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V. REPORT OF COMBINATION OF RISKS TASK FORCE

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The Combination of Risks Task Force (CORTF) was organized in the summer of 1983. It reports to the Society of Actuaries Committee on Valuation and Related Problems.

The charge of the Task Force as set forth in the SOA *Yearbook* is as follows:

> "To develop understanding and quantification of risks of loss from combination of C-1, C-2, C-3 and C-4 risks and implications on statutory reserves, contingency surplus needed on inforce business, valuation statutes, early warning tests, and corporate planning."

The Task Force held its organizational meeting on November 10, 1983, and met several times in subsequent years. Most of the work of the Task Force has been performed at Aetna Life & Casualty where many individuals who are not official Task Force members were involved.

While the Task Force has generally embraced the concept that analysis of cash flows is the key to risk analysis and understanding combination of risks, there is a divergence of opinion on the details. Discounting of cash flows, in particular, is a troublesome area, and the statistical work on combination of risks, which is quite complicated, reflects the efforts of just a few individuals and is not necessarily supported by the Task Force members.

Given the difference of opinion on many of the details with respect to cash-flow analysis and combination of risks within the Task Force, it was not possible to issue a report that the entire Task Force would endorse. What follows, therefore, is a series of papers prepared by various individuals responsive to the charge of the Task Force. There is no difference of opinion within the Task Force as to the value of this material, but the various papers should be regarded more as the personal opinions of the authors rather than the opinions of the Task Force.

SUMMARY OF MAJOR FINDINGS AND CONCLUSIONS

This report is organized into five major sections that are closely related. The importance of cash-flow analysis can be seen to be a common theme uniting the different sections.

I. Risk

- The understanding and quantification of combination of risk requires an understanding of the underlying individual risks and an appreciation of the overall risk management process of an insurance company.
- Deviations from expected cash inflows and outflows are the essence of risk in an insurance company.
- Risks, both individually and in combination, are difficult to quantify with any degree of precision, particularly at the extremes (catastrophes).

II. Cash-Flow Analysis

- Insurance is fundamentally a cash-flow business, and therefore, understanding cash flows is essential to understanding the risks associated with the insurance business.
- Cash-flow analysis reveals unexpected characteristics of combinations of risk.
- Analysis of cash flows requires a computer model because of the volume and complex interrelationships of the data.

III. Cash-Flow Valuation

- Cash-Flow-Based Surplus (CFS), which is described in detail in Appendix C-1, provides the means of reducing complex cash-flow information to manageable proportions.
- As a measure of true economic strength, CFS is much more reliable than statutory surplus.
- By calculating CFS under a variety of scenarios, the valuation actuary can assess the adequacy of statutory reserves and surplus.
- Cash-flow analysis can be combined with utility theory as described in Appendix C-2 to provide a different perspective on risk analysis suitable for use in a management decision-making context.

IV. Application of Cash-Flow Analysis to Mismatch Risk

- The selection of an interest crediting strategy has surprisingly little effect on the ultimate economic cost associated with a change in interest rates for a typical SPDA product. There is, however, significant impact on levels of required surplus and the emergence of the statutory losses.
- The CORTF model has provided insight into the relationship between required surplus and other operating and management practices that affect cash flows, for example, FIT credits, dividend policy, reserve conservatism, that heretofore have not been recognized as determinants of surplus.

- Regression techniques provide a simple and intuitively satisfying means of expressing required surplus in terms of asset and liability durations, the change in the interest rate, and the initial earnings margin.

V. Combination of Risks

- Cash-flow analysis techniques are useful to understand and quantify deterministic scenarios where there is a *specified* level of the various risks. Such analysis, however, can only be undertaken with a fairly sophisticated computer model.
- Probabilistic analysis of combinations of risk using cash-flow analysis techniques is very complex, but simple tests suggest that cash-flow results can be materially different from those developed using "traditional" statistical formulas.
- Analysis of surplus requirements associated with extremes of risk must be regarded with some skepticism whether based on cash-flow or statistical techniques. Perhaps the greatest benefit of cash-flow techniques is improved understanding of the risk management process within an insurance company.
- Research on combinations of risk completed to date represents only a humble beginning. There is opportunity for much additional work in this area.

