, have been shown to materially affect required surplus in a C-3 risk situation. These variables include the interest crediting strategy, owner dividend policy, treatment of FIT credits, and earnings margin as determined by pricing policy. The effect of these variables greatly complicates the problem of developing appropriate surplus guidelines for management of C-3 risk.

Finally, it has been shown that valuation standards materially affect required surplus. In general, any conservatism in valuation reserves produces a net increase in the total surplus, i.e., reserve conservatism plus required surplus, needed to assure solvency. This suggests that conservatism in valuation reserves may not be the best means to assure solvency.

Regression analysis has revealed a strong relationship between durations, earnings, and required surplus. This relationship can be useful for developing an intuitive sense for the effect on risk levels of alternate assumptions.

APPENDIX 1

SPDA and MODEL ASSUMPTIONS

The final report of the CORTF will outline the mechanics of the computer model used in the testing.

Following is a list of the parameters used to develop the "base case" of SPDA business:

- initial assets invested at 14 percent
- FIT = 36.8%
- 50 percent of positive GFO paid out as dividends
- initial earnings margin = 1%
- a minimum owner dividend of 32 basis points applied to the beginning of year liability balance
- utilization of the negative FIT credit
- underlying asset is a 15-year mortgage
- lapse rates from the intermediate annuity cash-out formula (see Appendix 2)
- insurance cash flows of the prior ten years in the proportion 100, 103, 107, 110, 112, 115, 117, 118, 119
- statutory reserve equals cash value, i.e., no margin for adverse deviation

The minimum owner dividend factor was based on the base case, where 50 percent of the after-tax earnings margin is assumed to be paid to shareholders. Thus,

$$50\% \times (1 - .368) \times .01 = 31.6 \text{ bp} = 32 \text{ bp}.$$

The insurance cash flows for the 10 years prior to issue that we used to develop the initial block of assets were derived assuming the SPDA product was first sold ten years ago, sales grew by 10 percent each year, and annual lapses were at 5 percent.

Since interest rates are assumed to immediately rise to some high level and remain there, the future reinvestment or borrowing assumptions have no effect on required surplus. Thus, the debt instrument was arbitrarily chosen to allow 10 percent of the debt to roll over each year for 10 years.

In presenting the test results, an effort has been made to note all deviations in assumptions from those of the base case.

APPENDIX 2

LAPSE ASSUMPTIONS

Lapse Formula

We initially considered using a lapse formula based upon formulas presented in the C-3 study by Messrs. Feldman and Kolkman. (Record of the Society of Actuaries, Volume 8, Number 4, page 1557.) Their formula for single-payment annuities, "moderate" lapse rates, is as follows:

$$\text{Lapse rate} = .05 + (.01)\,(100 \times d)^{1.5},\ d = [0,17\%],$$
$$= .75 \text{ for } d > 17,$$

where

$$d = (\text{new-money rate}) - (\text{credited rate}) - .01.$$

Messrs. Feldman and Kolkman also tested "extreme" withdrawal rates, defined as the moderate rates times 2, but not higher than 75 percent.

After extensive testing, and further thought, we decided the moderate formula was unsatisfactory. In particular:

1. The formula implies a greater increase in lapse rates for a given increase in the new-money rate/credited rate spread as this spread reaches higher levels. Our intuition and limited experience suggests an opposite result, i.e., that at some point, further increases in the interest spread will lead to only relatively small increases in lapse rates.

