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The Cost of Capital Assumption in Actuarial Appraisals: An Application of Fair Value of Liability Concepts

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A critical assumption in actuarial appraisals is the rate at which free statutory cash flows are discounted, a rate referred to as the cost of capital. The cost of capital in part reflects the risk inherent in the deal, but there are many considerations in setting the assumption, and there is no consensus about what theory to use. Reflecting this lack of consensus, appraisal values are typically calculated for a range of assumptions (e.g. 10% to 14%). However, negotiations seldom center on the cost of capital as an input to the valuation. Rather, the cost of capital usually serves only as a way of quoting the appraisal value that a particular counterparty to the transaction has arrived at through other considerations.

In his pivotal paper, "Market Value of Insurance Liabilities: Reconciling the Actuarial Appraisal and Option Pricing Methods," Luke Girard demonstrates an algebraic connection between the cost of capital used in actuarial appraisals and the degree of leverage implied by the valuation of asset and liability cash flows using the option pricing method (OPM). His work provides us with an intriguing theory for setting the cost of capital assumption. Further, the theory ties into the leverage equations familiar in finance from the work of Modigliani and Miller.

In the following, we explore the implications of Luke's work in setting the cost of capital assumption in actuarial appraisals. Of course, the process of appraising value will continue to involve many considerations, but any theory that helps pin down the cost of capital assumption could potentially become a useful negotiating tool.

The Case Ignoring Taxes

The theory presented in Luke's paper is easiest to grasp when taxes are ignored. When taxes are introduced, results are similar, but adjustments are required. It is also easier for this discussion to think of all cash flows as static, but this assumption may also be relaxed.

If taxes are ignored, the following equation expresses the relationship that should hold between the valuation of distributable earnings and the valuation of the asset and liability cash flows underlying those earnings:

1) $$DDE \equiv RS + MVA - MVL$$

In (1), "DDE" is the discounted value of distributable earnings, "RS" is required surplus, "MVA" is the market value of assets backing operations, and "MVL" is the fair value of liabilities. (The separation of assets into those backing operations and those comprising required surplus is only a convenience.)

Luke's paper presents algebra that allows us to work this equation in two ways. On the one hand, we can start with a given cost of capital assumption that produces a value for DDE. We can then derive implied liability discount rates which give us a value of MVL fitting the equation. On the other hand, we can start with the liability valuation and back into a cost of capital assumption that produces a value for DDE fitting the equation[1].

Pursuing the second approach mentioned above, Luke shows that if $\{j^t\}$ are the discount rates that apply in deriving RS, $\{i^t\}$ are rates that apply to asset cash flows, and $\{d^t\}$ are rates that apply to liability cash flows, the cost of capital assumptions that fit the equation are as follows:

2) $$k^t = \frac{j^t RS_t + i^t MVA_t - d^t MVL_t}{RS_t + MVA_t - MVL_t}$$

The $\{d^t\}$ used in discounting liability cash flows would be derived per OPM, viewing the liabilities as if they were debt cash flows. Note that the cost of capital changes with duration t as the relationship between RS_t, MVA_t and MVL_t changes.

We recognize that the cost of capital rate in (2) is just the weighted average of the asset and liability discount rates. In other words, the cost of capital assumption that ensures consistent valuation of assets, liabilities, and free cash flows is an asset-based rate levered by the liabilities. Dropping subscripts and pooling RS and MVA, (2) is closely related to Modigliani and Miller's proposition II for leverage adjusted capital:

3) $$k^L = \frac{kA - dD}{A - D}$$

In (3), "A" are the assets of the a firm, "D" the firm debt, "k" the unlevered cost of capital, "d" the cost of debt, and "k^L" the levered cost of capital of the firm. Luke has applied the same concept in the appraisal context. Liabilities play the role of "D," and the asset rate plays the role of "k."

Implications

Equation (2) gives us the following algorithm for backing into a cost of capital rate: