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III. C-3 TASK FORCE REPORT

MODELING THE C-3 RISK

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I. MODELING MECHANICS

The purpose of this paper is to set forth a detailed description of how the actuary can analyze the C-3 risk. The paper discusses the assumptions that the actuary needs to make, as well as how the actuary might go about setting those assumptions. In addition, the paper establishes an analytical framework for the actuary to use in tackling the C-3 question. Finally, the paper discusses how the actuary might build a model, both in the sense of the formulas needed for a projection system and in the sense of how the actuary should group assets and liabilities to simplify calculations.

The first section of the paper addresses the issues that the actuary needs to resolve and discusses how assumptions might be chosen for traditional life, universal life, and deferred annuities. The second section presents four case studies, including detailed discussion of how the assumptions were selected. The third section presents detailed calculations and formulas for a simple example, so that the actuary can see how a model for analyzing the C-3 risk should fit together.

The work required to perform C-3 risk analyses is essentially the same as traditional actuarial work. On the liability side, the actuary will need to relate crediting rates, lapse rates, premium suspension rates, and policy loan utilization rates to the interest scenario being tested, but the remaining assumptions will be unchanged from traditional analyses. However, the actuary will have to spend a good deal more time analyzing assets than actuaries have traditionally spent on assets. In particular, the actuary will have to project the impact of calls, mortgage prepayments, and cash shortfalls on investment income and capital gains.

A. Liability Modeling

Building a liability model to help evaluate the C-3 risk is not much different from the traditional modeling that actuaries have been doing for years. Several of the assumptions will be tied to interest rate movements, but the majority of the assumptions will be independent of interest rates and will be no different from the assumptions used in any pricing model. However, it is important that the model be flexible enough to accommodate those assumptions that will vary with interest. The model also should be flexible enough to accept reserves and cash values that vary from one interest scenario to the next. Because the reserves and cash values will generally vary from one scenario to the next, it is typically easiest to generate them right in the projection system, so that it is not necessary to rerun separate values each time a new scenario is desired.

Once the interest-sensitive model is in place, it is necessary to develop in-force data. For the liabilities, the actuary will need the beginning reserve, account value, cash value, and face amount for each cell. The actuary also will want some data on the recent premium-paying history of each cell, as well as the current average crediting rate and the period for which that credited rate is guaranteed. The criteria for selecting model cells may be somewhat different from the criteria for more traditional models. The surrender charge and credited rate will be the most important criteria for selecting model cells, followed by age or mortality characteristics. Depending on the modeling method used, it is quite likely that separate cells will be necessary for the last several issue years. Depending on the actuary's judgment and the distribution of expected mortality gains across ages, it is possible that no distinction will be made between different issue ages, because the model will already be extremely complex just from the consideration of groupings by plan, current surrender charge level, and credited rate (often groupings in accordance with the last two can be accomplished by grouping by issue year).

Most companies already have systems in place that will provide the necessary liability in-force data. If the systems to gather the data are not in place, it would be advisable to design the necessary systems even if there were no valuation actuary requirements. The liability data that are needed to perform the valuation actuary analyses also are needed to make realistic corporate planning projections and to perform the types of analyses that can be used to compare different asset and liability strategies.

B. Establishing Liability Assumptions

Commissions, expenses, death benefits, mortality, and initial premiums are all assumptions that need to be made both for traditional models and for C-3 risk models. Occasionally, expense inflation will be tied to the interest scenario, but since inflation is a minor factor in this type of analysis, this is typically an unnecessary refinement. Death benefits for universal life may change as account values grow, but they are essentially independent of the interest scenario. Theoretically, antiselection should cause mortality experience to deteriorate under scenarios in which lapses are substantially higher than normal, but this is a fairly complex refinement that many companies choose to ignore. At some point in the future, it will probably be common practice to reflect deteriorating mortality under high lapse scenarios, but for now most companies are giving this refinement a far lower priority than developing the basic analytical structure.

Product characteristics, such as mortality charges, expense loads, and surrender charges will be needed to develop cash values, reserves, and possibly death benefits under the different scenarios. The most obvious liability assumption that will vary with the interest scenario is the credited rate. The first step in projecting credited rates is to determine what the crediting strategy will be, that is, how will management change the credited rate in response to changes in the interest rate environment. Since many managements have not developed a clearly defined crediting philosophy, developing this assumption may require substantial discussion with and education of management. There are an infinite variety of possible crediting strategies, but most crediting strategies can be placed into several broad classes of strategies (many strategies are hybrids of two or more classes of strategies). Some of the broad classes of crediting strategies are:

1. Credit a fixed rate that does not vary with market conditions.
2. Credit a competitive market rate at all times.
3. Lag the market.
4. Lag the market in one direction only.
5. Credit the market rate plus (or minus) a predefined number of basis points.
6. Follow one of the above strategies, but do not pierce the bailout until the surrender charge disappears.
7. Credit a fixed rate during the period that the surrender charge is in effect and then credit the market rate.

One of the questions that is immediately apparent upon looking at this list of strategies is: What is the market rate? Ideally, the market rate will be the rate a policyholder can get by lapsing the policy and buying a comparable new policy from a competitor. Since market rates are largely a factor of what insurance companies can earn, typically the market rate would be equal to the rate on new bonds less some spread. For example, the market rate might be equal to the 7-year corporate bond rate less 150 basis points, or just the 7-year Treasury rate, or it might be set equal to the greatest rate from among 1- to 15-year bonds, less 175 basis points. Since many companies credit interest based on their portfolio rate, the market credited rate might lag the market rates on investments. Perhaps the most realistic assumption would be that portfolio rate companies drive the market as rates fall and that new money rate companies drive the market as rates rise; thus the market rate would be based on the greater of current and recent rates. For example, the market rate might be set at 200 basis points less than the greater of the current 10-year corporate rate and 3-year average of 10-year corporate rates.

Once formulas for determining the market rate and credited rate have been determined, it is necessary to develop lapse rate assumptions. The lapse assumption will be largely based in intuition and the actuary's judgment, for two reasons. First, there is little experience about how lapses respond to interest rate movements. Second, it is uncertain how valuable experience will be as a predictor of future lapse rates given the recent rapid changes in the insurance market. Any experience prior to 1980 is probably invalid, because it reflects an entirely different environment.

Both intuition and the limited amount of experience that we do have indicate that lapses are dependent on the difference between a policyholder's actual credited rate and what the policyholder can get in the market on a similar policy. As the differential between market and actual credited rates widens, the policyholder's incentive to lapse increases dramatically. Many companies experienced annual lapse rates in excess of 30 percent or even 40 percent during the interest rate spike of the early 1980s. Several companies experienced annual lapse rates in excess of 60 percent.

The other major factor in determining lapse rates is the policy's surrender charge. The surrender charge provides a disincentive to lapse, particularly if it will disappear within the next several policy years. There are number of intangible factors that also affect lapses. Some of these factors are the sophistication of policyholders and agents, the affiliation and loyalty of the agents, and policy characteristics. These factors are probably best dealt with by considering them when choosing the parameters for the lapse formula.

While there is no right or wrong formula, the lapse formula should conform generally to the limited experience that we have, as well as appealing the intuitive sense of how lapses will behave. That is to say that if the credited rate remains constant and the market rate rises, lapses should increase. Similarly, if the surrender charge is decreased, lapses should increase. There also may be one-time jumps in the lapse rates when certain events occur. For example, the lapse rate might be increased to reflect "shock" lapses the first time that the credited rate is lowered or when the bailout is pierced. Note that triggering the bailout will cause the surrender charge to be zero, so that if the surrender charge is a component of the lapse formula, lapses will automatically increase. A decreasing surrender charge will probably provide more of an incentive not to lapse than a flat or increasing surrender charge.

As an example of the type of formula that could be used to generate lapse rates, consider: $15\% + 2 \times Z \times (MR-CR)^2 - 3 \times SC$, with a minimum of 3 percent. SC equals the surrender charge as a percentage of the account value. MR equals the market rate as a percentage. CR equals the credited rate as a percentage. Z equals -1 if $MR-CR$ is negative and 1 if $MR-CR$ is positive. This formula should not be viewed as "the" lapse formula, rather it is important because of the principles it embodies. The impact of the formula is illustrated in Figure 1. In this formula, lapses increase rapidly if the market rate increases relative to the credited rate. On the other hand, lapses are substantially reduced by the surrender charge. When the formula has a large surrender charge component, lapses will tend to be low until the surrender charge disappears, unless the market rate moves substantially. After the surrender charge disappears, the lapses will be much higher even if $MR-CR$ equals zero. The lapse formula will not necessarily involve $(MR-CR)^2$; the formula could just as easily be linear or have some other relationship with respect to $MR-CR$. Figure 2 illustrates the formula: $10\% + 7 \times (MR-CR) - 2 \times SC$, with a minimum of 3 percent. Even the form of the lapse rate function could be different from the formulas shown so far. For example, the surrender charge could be incorporated into the $MR-CR$ term: $5\% + 2 \times Y \times (MR-CR - 0.5 \times SC)^2$; where Y equals 1 if $MR-CR$ is positive and 0 otherwise. Figure 3 illustrates this formula.