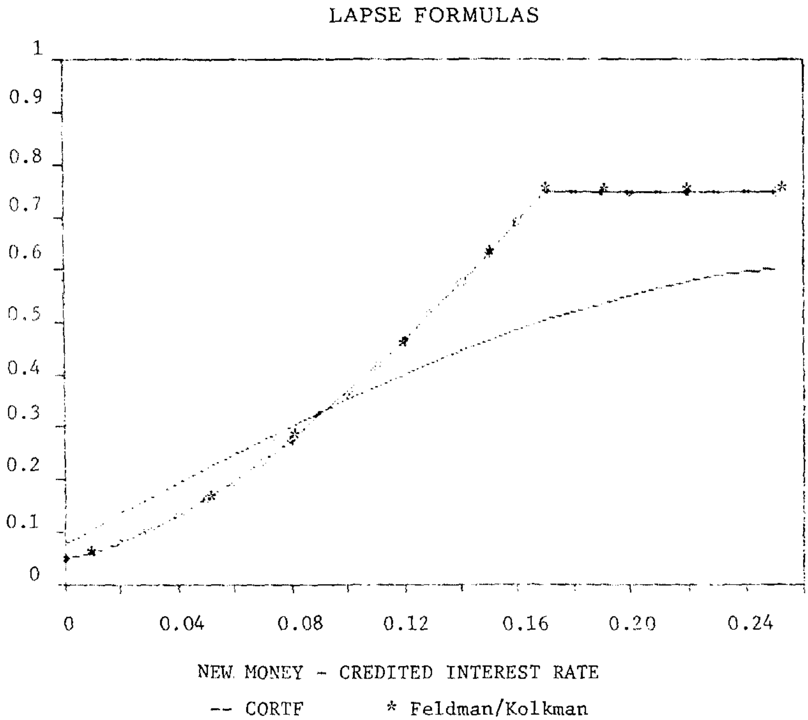
2. The lapse rate of 5 percent for a 0 percent spread appeared low; we felt 7.5 percent was more appropriate.

3. The moderate formula implied lapse rates of 19.8 percent for our base case of 20 percent new-money rates, 13 percent credited, versus our usual assumption of 25 percent.

We developed the following formula, which we believe produces more consistent and more reasonable results for different interest and crediting assumptions.

Lapse rate = $.075 + 3.0d - 1.5d^2 - 8d^3$, $d = [.00, .25]$
= $.60$, $d > .25$

The following graph compares the two formulas.



For some tests of extreme lapse rates, we continued with the Feldman/Kolkman extreme formula.

RAW DATA FOR REGRESSION FORMULAS

Regression Analysis for Various Future Interest Rate Levels

Table 15

	17%			20%			25%			30%		
	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.
5 yr. bond	--	-1.375	-2.028	--	-1.655	-2.475	--	-2.272	-3.687	--	-3.716	-5.093
7 yr. bond	--	-.390	-.927	.261	-.575	-1.252	1.292	-.739	-1.478	2.341	-.853	-1.648
10 yr. bond	.525	.549	.257	2.270	.593	.334	5.179	.596	.339	8.072	.596	.339
15 yr. bond	4.481	2.350	2.514	10.442	2.350	2.514	20.611	2.350	2.514	30.903	2.350	2.514
20 yr. bond	7.486	3.261	4.012	16.545	3.261	4.012	31.726	3.261	4.012	46.720	3.261	4.012
30 yr. bond	11.054	3.980	5.847	23.450	3.980	5.847	43.256	3.980	5.847	61.615	3.980	5.847
10 yr. mort.	--	-.345	-.790	--	-.489	-1.055	.110	-.805	-1.672	.604	-1.132	-2.385
12 yr. mort.	--	.233	-.083	.851	.249	-.042	2.536	.304	.055	4.214	.304	.055
15 yr. mort.	1.218	1.064	1.006	3.708	1.064	1.006	7.828	1.064	1.006	11.869	1.064	1.006
20 yr. mort.	3.864	2.113	2.454	9.063	2.113	3.384	17.530	2.113	2.454	25.638	2.113	2.454
30 yr. mort.	8.315	3.384	4.662	17.893	2.454	4.662	33.183	3.384	4.662	47.408	3.884	4.662

	17%	20%	25%	30%
Regression Equations:				
Pre-tax:	y = 3.019x - 1.926	y = 5.847x - 1.894	y = 10.694x - 2.563	y = 15.179x - 2.800
Post-tax:	y = 1.924x - .427	y = 3.906x + .288	y = 7.009x + 1.930	y = 9.916x + 3.663
Correlation Coefficients:				
Pre-tax:	.9855	.9880	.9910	.9925
Post-tax:	.9965	.9968	.9965	.9948
Points Excluded:*				
Pre-tax	Bonds 5,7/Mortg. 10,12	Bonds 5,7/Mortg. 10	Bonds 5,7/Mortg. 10	Bonds 5,7/Mortg. 10
Post-tax	Bonds 5,7/Mortg. 10,12	Bonds 5,7/Mortg. 10,12	Bonds 5,7/Mortg. 10	Bonds 5,7/Mortg. 10

* Points excluded because of negative duration differences or zero required surplus.