More detail is presented below and in the various attachments to this report.

I. RISK

Risk by its very nature is difficult to understand and not easily quantified. Any quantification process is necessarily limited by assumptions and the inherent impossibility of replicating the real world in mathematical formulas or models. The process is further clouded by the presence of catastrophic risk, which is basically unpredictable both as to timing and severity.

It must also be recognized that at an individual insurer level, there is great diversity in specific product terms, investment programs and management practices, which suggests the potential for material variation in the degree of risk assumed by different insurers. Thus, in any analysis such as that undertaken by the Task Force, it is very difficult to make generalizations about risk that can be extended to individual companies.

These considerations suggest that there must necessarily be uncertainty associated with any effort to quantify individual risks or combination of risk. This conclusion constitutes a word of caution to all those who in some way may rely on the findings of the Task Force. The immediate practical effect of these considerations was to focus the initial efforts of the Task Force on developing a sound understanding of risk and a practical and understandable methodology to quantify risk.

Appendixes A-1 through A-4 present some thoughts about risk management in an insurance enterprise and develop the following major points:

- Insurance is fundamentally a "risk-sharing" rather than a "risk-taking" function.
- Finite surplus requires insurers to be mindful of the level of risk they assume and to avoid unreasonable levels of "risk-taking" by appropriately controlling risk.
- Financing risk, that is, the ability to pay for losses associated with the manifestation of risk, is the primary area of concern to the CORTF.
- Deviations from expected cash inflows and outflows are the essence of risk in an insurance company.

II. CASH-FLOW ANALYSIS

As mentioned above and discussed in more detail in the A appendixes, deviations from expected cash inflows and outflows are the essence of risk in an insurance company. This concept was first explored by the C-3 Risk Task Force, which focused on the problem of understanding how cash flows shift in response to changes in the interest rate environment. The COR Task Force carried this analysis further and attempted to understand how cash flows are affected by *all* types of risks and by doing so gained a better understanding of the risks themselves. Also, a study of cash flows and their deviations in the presence of risk allowed us to better understand the effect of combinations of risk.

Once the expected cash flows of an insurance company are defined and modeled, deviations from expected cash flows can be illustrated for all types of risk. An asset default, for example, is the failure of a borrower to meet a scheduled interest or principal payment (that is, a cash inflow) under the terms of a debt instrument. Similarly, mortality or morbidity risk is evident in actual claims (that is, cash outflows) exceeding expected claims. It quickly becomes clear that cash-flow deviations are the common denominator of all risk and that an understanding of cash flows can provide a basis to understand the combinations of risk.

This line of inquiry led to the conclusion, which is more fully developed in Appendix A-2, that C-4 risk is realized ultimately in terms of cash flows that can be directly attributed to C-1, C-2 or C-3 types of cash-flow deviations. While C-4 risk is not directly addressed in this report, it should be understood that it is conceptually possible to model C-4 using cash-flow deviations with respect to the basic insurance risks: C-1, C-2 and C-3.

Recognition that insurance is fundamentally a cash-flow business is the basic premise upon which the work of quantifying and combining risks proceeded. But it quickly became apparent that the volume of data involved in this analysis is so large and the processing necessary to develop the data is so complex that one must use a computer model of an insurance company to fully understand the interrelationships. Similar models have been used in recent analyses of C-3 risk. The model underlying the work of the CORTF is similar to C-3 models but incorporates the ability to recognize the effect of C-1, C-2 and C-3 risks on future cash flows and balance sheets and to translate that effect into current statutory financial strength requirements.

A conscious attempt was made to keep the COR model as simple as possible. Complexities were introduced only where it was thought this was absolutely necessary to accurately measure and quantify risk. Experience with past modeling has demonstrated that if the model is kept simple, it is easier to understand the results, communicate them to others, and allow others to both duplicate results and adopt the model for risk measurement in their own companies. A complete description of the model is presented in Appendix B-1, but it is described briefly here.

