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Chairperson's Corner

by Josephine E. Marks

The Investment Section had 4,127 members and a continued strong financial position with a fund balance of \$191,888 as of March 31, 2000. A future article will discuss the source and uses of our Section's financial surplus. The primary objective is to use Section assets to fund investment research and enhance the calibre of investment seminars and meeting sessions to provide educational services to our members. Currently approximately \$ 30,000 of the Section's fund balance is committed to support research and education.

Elections for Section Council are taking place in this month. We have a strong slate of candidates for the Section election this year. Three are to be elected. The candidates are: Tony

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When Is It Right To Use Arbitrage-Free Scenarios?

by Stephen Britt

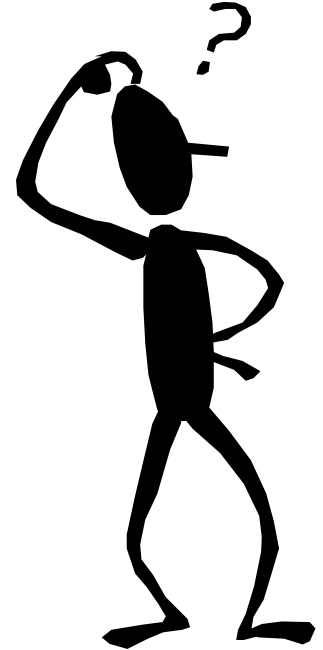
One of the questions often asked of economic scenario generators and their simpler siblings — interest rate generators — is "Is it arbitrage free?" Curiously, this is not the question that should be asked, which is "Is the generator risk neutral or realistic?"

There is no right or wrong answer to this question, as the type of generator must be appropriate to the use of the simulation. In general, simulations to price securities that can be hedged require risk-neutral scenarios. Scenarios to assess the distribution of results of holding instruments (some of which may not be able to be hedged) require realistic scenarios. Simulations that will be used to price securities that cannot be hedged (such as a catastrophe reinsurance program) require realistic scenarios.

The Debate (re) Defined

Model builders in finance and insurance companies often need to build models of interest rates, and other

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financial variables like equity returns. These scenario generators are used in simulations for a number of purposes. Scenario generators come in two flavors:

- Risk-neutral
- Realistic

There is often discussion, more heated than informative, on the relative merits of these two approaches to modeling interest rates and other variables. Our observation is that the religious zeal with which some practitioners praise one or the other is often in inverse proportion with their understanding of what should be a relatively straightforward issue.

Confusing the issue further is the question of “arbitrage free” scenarios.

In this article we discuss the issue of arbitrage-free scenarios, risk-neutral scenarios and realistic scenarios in the context of a Monte-Carlo simulation approach to solving financial problems. We start with the concept of “arbitrage-free” models. We assume the need to perform Monte-Carlo simulation of bond yields and other variables and address the issue of whether these simulated rates should be arbitrage-free, risk-neutral, or realistic.

Arbitrage and Arbitrage-Free Yield Curves

A set of scenarios allows arbitrage if it is possible to construct a portfolio of securities that offers a “free lunch.” Either:

- The portfolio can lock in possibility of profit, at no initial cost to the investor; or
- The portfolio generates an immediate profit with no residual risk to the investor.

A clear example of arbitrage — imagine it were possible to borrow from the government at a rate of 5% for a

period of 10 years, with no payments due until the end of year ten (an example could be a low-interest loan for study, funded by a government agency).

Imagine that at the same time the investor could lend to the government (buy a ten year zero) at 6%. A rational person would borrow as much as possible — and invest at the higher rate, locking in a significant profit on each dollar invested. This generates a locked-in profit at no cost, representing arbitrage. (It is customary in discussions such as these to assume that there is no limit to the amount that investors could borrow or buy).

Arbitrage opportunities are most often discussed in the context of the fixed income market; in particular, the treasury yield curve is assumed to offer no arbitrage opportunities.

Not surprisingly, arbitrage opportunities are rare in the real world, and should be rare in the models we build. How rare? At the extreme there should be no arbitrage opportunities for a portfolio of any securities, regardless of whether:

- The securities exist or not in the real world, or
- The securities are utilized in our models.

