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## The Investment Models of a Finnish Pension Company

by Antero Ranne

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The Mutual Pension Insurance Company Ilmarinen is one of the companies managing the statutory employment pension in Finland. The investment models for actuarial use have been developed in Ilmarinen starting from the year 1993. The main parts are:

- financial time series data management
- time series analysis programs
- investment simulation models
- portfolio optimization models.

The models have been used for the following purposes:

- Developing the statutory solvency requirements for the Finnish pension companies and other insurance companies
- Developing new rules for determining the technical interest rate of Finnish pension companies
- Analyzing the risks connected with the investment strategy of the company.

### Financial Time Series Data

The models are based on financial time series from 12 countries. These include inflation, GNP growth rate, interest rates, share price indices, and dividend yields. The longest of the series start from the year 1950. The data are saved in an three-dimensional APL array. There are programs for adding new data and for output as tables or figures.

### Time Series Analysis Programs

Various time series equations are defined for the variables in the investment model. Programs exist in APL to estimate the parameters of these equations. Although commercial statistical programs can be used for more thorough analyzes, it is convenient to be able to make calculations in the same environment where the data are.

As an example it is shown how the parameters of the inflation model are estimated. The inflation rate may be influenced by external shocks which in the model are represented by oil price movements. The inflation  $i(t)$  in year  $t$  may be modeled by the equation

$$i(t) = m + a_1 [i(t-1) - m] + a_2 [i(t-2) - m] + b_0 o(t) + b_1 o(t-1) + e(t)$$

where  $o(t)$  is the oil price inflation and  $e(t)$  are the residuals or the stochastic part of the model. The parameter values are obtained by iteration. For the intermediate parameter values  $m$ ,  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  we calculate the residuals  $e(t)$ , as well as the partial derivatives

$$\frac{\partial e(t)}{\partial m} = -1 + a_1 + a_2, \quad \frac{\partial e(t)}{\partial a_1} = -x(t-1) + m, \quad \text{etc.}$$

By ordinary linear regression we find the coefficients  $d_1, \dots, d_5$  in the equation

$$e(t) = d_1 \frac{\partial e(t)}{\partial m} + d_2 \frac{\partial e(t)}{\partial a_1} + d_3 \frac{\partial e(t)}{\partial a_2} + d_4 \frac{\partial e(t)}{\partial b_0} + d_5 \frac{\partial e(t)}{\partial b_1}$$

The new parameter values are calculated by subtracting the d-coefficients:

$$m' = m - d_1, \dots, b_2' = b_2 - d_5.$$

(continued on page 20, column 1)

## The Investment Models of a Finnish Pension Company

*continued from page 19*

The solution is easy in APL because the linear regression part can be calculated directly using  $\tilde{Z}$ , as is shown in the following programs. The variable "inf" is the inflation and "oil" the oil price inflation.

```

par,,Infparam; p2; e; d; c
p2,,1+par,,51/2
: while 1E-3<- /|par-p2
  e,,p2,,par
  e,,p2 Resid inf, [1.1]oil
  d,,p2 Deriv inf, [1.1]oil
  c,,e2d
  par,,p2-c
: endwhile
e,,par Resid vars; x; y; m; a1; a2; b0; b1; i
(x y) ,,> [1]vars
(m a1 a2 b0 b1) ,,par
x,,0 0, x-ma y,,0 0, ya e,,(1/x)1/2
: for i :in 21/21/e
  e[i] ,,x[i]+(-a1*x[i-1]) +(-a2*x[i-2])+(-b0*y[i])+(-b1*y[i-1])
: endfor
e,,21/2e

d,,par Deriv vars; x; y; m; a1; a2; b0; b1
(x y) ,,> [1]vars
(m a1 a2 b0 b1) ,,par
d,,((1/x), 1)1/2-1+a1+a2
d,,d, -111/20, x-m
d,,d, -211/20 0, x-m
d,,d, -y
d,,d, -111/20, y

```

The calculation takes only a few steps. The same principles can be used for other quite complicated equations.

