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A CASH-FLOW SCENARIO METHODOLOGY FOR C-1 RISK: PRELIMINARY REPORT

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ABSTRACT

This paper presents a way to study C-1 risk (default risk) for fixed-income assets. The context is asset/liability management for life insurance companies. In this approach interactive cash-flow projections of assets and liabilities are computed along a set of scenarios of default rates. This part of the methodology is similar to and compatible with cash-flow projections along sets of interest rate scenarios. To reflect the effects of portfolio diversification, this C-1 risk methodology applies Monte Carlo sampling to periodically test for default each of the separate assets involved in the projections. The methodology presented in this paper is distinct from any particular set of default rate scenarios that might be used with the methodology. This paper presents an illustrative product analysis based on some representative assumptions.

INTRODUCTION

In the context of life insurance company asset/liability management, C-1 risk is defined as "asset defaults and loss of market value of common stock and related reductions in investment income." This paper concentrates exclusively on a way to study C-1 risk for fixed-income assets, such as bonds and mortgages.

THE PROBLEM OF C-1 RISK

The following points generally outline the nature and scope of the problem of C-1 risk:

- Default rates in a given period vary because of the different characteristics of different assets. For instance, quality (investment-grade or high-yield), industry (oil and gas, airlines), and coupon and time-to-maturity can all influence the default experience. Bonds and mortgages may behave differently.

- Future default rates are difficult to predict. They almost surely will vary over time.
- The extent of diversification of an asset portfolio affects its exposure to C-1 risk. Holding a few big bonds is not the same as holding a lot of little bonds. It seems intuitively apparent, for example, that a $100,000,000 portfolio of ten $10,000,000 bonds is more exposed to larger default losses than is one made up of one hundred $1,000,000 bonds, everything else being equal.
- Bonds in default are seldom completely worthless. They have a "salvage value" because some payments by the debtor will probably be made eventually. These payments can be commuted by selling a bond when it defaults.
- Investment managers can plan to select bonds that are good buys relative to their credit ratings, and they can plan to sell off bonds quickly if the issuer's financial standing begins to deteriorate. However, it is difficult to "beat the odds" or outperform the market with any consistency. After all, only half of all portfolios can outperform the median. If one is to sell a bond that has deteriorated in credit standing, then someone else must agree to buy it. Since the life insurance industry owns more than one-third of outstanding bonds, *some* life companies will probably own bonds when they default. Widespread use of early warning tests like Zeta [2] may make the problem of unloading a deteriorating credit more difficult.
- C-1 risk cannot be completely mastered in isolation from C-3 risk. Economic conditions affect both default rates and interest rates. Default rates and salvage values depend in part on interest rate levels. Cash-flow matching is affected when assets go into default. Spreads between the yield on different assets, hence their market values, may depend on the levels of interest rates and default rates.

WHAT SHOULD A C-1 MODEL DO?

To begin the development of the C-1 risk methodology presented in this paper, the author made some general observations about the utility of such a methodology:

- A model of C-1 risk would be useful for valuation actuaries if it were consistent with scenario-testing cash-flow simulations methods already developed for C-3 risk. This approach also might make it more straightforward to incorporate C-1 risk into existing asset/liability management information systems.

- A good C-1 risk model should be able to handle existing portfolios of insurance product liabilities and accompanying assets. The model should reflect the effects on C-1 risk exposure of recurring premiums and open blocks of business, in particular, the timing of default events over the projection period.
- The output of the model should help to answer questions about risk charges (pricing) and reserve and surplus requirements (valuation).
- The model should accept different assumptions about yield spreads, asset allocations by quality grade, and diversification rules and then be able to calculate the distribution of profit or loss across a universe of default rate scenarios. This would permit effective quantification of the risk/return position created by different asset portfolio strategies. Such a model would be of value to actuaries and investment officers and to senior management as a management information tool.