For nontraditional products, the policyholder's premium-paying pattern is not fixed in advance. The rate at which policyholders stop paying premiums on policies that remain in force is called the premium suspension rate. The premium suspension rate should vary in much the same manner as the lapse rate, although the parameters will be different. The surrender charge will

FIGURE 1

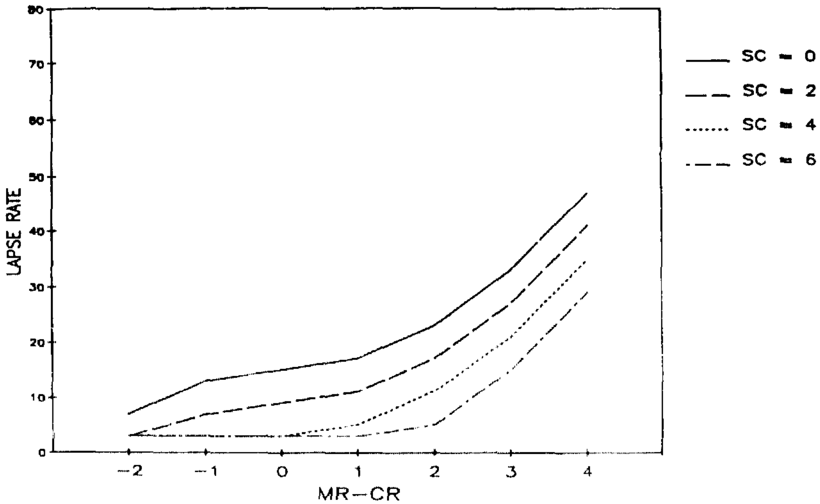


FIGURE 2

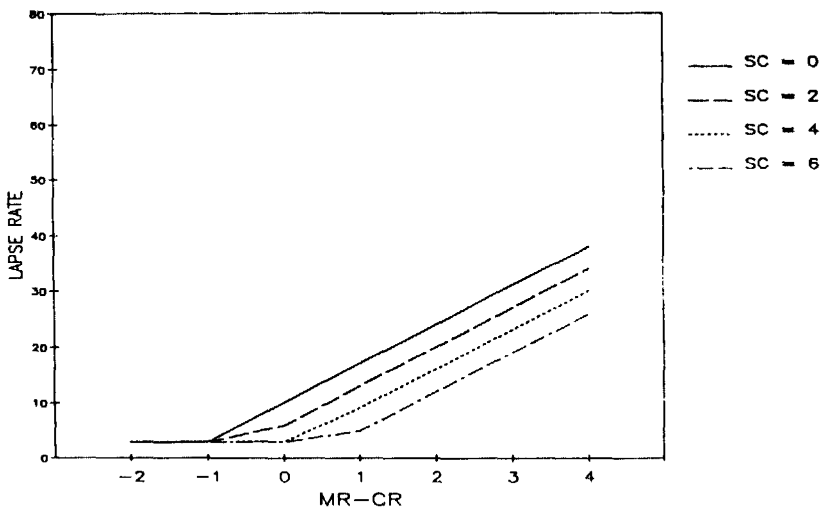
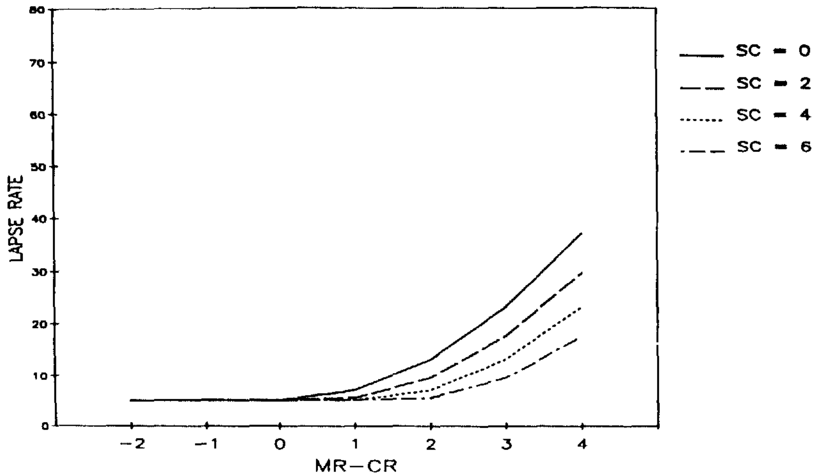


FIGURE 3



probably increase premium suspension rather than decreasing it, because unhappy policyholders may be more likely to stop paying premiums than to lapse outright if they incur a surrender charge by lapsing.

Policy loan utilization also should be an interest-sensitive variable, much like premium suspension. Policy loan utilization also will reflect how the policy is designed and marketed. Policy loan utilization may be the most critical assumption for traditional products with fixed-rate loans. An alternative to having separate lapse, premium suspension, and loan utilization formulas would be to have a generalized withdrawal function with no surrender charge component. Withdrawals would then be allocated among lapses, premium suspension, and policy loan utilization using a formula that incorporates the surrender charge.

In setting lapse, premium suspension, and policy loan utilization rates, the actuary often will have to work more from an intuitive sense than from hard data. In general, the formulas should result in increasing rates as the policyholder's economic incentive to lapse/suspend/loan increases. The following discussion focuses on lapses, but the same concerns apply to all three assumptions. In setting these assumptions, the actuary should keep in mind the characteristics of different blocks. Business sold through captive agents will tend to have lower lapses than brokered business. Similarly, business sold in less sophisticated markets will tend to be less sensitive to interest rate changes. Salary deduction plans will tend to have somewhat lower lapses and substantially lower premium suspension than other plans, because the policyholder does not have to do anything to continue paying premiums. There are a multitude of other factors that affect lapses, suspensions, and loan utilization.

If the actuary can get useful data on lapses, the ideal way to perform a lapse study would be as follows:

1. Group the experience by policy year, plan, issue age, policy size, crediting rate, calendar month, premium-paying pattern, and any special considerations (for example, data from a period where surrender charges were waived because of a bailout provision should be treated separately).
2. If this grouping provides experience pools that are too small, the actuary should combine experience across the less important parameters.
3. Average investment market rates of insurance should be entered by calendar month and compared to the contract crediting rates.
4. Regressions of lapse rates against various parameters should be performed. For example, for an SPDA, the regression would probably involve lapses, the differences between contract and market interest rates, and surrender charges. For universal life, the regression would probably also involve policy year and quite possibly issue age and average size. The actuary should be creative in performing the regressions. For example, the regression might involve the square of the difference between market and contract rates instead of just the difference. Similarly, the actuary could use a "dummy" variable where the data seems to have a discontinuity, such as that between first-year and renewal lapses.
5. The formula that the actuary develops should be tested for reasonableness. The three key tests are:
   a. Statistical measures of the formula's "fit" to the data.
   b. Does the formula fit the actuary's intuitive sense of the underlying relationship of lapses to the parameters?
   c. How closely do the rates produced by the formula track actual rates? This test is important where the results of different regressions were used to develop the parameters for different factors.

Cash values and reserves on traditional products will not vary by scenario. However, dividends, policy loan utilization, and lapses of traditional products will vary by scenario. The actuary must develop a dividend strategy that indicates what the dividend will be at any point. This formula will probably be similar to the crediting formula in that it will usually be based on an underlying dividend amount plus an additional amount related to current market conditions and/or the company's earned rate. The lapse rate should be linked to the dividend scale somehow, although the relationship will probably be harder to derive than the relationship of lapses to the market and actual crediting rates for interest-sensitive products. Most other assumptions for traditional products will be just like the assumptions actuaries have always made for traditional analyses.

C. Asset Modeling

Most asset portfolios contain a much smaller number of holdings than the typical liability portfolio. Therefore, it may not be necessary to group the assets into model cells. If the actuary decides to group the assets, the first criterion will be maturity date. The second criterion will be coupon. The third criterion will be call characteristics. As an example of an asset model, bonds could be grouped by maturity year in 2 percent coupon intervals. Noncallable bonds would be kept separate from callable bonds, and callable bonds would be grouped by call date in one-year intervals, with bonds with more than 5½ years of call protection maintained as a single cell.

The following items are needed as of the valuation date for each security or group of securities to model existing assets:

1. Book value
2. Market value
3. Par value
4. Book yield
5. Coupon
6. Call date
7. Call price
8. Maturity date
9. Special features of the security (for example, sinking fund).

Most of these items are readily available from the data used to generate Schedule D for the annual statement. However, many companies do not have accurate call data, because all that is required for Schedule D is the call date and price for those bonds that are being amortized to call. Getting the necessary call data may require a special project, because although the investment department usually has the data, the data are not always in an easily usable form.

D. Establishing Asset Assumptions

To project asset cash flows, it is necessary to project investment income, maturities, payment of mortgage principal, calls, prepayment of mortgage principal, and the market value of any liquidations. These items will need to be projected both for the existing assets and for assets that are assumed to be purchased during the projection period. Projecting investment income simply involves keeping track of each security's outstanding par value and coupon and multiplying the two. Similarly, projecting maturities requires that each security's maturity date and outstanding par value be known. Projecting mortgage principal payments requires that the amortization schedule be calculated.

Projecting calls and mortgage prepayments is somewhat more difficult because they will vary with interest. It will be necessary to develop assumptions of when bonds will be called and how much of a given block of mortgages will prepay under different interest environments. Bond calls are somewhat simpler than mortgage prepayments because the decision to call a bond is typically all or nothing, while mortgage blocks tend to have prepayments of part of the block.

There will be no calls prior to the bond's call date. Thereafter, calls will be a factor of call price, years to maturity, and prevailing interest rates (in projecting the value of calls, it is important to remember that call prices generally start above par and decline linearly to par at maturity). A simple approach would be to say that a bond will be called when rates on comparable new bonds are X percent below the coupon rate on the bond, where X percent might, for example, be 1.5 percent or 2 percent. More complex approaches might have only a portion of bonds being called when the difference in rates is less than, say, 4 percent, but 100 percent being called when the difference exceeds 4 percent. It is also possible to adjust X to reflect the bond's current call price and current years to maturity. The extra sophistication of these more complex approaches is probably not worth the effort for companies that are just getting started in valuation actuary analyses. There will be a book gain or loss at the point of call, depending on the book value and call price of the bond.

