Article from:

Risks and Rewards Newsletter

February 2001 – Issue No. 36

RISKS and REWARDS

The Newsletter of the Investment Section of the Society of Actuaries

NUMBER 36 — *SPECIAL INVESTMENT ACTUARY SYMPOSIUM ISSUE* — FEBRUARY 2001

The Bullet GIC as an Example

by David F. Babbel, Jeremy Gold, and Craig Merrill

There has been considerable discussion of a variety of issues related to fair value in the actuarial literature, in conferences, and among individuals interested in this topic. Unfortunately, we seem to be failing to communicate due, in part, to inconsistent use of terminology. The goal of this discussion paper is to present a few concepts that we hope will be of use in the broader discussion of fair value of liabilities.

Fair Value from the Perspective of FASB

Current practice dictates that corporate liabilities (specifically, bonds) are listed on the balance sheet on a book value basis. The liability changes only if the company actually refunds or retires the bond. FASB is moving toward a requirement that the market value of the bond be reported in place of, or in addition to, the book value of the bond. The reasons for this change are covered in some detail in document number 204-B of the Financial Accounting Series (December 14, 1999) entitled, "Preliminary views on Major Issues Related to Reporting Financial Instruments and certain related assets and liabilities at fair value."

In the preliminary views document they indicate that "fair value" should be determined based on observable market prices.

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Beyond the Bullet GIC

by Stephen J. Strommen

In a separate article in this edition of Risks and Rewards, Babbel, Gold and Merrill provide an excellent exposition of three approaches to present-valuing a series of risky cash flows and provide several insights into the way modern finance theory deals with risk. The purpose of this article is to bring these insights to the world of insurance risks and view them in relation to existing actuarial techniques.

All three of the valuation approaches presented by Babbel, Gold and Merrill involve direct discounting of liability cash flows. However, current actuarial practice for determining liability exit value (i.e. fair value) is embodied by the actuarial appraisal method, an indirect method under which the value of the liability is computed as the market value of assets supporting the liability less the present value of future distributable earnings at a hurdle rate.

Many observers feel that direct discounting and the actuarial appraisal method produce different values. However, Luke Girard demonstrated that these two methods produce identical values when identical assumptions are used. The source of confusion is that many observers find it hard to justify the assumptions that must be used under a direct discounting

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In a thinly traded market, "exit value" might be used as an indicator of "fair value." In this context, exit value is "an estimate of the amount that would have been realized if the entity had sold the asset or paid if it had settled the liability on the reporting date." FASB also allows that in some cases the present value of projected liability cash flows may be used as an estimate of the fair value of liabilities. This is the current practice in the pension area. Use of the present value method is discussed in the Financial Accounting Series document FASB Concepts Statement No. 7, "Using Cash Flow Information and Present Value in Accounting Measurements."

Approaches to Valuation

There are at least three theoretically correct methods for estimating the value of a series of (potentially risky) future cash flows. One, discount the future cash flows using a discount rate that is the sum of a risk-free rate and a risk premium. Two, modify the probabilities of the risky future cash flows to account for risk and discount at risk-free interest rates. Three, modify the risky cash flows to account for risk and discount at the risk free rate. We will discuss each briefly in the form of an example. [1]

Consider a security with price S, that will pay either S_u or S_d in one year. We can apply the three methods of valuation as follows. First,

$$S = \frac{[p\,S_u + (1-p)\,S_d]}{(1 + r + \lambda\sigma_S)} \qquad (1)$$

where r is the one-year risk-free rate, p is the "true" probability of the payoff being S_u, λ is the market price of risk associated with the uncertainty about the security's payoff, and σ_S is a volatility parameter associated with the uncertainty of the security's payoff. [2]

Second,

$$S = \frac{[\pi\,S_u + (1-\pi)\,S_d]}{(1+r)} \qquad (2)$$

where $\pi = p - \lambda\sqrt{p(1-p)}$

is the risk-neutral (martingale) probability. Or, third,

$$S = \frac{[p\,S_u + (1-p)\,S_d] - Z}{(1+r)} \qquad (3)$$

where Z is a quantity that makes the numerator of (3) equal to the certainty equivalent of the risky expected payoff in the numerator of (1).

In order to illustrate how pricing with martingale probabilities compares to pricing with the "true" probabilities or using a certainty equivalent, consider the problem of valuing a simple one-year interest rate contingent claim. This claim will pay $110 if the short rate goes up and $90 if the short rate goes down. This claim can be valued using the "true" probability, $p = 0.51$, and a risk-adjusted discount rate. The risk-adjusted discount rate is

$$r + \lambda\sigma_s = 0.0520995$$

where $\lambda = 0.02$ and $\sigma_s = 0.104979$.