SPECIFICATIONS FOR THE C-1 RISK METHODOLOGY

The methodology presented in this paper:

- Uses scenarios of nonconstant future default rates. These can be varied by asset type (bonds, mortgages, etc.), by quality grade (AAA, B, etc.), and by industry (oil and gas, airlines, etc.).
- Uses Monte Carlo sampling to quantify the statistical effects on profit variance of different approaches to diversifying the portfolio. This applies to the starting asset portfolio *and* to the reinvestment strategy.
- Sells a bond when it defaults, for a salvage value that is a user-specified percentage of par, adjusted for C-3 risk.
- Projects asset and liability cash flows for a user-specified data base of existing assets and in-force policies. Permits inclusion of new business. Can combine different products.
- Summarizes key financial output such as annual cash flows, book profits, and accumulated surplus, reflecting impact of C-1 experience.
- Uses a flat constant yield curve to have a "pure" C-1 risk model.
- Uses computer programming developed from that of a C-3 scenario-testing model, to allow combination of C-1 and C-3 risks into an "asset adequacy model." This model uses multirisk scenarios, specifying interest rates and default rates together.

The author stresses that other approaches to modeling C-1 risk are certainly possible. Some are discussed in the references cited in the Bibliography. Additional research may lead to new techniques in the future.

MONTE CARLO SAMPLING

Monte Carlo statistical sampling techniques are an "empirical" way to derive the probability distribution of a complicated financial variable. These methods are covered in Part 3 of the Society of Actuaries Examination Syllabus [6]. Rick Sega applied them to study C-1 risk in his recent *Transactions* article about C-1 [8]. I use them somewhat differently, to follow Jim Tilley's approach to C-3 of projecting profit along a "worst-case" scenario, then measuring the surplus needed at the beginning to assure solvency at the end [9].

Professors Elton and Gruber of the New York University Business School, in a report to the Life Insurance Council of New York (LICONY), which they co-authored with Professors Altman and Sametz [1], provide tables of one-period default distributions for portfolios with different numbers of bonds. These tables were derived using closed-form combinatorial formulas. Since the present author wanted a C-1 model that was able to project wealth to the end of a multiyear period, for bonds of heterogeneous sizes, with varying year-by-year default rates, and with cash inflows and outflows, I decided to use Monte Carlo methods in my model.

The problem is that any one individual bond either defaults or doesn't default, in entirety, during a given period. If we assume "an overall default rate of 2 percent," the individual bonds all exposed to that 2 percent must be *separately* tested for actual default, one by one.

Were we to apply an aggregate 2 percent decrement to the investment yield or to the par on *all* the bonds, the situation is oversimplified. So a model of C-1 risk needs a way to reflect the effects on risk exposure of portfolio diversification. There will also be period-by-period statistical noise in the actual experience, according to the number of bonds in the portfolio, just like for a portfolio of life insurance risks. In fact, because the entire market of bonds has just a few thousand issues and default rates in the aggregate are usually rather low, the entire market's annual default rate can be thought of as being subject to sampling variance. (The author emphasizes that other approaches to modeling diversification are possible.) Monte Carlo testing within a cash-flow projection model takes all this into account.

In fact, we need to take a number of Monte Carlo samples through each default rate scenario to get a good feel for the gain and loss distribution. Monte Carlo sampling assumes independence of the trials (defaults of each bond). However, in some environments we might expect a correlation between defaults of separate bonds (contagion)—for instance, because of a depression in one industry. This is meant to be taken account of by the default scenarios, not by the Monte Carlo sampling within a scenario. If we wanted to examine the effects of a depression, we could use high default rates in a scenario. The Monte Carlo sampling then quantifies some of the inherent uncertainty of actual future experience. In general, I think a probabilistic approach to the future is useful for actuaries, to complement the credit analyst's concern with the "industry fundamentals," which are shorter term in nature. Finally I would note that some industries (such as airlines, entertainment, food, etc.) may have a limited number of major junk bond issuers, in which case a "2 percent average default rate" (or whatever one assumes in a scenario) becomes technically meaningful in the context of Monte Carlo sampling.