Mortgage prepayment rates behave much like lapse rates, except that the relationship to market rates is reversed. For mortgages, what we mean by market rate is the rate at which a mortgagor could borrow today. The lower the mortgage market rate falls, the higher the prepayment rate will be. However, because mortgagors typically are affected by non-economic factors more than bond issuers are, the prepayment rate will be less of an all-or-nothing affair. As with bond calls, there will be a book gain or loss at the point of a mortgage prepayment.

As an example of a prepayment formula, consider: $P = 5\% + 7 \times (C-M)$, with a minimum of 2 percent and a maximum of 50 percent. P equals the annual prepayment rate; M equals the mortgage market rate; and C equals the coupon rate for the mortgage. This type of formula reflects the incentive to prepay the mortgage as rates fall and also reflects the non-economic factors that keep mortgagors from being completely responsive to economic conditions. The sample formula is designed to demonstrate a concept, so the focus should not be on the numbers, but on the approach. It is quite reasonable to use other formulas, perhaps including a $(C-M)^2$ term. Most investment managers will be willing to make a guess at what the prepayment formula and call formula should be if the actuary shows them the type of framework that the formula should fit.

Many people avoid the question of the market value of liquidations by assuming that negative cash flows will be met by borrowing. However, even if this assumption is made, it is still necessary to calculate the market value of assets at the end of the projection period. Whether the market value of the portfolio is being calculated to determine the residual value of assets and liabilities or to calculate the market value of liquidations, the same considerations apply. There are formulas in any interest textbook that tell how to calculate the market value of a bond or mortgage given the market interest rate. However, those formulas assume that the security's cash flows are fixed and do not assign any value to the issuer's option to call the security. Ideally, the market value of the bond or mortgage should be reduced by the value of the issuer's option. In practice, the value of the option is often approximated by assuming that the market value of bonds is the lesser of the market value calculated assuming that the bond will be held to maturity and the market value calculated assuming that bond will be called. This in effect places a cap on the value of the bond and tends to moderately overstate the market value of the bond. Similarly, for mortgages, it is common to value the mortgage assuming that the prepayment rate will always reflect the then-current market rate. However the market value is calculated, you may want to reduce it by a small amount (for example, 0.25 percent) to reflect transaction costs.

If the analysis is performed strictly for valuation actuary purposes, then the book value and book yield are irrelevant. However, if profitability is being measured or if it is desired to compare the book value of assets to reserves in the future, then the model must be able to amortize the difference between book and par. Frequently, par and book are close together, in which case the actuary may choose to set the two equal to each other and avoid this issue.

Although defaults are accorded a separate classification (C-1 risk), the valuation actuary may wish to incorporate defaults in the C-3 risk model. Defaults could be incorporated into the model as a random variable. Alternatively, a fixed deduction from yields could be made to reflect expected defaults with a reserve determined externally to cover the value of possible random fluctuations in defaults. The major disadvantage of the first approach is that it would require many more projections of the entire model to gain the same degree of C-3 information. The second approach would allow a sophisticated analysis of the default risk to be made without requiring redundant C-3 analyses. However, under the second approach, it would be difficult to reflect any potential interaction of the C-1 and C-3 risks.

E. Other Assets

There are a wide variety of other assets that insurance companies purchase besides bonds and mortgages. The performance of some of these securities (for example, futures contracts) are easily related to Treasury yields. Other securities (for example, CMOs) are extremely complex, with each individual security having unique characteristics. The best way to model these securities is to sit down and talk to the investment department about how the securities will perform in different interest scenarios. If fixed scenarios are used, the investment department may be able to provide a schedule of projected cash flows on the unique assets for each scenario. Typically, the market value of these securities is more difficult to project than the cash flows, so it often makes sense to assume that the securities will be held to maturity.

Projecting the cash flows and market values of equity-type securities (for example, common stocks and real estate) presents a thorny issue since returns on these securities are, at best, loosely related to changes in the interest environment. Given the high degree of risk and unpredictability associated with the returns on equity securities, prudence and conservatism seem to require that an extremely low return be assumed on these securities. In fact, it could be argued that conservatism dictates an assumption of negative returns, except where the investment department is willing to provide strong opinions as to the safety of principal on a given security. Where the securities in question do not have a readily determinable market value (for example, stock in a closely held affiliated company), extra care and caution are dictated.

F. Reinvestment Considerations

After an asset model has been built, it will be necessary to make assumptions about what the new investments will look like. Each year's new investments can be appended onto the model, but determining what the new investments will look like requires that the investment department make assumptions of what the characteristics of the universe of potential investments will be and what the investment strategy will be. It is necessary to specify the maturity dates of potential new securities, the call dates (unless the securities are to be noncallable), the call prices, and the yields. Typically, the interest scenarios to be tested do not specify the interest rate for every security, but rather specify some smaller group of rates, such as the Treasury yield curve. Thus, rules must be developed to relate yields on other assets to yields on the base yield curve. The most common way to link the yields is through additive or multiplicative spreads. That is, the yield on each bond equals the yield for a comparable maturity on the base curve plus or times X, where X will be different for each security. The investment strategy can be either dynamic or fixed. In dynamic strategies, investment and liquidation percentages change in response to changes in the environment. In fixed strategies, the investment and liquidation percentages for a given year are not affected by the interest environment. Duration-matching strategies for fixed liabilities or pseudo-duration-matching strategies for interest-sensitive products are examples of dynamic strategies.

G. Defining Interest Scenarios

One of the most difficult issues facing the valuation actuary is what scenarios to test. If the scenarios are not dictated by the applicable valuation actuary regulation (or by senior management if it is pricing or internal valuation actuary work), the actuary will have to use judgment on how volatile the scenarios to test should be. The interest scenario represents the single most important assumption the actuary will make, yet it is probably the assumption for which there are the least data on which to base the assumption. The actuary should work closely with the investment department to determine the interest scenarios to test.

The type of scenarios that are tested will depend on the purposes of the analysis. The actuary will want to use a stochastic process to develop interest scenarios if the purpose of the analysis is pricing or the comparison of alternative investment or crediting strategies. In a stochastic process, a number of events are assumed to be possible and each event is assigned a probability of occurring. The events are then simulated a number of times by using a random number generator to select the event that will be assumed to occur. The selection process is based on the underlying probabilities. This approach is called Monte Carlo testing. By projecting results in 50 or 100 randomly generated interest scenarios, the actuary can produce a distribution of potential results. From this distribution, the actuary can determine the anticipated mean and dispersion of results, and these values can be used to help management evaluate the risk/reward trade-off of various alternatives.

One approach to developing scenarios for valuation actuary work would be to create a number of extreme interest scenarios. This is the approach taken in New York Regulation 126. The actuary would then be able to tell management whether the company could survive scenarios as extreme as those tested.

In the familiar risk theory analytic structure, it is determined that there is an X percent probability of ruin given certain assumptions. It is possible to develop similar conclusions for the C-3 risk. In this approach, the actuary would use a stochastic process to develop a large cross section of interest scenarios to test and then determine how many of those scenarios result in ruin. An important aspect of this approach is the degree of confidence that management and the actuary have in the stochastic process used to generate the scenarios. For example, the actuary might present the results in the following way: "In 4 of the 200 scenarios that I tested, there were losses at an unacceptable level. I feel 99 percent confident that the scenarios produced by the stochastic process involved tend to be at least as volatile as the scenarios that will unfold in reality."

One easy way to develop a set of yield curves for use in stochastic modeling is to determine how low and how high rates can go. Yield curves are then developed at the low and high points. Typically these yield curves will reflect the common belief that the yield curve tends to steepen as rates decline and invert as rates rise. The actuary can then pass a quadratic through the two extreme curves and the current curve to get the other curves in the universe. When this type of approach is taken, the movement probabilities are typically developed by using some symmetrical probability distribution, such as a discrete approximation to the normal distribution. At the ends of the yield curve universe, the probabilities will have to be asymmetric.

A key parameter in this approach is the location of the current yield curve. If the current yield curve is in the middle of the universe, then in addition to there being an equal probability of rates rising or falling over the coming year, at any point in the future there is an equal probability of rates being above or below the initial curve. If the current yield curve is not in the middle of the universe, then in the short run there will be an equal probability of rates rising or falling, but in the long run there will be a bias in one direction or the other. If the current yield curve is placed so that the curve drawn though the three yield curves is a line, then the mathematical expectation of rate will generally be no change in rates. More typically, the current yield curve will be placed so that the size of the moves from one curve to the next gets larger as one moves to higher yield curves. If the current yield curve is placed in the middle of the universe, but rates can rise further than they can fall, this latter phenomenon will occur.

Some actuaries desire that it be possible for there to be different yield curve slopes at the same general level of yield rates. In such a case, the easiest way to generate a set of curves and probabilities is to follow a two-step process. The first step is to generate a set of probabilities of moving from one yield level to another and the second step is to generate a set of probabilities of yield curve slope changes and then cross-multiply the two probability sets. Ideally, the probabilities of slope changes will be partially dependent on the change in the level of rates and the probabilities of level changes will be partially dependent on the current slope of the yield curve. Typically, the level of the yield curve would be defined by rates for one particular security, for example, 20-year bonds. It makes sense to use long bonds as the slope pivot point, because long bond rates have historically been less volatile than short rates. For many analyses, using multiple sets of yield curve slopes represents an unnecessary level of complexity.

H. Other Considerations

Several other issues must be considered. Depending on the company's tax situation, it may be appropriate to perform the projections on an after-tax basis. Under the new tax law, taxes are fairly straightforward to calculate, although it may be necessary to provide for the separate calculation of tax reserves. If projections are done on a line by line basis for a mutual company, the surplus tax will need to be allocated.