Regression Formulas for Various Earnings Margin Levels

Table 17

	0 Basis Points			50 Basis Points			100 Basis Points			150 Basis Points		
	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.
10 yr. bond	3.235	.493	.195	2.322	.541	.262	1.464	.555	.271	.689	.522	.187
15 yr. bond	11.572	2.247	2.370	10.637	2.299	2.512	9.735	2.350	2.514	8.890	2.399	2.583
20 yr. bond	17.789	3.158	3.868	16.901	3.210	3.941	16.048	3.261	4.012	15.228	3.310	4.081
30 yr. bond	24.886	3.877	5.703	24.043	3.929	5.776	23.234	3.980	5.847	22.456	4.029	5.916
12 yr. mort.	1.794	.201	-.089	.895	.189	-.130	.092	.149	-.256	--	.206	-.171
15 yr. mort.	4.714	.961	.862	3.779	1.013	.935	2.907	1.064	1.006	2.099	1.113	1.075
20 yr. mort.	10.168	2.010	2.310	9.240	2.062	2.383	8.384	2.113	2.454	7.578	2.162	2.523
30 yr. mort.	19.193	3.281	4.518	18.330	3.333	4.591	17.502	3.384	4.662	16.728	3.433	4.731

	0 Basis Points	50 Basis Points	100 Basis Points	150 Basis Points
Regression Equations:				
Pre-tax:	$y = 6.024x - .551$	$y = 5.977x - 1.617$	$y = 5.867x - 2.441$	$y = 6.147x - 4.375$
Post-tax:	$y = 3.992x + 1.772$	$y = 4.011x + .489$	$y = 3.968x - .499$	$y = 3.913x - 1.267$
Correlation Coefficients:				
Pre-Tax:	.9887	.9873	.9848	.9837
Post-Tax:	.9969	.9969	.9963	.9946
Points Excluded				
Pre-Tax:	12 yr. mortg.	12 yr. mortg.	12 yr. mortg.	12 yr. mortg.
Post-Tax:	12 yr. mortg.	12 yr. mortg.	12 yr. mortg.	12 yr. mortg.

Regression Formulas for Various Lapse Assumptions

Table 18

	Table 5 Data			Intermediate Lapse Formula				Table 8 Data		
	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Withdrawal Assump.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.
5 yr. bond	--	-1.655	-2.475				A	3.569	1.032	9.610
7 yr. bond	.261	-.575	-1.252				B	.363	-.167	-1.034
10 yr. bond	2.270	.593	.334				C	3.708	1.064	1.006
15 yr. bond	10.442	2.350	2.514	10.281	2.318	2.469	D	6.593	1.695	1.862
20 yr. bond	16.545	3.261	4.012	16.363	3.229	3.967	E	10.416	2.493	2.869
30 yr. bond	23.450	3.980	5.847	23.264	3.948	5.802	F	3.958	1.035	1.303
							G	5.991	1.504	1.809
10 yr. mort.	--	-.489	-1.055				H	7.657	1.873	2.208
12 yr. mort.	.851	.249	-.042							
15 yr. mort.	3.708	1.064	1.006	3.569	1.032	.961				
20 yr. mort.	9.063	2.113	2.454	8.894	2.081	2.409				
30 yr. mort.	17.893	3.384	4.662	17.709	3.352	4.617				

	Table 5 Data	Intermediate Lapse Formula	Table 8 Data
Regression Equations:			
Pre-tax:	y = 5.847x - 1.894	y = 6.659x - 4.367	y = 4.604 - 1.049
Post-tax:	y = 3.906x + .288	y = 4.032x - .246	y = 3.618x - .227
Correlation Coefficients:			
Pre-tax:	.9880	.9981	.9909
Post-tax:	.9968	.9912	.9970
Points Excluded:			
Pre-tax:	Bonds 5,7/Mortg. 10	(B)	
Post-tax:	Bonds 5,7/Mortg. 10,12	(B)	