In theory (the theory going by the grand title of “The Fundamental Theory of Asset Pricing”), there should be no arbitrage opportunities within the span of available traded assets. That is, there should be no arbitrage for any securities (e.g., options, swaps, forward contracts) e.g., based on traded underlying securities, regardless of whether those securities exist.

This is an extreme restriction on arbitrage. Present technology is such that it is not possible to build models that restrict arbitrage to this extent and still faithfully reproduce the characteristics of

yield curves and equity returns. For a discussion of this issue, see these references Cont R (2000), Christiansen (1998), Pliska (1997) and Tilley (1992).

For example, most models run at best at monthly rests; the need to interpolate for cash flows due mid-month open up opportunities for arbitrage for zero-coupon securities priced at less than monthly rests.

To create yield curves that truly look like yield curves, they must allow some limited arbitrage. In the real world, the presence of transaction costs will often make this arbitrage opportunity ineffective.

One option is to create yield curves that do not allow arbitrage among the securities and asset classes employed in the simulation. Call this the absence of model arbitrage. Model arbitrage can create critical errors in a simulation if the strategies that emerge from the simulation require this arbitrage to be effective, and the opportunities are not available in the real world.

Realistic Scenarios

For many purposes, we need to be able to place a distribution around a company’s holdings of assets and liabilities. For example, an insurance company wishes to know the likelihood of exhausting its available capital given its holdings of investments and book of liabilities.

Under these circumstances, we generally need our distribution of interest rates to be realistic. An inverted yield curve at the current time should not imply an inverted yield curve in the future, and equities do not have the same expected return as bonds.

Risk-Neutral Scenarios

Suppose we wish to value a claim on the S&P 500 and the 10 Year T-Note. It could be something explicit like “pay \$100 if the price of a 10 year T-Note falls below \$95 and the S&P falls below

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When Is It Right to Use Arbitrage-Free Scenarios?

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1500.” It could be something less clear, like the guaranteed minimum death benefit on a portfolio consisting of stocks and bonds.

It makes sense that if we could define a portfolio that exactly matches the payoff of our claim, we can work out the appropriate price for it in terms of the current price for the stocks, bonds, and perhaps cash. For simple claims (such as a forward contract), this is indeed the case. For more complex instruments (swaps, options, etc.), it can be proven (see a good textbook on option pricing) that there always is such an initial portfolio, although the portfolio will usually vary over time. That is, there is a dynamic hedging strategy that, given the usual assumptions relating to frictionless markets, reproduces the claim with no additional investment over time. The trick is to find the portfolio, or rather the initial portfolio and the replicating strategy.

It can be shown (again, see the text book) that the price of the claim and the portfolio to hedge the claim can be derived using the expected value of the claim in a “risk-neutral” world. The risk-neutral world is an odd place. It is identical to our own, except that expected return on risky assets (e.g., equities) is the same as the return on a riskless asset such as a government bond. The volatility and correlation structure of asset returns is the same; it is just the expected returns that are different.

When Is It Right to Use Arbitrage-Free Rates?

Paradoxically, the answer is “never” and “always.” It is possible to find arbitrage opportunities in any model, when implemented in computer code. These arbitrage opportunities exist in the real world, but transaction costs and other real-world impediments make it impossible to exploit them.

The Issue Is the Degree of Model Arbitrage Allowed

If your simulation results are biased because of exploitation of arbitrage in the model that is not exploitable in the real world, then your results are invalid. Remove this arbitrage either from the model, or remove the opportunity from the set of available strategies.

When Is It Right to Use Risk-Neutral Rates?

Consider a trader wishing to work out a fair price for an option on a bond. This will pay a fixed amount depending on the state of the 10-year bond yield. The secret to pricing this option has been known ever since the seminal article by Fischer Black and Myron Scholes (Black-Scholes 1973) that derived the deservedly famous Black-Scholes option pricing formula. The secret is to derive an investment strategy that will:

- Require an initial investment to establish the portfolio,
- Require no additional investment, and
- Will replicate the payoff of the option, regardless of what it actually is, at the maturity date.

Arbitrage considerations would mean that the price of the option must be the same as the price of the initial portfolio that would eventually replicate the option. Furthermore, we can hedge the risk of holding the option by always holding the replicating portfolio.

How do we determine this strategy that replicates the option and hence its price? We can cheat a little here because if the option is solely dependent on the price of one or more tradeable securities (stocks and bonds), the price will be the same as if we were working out the expected value of the

option using an arbitrage-free (or more precisely, risk-neutral) set of scenarios.

We can go further. A theorem in finance states that if a certain state-contingent claim (e.g., a security like an option or a bond) is solely determined by one or more tradeable securities, then there is a strategy that can replicate its pay-off. Furthermore, our trick of valuing using the risk-neutral scenarios can assist in finding the replicating portfolio and working out the price of the claim.

State-contingent claims can be more than just options. For example:

- A ten-year bond is a simple claim. (The payoff is simply the face value of the bond. The strategy is to hold the bond, and the value of the bond is, simply, the bond)
- The forward contract described above is a state-contingent claim. It can be replicated by holding a position in the nine and ten-year zero coupon bond
- An option on the S&P 500 is a state-contingent claim. It can be replicated by dynamically borrowing money to pay by some stock.
- Certain life-insurance products can be considered state-contingent claims. A pure endowment with a guaranteed surrender value is like an option. After making assumptions on the behavior of policyholders regarding surrender, it can be replicated by holding positions in bonds.

Not all claims are state-contingent claims, however, in the sense that they can be replicated by a strategy solely involving tradable securities:

- The pay-off from a lottery ticket cannot be replicated in this way.
- The pay-off from an insurance policy on a car cannot be replicated by holding positions in tradable securities.
- The pay-off from a term life insurance policy cannot be replicated in this way.

Putting all this together, then arbitrage free scenarios can be used:

- Where the aim of the simulation is to find the price of certain claims (e.g., an option); and
- Those claims can be replicated using an investment strategy solely employing tradeable securities.

Not surprisingly, financial markets almost by definition deal with tradable securities, so there is a great deal of interest in being able to price these securities. Arbitrage-free scenarios are used commonly in finance to price and hedge these types of claims.

When Is It Right to Use Realistic Scenarios?

The short answer is “in almost all other cases.” In particular:

- When the claim we are trying to price cannot be fully replicated using tradable securities.
- When we are not required to calculate the current price of a claim.

- When we are interested in the range of likely values of some claims in the future (what is the 5th percentile worst value for the S & P in five years).

These cases abound in insurance, but are less frequent in financial markets (although Value-At-Risk is an example of the last case). Examples include:

- Setting the asset mix for a pension fund or property-casualty insurance company
- Investigating the required capital for an insurance company and allocating it to lines of business
- Investigating certain reinsurance contracts

Summary and Conclusion

Risk-Neutral scenario generation is an elegant and useful tool for pricing certain securities and claims. It is right that these techniques and tools should be used in cases where they are appropriate. It is also incorrect not to use them when arbitrage consideration implies their use.

However, like any tool they cannot be used for many tasks. For tasks where we

need to reproduce the statistical distribution of interest rates, inflation, and equity markets, we need to use different, realistic scenarios.

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- 3) Cont R (2000), “Modeling term structure dynamics: an infinite dimensional approach,” Working Paper, Centre de Mathematiques Appliquees, Ecole Polytechnique
- 3) Pliska S (1997) *Introduction to Mathematical Finance*, Blackwell Publishers.
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Chairperson’s Corner

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Dardis, Craig Fowler, Doug George, Charles Gilbert, David Ingram and Ken Mungan. Those leaving the Council this year are Doug George (after a one-year term), Christian-Marc Panneton (after a two-year term) and Josephine Marks.

The Section will be sponsoring 10 sessions at the Annual Meeting in Chicago to be held from October 15 to 18, 2000. In Peter Tilley’s article, he discusses the planning process for spring and annual meetings in a behind-the-scenes description of how these

sessions are planned and organized. The Finance Practice area, with support from the Investment Section, is organizing several investment seminars to be held later this year, including one with a new concept — an Investment Actuary Symposium to be held November 13 – 14 in Boston. (See the article in this issue for more details).

Our Section Web site is now up and running. Refer to the SOA Web site (www.soa.org) to access the site under Special Interest Sections — Investments.

The list serve is now active, and members are encouraged to join the list serve and use it wisely.

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