Federal income tax will generally be 34 percent of the gain from operations calculated using tax reserves instead of statutory reserves for a stock company, under the 1986 tax law. Any projected capital gains also should be taxed at 34 percent under the 1986 law. Note that capital gains and losses cannot be offset against operating gains and losses. For a mutual company, the surplus tax also will need to be calculated. The rate at which surplus will be taxed will vary from year to year, so that an assumption on future surplus tax rates needs to be made. Many companies simply assume that the surplus tax rate will remain at current levels. The actuary also needs to be aware of other issues that can affect taxes, such as loss carryforwards/backwards and the small-company deduction.

Often the impact of taxes does not change the conclusions that will be reached, just the magnitude of the results. If that is the case, taxes could be ignored. Taxes also might be ignored if the actuary is uncertain about future tax rates, in which case incorporating taxes might be an undue complication with little value added to the analysis.

Typically, in performing valuation actuary analyses, future issues are ignored for several reasons. First, there is a great deal of uncertainty about both the amount and characteristics of future issues, so they may impair the validity of the analysis. Second, the concept of the valuation actuary is to test the adequacy of current reserves to cover benefits on the policies currently in force. Third, if aggressive growth assumptions are used, the value of future issues often dwarfs the value of existing business so that the valuation actuary's analysis will not reflect the current state of the company. Fourth, projecting 20 years of future issues when so many of the assumptions are scenario-dependent may unduly complicate the analysis. Fifth, any time that valuation actuary analysis shows that the combination of surplus and reserves is getting low, future sales can be discontinued. Thus, it seems burdensome to reserve for future sales that could be discontinued if they become a problem. Much the same reasoning applies to stockholder dividends. However, in work done for internal purposes to determine how much the company can safely grow or how much capital the company will need, it is often appropriate to include both future issues and shareholder dividends.

Another issue is whether the analysis should incorporate any reinsurance treaties in place. The most realistic approach seems to be to look at everything net of reinsurance, particularly since that is the way reserves are measured. Under some coinsurance treaties it may be feasible to ignore reinsurance if the reinsurer simply accepts a pro-rata share of the insurer's gain or loss.

Two projection methods currently are general use among actuaries. The first method is known as the profits-released method, or Anderson's method. Under this method, the excess or shortfall of assets against reserves is assumed to be paid to or borrowed from surplus each year. The block is then evaluated on the basis of the present value of these payments to or from surplus.

The second method is known as the profits-retained method, or Hoskin's method. Under this approach, there are no payments to or from surplus (that is, profits are retained in the block), and the block is evaluated on the basis of ending surplus or the ending market value of assets relative to the market value of liabilities.

The two major advantages of the profits-retained approach are that it is simpler and that it is not necessary to calculate interim reserves and book values of assets. The major advantages of the approach are that it provides substantially more information and that it more accurately reflects the way that most companies manage their business. Under the profits-released approach, if profits are accumulated at the asset earning rate (net of taxes if appropriate), the result will be close to that obtained from the profits-retained approach (for mutual companies, an appropriate adjustment must be made for the surplus tax). In addition, it is possible under the profits-released approach to determine what the impact of other uses of surplus would be by using other discount rates or a series of discount rates. Also, under a profits-released approach, potential statutory problems can be identified from the pattern of future profits. For example, in the profits-retained approach, a prolonged period of losses could be masked by an earlier or later series of gains. This is particularly distressing where large gains at the end of the projection period mask early losses, since the early losses could result in statutory insolvency.

In using the profits-released approach, an appropriate discount rate or set of rates must be selected. As a general rule, it is preferable to look at several sets of discounted values, because there is no single "right" approach to discounting the profits.

In valuation actuary work, the present value or accumulated value at the earnings rate should definitely be included since that value reflects what will happen if it is decided that future sales cannot be supported and the block should be run off as a closed block. The actuary may want to look at sets of discount rates that are somewhat adverse. For example, the actuary could use low rates when accumulated profits are positive and high rates when accumulated profits are negative. Ths would allow the actuary to look at the impact of poor performance in the investment of surplus. This discounting approach may be appropriate for valuation actuary work, but it is not appropriate for pricing work.

It may also be useful to look at discount rates that vary with the interest scenario—this implies that the investment of surplus is a short-term investment. Since most companies view surplus as a long-term investment, the actuary also should probably look at a set of discount rates that do not vary by scenario. This discounting could be based on a single rate. Using a single rate is consistent with traditional actuarial work and with the way bonds and other investments have typically been evaluated. Recently, considerable interest has developed in using a rate that varies depending on the time that a cash flow occurs (that is, a zero coupon rate). While this approach makes theoretical sense, it may be difficult to explain to management. In addition, zero-coupon yield curves are not always readily available. Whatever discounting approach is used, the actuary must be prepared to explain the approach to management or the other audiences for his/her work.

The last issue is the length of the projection period and how to end the study. The appropriate projection period will vary with the contract being studied. For GICs and some deferred annuities, 5 or 10 years is appropriate, while some structured settlement annuities may require a 50- or 60-year projection period. The key is to use a projection period that is long enough so that the combination of lapses and mortality diminishes the remaining block to an insignificant size. At the end of the projection period, it is appropriate to compare the market value of assets to the present value of remaining liabilities at the then-prevailing rates. If the liabilities are interest-sensitive, the present value of the future benefits is probably best approximated by the cash surrender value, unless rates have fallen low enough that contract guarentees exceed the interest rates prevailing in the market. In determining the projection period for deferred annuities, it should be kept in mind that, after age 60, in force will decline rapidly as policyholders retire or die.

II. ILLUSTRATIVE ANALYSES—CASE STUDIES

The easiest way to understand this sort of material is through examples. The following case studies are based on actual company situations, although the numbers have been changed to protect the identity of the companies involved.

The first example involves a GIC writer that had $1.4 billion of assets and liabilities as of June 30, 1985 (see Table 1). The company's portfolio was duration-matched, and the company intended to rebalance the portfolio every quarter to maintain the duration match. The company was interested in determining how much risk it faced under different interest scenarios, so it selected the 12 scenarios shown in Table 2 for testing purposes. Figure 4 displays the results of the analysis. If interest rates do not change, as in scenario 1, the company would have a small surplus at the end of the projection period. Under the worst of the 12 scenarios (scenario 5, which was an extremely severe rising interest scenario), the company would have a shortfall of $37.2 million at the end of the projection period. Given the severity of scenario 5, $37.2 million is not a great deal to put at risk for a $1.4-billion portfolio, and the result indicates how effective a duration-matching strategy can be in *minimizing* risk for fixed liabilities. However, $37.2 million is substantial enough to indicate that duration-matching *does not* by any means *eliminate* interest rate risk. The company concluded from the work that GIC reserves were inadequate by $15 million, which was the present value of the ending shortfall in scenario 4 (scenario 5 was considered beyond the bounds of "reasonable" scenarios).

TABLE 1

GIC ASSUMPTIONS

Deposit	Initial deposit of $1.2 billion has grown to $1.4 billion by 6/30/85
Book Value Withdrawals	Death benefits
Annuitization	$100 million
Compound GICs	$1.0 billion
Simple GICs	$300 million

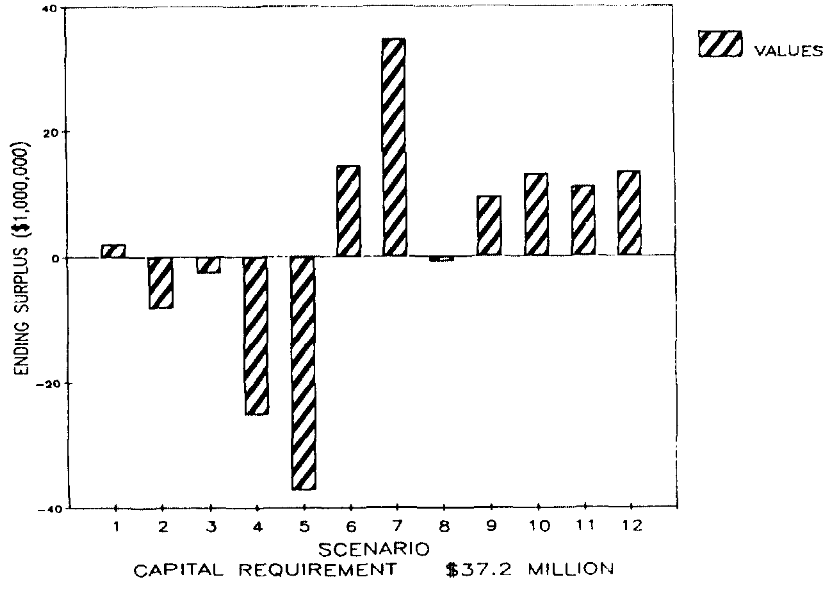
TABLE 2

SCENARIO DESCRIPTIONS FOR PROJECTIONS OF "IMMUNIZED" GIC PORTFOLIO

1.	Level
2.	Level. Yield curve steepens.
3.	Valley. Rates fall then return to original level. Yield curve steepens.
4.	Mountain. Rates rise then fall back to original level. Yield curve steepens.
5.	Rising. Yield curve steepens.
6.	Falling. Yield curve steepens.
7.	Falling. Yield curve becomes less steep.
8.	Rising. Yield curve inverts.
9.	Mountain. Rates rise then fall back to original level. Yield curve becomes less steep.
10.	Valley. Rates fall then return to original level. Yield curve becomes less steep.
11.	Deep valley. Rates fall further then in scenario 10. Yield curve becomes less steep.
12.	Level. Yield curves becomes less steep.

FIGURE 4

TWELVE PROJECTIONS OF A $1.4-BILLION "IMMUNIZED" GIC PORTFOLIO



Note that scenarios 1, 2, and 12 are all described as being "level." In each of these scenarios, the 5-year bond rate never changes, but in scenario 2 long-term rates increase and short-term rates decrease, while in scenario 12 long-term rates decrease and short-term rates increase. The ending surplus for scenario 1 is about $1 million; for scenario 2, about $ − 8 million; and for scenario 3, about $12 million.