Regression Formulas for Different Interest Crediting Strategies

Table 19

	Low			Intermediate			High		
	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.	Req. Surp.	Pre-Tax Dur.	Post-Tax Dur.
5 yr. bond				.298	-3.862	-8.190	1.692	-8.129	-14.880
7 yr. bond				1.999	-2.955	-7.188	4.599	-7.222	-13.878
10 yr. bond	1.464	.555	.271	2.981	-2.122	-6.180	6.329	-6.389	-12.870
15 yr. bond	9.735	2.350	2.514	12.222	-.368	-4.005	16.683	-4.635	-10.695
20 yr. bond	16.048	3.261	4.012	16.733	-.543	-2.507	19.578	-3.724	-9.917
30 yr. bond	23.234	3.980	5.847	24.183	1.262	-.672	27.423	-3.005	-7.362
10 yr. mort.				1.344	-2.944	-6.833	4.300	-7.211	-13.788
12 yr. mort.	.092	.149	-.256	2.746	-2.414	-6.464	6.430	-6.681	-13.154
15 yr. mort.	2.907	1.064	1.006	4.144	-1.654	-5.513	7.894	-5.921	-12.203
20 yr. mort.	8.384	2.113	2.454	10.843	-.605	-4.065	15.059	-4.872	-10.755
30 yr. mort.	17.502	3.384	4.662	17.481	.666	-1.857	19.471	-3.601	-8.547

Regression Equations:

	Low	Intermediate	High
Pre-tax:	$y = 5.867x - 2.441$	$y = 4.773x + 15.377$	$y = 4.731x + 38.174$
Post-tax:	$y = 3.986x - .499$	$y = 3.293x + 24.644$	$y = 3.409x + 51.456$

Correlation Coefficients:

	Low	Intermediate	High
Pre-tax:	.9848	.9545	.9792
Post-tax:	.9963	.9836	.9834

Points Excluded:
Pre-tax: 12 yr. mortg.
Post-tax: 12 yr. mortg.

APPENDIX 4

CASH-FLOW SURPLUS AS FUNCTION OF ASSET/LIABILITY DURATIONS

A similar regression analysis was applied to CFS as has been described in the main text for required statutory surplus. For example, for various assets for different interest rates, we derived the following formulas:

	CFS		Required Surplus	
Interest	Formula*	Correlation Coefficient	Formula	Correlation Coefficient
17%	-1.44%x + 1.54%	.9970	1.92%x - .43%	.9965
20	-2.40%x + .60%	.9932	3.91%x + .29%	.9968
25	-3.16%x - -1.39%	.9848	7.01%x + 1.93%	.9965
30	-3.40%x - -3.19%	.9745	9.92%x + 3.66%	.9948

* x is the difference between asset and liability durations computed with post-tax interest rates.

Note that the coefficients of the duration difference in the CFS formula do not increase with increasing interest nearly to the extent that they do with the required surplus formulas. Also, the correlation coefficients for the CFS formulas imply a somewhat worse fit than the required surplus formulas. However, the difference in fit is not great, as can be seen in the following table, which compares the formula to actual values for the two variables.

Interest Rates to 25%

Asset:	10-Yr. Bond	15-Yr. Mortg.	20-Yr. Mortg.	15-Yr. Bond	20-Yr. Bond	30-Yr. Mortg.	30-Yr. Bond
$D^A - D^L$	.339	1.006	2.454	2.514	4.012	4.662	5.847
CFS							
Formula	- 2.5%	- 4.6%	- 9.1%	- 9.3%	-14.1%	-16.1%	-19.9%
Actual	- 1.5	- 4.0	- 9.5	-10.9	-15.4	-15.8	-18.6
Difference	1.0	.6	.4	1.6	1.3	.3	1.3
% Difference	67%	15%	4.2%	14.7%	8.4%	1.9%	7.0%
Required Surplus							
Formula	4.3%	9.0%	19.1%	19.6%	30.1%	34.6%	42.9%
Actual	5.2	7.8	17.5	20.6	31.7	33.2	43.3
Difference	.9	1.2	1.6	1.0	1.6	1.4	.4
% Difference	17.3%	15.4%	9.1%	4.9%	5.0%	4.2%	.1%

The average absolute difference for CFS is 0.9 versus 1.2 for required surplus. However, as a percent of average CFS and average required surplus, respectively, the comparison is 8 percent for CFS, 5 percent for required surplus.

PANEL DISCUSSION

MR. DONALD D. CODY: Mr. Mateja and Mr. Geyer have done a superb job of describing significant research findings by the CORTF. We now understand the essentials of risk taking and have enough tools to begin to measure risk. I am going to discuss the relationship of designated surplus, which is a statutory financials concept, to benchmark or target surplus, which is a similar concept in pricing and in internal-management-based financial systems. I will also discuss the relationship among the valuation actuary, the pricing and product actuary, and the corporate actuary in an expanded structure of valuation practice.