Thus, this security's value is

$$[p\$110 + (1-p)\$90]/(1 + r + \lambda\sigma_s) = \$95.24.$$

Similarly, this security can be valued using the martingale probability, $\pi = 0.5$, and discounting at the risk-free rate, $r = 0.05$.

$$[\pi\$110 + (1-\pi)\$90]/(1 + r) = \$95.24$$

Finally, using the certainty equivalent approach with Z = 0.2, the value would be

$$[p\$110 + (1-p)\$90 - 0.2]/(1 + r) = \$95.24$$

The conclusion is that the valuation process can account for risk, either by using the "true" probabilities and discounting by a risk-adjusted discount rate, or through converting the "true" probabilities into martingale probabilities and discounting by the risk-free rate, or by adjusting the cash flows to a certainty equivalent level and discounting at the risk-free rate.

Each of these three approaches is theoretically correct. Practical considerations dictate the choice between the three approaches. Equation (1) is the traditional discounted cash flow model. It is most often used for capital budgeting and net present value type of analysis. It is also the traditional method of choice for non-traded or thinly-traded securities. Equation (2) is a one-period lattice version of the option pricing model. The existence of the martingale probabilities arises from the ability to create a hedge portfolio in a complete market. The hedge portfolio exactly replicates the cash flows of the security under consideration. In fact, the ability to create a hedge portfolio is synonymous with markets being complete. This approach is used when pricing interest-sensitive financial instruments and other derivatives in a complete market. Equation (3), the certainty equivalent method, is not often used because the certainty equivalent adjustment, Z, is dependent on the form of a utility function. It has, however, been successfully used in capital budgeting problems.

Some Applications of the Option Pricing Model

There are examples where the option pricing model has been successfully applied to thinly traded securities.

Probably the most prominent are the mortgage-backed securities (MBS). The underlying prepayment risk was not actively traded until the creation of MBS. The uncertainty surrounding the prepayment risk was accounted for using an option-adjusted spread (OAS). The OAS was, essentially, a fudge factor added to the discount process that reconciled the models with the market. Over time as market participants understood the prepayment risk better, and active trading emerged, the OAS shrunk drastically on vanilla MBS when valued using properly calibrated, adequate models.

Another example of an application of the option pricing model to thinly traded assets is the pricing of corporate bonds. Merton, as well as Black and Scholes, suggested that corporate securities could be viewed as options on the underlying assets of the company.

The underlying assets include plant and equipment, franchise value, customer relationships, etc. These parts of the asset value are difficult to observe and price. The model has still been used successfully in pricing credit derivatives. The inability to observe the value of assets is less of a concern for insurance liabilities where the vast majority of assets are financial and easily observed.

Consider a simple company with equity holders and a single bond issuance. Note that the bondholders are entitled to the value of the assets up to the face amount of the debt and that the equity holders are entitled to the value of the assets in excess of that amount.

This means that we can view equity as a call option on the assets with a strike price equal to the face value of the debt. For a zero coupon bond, the value of equity is given by the Black-Scholes call option formula. Extensions for coupon bonds have also been derived. The value of the bond is given by subtracting the equity call option from the underlying assets.

Thus, the bondholders are described as owning the assets and selling a call option to equity holders.

Recall the Black-Scholes call option formula

$$C = AN(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(A/X) + (r + \sigma^2/2)/T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and where

C = call option value = value of equity in the Merton model,
A = current asset value of the company,
$N(d)$ = standard normal density evaluated at d,
X = exercise price = face value of debt,
r = risk-free rate,
T = time to maturity for the option,
σ = standard deviation of the annualized continuously compounded rate of return on the assets.

Then the value of the bond is $A - C$.

There are three key observations that can be made at this point. First, the bond value converges to a risk-free bond value as the asset value of the company increases. Second, the value of the bond decreases as the volatility of assets increases. And third, the expected return on assets is not an explicit component of the value of the bond. We will comment on each point in turn.

An increase in asset value increases both the value of equity and the value of debt, up to a limit. The most that the bond can be worth at maturity is X, the face value of the debt.

As the value of assets increases, the value of the equity converges to $C = A - Xe^{-rt}$. This can be seen by observing that as A grows large relative to X, d_1 and d_2 increase and the call option (equity) value increases toward an upper limit of $C = A - Xe^{-rT}$. Then, the value of the bond is $A - C = Xe^{-rT}$. Thus, for very large asset values, the bond is risk free and the price of the bond is the promised cash flow discounted at the risk-free rate. Notice that this result holds for relatively conservative assets with a low standard deviation or for very risky assets with a large standard deviation. For any given risk level (standard deviation of assets), the bond will be risk free for a sufficiently large asset level.

The second point deals with volatility. It is a standard result in option pricing that an increase in volatility increases the value of an option. This can be seen by taking the derivative of the option pricing formula with respect to σ.

Therefore, all else being equal, the value of the bond decreases when volatility increases. This is an intuitive result. Higher volatility in the assets leads to a greater probability of the firm defaulting and the bond holders receiving the assets of the firm as partial payment of their claim. Thus, our first point does not violate the simple intuition of this second point.

Finally, many students of the mathematics of finance find it troubling that the expected return on assets is not an explicit component of the equity or bond value. While the option pricing formula involves discounting at the risk-free rate, the relationship between the martingale probabilities inherent in the option-pricing formula and the “true” probabilities depends on the risky return on assets.

Recall equation (2) above. The martingale probability, π, is a function of the “true” probability and the market price of the underlying risk.

The same intuition holds in the more complex Black-Scholes option pricing formula. The market price of the asset risk of the company enters into the relationship between the martingale measure, N(d) in the option pricing formula, and the “true” probability density.

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There is an alternative representation of the value of a corporate bond in the option pricing framework. Recall the put-call parity relationship

$$P = C + Xe^{-rT} - A$$

where *P* is the price of a put option written on the same assets, *A*, having the same strike price, *X*, and the same time to maturity, *T*, as the call option, C. The put-call parity formula can be rewritten as

$$A - C = Xe^{-rT} - P. \qquad (4)$$

Notice that the left hand side of (4) is the value of the bond, as described above. The right hand side of (4) is the price of a risk-free bond minus a put option. Thus, a corporate bond value can be decomposed into a risk-free bond and a put option on the assets of the firm. For convenience, we will refer to the value of the bond cash flows, discounted at risk-free Treasury rates, as the synthetic Treasury value of the bond. Thus the decomposition involves two terms: the synthetic Treasury value of the bond and the put option.[4] This is a useful decomposition, as we can now observe the relative impact of interest rate changes and credit quality changes. Interest rate changes will impact both terms, but the price of the risk-free bond will capture the pure time value of money. When the creditworthiness of the firm changes, that will be captured by the put option value.

It is important to note that the option pricing approach differs from simply discounting liability cash flows at Treasury rates and calling the resulting present value the fair value of liabilities. As has been pointed out repeatedly and forcefully, there must be some accounting for risk. The accounting for risk is done properly in our decomposition approach. Notice, though, that simply using the asset portfolio return as a discount rate would be a mistake. The asset portfolio return is not the key to the risk in the liabilities. The keys are the degree of overcapitalization ($A - X$) and the volatility of asset returns.

Fair Value from a Finance Perspective

Consider a bullet GIC as a simple insurance company liability. In its simplest form, the bullet GIC is little more than a zero-coupon bond. The fair value of the bullet GIC could be determined using any of the valuation approaches discussed above. There are several reasons, however, that we suggest it should be valued as a risk-free zero-coupon bond minus a put option. As before, no correctly implemented valuation approach is more theoretically correct than any other correctly implemented valuation approach. The choice of valuation methodology is often driven by practical considerations.

If the bullet GIC were the only type of liability issued by an insurance company, we could just calculate the market value in the most convenient way possible. We could simply look to the secondary market, thin though it might be, and price accordingly. Alternatively, we might look to the creditworthiness of the issuer and add a spread to Treasury STRIP rates to discount the promised cash flow from the bullet GIC. The liabilities of an insurer, however, are much more complex than a simple bullet GIC. It is when we turn to the more complex liabilities that the decomposition into a risk-free liability and a put option become particularly desirable.

The key benefit of the decomposition approach is that it increases transparency. Insurance liabilities are far more complex than corporate bonds. Any reasonably competent analyst, given a market price and the details of a corporate bond (coupon rate and maturity date), could use Treasury bond data to figure out the synthetic Treasury value of the corporate bond, and the value of the put option. The put option is just the difference between the synthetic Treasury value of the bond and the market price of the bond. The relative ease of this decomposition is due to the limited information required to fully describe the cash flows of a corporate bond. Thus, for a corporate bond, it is fully adequate to report only its fair value.

> *"The synthetic Treasury value of liability is like a defeasance value of the liabilities. The put option value captures the risk inherent in the company backing the liabilities."*

The relative impact of interest rate changes and credit quality changes is easy to discern. Similarly, for a GIC, it would likely be adequate simply to report the market (fair) value of the liability. For more complex insurance liabilities, the decomposition approach has advantages.