Basically, the model comprises three parts:

- Cash Flows. As developed in this report, the cash-flow implications of the various risks represent the key to understanding and quantification of the risks.
- Gain from Operations. This provides the linkage between actual cash flows and successive balance sheets.
- Balance Sheet. The balance sheet consists of the usual statement of assets, liabilities and surplus. The ultimate effect of the actual cash flows on the surplus account can be tracked.

The model calculates "required surplus," that is, the minimum amount of statutory surplus, if any, that is required initially to assure that statutory solvency is maintained in all subsequent years. By varying the level of the statutory reserves, it is also possible to develop insight into the relationship between solvency and valuation standards. The model also calculates Cash-Flow-Based Surplus (CFS) which is described in greater detail below.

Thus, analysis of cash flows was clearly demonstrated to be useful and revealing. The sophistication of a computer model allows us to understand the multiple relationships of all the various components. Cash-flow analysis used with the computer model provides insight into the characteristics of individual risks, the interaction of multiple risks, and the impact of accounting methodologies on the assessment of risk.

III. CASH-FLOW VALUATION

With the development of a computer model that projects cash flows corresponding to specific risks and investment/management policies, the need for interpretation and quantification became apparent. Dealing with many cash-flow streams proved awkward and cumbersome. The Combination of Risks Task Force addressed this need by developing a new concept of insurance company surplus—Cash-Flow-Based Surplus (CFS). CFS reduces a series of cash flows to a single number and provides discipline for comparing different cash-flow streams.

A report on CFS was submitted to the CORTF on July 2, 1985. This report, with minor modifications, appears as Appendix C-1 to the present report.

CFS is defined to be the present value of anticipated asset cash flows, derived from the existing assets, less the present value of anticipated liability cash flows for a specified interest scenario. If computed properly, CFS can be interpreted as:

- The amount of *cash* that could be removed currently such that the remaining assets would be sufficient to mature all benefits on a true economic basis (for example, ignoring statutory accounting conventions for assets and liabilities) under the assumed interest rate and experience scenario.
- The present value of amounts removed over time such that the remaining assets can mature the benefits, given the same qualifications noted above. The amounts removed over time can be interpreted as shareholder dividends.

Thus, by calculating CFS, we can reduce a complex set of data to manageable proportions and determine a measure of the "true" financial strength of a block of business for a given interest rate scenario. While CFS is conceptually simple and straightforward, its calculation requires care. Attachment C-1(a) to Appendix C-1 provides considerable guidance in this direction. In particular, it is observed that the most useful results are obtained by tax-affecting the cash flows and discounting at an after-tax rate.

Viewing CFS as a measure of "true" financial strength, one may be tempted to assign meaning and importance to a single calculation as an absolute measurement. This use of the CFS concept is discouraged; each CFS result is meaningful only for the specific interest rate scenario assumed. The details on CFS given in Appendix C-1, therefore, concentrate on multiple calculations and the value of CFS as a *relative* measure of financial strength and earnings potential.

The problem with using CFS as an absolute measure is that it is impossible to reliably model all the factors bearing on the future cash flows for even a simple block of business. On the other hand, by examining CFS under various scenarios, it is possible to form an opinion as to a company's ability to mature its obligations and its real financial strength.

A comparison of CFS with statutory surplus is useful in assessing the true economic strength of an insurance organization. For example, suppose that CFS is negative and statutory surplus is positive. Imagine a situation in which assets are long and liabilities contain long-term interest guarantees. If interest rates rise substantially after the products are priced and sold, CFS may turn negative as a result of the mismatch and yet statutory surplus, which does not reflect changing interest rates, will remain positive. The picture of strength given by statutory accounting is illusory: this company is headed for insolvency unless there are sufficient profits in future new business to offset the impending losses on the existing block.

Interpretation of other possible relationships between CFS and statutory surplus is contained at the end of Appendix C-1. The message of these examples is that CFS does a good job of overcoming the shortcomings of statutory accounting (although neither CFS nor statutory surplus reflects the value of a company's future business). Thus, the relationship of CFS to statutory surplus can provide useful information about solvency and can be a valuable tool for the valuation actuary preparing an actuarial opinion.

Readers may also find of interest Appendix C-2, prepared by Oakley Van Slyke, which introduces utility theory into the problem of cash flow and risk analysis.