As I proceed, ask yourselves what part of this structure can be deleted without destroying the objectives? Also, who has the knowledge, skill and stature to do the total job better than the actuary? I recognize that statutes and regulations need to be changed; support of management and regulators is necessary; and additional professional education and research by committees and our membership are needed. But let us put aside these concerns for a few minutes and concentrate solely on the need for the enhanced role of the valuation actuary--for if the need exists, the other items will be managed.

To enable an overall view of the structure of valuation practice, see exhibit 1. This exhibit covers not only the role of the valuation actuary envisaged in the Report of Recommendations by the Joint Committee, but also the extended implications of the role of the valuation actuary on pricing, new business, growth and change, internal management financials, financial planning, and surplus management.

If you think that the exhibit suggests that the valuation actuary belongs most naturally in the office of the financial officer, you have perceived a major point of mine.

The top section of the exhibit digests the role of the valuation actuary as recommended by the Joint Committee. Assets are emphasized and are equal to the sum of statutory reserves, designated surplus for capacity utilized, and the balance of surplus, including the mandatory securities valuation reserve (MSVR) and other such contingency reserves, referred to as vitality surplus for growth and change.

Reserves are intended to be adequate for reasonable deviations from expected. Reserves plus designated surplus are intended to be adequate for plausible deviations from expected. Vitality surplus, of course, ought to be positive, but more importantly, the size and dynamics of vitality surplus for at least several years is a means of judging the ability of the company to finance its planning goals. While vitality surplus is mentioned in Appendix B of the Joint Committee Report, it was not explored by the committee. But I can assure you that it is of deep interest to regulators.

I have listed C-1, C2, and C-3 risks with equal billing. C-3 risk from changes in the interest rate environment is the most complex risk and, on interest-sensitive products, the most dangerous one. Cash-flow models based on interest rate scenarios are absolutely necessary to appraise C-3 risk.

But, C-1 risk from defaults and changes in market values of equity investments occurs in a deflationary episode like the Great Depression or in a serious inflationary episode with ongoing stagflation with which we recently had a close encounter. And now we must consider a new gamut of various kinds of ventures and subsidiaries. Also, there is the C-1 risk associated with low quality securities used to upgrade interest income on investments held for interest-sensitive products. There is also a correlation between C-1 risk and C-3 risk to be considered.

Finally, there is the classic actuarial risk--C-2 risk from deviations in claims and expenses due to unwise underwriting, concentration of risk, inflation, earthquake, weak reinsurers, an epidemic, medical cost shifting, inadequate premiums, and the like.

The effects of combination or risks must be determined. Fortunately, a dollar in surplus is more efficient than a dollar in reserves and risks are not additive. However, there are correlations between C-1 and C-3 risks and between C-1 and C-2 risks for disability income, for example. My fellow panelists have exhibited cash-flow models combining all risks. Unfortunately, the results of pure cash-flow models are myriad and make drawing specific conclusions difficult. Fortunately, it appears that reasonable results can be obtained by confining cash-flow modeling to C-3 risk, where it is absolutely necessary and measuring other risks by methods natural to those risks. Then, the results for each risk for the same level of probability of ruin can be combined by formulas which are simple, even though based on complex mathematics, to produce combined results for the same probability of ruin. These formulas incorporate correlations.

C-4 risk, due to happenings unforeseeable by management or due to management ignorance, foolishness, or fraud, is not in the quantitative domain of the valuation actuary, even though the path to insolvency frequently starts with C-4 risk, eventually emerging as C-1, C-2, or C-3 risk. Analysis of the dynamics of vitality surplus probably will first begin to disclose C-4 risk.

Let us now turn to Financial Planning in the lower part of exhibit 1. Vitality surplus is the key to the financial ability of the company to pursue its plans for new business, growth and change. A central concern of regulators today is whether the capacity of some companies is already so overextended that such plans endanger solidity.

There are similarities and differences among various forms of contingency surplus for capacity utilized, which are used, respectively, by valuation actuaries, pricing actuaries, and financial planning actuaries:

1. Such surplus is called designated surplus by the valuation actuary. Its determination on each valuation date accepts the existing pricing, product, and investment structure and examines the implications of the range of future scenarios of the various risks. The determination can change annually. It is for the company as a whole.