### Investment Simulation Models

When the parameter values have been established, the time series equations can be used to simulate the variables. The residuals are then generated as random numbers using the Wilson-Hilferty algorithm (see Daykin et al., 1994). The numbers are calculated using the following program:

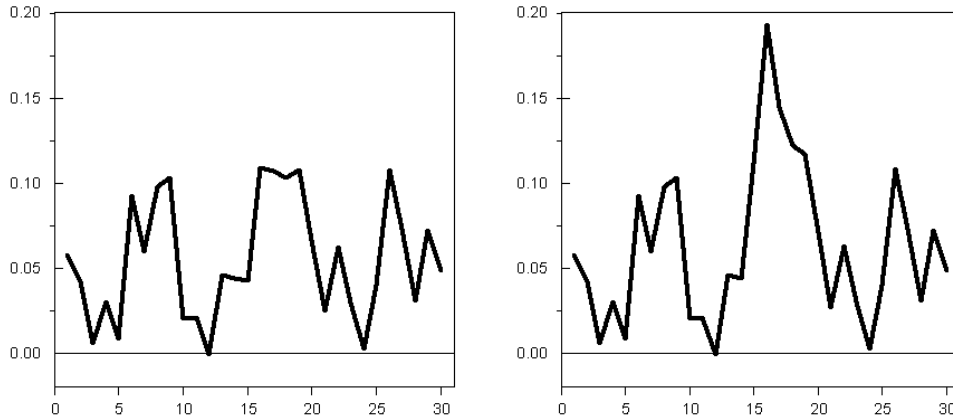
```

Z,,V Random N; M; S; A; B; C
© N random numbers from distribution with
© mean = 0, std deviation = 1 and skewness = V
© first numbers from normal distribution
S,,(÷1+M)×?(N, 2)1/2M,2147483646
Z,,(2±±2×S[; 2])×(-2×μS[; 1])*0.5
... (0=V) /0
© skewness is added by Wilson-Hilferty algorithm
(A B C) ,,((V*2)÷108), ((V÷6)-6÷V), 2÷V
Z,,(A×(Z-B)*3)-C

```

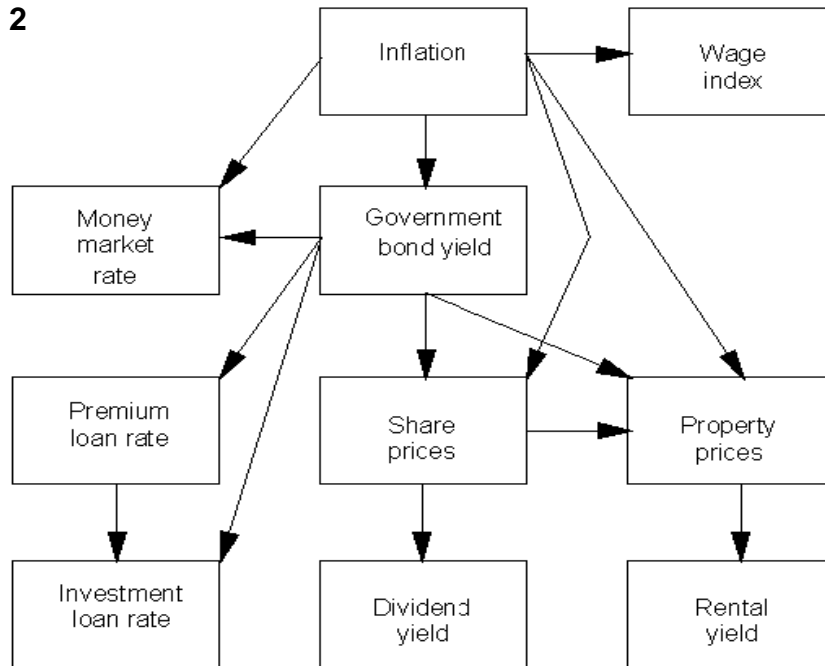
Figure 1 shows an example of simulated inflation numbers using the parameters obtained by the programs shown above. The second version contains in year 15 an inflation shock generated by the oil price variable.

**Figure 1**



The structure of the whole investment model is shown in Figure 2. A detailed description can be found in Ranne (1998).