SOME POINTS OF ELABORATION

This section covers a few miscellaneous areas in which the basic approach presented above can be applied with more flexibility or detail:

- If we want to examine reserving for 90 percent likelihood of solvency, and holding additional surplus for, say 99 percent likelihood of solvency, then we really need to look hard at the end of the loss tail for investment risk. Scenario/Monte Carlo testing is a way to get at this information for C-1 risk.
- Some actuaries may want to study situations in which bonds are sold before they enter default. This could be done by specifying a rule about when such a sale is triggered, for instance, when the bond is trading at, say, 80 percent of what otherwise similar bonds (of the same public rating and in good standing) are selling for. We can make up scenarios showing rates of credit deterioration assumed to produce this level of price depreciation and combine this with a salvage value assumption of 80 percent.
- The C-1 model's investment diversification rule can include two components: how much of the portfolio goes into one rating class, and how much money is allowed to be put into any individual bond in that rating

class. Note that this can increase the model's computer running time because the diversification rule can lead to the creation of a large number of different bonds. Also, note that under some diversification rules the model could try to buy more different bonds, if lots of assets accumulate, than might actually exist in a real-world market. This kind of "market saturation" can be handled by switching to other classes of assets (there are more investment-grade bonds than junk bonds now outstanding) or by increasing the holdings of bonds the model already owns (that is, adding to their par). This can be done under computer control, including a projection of the future size of the junk bond (and other) markets used to trigger the switching of asset purchase rules. Alternatively, the output can be scanned for unreasonable outcomes and the assumptions adjusted as needed.

- Some investment-grade bonds *do* default now and then. They may undergo a gradual downgrading of their credit rating before the default, but this doesn't always happen first. Such downgraded bonds are called "fallen angels." Bond quality ratings sometimes are upgraded. A C-1 model could take account of these quality class "transfers" through the Monte Carlo sampling process. Even with a small default rate, some defaults will sometimes occur because of the sampling process. These defaults are fallen angels. We could consider upgrades to be encompassed within those lower-quality bonds that, during the Monte Carlo sample through the default rate scenario, don't default.

MONTE CARLO EFFECTS IN AN ASSET ACCUMULATION EXAMPLE

The impact of the Monte Carlo sampling process as a way to examine portfolio diversification can be illustrated with a simple asset-side-only example. Such a case illustration was presented in my panel discussion writeup "Quantifying the C-1 Risk" [3]. In this section I briefly recap that work. I projected a starting portfolio of $100,000,000 for 20 years. I assumed two default rate scenarios, 2 percent per year constant and 4 percent per year constant. I tested three diversification rules, namely, $1 million per bond (start with 100 bonds), $5 million per bond (start with 20 bonds), and $10 million per bond (start with 10 bonds). The diversification rules also applied to the reinvestment of cash flows during the 20 years. One observation I made is that, because of the statistical uncertainty of actual default experience over 20 years for portfolios with a limited number of bonds, a few times out of 100 the ending wealth under the 2 percent scenario was very near the mean of the 4 percent scenario, and vice versa. Since valuation actuaries are concerned about events with low probabilities of occurrence, these interesting results cannot be ignored. C-1 risk is not necessarily quantified by "deterministic" default rate assumptions.

I would like to make an observation about the C-1 risk scenario/Monte Carlo model's conclusion that, for an asset-side-only accumulation, diversification reduces the loss tail more than the gain tail [3]. Richard Bookstaber and David Jacob [5] studied total return over five years for different junk bond portfolios, using actual historical data. They concluded "the drop in variation of return comes more from a truncation in the risk of substantial underperformance than it does from a diminished opportunity for extraordinary gains" as diversification increases. This is a useful independent confirmation of this C-1 model. Furthermore, it suggests that strategy decisions derived from using the C-1 model ex ante might be borne out by actual results ex post.

SPDA CASE EXAMPLE

The next few sections of this paper present the assumptions, default scenarios, and results for an illustrative application of this C-1 risk scenario projection methodology. The sole purpose of this example is to show:

1. What sort of assumptions are needed by the methodology.
2. How default rate scenarios can be defined for use by the model.
3. What the output is like, and one way it may be summarized.