IV. APPLICATION OF CASH-FLOW ANALYSIS TO MISMATCH RISK

We have already noted that an understanding of cash flows is essential in any study of risk and that an early response to this realization was the development of techniques for dealing with cash flows within the COR computer model. As a useful by-product of these early CORTF achievements, it became feasible to examine the C-3 or mismatch risk in some depth. In particular, we studied how this risk was affected by factors that affect cash flows in an insurance company. These include the recognized risks associated with interest rate movements, varying asset length and changes in lapse rates, and also such company practices as shareholder dividends, reserve conservatism, rules for crediting interest to existing policies and FIT credits.

Mismatch risk is present in any product with long-term interest guarantees. A high degree of risk appears in the single-premium deferred annuity, since here the policyholder exercises considerable discretion in determining the liability cash flows: surrenders are highly sensitive to the difference between available new money rates and the rates credited on the contracts. In contrast, the typical GIC does not allow withdrawals at book value. Benefit accumulation contracts, such as those used with corporate profit-sharing plans, impose severe restraints on the policyholders' right of withdrawal, so these withdrawals are largely predictable and are only modestly sensitive to changes in market interest rates. In short, the SPDA product allowed the most interesting and general application of the new CORTF tools, and this was where we concentrated our attention.

The SPDA work was documented in a July 5, 1985 paper written by Mateja and Geyer. A slightly modified version of this paper is attached as Appendix D-1 to the present report. Before turning to this appendix, the reader may find it useful to review the operation of the COR computer model (Appendix B-1) and also the concepts of required surplus and cash-flow-based surplus (Appendix C-1).

The major findings of the C-3 risk study are as follows:

- The strategy for determining the level of interest credited to policyholders has a significant impact on levels of required surplus and the emergence of the statutory losses. There is, however, little impact on economic cost.
- There are several critical variables that affect the required surplus result. These include:
  - Investment strategy
  - Assumed future interest rates
  - Assumed future lapse rates
  - Owner dividend policy
  - Reserve conservatism and earnings margins
  - Federal income tax assumptions.

- One can determine a formula for estimating required surplus that takes into account the first three items above. This formula is defined and described in Appendix D-1.

This study also reinforced our views concerning the importance of cash-flow analysis and clearly demonstrated the interrelationship between reserves and surplus. This interrelationship is not clear in statistically based methodologies.

Since this study was completed, much has been learned about the use of duration as an immunization device. While the study correctly establishes that the relationship between asset and liability durations is a determinant of the level of risk, it does not appropriately recognize that where interest-sensitive cash flows are concerned, the calculation of duration must reflect the options associated with the cash flows. The study contained here is valid under the assumption that cash flows are fixed.

The report in Appendix D-1 is self-contained and can be referred to for additional details of our study of mismatch risk and SPDAs.

V. COMBINATION OF RISKS

Understanding and quantification of *combinations* of risk was the basic charge to the Combination of Risks Task Force. As noted earlier, the Task Force chose to pursue this charge using a cash-flow-based methodology. This proved to be a logical extension of the work of the C-3 Risk Task Force, which pioneered the application of cash-flow analysis to the problem of risk quantification.

The work of the C-3 Risk Task Force focused more on the methodology to quantify a specified C-3 risk scenario rather than on an objective measure of the surplus required to manage the full range of C-3 risk. Similar results were quickly developed for combinations of risks as well. But while a surplus amount for a specific combination of risks provided interesting and useful insight, it did not provide information into the probability of occurrence of the specified combination or the surplus level appropriate to manage the risks involved.

Appendix E-1 presents details of a cash-flow-based methodology developed at Aetna addressing the problem of associating a probability with specific levels of various risks. Donald Cody, Chairperson of the Committee on Valuation and Related Problems and an ex-officio member of the CORTF, has developed a second methodology in which risks are combined by means of a multivariate normal model. These two methodologies provided the opportunity to compare results based on differing methodologies. The Task Force is grateful to Mr. Cody for his contribution to our overall results.