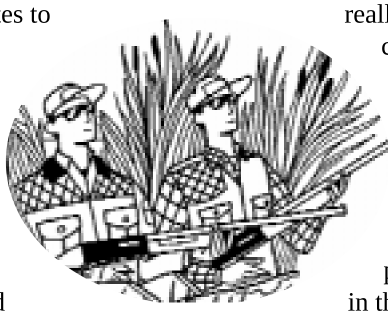
The increased transparency of the decomposition approach is valuable for analysts, regulators, investors, and management. Analysts would be able to compare the structure of liabilities from one company to another more easily because of the consistent use of Treasury rates in calculating the risk-free present value of liability cash flows (the synthetic Treasury value of the bond). Then, a contra-liability (the put option) would summarize the condition of the company backing the liabilities.

If the liabilities were to be transferred from one company to another, the contra-liability would change, not the present value of liability cash flows. This would aid in mergers and acquisitions analysis and decision making as well as for sales of a block of business. Regulators would also benefit from this decomposition.

The synthetic Treasury value of liabilities is like a defeasance value of the liabilities. The put option value captures the risk inherent in the company backing the liabilities. Similar reasoning applies to investors and managers who are concerned with the condition of the company.

The put option value is relatively easy to compute. The same projected cash flows that are discounted at Treasury rates to arrive at the synthetic Treasury portion of the decomposed liability value can also be discounted at risky interest rates. A spread, with appropriate maturity and risk dependencies, can be added to the Treasury interest rates to discount the projected liabilities.

The difference between the two present values is the value of the put option. While it might seem that it would be easier just to discount with a spread and call that the fair value, the decomposition is valuable for the reasons listed above.



Concluding Comments

There is a lot of work still to be done to extend the reasoning in this note to more complex liabilities. In fact, it may well be that the best we can do at this point is to estimate future possible cash flows with their interest rate contingencies and discount them by Treasury interest rate processes and then by interest rate processes that incorporate appropriate spreads. In this way, we can estimate the two pieces of the decomposed value of insurance liabilities.

It could be argued that reserves are analogous to the Treasury rate discounted insurance liabilities. If reserves are estimated according to consistent actuarial and statutory standards, it is asserted, they can be compared to fair value estimates, and out pops a default risk premium.

We think not, for two reasons. First, for more general corporate bonds, the construction of a synthetic Treasury captures properly all of the interest-sensitive elements in the bond. Stochastic interest rate valuation models then capture the option value.

In contrast, reserving methods either ignore options or render their value at the current exercise price. Either treatment greatly misvalues the option. This is particularly ironic in light of the modern trend to view the life insurance policy as a package of options.

Second, reserving methods typically are conservative and embed margins designed to provide security that insurance promises can be kept. To the financial economist, these margins are more properly considered a part of surplus, not liabilities. What is really needed by the financial community, investors, and regulators is analogous to the synthetic Treasury used to analyze corporate bonds, and this measure is not currently produced by life insurers in their financial reports.

Regarding the issue of risk-based interest rate spreads, it has been suggested that insurance liabilities be discounted by rates that reflect the "claims paying rating" spreads associated with Moody's or Standard & Poor's ratings. We have two concerns with such procedures. First, the resulting estimation could hardly be called a "market value," because a rating agency's claims paying rating is not a market rating. Second, there is far more variation within a given rating than there is across rating categories.

For instance, Moody's chief economist, Jerome Fons, demonstrated that even with bonds, where the rating agencies have decades of experience, there are large disparities in yields. He showed how on a single day you can observe bonds in the same rating category with the same maturity commanding yields that are 50 to 800 basis points apart, depending on which of the investment grade categories one is considering. By way of contrast, the variation in average yields across categories is less than one-fourth as large. Clearly, such large disparities are forcing claims paying ratings to shoulder too heavy a load when it comes to valuing insurance liabilities.

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Footnotes

1) This example is drawn from the monograph, "Valuation of Interest-Sensitive Financial Instruments." Babbel and Merrill, SOA Monograph M-FI96-1, pp. 43-44.

2) The market price of risk is the equilibrium excess reward to risk ratio,

$$\lambda = \frac{\mu_S - r}{\sigma_S}$$

where μ_S is the expected return and σ_S is the standard deviation of return for the security, *S*. In equilibrium the reward to risk ratio is constant for all securities. In a CAPM framework λ would be defined with β in the denominator. In a multi-factor setting there would be a market price of risk for each stochastic factor.

3) Other names applied to this model include the martingale measure, risk-neutral probability, or hedging model.

4) In Merton's original derivation of this model the only risk captured by the option was default risk. In an insurance liability application it would need to capture other risks such as illiquidity.

5) In this context "defeasance value" means the value of a portfolio of Treasury securities that fully funds the expected cash flows, including interest rate contingencies, of the insurance liabilities being considered.