Additional research is needed, especially on good ways to generate sets of default rate scenarios. One approach may be to use a model that produces overall economic scenarios, from which specific parameters such as default rates and interest rates follow. Another may be to draw on the research by Irwin Vanderhoof into modeling default rates with a beta distribution [10].

Note also that these illustrative calculations are from a "pure" C-1 risk version of a more general model (that is, a constant yield curve is used throughout and neither disintermediation nor reinvestment risks arise). This general model combines C-1 risk and C-3 risk into an integrated set of cash-flow projections by using joint default rate/interest rate scenarios. This approach is discussed in "Asset/Liability Matching" [4]. This general methodology in part extends work by Geyer and Mateja [7].

Product Specifications and Assumptions

- Single-Premium Deferred Annuity
- Projecting a single block of new issues
- 5,000 policies, average size $20,000
- Total premium $100,000,000
- Commissions 4 percent first year
- Expenses ignored
- FIT rate 34 percent, no surplus tax
- Reserve equals full fund value
- No market value adjustment
- No bailout
- Surrender charge 5 percent of fund value first five years
- 10 percent per year free partial withdrawals
- 20-year projection, quarterly cash flows
- Policy credited interest rate, reset quarterly: Portfolio earnings minus 1.5 percent but never below competition minus 1 percent, more greater than competition rate. Minimum guaranteed policy rate is 4 percent.
- Competition credited interest rate: 10-year Treasury rate minus 1 percent, minimum 4 percent.
- Lapse rates: 5 percent annual base lapses, plus interest-sensitive lapses, which are 2 times square of (competition rate minus policy rate, if positive).

Investment Assumptions

- For a C-1 risk illustration the model was run with a flat 8 percent Treasury yield curve.
- All assets purchased are 10-year par bonds, callable at par after five years.
- All funds are invested 80 percent in A-rated "investment-grade" bonds and 20 percent in B-rated "junk" bonds.
- The B-rated bonds have yields 2½ percent above A-rated bonds.
- Diversification rules:
  - At most $10,000,000 face amount in any one A-rated bond.
  - At most $2,000,000 face amount in any one B-rated bond.
- Salvage value rule: Defaulted bonds are sold for 40 percent of *lesser* of par value, and market value of a comparable bond in good standing. Salvage proceeds are reinvested with other cash flows.
- Negative cash flows: Negative cash flows are treated as negative investment. For "junk bond" (B-rated bond) negative investments a 2 percent bid/asked spread is used to simulate illiquidity. 0 percent defaults on the "negative assets."

- Investment expense:
  - 10 basis points (0.1 percent) per year A bonds.
  - 20 basis points (0.2 percent) per year B bonds.
- Bond calls: Bonds are called when new money rate declines enough to make prepayment economically beneficial. Refinancing costs were 1 percent for A-bonds and 1½ percent for B-bonds. A sluggishness threshold of 2 percent was assumed.
- Interest rate scenarios: Treasury bond rates are a flat 8 percent. Spreads off Treasuries for investment-grade (A-bonds) are 20 basis points at 90 days grading to 70 at 5 years, 110 at 10 years, and 135 basis points at 20 years. B-bond rates equal A-bond rates plus 250 basis points.
- Default rate scenarios: Appendix A gives the "junk-bond" (B-bond) default rates. The A-bond "investment-grade" default rates were assumed to be one-tenth of the B-bond default rates, for illustration purposes only.
- Monte Carlo Samples: 40 Monte Carlo samples of the bond-by-bond default testing each period were run for each of the seven illustrative scenarios examined in this study.

Note: The combined effect of the above assumptions is to remove the effects of C-3 risk from the calculations. The assumption details are given because they are all needed by the software used in these calculations. See [4] for a more general application of the software.