While the work presented in Appendix E-1 admittedly is embryonic, it clearly establishes that a cash-flow-based methodology is a viable approach to the problem of combining risks. Its major shortcoming, which at the same time is perhaps its major strength, is that it requires considerable detail and discipline. The reward for overcoming the obstacle of the detail is enhanced understanding of risk and the financial operation of an insurance company. Above all, one begins to appreciate the sheer complexity of the problem of combining risks in practice.

In the process of working with these methodologies, considerable insight was developed into issues that must be considered when combining risks. Some of these issues are discussed briefly below:

- *Earnings Margin.* The earnings margin has long been recognized as the first line of defense in the management of risk. Obviously, the size of this margin will affect any surplus requirements. Also, cash-flow analysis techniques have made it clear that this margin can be used only once. When risks are tested individually, the earnings margin can be applied against the surplus needed for each risk, but when the risks are combined, it is applied only once against the total combination. Care must be taken, therefore, when combining required surplus results that have been analyzed individually first.
- *Magnification of Risk.* When combining risks, the manifestation of a second or third risk produces an additional surplus requirement. The total surplus thus required to manage the combination of risks can be greater than or less than the sum of the surplus required to manage each risk alone. Magnification of risk, that is, when the total is greater than the sum of the individual risks, has been demonstrated with C-3 risk in particular, due to its unique characteristics, but it also is true at times with the other types of risk as well.
- *Frequency Distribution of Risks.* Surplus required for both individual and combinations of risk is extremely sensitive to the assumed frequency distribution of the individual risks. This sensitivity is particularly acute, as would be expected, in the tails of the distribution where the extremes of risk occur. This phenomenon can be seen most easily with C-3 risk. Even with the other risks, a simple assumption of normalcy can produce results with material error.
- *Statutory vs. Economic Loss.* Statutory accounting greatly overstates the economic loss associated with asset default and greatly understates the loss associated with mismatch risk. Combinations of these risks expressed in terms of statutory required surplus must be interpreted with great care.

- *Required Shareholder Dividends.* In general, even as losses from the presence of risk manifest themselves, a certain level of shareholder dividends are paid each year. Dividends represent an important cash flow and need to be treated with care in cash-flow analysis. We believe they must be somehow reflected in statistical analysis as well.

When studying methodologies for combining and quantifying risk, it quickly becomes clear that dealing with a myriad of investment vehicles and insurance products, each with unique cash flows and each with its own sensitivity to economic changes, presents a range of risk beyond comprehension. The interrelationship of valuation, investment, pricing, dividend, and similar financial policies and their effect on risk management capability adds another dimension of complexity. Perhaps the most lasting impression of the effort to quantify combinations of risk is that any effort, whether based on a cash flow, a statistical or another type of analysis, must be regarded as a crude approximation to the "true" results and that, in the end, it is the process of studying risk that is the most valuable rather than any single answer one might develop. Understanding risk is the first step to successfully managing it.

CONCLUSIONS

The following major conclusions flow from our work.

- Deviations from expected cash inflows and outflows are the essence of risk in an insurance company. An in-depth analysis of cash flows appears essential to the understanding of both individual risks and combinations of risks. Such an analysis exposes the intricate interrelationships among the many variables that somehow get lost in statistical methodologies.
- Complete quantification of risks, whether by cash-flow, statistical or other analysis, is difficult, if not impossible, with any degree of confidence. The difficulty stems from the potential for catastrophe, the gulf between models and the realities of operating companies and the overriding presence of management risk (a C-4 risk), all of which are unquantifiable. Any effort to quantify individual risks or combinations of risks must in the final analysis be regarded as somewhat, if not highly, subjective.

ACKNOWLEDGMENT

The Task Force wishes to acknowledge with thanks the efforts of Linda Dinius and Joel Thomison, who were primarily responsible for preparing this summary report and editing the appendixes to be suitable as attachments.

APPENDIX A-1

RISK MANAGEMENT IN AN INSURANCE ENTERPRISE

JAMES A. GEYER AND MICHAEL E. MATEJA

Risk management is the cornerstone upon which the insurance business has been built. An insurance company, in fact, may be viewed as a special kind of risk manager.