These results indicate that the slope of the yield curve is almost as important as the level of the yield curve, especially for duration-matching strategies. We treated all the bonds as being noncallable in this first example, because the company makes an appropriation of surplus for the call risk. Thus, the fluctuations in ending surplus in this example are strictly due to interest rate risk and do not reflect call risk at all.

We built our model of existing assets from a tape that the company's investment department provided. The tape included book value, par value, market value, book yield, market yield, coupon, and maturity date for each security. Because the portfolio was extremely short, the client's investment officer felt that we could ignore calls. We assumed that the spread between market yield and Treasury yields would not change for each security through time. The investment department also told us that we could assume that they would earn 50 basis points more than Treasury yields on new 1- to 5-year bonds (no new investments in assets longer than 5 years were contemplated). The interest scenarios to be tested were developed by the investment department in consultation with senior management. The company also provided a listing of the reserves in force by block. There were not a lot of assumptions to make in this analysis, because it was a simple block of liabilities with withdrawals so severely penalized that we felt it was reasonable to assume no withdrawals. Because deaths were not a major factor in this analysis, we did not spend a lot of time in developing our mortality assumption. Mortality for the GIC block was based on 85 percent of the 1965–70 Ultimate Table for a male age 55. However, mortality was significant for the annuitized block, so we spent some time in discussions with the company's actuary before settling on the 83 basic table. Our model also separated the annuitants by age and certain period.

In this case, we ignored taxes because tax and statutory reserves were close together and because the company had substantial gains to offset against future taxable losses, so that the after-tax present value of any surplus or deficit would be roughly equal to the before-tax present value. The other analyses shown here were also run on a before-tax basis for simplicity.

In all these examples, we assumed that negative cash flows would be met by liquidating assets for their market value as determined by the model. In the first example, the investment strategy was to match the durations of assets and liabilities. Thus, a constant rebalancing of the duration of assets was necessary, and from time to time liquidations were necessary to maintain the duration match. Therefore, we used a linear programming routine to select which assets should be kept, which assets should be liquidated, and which new assets should be purchased. The method for selection of assets was to maximize the average market yield of the portfolio while keeping the duration of assets equal to the duration of liabilities.

At the end of the projection period in each of our analyses, we compared the ending market value of assets to the ending market value of liabilities and added the difference to the profit in the final year. In the first example, the ending differential by itself was the result of the analysis because we used a profits-retained approach. In making the comparison between ending assets and liabilities, we had to define the ending market value of liabilities. The policyholder account value seemed appropriate for the market value of liabilities, since the policyholder has the right to demand that value at any time and since the company did not have any guarantees that seemed to pose a threat. In addition, there generally was not much liability left so that more complex approaches were not justifiable. The one exception to this approach was the policyholders who had annuitized. Since these policyholders did not have an account value, we needed another measure. We considered the statutory reserve as a measure of the market value of the liabilities, but discarded that measure since the statutory reserve reflects interest rates at the time of issue rather than current rates. We settled on the present value of the remaining projected annuity benefits at the 30-year bond rate prevailing at the end of the projection, net of investment expenses, and a 20-basis-point risk charge.

The second example involves a company that we have called "Make Your Spread Life," which issues SPDAs. The company was interested in issuing SPDAs and wanted to know how much risk capital it should allocate if it issued $200 million of SPDAs. Table 3 shows the investment assumptions that Make Your Spread Life used for its analysis. This table describes the characteristics of the securities that Make Your Spread Life will be assumed to be able to buy for the projection. Table 4 shows a sampling of the 21 yield curves that were assumed to be possible for this analysis. A probability distribution was used to randomly generate interest scenarios based on this yield curve universe. Examples 3 and 4 are also based on Tables 3 and 4.

TABLE 3

INVESTMENT ASSUMPTIONS

- Spreads to Treasuries
  - 50 basis points for noncallable bonds
  - 150 basis points for callable bonds
- Bonds are called if rates fall 150 basis points
- 7-year callable bonds are callable in 4 years at par
- 10-year callable bonds are callable in 5 years at par
- 20-year callable bonds are callable in 5 years at 106% of par
- 30-year callable bonds are callable in 5 years at 106% of par

TABLE 4

YIELD CURVE UNIVERSE

Curve Number	1-Year Treasury	5-Year Treasury	10-Year Treasury	20-Year Treasury
1	1.41	2.27	2.38	2.78
3	2.41	3.27	3.38	3.78
7	4.41	5.27	5.38	5.78
11*	6.41	7.27	7.38	7.78
15	10.41	11.27	11.38	11.78
19	15.34	15.27	15.15	15.10
21	18.27	17.27	16.92	16.42

*Current curve.

Table 5 shows the other major assumptions for Make Your Spread Life. The company's intent was to buy 20-year callable bonds initially and thereafter to buy 7-year callable bonds. The credited rate would be equal to the earned rate less 150 basis points, because the product had been priced by assuming that the company would earn 150 basis points more than it credited. Figure 5 shows the results for this analysis. Based on 50 randomly generated trials, Make Your Spread Life had an expected present value of profits at 15% of about $2 million (see the dashed line). Under the worst of the 50 scenarios, the present value of profits was $ − 39 million, and there were 9 other scenarios for which losses exceeded $20 million. Based on this work, the company concluded that this strategy would require that $35 million of surplus to be set aside for every $200 million of premium issued.

Note that, in example 1, we used a profits-retained approach, so the results are displayed as ending surplus numbers, while in examples 2 through 4, the results are displayed as the present value of profits. Thus, if all other

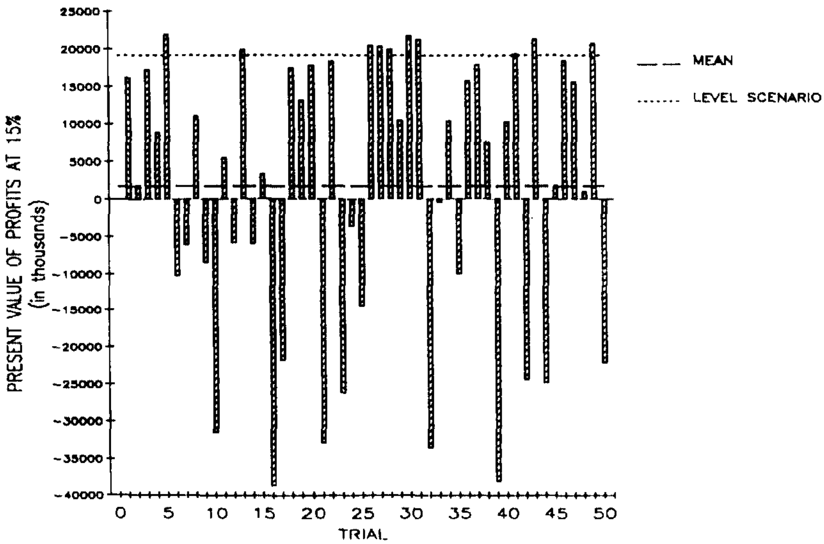
TABLE 5

MAKE YOUR SPREAD LIFE:
SPDA NEW ISSUES

Premium	\$200 million
Average Size	\$25,000
Surrender Charge	7,7,7,6,5,4,3,2,1,0%
Investment Strategy	Twenty-year callable bonds Initially; thereafter seven-year callable bonds
Credited Rate	Earned rate less 150 basis points
Market Rate	5-year Treasury
Lapse Rate	$15\% + 2 \times (MR - CR)^2 - 3 \times SC$ minimum $= 3\%$
Issue Expense	\$50 per policy
Maintenance Expense	\$25 per policy, inflated at 3%
Commission	5%
Investment Expense	0.2% of fund
Bailout	0
Guaranteed Interest	4%

FIGURE 5

MAKE YOUR SPREAD LIFE:
IMPACT OF INTEREST RATE SWINGS ON EXPECTED PROFITS



things are equal, a loss of $1 million in example 1 will require less initial surplus than a loss of $1 million in the other examples. Note also that, in example 1, the scenarios were selected by management and several were chosen to be severe, while in examples 2 to 4, the scenarios were chosen through a random process to produce what management hoped was a reasonable cross section of scenarios.

One interesting result of the analysis for Make Your Spread Life is that when interest rates never change (the level scenario result is indicated by a dotted line), the present value of profits is considerably higher than the mean present value of profits over the 50 scenarios and is almost as high as the best of the 50 trials. Since the company's actuaries had priced the product assuming a 150-basis-point spread with no adverse lapse deviations, their pricing result was close to the result under the level scenario. It is quite common, as in this case, for analysis of the impact of swings in interest rates to indicate that expected results will be far worse than would be indicated by conventional pricing analyses. Interest rate movements have an adverse impact on profitability, because insurers typically grant options both to their policyholders and to bond issuers. In a level-interest scenario, those options have no value, but when interest rates are volatile, insurers face heavy antiselection as those options are exercised, either through higher lapses in rising-rate environments or through calls in falling-rate environments.

The investment assumptions shown in Table 3 were developed by the company's investment department after we described what we needed. We told the investment department that we needed to know what new bonds of different maturities would yield, as well as what types of call provisions the securities would have. The investment department then told us that noncallable bonds of the quality they typically buy yield 50 basis points more than Treasuries and that comparable callable bonds yield 150 basis points more than Treasuries. They also gave us typical call dates and call prices for the various maturities we intended to include in our universe of potential investments. Finally, they told us that bonds would be called when rates fall 150 basis points from the level at which they were issued (only if the bond was past its call date). Note that the 50- and 150-basis-point spreads were between *nominal* yields on corporate bonds and *nominal* yields on Treasury bonds. Investment departments typically speak in terms of nominal yields, so the actuary will probably want to clarify what type of yield the investment department is talking about, for example, nominal or effective annual.

We developed the yield curve universe shown in Table 4 based on the following criteria from the investment department:

(1) Rates were equally likely to rise or fall, but they could rise as much as 10 percent from then-current levels and they could only fall 5 percent.
(2) The slope of rates was not particularly likely to increase or decrease, except at higher levels where the investment department expected rates to gradually invert.
(3) The universe should be a simple one that would be easy to understand and explain.
(4) The probability grid should be roughly normal, and it should be possible to move plus or minus four curves at a time.

This universe was not developed by passing a quadratic through the intial yield curve, lowest yield curve, and highest yield curve, but rather by assuming that movements downward from the initial curve would be in 50-basis-point intervals, while movements upward would be in 100-basis-point intervals. This approach leads to a substantial discontinuity at the initial curve and also has funny implications for the mathematical expectation of rate movements from each yield curve. The probability grid assumed a 3 percent chance of moving 4 curves in either direction, a 7 percent chance of moving 3 curves in either direction, a 15 percent chance of moving 2 curves, a 15 percent chance of moving 1 curve, and a 20 percent chance of no change in rates.

The investment strategy shown in Table 5 (buy 20-year callable bonds initially and 7-year callable bonds in future years) was one of several proposed by different groups within the company and was directly related to the proposed strategy of crediting the earned rate less 150 basis points (after crediting the market rate initially to be competitive).

The average size, issue expense, maintenance expense, commission, bailout, and guaranteed interest were all provided by the pricing actuary. We ignored death benefits here, although typically we would probably use something simple, such as 80 percent of the 1965–70 Ultimate Table. The assumption of $200 million of premium reflected the company's marketing plan. We assumed that the market rate was equal to the 5-year Treasury rate, because the client felt that the SPDA market is driven by the rates available on intermediate bonds and because the company's then-current SPDA rate happened to equal the 5-year Treasury rate. As a general rule, the formula for the market rate should reproduce the company's current rate on new issues, unless there are strong reasons not to do so, such as the company feeling that its current rate is not competitive or that its rate is above the market.

The lapse rate formula was chosen to fit both the pricing actuary's intuitive sense for how lapses would behave and the company's limited experience. In the 2 years that the company had been issuing the product, lapses had been between 2 and 4 percent, which partially reflected the fact that the company had not been more than 2 percent off the market in either direction during that period. During the early 1980s, a predecessor product had experienced lapses between 40 and 50 percent when it had been 4 to 5 percent off the market and had a 7 percent surrender charge.

Many times, we begin increasing the lapse rate after 7 or 8 years to reflect annuitizations. In this study, we chose instead to assume 100 percent lapses after 10 years.

Example 3 involves an SPDA writer that we have labelled "Matched Life." Table 6 shows the company's major assumptions. Note that although Matched Life believed that it was duration-matched, the duration of the typical interest-sensitive liability changes as interest rates change and there are not any assets that behave comparably. Since Matched Life intended to credit a market rate of interest, its liability would behave more like a series of 1-year bonds rolling over than anything else, but if it invested in short assets, earnings would be insufficient to fund the credited rate. Nonetheless, Matched Life concluded that its liability had a duration that is comparable to that of a 7-year bond, so it would buy 7-year bonds initially. Thereafter, Matched Life would buy shorter and shorter intermediate bonds in an attempt to maintain the so-called match. Figure 6 shows the results for Matched Life. Of 50 scenarios, the mean present value of profits is \$2 million and the low is \$ − 22 million. In 5 of the 50 scenarios, Matched Life had losses in excess of \$20 million. Based on this work, the company concluded that \$20 million of surplus would be required if \$200 million of SPDA premium were issued. Although Matched Life has lower risk than Make Your Spread Life, it clearly did not come close to eliminating the interest rate risk. It is generally not possible to eliminate or nearly eliminate the interest rate risk for interest-sensitive products. As in example 2, the level scenario produced a result that is considerably better than the mean. The assumptions for the third example are the same as for the second example, except that the company wanted to try a different investment and crediting strategy.

The final example, example 4, involves a universal life writer. Because universal life is a more complex product than an SPDA, the assumptions

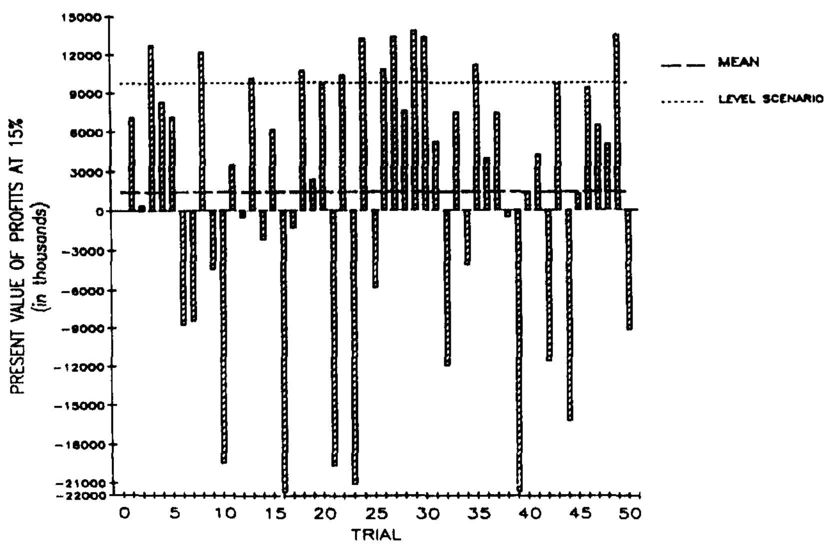
TABLE 6

MATCHED LIFE:
SPDA NEW ISSUES

Premium	$200 million
Average Size	$25,000
Surrender Charge	7,7,7,6,5,4,3,2,1,0%
Investment Strategy	Intermediate bonds; initially 7-year callable bonds
Credited Rate	Market
Market Rate	5-year Treasury
Lapse Rate	15% + 2 × $(MR-CR)^2$ − 3 × SC Minimum = 3%
Issue Expense	$50 per policy
Maintenance Expense	$25 per policy, inflated at 3%
Commission	5%
Investment Expense	0.2% of fund
Bailout	0
Guaranteed Interest	4%

FIGURE 6

MATCHED LIFE:
IMPACT OF INTEREST RATE SWINGS ON EXPECTED PROFITS



that are shown in Table 7 are more extensive than for the SPDAs. However, the basic approach and principles are unchanged. This company intended to buy 10-year callable bonds and credit the market rate, which it defined as the 7-year Treasury rate. The results, which are shown in Figure 7, are fairly favorable: the mean present value of profits at 15 percent is $450,000 and the low of the 50 trials is only $ − 150,000. Again, the result under the level scenario is considerably better than the mean result. In this case, the client concluded that reserves were adequate.

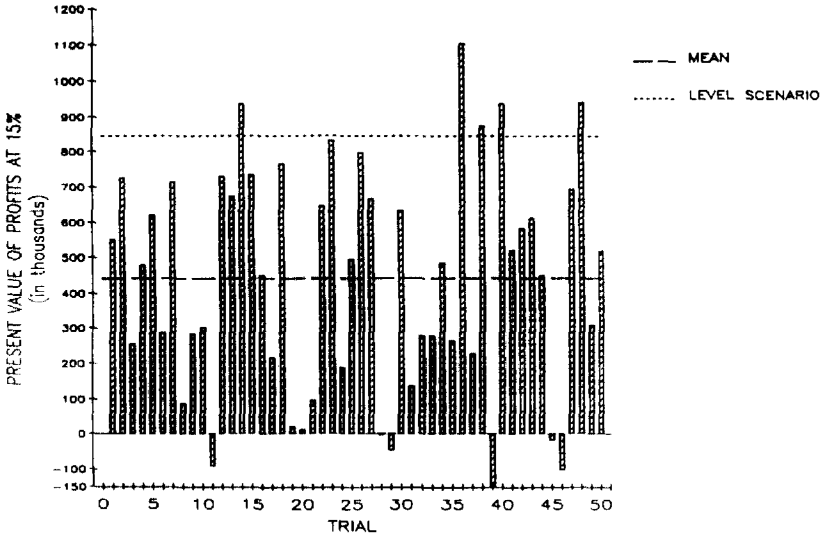
TABLE 7

UNIVERSAL LIFE COMPANY:
UNIVERSAL LIFE NEW ISSUES

Premium	Initially $5 million per year
Average Size	$100,000
Investment Strategy	10-year callable bonds
Surrender Charge	100% of first-year target premium grading to zero in year 16
Credited Rate	Market
Market Rate	7-year Treasury
Lapse Rate	17% + 1.5 $(MR-CR)^{2.5}$ − SC minimum = 4%
Suspension Rate	15% first year, 5% thereafter
Issue Expense	$150 per policy plus $1.75 per $1000
Maintenance Expense	$50 per policy
Investment Expense	0.15%
Premium Tax	2.25%
Commission and Allowances	115% of first-year target premium; 15% of second-year target premium; 5% of all other target premiums and excess premiums
Loads	4% of target premium
Mortality Charge	85% of 65/70 ultimate
Actual Mortality	75% of 65/70 Select and Ultimate
Guarantee	80CSO and 5.5%
Reserve	CRVM; initially $10 million

FIGURE 7

UNIVERSAL LIFE COMPANY:
IMPACT OF INTEREST RATE SWINGS ON EXPECTED PROFITS



The company evaluated the risks associated with the existing universal life block at the same time that it performed the SPDA pricing analysis. Because the investment department bought the same quality of bonds for SPDAs and universal life, we did not need new asset assumptions or interest scenarios. However, we did need assumptions on the behavior of the liabilities, as well as in-force data.

