SOCIETY OF ACTUARIES

Article from:

Risks and Rewards Newsletter

February 2001 – Issue No. 36

Investment Actuary Symposium

Risk-Neutral Pricing for Insurance Contracts

by Stephen Britt

This article discusses is about the pricing of life insurance contracts in the risk-neutral world. Specifically it deals with three aspects:

- The motivation for pricing contracts using a risk-neutral methodology in valuing life insurance contracts
- Development of some 'intuition' behind the risk-neutral valuation techniques
- Some caveats which need to be recognized before Risk-Neutral Valuation can be more widely accepted

The Motivation for Pricing Using a Risk-Neutral Methodology

Question :
How is an actuary unlike a car salesman?

Answer :
Sometimes an actuary will actually give away free options.

Life insurance contracts have always had implicit and explicit guarantees. A simple whole of life or endowment contract contains an implicit guarantee of a minimum guaranteed interest rate underlying the growth of the policy reserve. Minimum surrender values offer 'harder' and more explicit guarantees.

More recently, SPDA and variable annuity contracts have offered explicit guarantees to what would otherwise be investment products.

If it is true that these guarantees have always existed in these products, it is equally true that their existence has not always been accepted in general by the actuarial community, and even now, it is by no means common to see these guarantees explicitly costed in the product development stage.

Risk-neutral pricing allows the pricing actuary to develop a relative pricing strategy, in that the price is calculated relative to similar traded instruments like swaps and options. It also opens the opportunity that these products may be hedged using these instruments.

Risk-Neutral Valuation Techniques

In this section, we investigate cases whether knowing the probability of an adverse event does not assist in pricing it.

We consider the case of a bookmaker offering odds on a horse race. There are two horses, and the chance of each winning is 50%. Due to some popular sentiment on the part of the crowd, the current bets are not split evenly. Scenario 1 shows the outcome, should the bookmaker offer even money odds on the part of both horses.

Scenario 1: Bookmaker offers the probabilities

						Expected
	Bets	Odds	Payoff	Profit/Loss	Probability	P/L
A Wins	10,000	one to one	20,000	-5,000	0.5	-2,500
B Wins	5,000	one to one	10,000	5,000	0.5	2,500
	15,000					0

Under this scenario the bookmaker expects to neither win nor lose money, but may be down $2,500 if A wins.

Consider the next scenario, Scenario 2, where the bookmaker offers a different set of odds, with different implied probabilities.

Scenario 2: Bookmaker follows the money

						Expected
	Bets	Odds	Payoff	Profit/Loss	Probability	P/L
A Wins	10,000	one to one	15,000	0	0.5	0
B Wins	5,000	two to one	15,000	0	0.5	0
	15,000					0

In this case, the bookmaker is in the pleasant situation of not caring which horse wins the race; they are fully hedged in either case.

The applications to both pricing and hedging in finance are clear — if it is possible to completely hedge a claim, then the price of the claim must be equal to the initial cost of the hedge.

Consider a simple case where the hedge portfolio is set at the start of the period and remains unchanged. This is a simple forward contract. An example would be to receive an amount X and pay $1 times the value of the DOW index (currently at 10,000) in three months time. We assume the risk-free rate is 6% per annum.

The approach is to:

- Borrow $10,000
- Invest the proceeds in the DOW

The replicating strategy is then:

- $10,000 the cash bond
- $10,000 long the DOW

On expiry, we can sell our DOW and receive X. We need to repay the loan, now standing at $10,150. For this replicating portfolio to have same value as the forward, we need to receive exactly $10,150 — the cost of the claim.

Applications to Insurance

The examples given are relatively simple and give no hint as to whether a replicating strategy exists to allow pricing of insurance products products. The good news is:

- There is a financial theory which provides a methodology for this valuation process; and
- There is a market for liquid securities, which are 'similar' to interest rate sensitive life products in many ways.

The theory goes by the grand title 'Fundamental Theorem of Asset Pricing.' Stripped of detail, the relevant part is that where a claim can be replicated, it can be valued as the expected value using a set of 'risk-neutral' probabilities.

The market which shows similar risk patterns to interest rate-sensitive life products is the mortgage backed security market.

The Mortgage-Backed Security Market

The MBS market is similar to the life market in several ways:

- Both deal in long-term financial instruments
- Both deal in cash flows emanating from the same group of consumers — policyholders also hold mortgages
- These policyholders do not always behave in a way which is completely 'rational' in the economic sense of the word — their reaction to changes in interest rates etc. needs to be estimated

There are also some salient differences:

- The MBS market is one of the largest and liquid physical markets in the world
- Mortgage-backed security holders are not subject to risk from expense overruns etc. — these are borne by Fannie Mae, etc. who administer the securities
- The MBS market is not subject to event risk of wholesale surrender by mortgage holders — the event of adverse publicity, as may happen to an insurance company.

With these caveats, risk-neutral valuation has been successfully used in pricing MBS securities for many years now. The market has developed a mechanism for dealing with the approximations needed to cope with mortgage holder behavior (Option Adjusted Spread), and while not perfect, these valuation tools are proving their worth.

Interest Rate Models

The academic literature on interest rate models is enormous, as is the amount of money spent by investment banks and others to implement models. The ability to better price a security is key to the solvency of these institutions.

Unfortunately, no single interest rate model serves all needs for all investors, and it would not be uncommon for some investment banks to use different interest rate models to value different instruments. Interest rate models are usually judged on their ease of use and, most importantly, on their ability to accurately price the relevant financial instruments.

This creates a chicken and egg situation for life companies. The way to test an interest rate model is to test how well it matches observed prices of life insurance products. As there is no liquid secondary market for life products, we must rely on our interest rate models to value the models.

More Research is Needed

Risk-neutral valuation opens up opportunities for actuaries to determine a market price, and in some instances to hedge the interest rate and other financial risks in their portfolios. However, there is still a need for some additional research.

There is a need for additional research on interest rate models. It is fairly certain that simple interest rate models (so-called one factor models such as the extended Vasicek model) will not make the grade — the spread seems to influence policyholder behavior, so at least two factors are preferred. Statistical analysis suggests that three or four factors are required, but these models have proven quite cumbersome to derive and manipulate in the past.

Finally, even the best interest rate models should not be expected to deal with all sources of risk perfectly. There will be a need to adjust the values to adjust for these risk factors — something akin to the option adjusted spread in the mortgage market.

Stephen Britt, CFA, is a consultant at Tillinghast-Towers Perrin in Weatogue, CT. He can be reached at (860) 843-7071.