2. Such surplus is called target surplus or benchmark surplus by the pricing actuary who looks forward from the issue date, just as the valuation actuary does on each valuation date. But, the pricing actuary deals with the full hypothetical range of future plans for management of investments and product. The pricing actuary at renewal date of participating and nonguaranteed contracts will reflect the existing investment and product structure as well.

3. Such surplus in internal-management financial structures is also called target surplus or benchmark surplus. Here there should be close correspondence with the designated surplus of the valuation actuary, but there may be some accounting conventions which will cause timing differences. Ideally, the contingency surplus required for capacity utilized in the internal-management financial structure should be equivalent to designated surplus.

4. The determination of designated surplus and the determination of target or benchmark surplus, despite such differences, should be based on consistent C-1, C-2, and C-3 risk appraisals.

Now, as to pricing: I have been very impressed in listening to discussions by pricing and product actuaries. These discussions show that most pricing actuaries are well aware of C-1, C-2, and C-3 risks and, in particular, are skilled in ways of reducing C-3 risk by coordinating investment policy with product policy. They call contingency surplus by terms such as target surplus, buffer surplus, or benchmark surplus. And, they realize that company surplus must be advanced to cover plausible deviations and that charges should be made in pricing to repay the company via risk charges for such use of company surplus. They also realize that profit charges must be made to enhance company surplus. Unfortunately, it seems to me that, frequently, proper pricing is incompatible with the competitive market. The job of the valuation actuary is to advise management of the effects of inadequate pricing on solidity. The job of management and regulators is to assure this solidity.

The next line in the exhibit concerns internal-management-based financials, based on an adequate management accounting, planning, and information system that enables monitoring and control of profit by product and line, financial planning, and surplus management with the objective of optimal use of capital. Invested assets by product and line on internal-management-based financials are the same as on statutory financials. On the internal-management basis, there are additional assets consisting of deferred acquisition expenses and other adjustments. It follows that designated surplus (benchmark surplus, target surplus) on the internal-management basis is equal to that on the statutory basis plus the excess of reserves on the statutory basis over those on the internal-management basis plus deferred acquisition expenses and other accounting adjustments. With this format, vitality surplus is the same on the internal-management basis as on the statutory basis. Including going-concern adjustments, like deferred acquisition expenses, in designated surplus preserves the identification of vitality surplus as capital available for new business, growth, and change and the identification of reserves plus designated surplus as assets behind in-force business is a desirable dichotomy for return on investment, monitoring, and other such purposes. Incidentally, reasonable arguments probably can be made for directing some accounting adjustments on assets, other than deferred acquisition expenses, to the vitality surplus, thereby changing its size from that on the statutory basis. However, the dynamics of the vitality surplus for future years, especially the near future, are its important characteristic, rather than its size, which, of course, should be positive on the statutory basis.

I wish that I could be precise about "reasonable" and "plausible" deviations. But, suppose "reasonable" means that reserves imply a 10 percent ruin probability, and "plausible" means reserves plus designated surplus imply a 1 percent ruin probability. Setting scenarios for C-1, C-2, or C-3 risk in order of danger, it is possible subjectively to determine a worst scenario for reserves and a worst scenario for reserves plus designated surplus. Your management can help here. Incidentally, the limit on "plausible" is when designated surplus equals statutory surplus plus MSVR; if your management agrees that the probability is more than 1 percent, you have exposed a problem. This type of exercise itself will go far toward attaining the ultimate objective of the valuation actuary.

Finally, it is impossible for most companies to calculate designated surplus like adding up CRVM 1980 CSO 6 percent reserves on life insurance. Models are used to develop approximate parameters and then applied to the in-force business. The name of the game is magnitudes. Judgment is basic. It calls for the ultimate in professionalism, not clerical procedures. In the end, a simply list of percentages will probably suffice, subject to change as material changes occur in pricing, product design,investment policy, and the environment. The dependability and credibility of the results will be clear in the Report to Management.

MR. THOMAS G. KABELE: Do the percentages vary by the size of the company? Also, does the presence of a holding company influence the amount of surplus you must maintain downstream?

MR. GEYER: I think the holding company would clearly have an effect on your FIT assumption. For example, if you are experiencing losses in a given line or in the subsidiary in a period of adversity and you have a diversity of other products in other companies, it is more likely that you will get cash tax credits for those losses. There may also be synergy among products. For example, if you're looking at C-3 risk, some products may experience losses when interest rates are high, while others would produce gains. Such synergy would affect how much surplus the company ought to have in total.