Asset in-force data were provided on tape in much the same form as the assets for the first example, except that we also needed call data for each bond. In-force on existing liabilities was provided in the form of valuation extracts. Our model included 2 issue year splits, 5 issue age splits, and 3 premium pattern splits. The premium pattern splits were by the size of dump-ins. Average sizes and premium in-force were derived from the valuation extract. The investment strategy (10-year callable bonds) was what the investment department described as the current strategy.

The expenses, premium tax, premium suspension rate, surrender charge, commissions, loads, mortality charge, reserve basis, and expected actual mortality were all provided by the pricing actuary. The lapse formula was suggested by the pricing actuary based on an intuitive sense for the business. The crediting strategy was what the company said it anticipated doing, and the market rate reflected the company's belief that it needed to credit at least the 7-year Treasury rate to be competitive.

We have not dwelled on the setting of assumptions such as expenses in this paper, because most actuaries have substantial experience in setting that type of assumption. Instead, we have focused on the setting of assumptions that are needed to model the C-3 risk but are not needed for traditional modeling methods.

If an actuary needs to set the other assumptions, many can be found in policy forms. The rest are usually determined by looking at the company's experience studies, although an actuary occasionally will not have access to experience studies, in which case the actuary will need to base the assumptions on a sense of what reasonable assumptions would be rather than on hard data.

In all these analyses, it is important to remember that the results are extremely sensitive to the assumptions used. The actuary often will want to perform sensitivity-testing to see what the results look like using alternative assumptions. If the results do not change much, there is no problem, but if they change dramatically, then the actuary will want to be sure that the audience for the work understands the implications of changes in the crucial assumptions. Some of the leading candidates for sensitivity-testing are the interest scenarios tested, the lapse assumption, and the relationship of the market credited rate to the investment market rate.

III. SAMPLE CALCULATIONS

In this section, we develop a simple projection of a simple SPDA new issue through a single-interest scenario. We show how each number is calculated so that the actuary will get a feel for how an asset and liability model fits together. This example illustrates the principles discussed in the preceding pages. We have intentionally chosen somewhat unusual assumptions both to make the calculations simple and to highlight certain key elements of the asset and liability modeling process. We have assumed that all cash flows occur at the end of the year, except for premiums and commissions, which are assumed to occur at the beginning of the year. In more complex modeling processes, most cash flows would be allocated throughout the year. For example, half of all deaths could be allocated to the beginning of the year and the other half to the end of the year. Similarly, policy years correspond with projection years in this analysis. Two alternative approaches would be to offset policy years from projection years by half a year or to treat some policies as being issued at the beginning of a projection year and some as being issued at the end of a projection year. Under this latter approach, each policy year would coincide with a projection year, but the average policy year for each year of issue could be offset by anything from 0 year to 1 year.

We have used a profits-released approach, or "Anderson's method," in these projections. In this approach, assets are always kept equal to reserves, either by paying excess cash to the surplus line or by borrowing from the surplus line to meet shortfalls. Results are evaluated on the basis of the present value of these payments (profits released) to and from surplus. It requires only minor adjustments to the methods shown here to use the profits-retained approach in which there is no borrowing from surplus nor payment of profits to the surplus line.

Table 8 shows the assumptions for this example. Tables 9 through 12 show various elements of our sample projection. Table 9 shows the development of the liability cash flows and reserves each year. Table 10 shows the sources and uses of funds each year. Table 11 shows profits and losses each year. Table 12 shows balance sheets for each year. As we go through the example, it becomes apparent that the various reports are interrelated. The interrelationships between the reports and the multiple definitions of various values can be used to help check that the formulas are consistent with each other. For example, the profit released each period equals the increase in the book value of assets less the increase in liabilities and less any profits retained in the line (profits retained are always zero in our example). The profit released also equals the sum of income items plus capital gains and less the sum of disbursement items and profits retained. It is a good idea to test that profits released is the same calculated on either basis. This test is especially important when a programming change is made.

We now start calculating the numbers that make up the reports in Tables 9 to 12. We start with the numbers at time 0 in Table 9. The initial premium of $100,000 (all values are shown in thousands) comes straight from Table 8. There are no death benefits, surrenders, expenses, or interest credited at time 0, because these are assumed to occur at the end of projection periods.

TABLE 8

ASSUMPTIONS FOR ANALYSIS

Product	SPDA New Issue
Premium	$100 million initially, zero thereafter
Projection Period	3 years
Death Rate	0.01, 0.015, 0.02
Surrender Charge	5%, 2%, 0%
Commission	2% of premium
Market Rate	7-year Treasury Rate
Credited Rate	Market rate at issue is credited in all years
Expenses	0.3% of beginning of projection year account value
Reserve	Account Value
Lapses	15% + 2 × $(MR-CR)^2$ − 3 × SC; minimum of 3% MR = Market Rate CR = Credited Rate SC = Surrender Charge If $MR-CR$ is negative, then $(MR-CR)^2$ is multiplied by negative one
Investment strategy	Buy 10-year callable bonds yielding Treasury plus 150 basis points. Bonds are callable in 5 years at par plus 2%.
Transaction Cost	0.25%

	Treasury Yield Curves						
Time	1-Year Rate	2-Year Rate	3-Year Rate	4-Year Rate	5-Year Rate	7-Year Rate	10-Year Rate
0	7.50	7.80	8.10	8.40	8.60	9.10	10.00
1	5.50	5.80	6.10	6.40	6.60	7.10	8.00
2	11.50	11.80	12.10	12.40	12.60	13.10	14.00
3	9.50	9.80	10.10	10.40	10.60	11.10	12.00

TABLE 9

LIABILITIES

	Time 0	Time 1	Time 2	Time 3
Premiums	100,00	—	—	—
− Death Benefits	—	1,091	1,715	1,449
− Net Surrenders	—	3,078	45,238	71,023
− Expenses	—	300	314	199
− Commissions	2,000	—	—	—
Insurance Cash Flow	98,000	−4,469	−47,267	−72,671
Beginning Account Value	—	100,000	104,769	66,427
+ Premiums	100,000	—	—	—
+ Interest Credited	—	9,100	9,534	6,045
− Death Benefits	—	1,091	1,715	1,449
− Gross Surrenders	—	3,240	46,161	71,023
Ending Account Value	100,000	104,769	66,427	—
Reserve	100,000	104,769	66,427	—
Cash Value	95,000	99,531	65,098	—

TABLE 10

INVESTMENT OF NEW FUNDS

	Time 0	Time 1	Time 2	Time 3
Sources of Funds				
Calls	—	—	—	—
+ Rollover	—	—	—	—
+ Liquidations	—	—	32,392	62,452
+ Investment Income	—	11,500	11,953	7,544
+ Insurance Cash Flow	98,000	−4,469	−47,267	−72,671
− Profits Released	−2,000	2,262	−2,922	−2,675
Total	100,000	4,769	—	—
Uses of Funds				
Short Term	—	—	—	—
5-Year Bonds	—	—	—	—
10-Year Bonds	100,000	4,769	—	—
Total	100,000	4,769	—	—
Rate	11.50%	9.50%	—	—

TABLE 11

PROFITS AND LOSSES

	Time 0	Time 1	Time 2	Time 3
Premiums	100,000	—	—	—
Investment Income	—	11,500	11,953	7,544
Total Income	100,000	11,500	11,953	7,544
Net Surrenders	—	3,078	45,238	71,023
Death Benefits	—	1,091	1,715	1,449
Commissions	2,000	—	—	—
Expenses	—	300	314	199
Increase in Reserve	100,000	4,769	−38,342	−66,427
Total Disbursements	102,000	9,238	8,925	6,244
Statutory Profit	−2,000	2,262	3,028	1,300
Capital Gains	—	—	−5,950	−3,975
Profits Retained	—	—	—	—
Profits Released	−2,000	2,262	−2,922	−2,675

TABLE 12

BALANCE SHEET

	Time 0	Time 1	Time 2	Time 3
Short Term	—	—	—	—
0–5 Year Bonds	—	—	—	—
6-Year Bonds	—	—	—	—
7-Year Bonds	—	—	—	—
8-Year Bonds	—	—	61,658	—
9-Year Bonds	—	100,000	4,769	—
10-Year Bonds	100,000	4,769	—	—
Total	100,000	104,769	66,427	—
Reserve	100,000	104,769	66,427	—
Surplus	—	—	—	—
Market Value	99,750	117,899	55,561	—
Unrealized Capital Gain	−250	13,130	−10,866	—

The $2,000 commission is 2 percent of the $100,000 premium, and the time 0 insurance cash flow of $98,000 equals the premium less the commission. There was no activity besides premium that affected the account value at time 0, so the account value at time 0 is simply $100,000. The cash value at time 0 equals the account value less the 5 percent surrender charge, and the reserve equals the account value.

After the liability calculations have been made, various values are transferred to the other three reports. Insurance cash flow is transferred to Table 10, and the various components of insurance cash flow are transferred to Table 11. In addition, the increase in reserve is calculated and transferred to Table 11. The ending reserve is transferred to Table 12.

At this point, we look at Table 12 to determine whether the book value of the assets that we intend to retain from the prior period is less than or greater than the reserve. In this case, since we had no prior assets, the prior assets do not exceed the reserve and we will not need to make any liquidations. Therefore, we can complete the profit-and-loss statement (Table 11). Because we are at time 0, there is no investment income. All the other income and disbursement items have already been transferred from Table 9, so we just add them up and arrive at total income of $100,000 and total disbursements of $102,000 for a statutory profit of $−2,000. Since there were no liquidations, we have no capital gains, and we are not retaining any profits, so the profit released at time 0 equals the statutory profit of $−2,000. The profit released is transferred to Table 10.