**Figure 2**

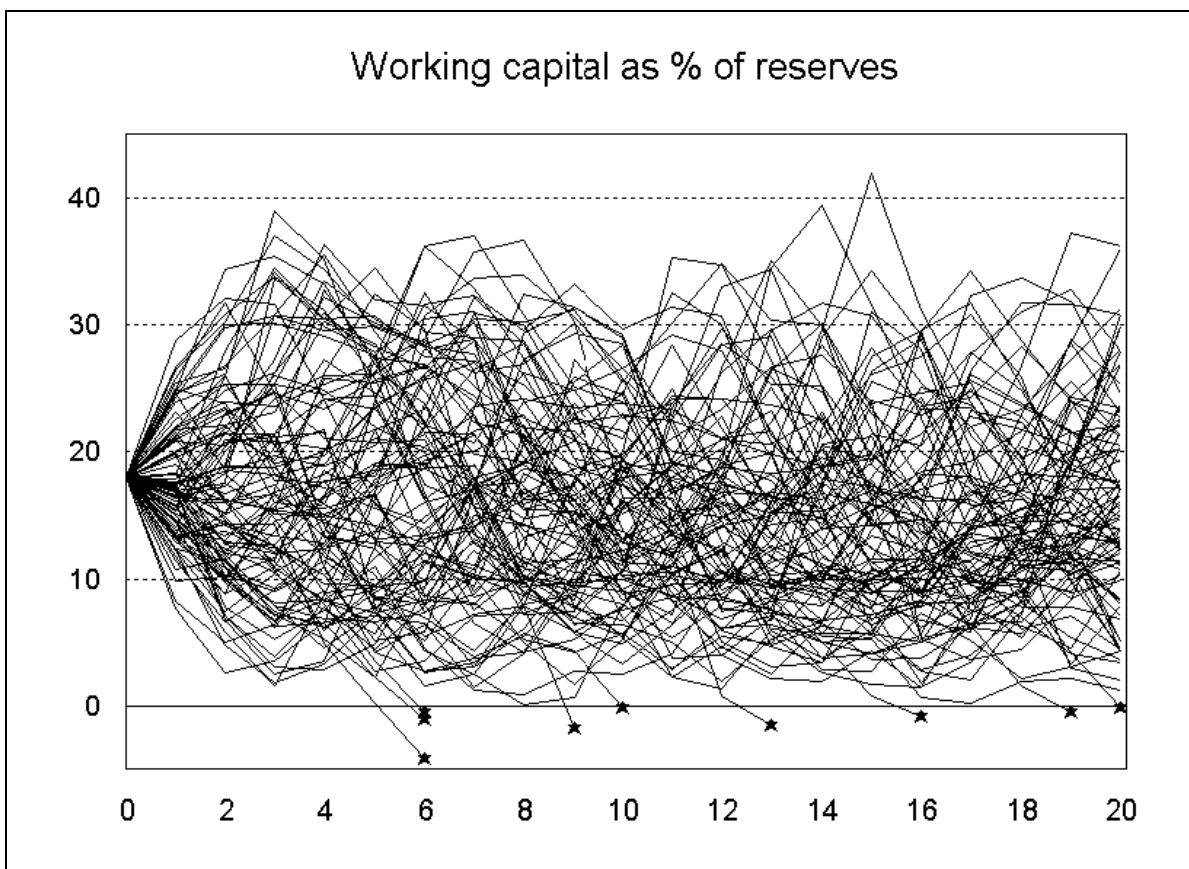


The purpose of the model is to generate investment yields that have realistic means, standard deviations and correlations in the long term. These are used to estimate the investment risks of a company.

## The Investment Models of a Finnish Pension Company

*continued from page 21*

The investment variables are combined with a forecast describing the development of the whole pension insurance company and various output variables are calculated (e.g. premiums, pension expenditure, reserves, bonuses, profits or losses). The most important output variable is the working capital, which is the difference between the total assets and the liabilities of the company and which acts as a buffer against variations in the investment results. The following picture below shows an example of simulated working capital as per cent of the reserves.



A thousand simulations for 20 years takes about 2 minutes 40 seconds on the present computer (300 MHz Pentium, 128MB memory).

### Optimization Models

In the optimization problem there are different investment categories (e.g. bonds, shares, property, cash). The investment returns have means  $\mu_i$ , standard deviations  $\sigma_i$  and correlation coefficients  $\rho_{ij}$ . Standard deviation is used to measure the risk level of the investments.

The proportion invested in category  $i$  is  $x_i$  ( $\sum_i x_i = 1$ ). The mean  $\mu$  and standard deviation  $\sigma$  of the whole portfolio is

$$\mu = \sum_i \mu_i x_i$$

$$\sigma = \sqrt{\sum_{i,j} c_{ij} x_i x_j} \quad \text{where } c_{ij} \text{ is the covariance } c_{ij} = \sigma_i \sigma_j \rho_{ij}.$$

The problem is to find the portfolio with the lowest risk  $\sigma$  corresponding to a given mean return  $\mu$ . (Alternatively one could find the maximum return corresponding to a given risk.) In practice there are often also minimum and maximum proportions for different investment categories.