The size of the company would affect your assumptions for risks such as excess mortality. For example, a small company would expect more fluctuations than a larger company, even without a widespread catastrophe.

MR. STANLEY B. TULIN: Looking at interest rates as if they're bullet interest rates as opposed to underlying yield curves is probably the

EXHIBIT 1

VALUATION PRACTICE STRUCTURE

	In-Force on Valuation Date				Valuation Date and Near Future
Assets=	Statutory Reserves+ (Solvency)		Designated Surplus + (Capacity Utilized)		Vitality Surplus (Solidity Dynamics)
Risks	Life	Health	Life	Health	
C - 1 :	R	R	P	P	-
C - 2 :	R	R	P	P	-
C - 3 :	R*	(R)	P*	(P)	-
(*Especially Interest-Sensitive GIC, SPDA, UL, Structured Settlements)					
C - 4 :	-	-	-	-	I
Financial Planning					
New Business Growth, Change:	-	-	-	-	R, P, I
Pricing:	I	I	I	I	I
Internal Management Financials:	I	I	I	I	I

R: Reasonable deviation scenarios (R): Group medical only

P: Plausible deviation scenarios (P): Group medical only

I: Included

Correlations: C - 1, C - 2 e.g. Disability income
C - 1, C - 3

Whole company: Adequacy of reserves, designated surplus and vitality surplus determined for all risks combined and all products and lines combined.

only way to get your arms around interest rates and, certainly, the only way to make a presentation like this. But one of the things that we are finding in our work is that the yield curves are important determinants of risk, even in relatively stable times, e.g., a period like the past couple of years where interest rates haven't behaved quite as wildly as they did, say, if you look at the last seven years. We've found that the variations in required surplus are enormous if you just assume an interest scenario that could be loosely characterized as level interest rates. For example, the ten-year bond rate doesn't move at all or moves very little, and short rates and the longer rates move around it. Do you have any results or yield curves in your work?

MR. MATEJA: We've done some work with yield curves, and basically, we've found that the models get too complicated. We will leave it to others to extend the research to more dynamic kind of yield-curve assumptions. I'm not surprised to hear that there is material risk under assumptions you described.

MR. GEYER: We have done some testing on yield curves but not in connection with CORTF work. For the CORTF work, our focus was on combination of risks. We were trying to understand how the various risks interacted, and it just didn't make a lot of sense to introduce that degree of complexity. We definitely agree that the yield curve is another area of sensitivity.

MR. JULIUS VOGEL: The models you have presented are very interesting. While the results are not easy to understand, they at least seem possible to grasp conceptually. It seems that you're working on developing surplus for a whole company. Yet these models were for fairly simple plans for one year of issue. Does this mean that you're giving us a simple way of looking at surplus for a company? Have you really developed something where you can add up the individual parts and get at the bottom line?

MR. MATEJA: Our emphasis is more on process rather than a finished set of numbers. When you start looking at risk in just the simple manner that we have described (focusing on three types of risk and trying to understand the variables and how they interact) it doesn't take long before you get overwhelmed. At a company level, the problem is more overwhelming. For example, mortality risk at the Aetna has to be different from mortality risk in any other company for reasons that we haven't even considered in our analysis. Mortality risk is influenced by underwriting, medical and nonmedical limits, reinsurance arrangements, and a host of other factors. Asset-related risks are even more overwhelming. It's unlikely that there are two portfolios in existence today that have the same level of risk exposure when you start comparing cash-flow mismatch possibilities in different interest environments. You can get very discouraged about the problem of combining risk when you start thinking of the problem in those terms. The only way we could get a handle on it was to oversimplify.

One message that we hope came through in our presentation is that it is useful to look at risk through cash-flow analysis. We're no cleverer than anybody out there in terms of postulating cash-flow deviations appropriate for the various risks. We've given you some ideas on where to start and how to discount cash flows, and now it's up to you to experiment yourself and get a handle on what this approach can tell you about your own business.

MR. GEYER: Our overall objective in our modeling work is to measure risk relativities among products. We do not have models where we can throw in all the assumptions and get the sum total answer for Aetna. We can, in various scenarios, measure relative riskiness of the products and use that to allocate total surplus that we actually hold.