CONCLUSIONS

Output from this sample run is summarized in Appendix B. These figures show book value surplus after 20 years. Since this is a preliminary report, I only offer these broad conclusions:

1. For the assumptions used here, the median ending-surplus results can vary by 10 percent or more from one scenario to another.
2. Since scenarios 4 and 7 are identical, the different results illustrate some sampling error between two sets of 40 Monte Carlo runs. This is most evident at the minimum.
3. Variation of results because of the actual default experience per Monte Carlo trial in a scenario is sometimes equal to or greater than variation of the medians between scenarios. This is in part because the asset portfolio used in this example is small (it starts with 8 investment-grade bonds and 10 high-yield bonds). Thus the level of diversification takes on significance along with the level of default rates.
4. Different assumption sets or scenario sets would produce different results.

This methodology is especially useful when it is applied to compare the risk/return effects of alternate strategies. That is beyond the scope of this paper, but the reader is referred to the *Record* [4] for a case study comparing two different asset quality allocation strategies.

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BIBLIOGRAPHY

1. ALTMAN, E., ET AL. *The Suitability of Low-Rated/High Yield Bonds for the Investment Asset Portfolio of Life Insurance Companies*. Report submitted to LICONY, November 1986.
2. ALTMAN, E., AND NAMMACHER, S. *Investing in Junk Bonds*. New York: John Wiley & Sons, 1987.
3. BUFF, J. "Quantifying the C-1 Risk," *Record* 13, No. 3 (1987): 1591–1622.
4. BUFF, J., "Asset/Liability Management," *Record* 13, No. 4B (1987): 2179–2207.
5. BOOKSTABER, R., AND JACOB, D., *Risk Management for High Yield Bond Portfolios.* New York: Morgan Stanley, January 1986.
6. HERZOG, T. "An Introduction to Stochastic Simulation," *Society of Actuaries Study Note 30–31–84.* Itasca, Ill.: 1984.
7. MAJETA, M. "Research of the Committee on Valuation and Related Areas," *Record* 13, No. 4B (1987): 2403–15.
8. SEGA, R. "A Practical C-1" *TSA* XXXVIII (1986): 243–79.
9. TILLEY, J., "Discussion Note—Preliminary C-3 Risk Calculation," *Record* 7, No. 4 (1981): 1368–77.
10. VANDERHOOF, I., "Quantifying the C-1 Risk," *Record* 13, No. 3 (1987): 1592–1600.

APPENDIX A

ILLUSTRATIVE SPDA CASE STUDY

SEVEN DEFAULT RATE SCENARIOS
JUNK BOND DEFAULT RATES
(ANNUALIZED RATES IN PERCENT)

Year	Scenario Number						
	1	2	3	4	5	6	7
1	2	2	2	2	2	2	2
2	2	2	1.5	3	2	4	3
3	2	2	1.5	4	2	6	4
4	2	1.5	1	3	2.5	8	3
5	2	1.5	0.5	2	2.5	10	2
6	2	1.5	0.5	1	2.5	8	1
7	2	1	1	2	3	6	2
8	2	1	1.5	3	3	4	3
9	2	1	1.5	4	3	2	4
10	2	1	2	3	3	2	3
11	2	1	2	2	3	2	2
12	2	1	2	1	3	2	1
13	2	1	2	2	3	2	2
14	2	1	2	3	3	2	3
15	2	1	2	4	3	2	4
16	2	1	2	3	3	2	3
17	2	1	2	2	3	2	2
18	2	1	2	1	3	2	1
19	2	1	2	2	3	2	2
20	2	1	2	3	3	2	3

APPENDIX B

ILLUSTRATIVE SPDA CASE STUDY

SAMPLE OUTPUT
20-YEAR BOOK SURPLUS
($ MILLIONS)

Scenario	Distribution of 40 Monte Carlo Samples per Scenario				
	Minimum	20th Percentile	Median	80th Percentile	Maximum
1	84	102	109	115	126
2	98	110	115	120	127
3	85	107	112	118	122
4	80	91	103	111	119
5	81	94	103	111	126
6	69	84	94	104	111
7	62	97	104	115	122