Mathematically the problem is therefore formulated as minimize the function  $\sum_{i,j} c_{ij} x_i x_j$   
using conditions  $\sum_i x_i = 1$

$$\sum_i \mu_i x_i = \mu$$

$$m_k \leq x_k \leq M_k, \quad k = 1, \dots, n.$$

By standard mathematical methods it can be shown that the solution may be found by solving the following group of equations:

$$2 \sum_i c_{ik} x_i + a + b m_k = \lambda_k, \quad k = 1, \dots, n$$

$$\sum_i x_i - 1 = 0$$

$$\sum_i \mu_i x_i - m = 0$$

where in each investment category i one of the following conditions is true:

a)  $x_i = m_k$

b)  $x_i = M_i$

c)  $\lambda_i = 0$ .

For each combination of these conditions the resulting system of equations is linear, and so it can be solved in APL using the function  $\tilde{Z}$ . One has to solve the equations for different combinations of the conditions a-c. The solution that fulfills all the conditions and has the smallest variance is finally chosen as the answer to the optimization problem. The part of the program solving the linear equations is shown in the following. The program uses the global variables:

*Mu* vector mean values for investment returns

*Cov* matrix covariances

*Min* vector minimum values

*Max* vector maximum values.

Since it is possible that a given system of equations has no solutions, there is a matrix division function that returns the empty vector instead of stopping in domain error:

```
Z,,X MatDiv Y; Gel x
© Matrix division
© Result is empty, if no solution
Gel x, 'Z,,D a ..0'
Z,,XZY
```

The second function finds the distribution for a given mean return "mean" and conditions that are based on the argument "list":

```
z,,list Optimal mean; n; N; MAT; C; k; i
© Finds the optimal distribution for given mean return "mean"
© Investment return means in "Mu", covariances in "Cov",
© maximum and minimum proportions in "Max" and "Min"
© list[k]=0: no restriction in investment category k
© list[k]=1: use minimum for category k
© list[k]=2: use maximum for category k
N,,2+2×n,,½Mu
MAT,,(2½N)½0
MAT[½n; ½n] ,,2×Cov
MAT[½n; n+1] ,,MAT[n+1; ½n] ,,1
MAT[½n; n+2] ,,MAT[n+2; ½n] ,,Mu
MAT[½n; n+2+½n] ,,-(½n)° . =½n
C,,(n½0), 1, mean, n½0
: for k : in ½n
i ,,n+2+k
: select list[k]
: case 0
MAT[i ; ] ,,i =½N
```

(continued on page 24, column 1)

## The Investment Models of a Finnish Pension Company

*continued from page 23*

```

      C[i]„0
: case 1
      MAT[i ; ]„k=¼N
      C[i]„Min[k]
: case 2
      MAT[i ; ]„k=¼N
      C[i]„Max[k]
: endselect
: endfor
z„n†C MatDiv MAT

```

One has to execute the program for different values of the variable “list” and find the solution that fulfils all the conditions and has the smallest variation. The part of the program doing this selection is not shown here.

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### References

- 1) Daykin, Pentikäinen, Pesonen: Practical Risk Theory for Actuaries, Chapman and Hall, London, 1994
- 2) Ranne, A.: The Finnish Stochastic Investment Model, Transactions of the 26th International Congress of Actuaries, Vol. 7, Birmingham, 1998.

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## *Letter to the Editor:*

### *Inspect Those Asset / Liability Optimizer Projections*

*by Douglas C. Doll*

I recently had an opportunity to review the results of an asset/liability optimization exercise. The objective was to choose a set of assets that would “perform” better than the existing assets, where performance was defined as profitability under each of a defined set of interest scenarios. At first glance, the results were quite impressive. The optimized assets provided higher profitability for every one of the scenarios, substantially higher for some of them.

Too good to be true? You bet! A closer review of the results showed that

the initial rebalancing of the assets produced capital gains on the assets sold. These capital gains went into the IMR, but the IMR amortization was not used in calculating portfolio yield for crediting rate purposes. The increased profitability was coming from a reduced crediting rate. In fact, it appears that a simple strategy to keep the original assets and reduce the crediting rate would be superior to the “optimized” strategy.

Optimizers can be worthwhile tools, but their results need to be inspected for possible weaknesses and biases. It seems

like a simplistic recommendation, but you should be wary of any optimizer result that show significant improvements until you have inspected the projections and figured out the “why” for the improvement.

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