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Bond Prices, Yields, and Convexity

by Macroeconomic Advisers

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The relationship between bond prices and the level of interest rates is nonlinear. More importantly, it is asymmetric. An increase in the level of interest rates lowers a bond's price by a smaller amount in percentage terms than a decrease in interest rates raises a bond's price. This asymmetry, which is described below and depicted in the accompanying chart, is strongest for long-term securities.

This asymmetry, sometimes referred to as convexity, is a reason why the yield curve sometimes flattens for longer maturities. Convexity also implies that lower interest rate variability reduces the slope of the term structure by less than would be suggested by the intuitive observation that long bonds are more risky than short ones. Risk in the latter sense is measured by another mathematical concept, duration, also explained below. Since inflation variability can feed through to interest rate volatility and bond prices, the concepts of duration and convexity suggest that lower inflation variability may reduce the term spread by less than is sometimes expected.

Duration and convexity may be easiest to explain with a security which pays $1 *n* periods from the present. It has a present value of $1/(1 + i)^n$, where *i* is the yield-to-maturity. Duration, defined as the lasticity of the present value with respect to a change in *i*, is equal to *n*. In other words, infinitesimal increases in the level of interest rates should lower the present value by *n*-times the percentage change in yields. Clearly, duration is larger for long-term securities. However, when changes in the level of interest rates are more than infinitesimal, actual changes in present value are not exactly equal to duration.

When $n = 30$ (a 30-year bond), a one percentage point increase in *i* lowers the bond's value by 24.75%, while a one percentage point decrease in *i* raises the bond's value by a larger 33.25%. The reason for this discrepancy follows from the convex relationship between present value and interest rate, depicted in the accompanying chart. This asymmetry is larger for long-term bonds.

Asymmetric changes in present value imply that in an environment of interest rate variability, the average present value is greater than the present value associated with the average level of interest rates. Furthermore, this effect is greatest for long-term bonds. If investors were perfectly risk-neutral, this asymmetry would imply that the market price of a bond should be greater than the present value associated with the average interest rate. Since yield-to-maturity is inversely related to price, this "convexity effect" implies that yields on long-term bonds could be below yields on short-term securities even in an environment where all expected future short-term interest rates are equal to current short-term interest rates. In reality, of course, investors are risk averse, so the increased duration of long-term bonds means they require an additional risk, or "term premium."

In such cases the convexity effect offsets the term premium, though perhaps by less than the full amount. This explains why yield curves often exhibit the following pattern: a pronounced positive slope at shorter maturities, a moderate positive slope through intermediate maturities, and finally a flat or even negative slope at the longest maturities. A nearby figure on the "convexity effect" illustrates its role in flattening the yield curve at long maturities.

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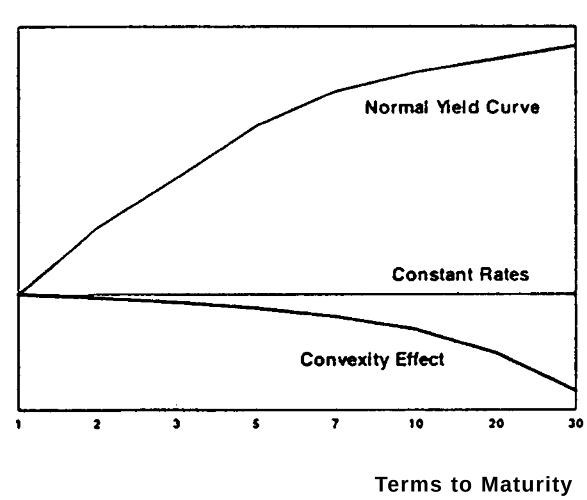
Convexity Effect and Term Risk Premium



Present Value vs. Yield-to-Maturity of $100 Zero-Coupon Bond