Because there were no existing assets, there are no calls, rollover, or investment income at time 0. There are also no liquidations. Therefore, the total funds available to invest are simply equal to the insurance cash flow of \$98,000 minus the profits released of \$ − 2,000, for a total of \$100,000. The available funds are then invested according to the investment strategy, which in this case calls for 100 percent in 10-year callable bonds earning the Treasury rate plus 150 basis points. Since the initial yield curve shows 10-year Treasuries earning 10 percent, the callable bonds will have a yield of 11.50 percent. For simplicity, we assume that all bonds are purchased at par. Although most bonds pay semiannual coupons and most investment personnel quote yields in terms of semiannual rates, we use only effective annual coupons and yields in this example.

The last thing to be done for time 0 is to finish filling out the balance sheet (Table 12). The \$100,000 of 10-year bonds from Table 10 is transferred to Table 12 and added to the bonds that were retained from the prior period (zero in this case). The total book value of bonds held equals the total reserve, which is a nontrivial test that the internal logic of the projection is consistent. Surplus (the difference between book assets and the reserve) equals zero, which is appropriate because we are using the profits-released approach. The market value of assets equals the present value of anticipated asset cash flows at currently prevailing rates, less the cost of selling the assets. Since the assets here were just acquired, the present value of cash flows equals the book value, but the market value is reduced by the assumed 0.25 percent cost to sell the bonds. The unrealized capital gain of −250 shown on the balance sheet equals the market value of assets (\$99,750) less the book value of assets (\$100,000).

At time 1, we again start with Table 9. Since the product is a single SPDA new issue, we have no premiums. The credited rate in this analysis always equals the market rate at the time of issue. The market rate is assumed to be the 7-year Treasury rate, which is 9.10 percent initially. Thus, the credited rate is 9.10 percent and interest credited equals \$9,100. Interest is assumed to be credited before deaths and lapses occur so that there does not need to be a reinvestment assumption for deaths and lapses (that is, investment income and interest credited should each be earned on the same base).

After interest is credited, death benefits are calculated based on the beginning account value plus the interest credited. In this case, the death rate for the first policy year is 0.01, so death benefits equal $0.01 \times (100{,}000 + 9{,}100) = 1{,}091$. We are assuming that the surrender charge does not apply to deaths, so that the death benefit has the same impact on the account value as on the insurance cash flow. If this were a universal life block, mortality charges for all policyholders plus the account values of policyholders who die would reduce the account value, while actual death benefits would reduce the insurance cash flow.

After deaths are calculated, surrenders are calculated. In this scenario, rates drop 2 percent at the end of the first year, so that the market rate is 7.10 percent as opposed to the credited rate of 9.10 percent. Thus, $MR-CR$ equals -2 percent and, when plugged into the lapse formula along with the surrender charge of 5 percent, produces an anticipated lapse rate of -8 percent.

The $2 \times (MR-CR)^2$ term of the lapse formula is multiplied by negative one since $MR-CR$ is negative and thus produces -8 percent. The term $-3 \times SC$ produces -15 percent and offsets the base rate of 15 percent. However, the lapse formula has a minimum of 3 percent, so 3 percent is used. The 3 percent is applied to the current account value of $108,009 (100,000 + 9,100 − 1,091) to get gross surrenders of $3,240. The gross surrenders reduce the account value by $3,240, but the 5 percent surrender charge reduces the impact on insurance cash flow to $3,078.

We have assumed that expenses equal 0.30 percent of the account value at the beginning of the projection year, although it is typical to base expenses on the average account value for the year, as well as on the number of policies in force and the amount of premium received. In this case, the account value was $100,000 at the beginning of the projection year, so expenses are $300. We now have all the items necessary to calculate the insurance cash flow and account value for time 1. As at time 0, the reserve equals the account value. The cash value equals the account value less the surrender charge, which does not change from 5 to 2 percent until the start of the following projection/policy year.

After the numbers from Table 9 are transferred to the other tables, we turn to Table 12 to determine whether liquidations are necessary. In Table 12, we see that the reserve at time 1 equals $104,769 and that there is only $100,000 of bonds left from prior years, so there will be no liquidations. We now turn to Table 11 and add the items that were not transferred from Table 9. Investment income equals 11.50 percent of the $100,000 of bonds held, or $11,500. There are no profits retained by definition and no capital gains, because there were no liquidations. The profit released thus equals total income of $11,500 less total disbursements of $9,238 for a net of $2,262.

We now transfer the profits released to Table 10, along with the investment income. There are no calls or rollover because the bonds purchased at time 0 had 10 years to maturity and 5 years to the first call date. We have already determined that there were no liquidations, so we can add up the sources of funds to arrive at a total available to invest of $4,769. According to our strategy, this money will be used to purchase 10-year callable bonds, earning 9.50 percent rather than 11.50 percent because rates have fallen 2 percent. The $4,769 is then transferred to Table 12, and it is reassuring to see that the total book value of bonds is $104,769, which equals the reserve.

The market value of the $4,769 equals $4,757, with the $12 difference between book and market reflecting the impact of transaction costs. By interpolating between the 7- and 10-year Treasury rates, we get 7.70 percent as the rate for a 9-year Treasury and add 150 basis points to arrive at the rate of 9.20 percent for the $100,000 of bonds purchased at time 0. This produces a market value for this block of $113,678 before accounting for transaction costs. However, we must also account for the impact of call provisions, so we calculate the market value assuming the bond will be called at the first call date (4 years from time 1) for the initial call price (102 percent of par). For this calculation, we use the 4-year Treasury rate (6.40 percent) plus 150 basis points as the discount rate. This calculation produces a market value of $113,426 before consideration of transaction costs. Since the market value based on the call provisions is lower than the market value without consideration of call, we will use the market value based on assuming the bond will be called. When transaction costs are reflected, the market value of the bonds purchased at time 0 is $113,142 at time 1 and the total market value at time 1 is $117,899. This results in an unrealized capital gain of $13,130.

At time 2, the interest crediting rate is still 9.10 percent and generates interest credited of $9,534. The death rate of 0.015 when applied to the account value plus interest credited produces death benefits of $1,715. The lapse rate reflects a market rate of 13.10 percent so that $MR-CR$ equals 4 percent. When this is put into the lapse formula, the $2 \times (MR-CR)^2$ piece of the formula produces 32 percent, which is added to the 15 percent base rate and reduced by $3 \times SC$ or 6 percent for a rate of 41 percent. The 41 percent is then applied to ($104,769 + 9,534 − 1,715) for gross surrenders of $46,161. Net surrenders equal 98 percent of gross surrenders because the second-policy-year surrender charge of 2 percent applies. Expenses equal 0.3 percent of the beginning-of-the-year account value ($104,769), or $314.

We now have the pieces to create the account value and insurance cash flow. The reserve equals the account value, and the cash value equals 98 percent of the account value. We then transfer the liability items to the other tables.

Looking at Table 12, we see that the book value of bonds remaining from the prior period is $104,769, so we have to liquidate. For simplicity, we liquidate only from the $100,000 block purchased at time 0. To determine the value of liquidations, we need to calculate the market value of the bonds. Since rates are now higher than they were when the bonds were issued, the call provision will not affect the market value. The discount rate for the block purchased at time 0 will be the 8-year Treasury rate (13.40 percent) plus 150 basis points. The present value of cash flows on this block is thus 84.69 percent of book, and the market value is 84.48 percent of book (reflecting the 0.25 percent transaction cost). Since the reserve is $66,427 and we are retaining $4,769 from the time 1 block, we must liquidate bonds with a book value of $38,342 and leave on the books bonds with a book value of $61,658. The market value of the bonds liquidated is $32,392, and the market value of the time 0 bonds that are retained is $52,089. The discount rate for the time 1 block is the 9-year Treasury rate (13.70) plus 150 basis points. This produces a present value of $3,481 and a market value of $3,472. The total market value of bonds is thus $55,561 and the unrealized capital gain is $ − 10,866.

The capital gain on Table 11 is $ − 5,950 ($32,392 − $38,342). The investment income at time 2 is $11,953 (11.50% × 100,000 + 9.50% × 4,769). Thus, total income is $11,953; total disbursements are $8,925; the statutory profit is $3,028; and the profit released is $ − 2,922.

Table 10 is completed by transferring the market value of liquidations plus the profits released and investment income from Table 11. As would be expected, the sources of funds add to zero, because the bonds retained from prior periods equal the reserve.

At time 3, the credited rate is again 9.10 percent, which produces $6,045 of interest credited. The death rate of 0.02 when applied to the account value plus the interest credited produces death benefits of $1,449. We are ending our study by lapsing all remaining policyholders, so the remaining account value of $71,023 is treated as a gross surrender. Since we are in the third policy year, the surrender charge is zero and net surrenders equal gross surrenders. There is no ending account value, cash value or reserve. Expenses are again 0.3 percent of the prior year's account value ($66,427), or $199.

Because the ending reserve is zero, all remaining assets are liquidated. There were no rollovers, so there is $61,658 remaining of the time 0 block and $4,769 remaining of the time 1 block. The discount rate for the time 0 block is the 7-year Treasury rate (11.10%) plus 150 basis points. The discount rate for the time 1 block is the 8-year Treasury rate (11.40%) plus 150 basis points. The present values of the two blocks are $58,621 and $3,988, respectively. The market values are $58,474 and $3,978.

Thus, the market value of liquidations is $62,452 and the realized capital gain is $ – 3,975 (the market value of liquidations [$62,452] minus the book value [$66,427]). Investment income is $7,544 (11.50% × 61,658 + 9.50% × 4,769). We thus have the pieces to finish Table 11. Total income is $7,544; total disbursements are $6,244; statutory profit is $1,300; capital gains are $ – 3,975; and the profit released is $ – 2,675.

There are no calls or rollover at time 3 in Table 10. As at time 2, when we add up the numbers in Table 10 at time 3, we get zero funds available to invest. All of the numbers in Table 12 are zero at time 3.