As the Task Force deliberated its charge, it became clear that at a conceptual level the Task Force was fundamentally addressing the responsibilities of a risk manager, which are as follows:

1. Identification of risks
2. Control of risks
3. Transfer of risks
4. Financing of risks.

Understanding and quantification of combinations of risks clearly require a thorough understanding of each of these responsibilities with respect to the risks assumed by an insurance company both individually and collectively.

Each of these responsibilities is discussed in the following sections.

Identifying Risk

The Society of Actuaries' Committee on Valuation and Related Problems originally identified three risks faced by an insurance company in conducting its business. They are:

C-1 The risk of loss of value of an asset because of default or some impairment of the earnings capacity or value of the property or organization underlying the asset.

C-2 The risk of inadequate pricing of insurance contracts for reasons other than those connected with the C-1, C-3, and C-4 risks. Inadequate pricing can be the result of such things as random adverse fluctuations in experience, an inaccurate estimate of some element of cost such as expenses, frequency or severity of claims, or some disaster such as an epidemic, explosion, windstorm, or earthquake.

C-3 The risk of loss because of variations in the level of interest rates. Variations in the level of interest rates change the present value of the future stream of income from an asset in the absence of default or impairment of the earnings capacity or value of the property or organization underlying the asset. These same interest rate variations change the present value of future payments to be made with respect to insurance contracts. Finally, they also change the timing and amount of the cash flows by triggering exercise of withdrawal rights under insurance contracts or options to repay debt, thus producing further changes in the present value of assets and liabilities.

As the discussion of risk proceeded, it became clear that insurance companies faced "other" risks, common to all business enterprises, which could produce ruin. These other risks have been called C-4 risks. Various definitions of C-4 risk have been offered from time to time, but the nature of this risk has never been completely clear. The Task Force has developed some insight into the nature of this risk, which leads to the following definition:

C-4 The C-4 risk includes all risks associated with the operation of an insurance company, except those specifically associated with C-1, C-2 and C-3 risks. These risks are common to all business enterprises and may be generally thought of as any "development" that adversely affects the future of the business as a "going concern." Such "developments" are associated with managerial, regulatory, sociological, technological, competitive, and similar changes.

The nature of the C-4 risk is further developed in Appendix A-2. As developed in Appendix A-3, the risk of management error or mistake is perhaps the most common type of C-4 risk.

Controlling Risk

While an insurance company may properly be considered a risk manager, dealing with risks as outlined in the previous section, it is essential to understand that the practical conduct of the insurance business has been built around the concepts of risk-sharing and risk avoidance.

An insurance company fundamentally provides a means for individuals subject to certain risks to share or pool their risks. The sharing mechanism operates to compensate those for whom the risk manifests itself out of the contributions of all exposed to the risk. The risk-sharing mechanism is most obvious in the case of participating or experience-rated contracts where policyholders as a class "participate" in experience results.

The risk-sharing mechanism works only if each individual's contribution is reasonably proportionate to his exposure to loss (for obvious reasons) and only if the insurance company, acting as a sort of risk broker, stands ready to assure that those who sustain a loss will be compensated (again for obvious reasons). Providing this assurance places some degree of risk with the company, so that the company effectively becomes a risk taker. If the risk-sharing mechanism has been appropriately designed, the company can expect to realize gain from its risk-taking activity; of course, by definition, there is some probability that the company will realize losses. An insurance operation, thus, fundamentally involves exposure to gain or loss. Historically, insurance products and insurance operations generally have been designed with very low exposure to loss to the insurance company.

It is important to clearly understand that the nature of insurance companies' risk-taking has its origins in the risk-sharing function. If the company acted solely as a risk taker, it would be considered a gambler. While there are similarities between an insurance company and a gambler, the fact that the insurance company operates within a position of public trust is sufficient distinction to set it apart. Operating within a position of public trust, an insurance company must be a prudent risk taker and limit its risk-taking activity to its fundamental risk sharing function. While there are no criteria that define the limits of an insurer's risk-taking, the traditions of the industry, the objectives of the regulators and the expectations of policyholders all combine to impose an obligation on management of insurance companies to manage their risk-taking so that there will be a very high probability that all obligations will be fulfilled. This, in part, explains why insurance products historically have been designed with low exposure to loss. Risk-sharing, not risk-taking, is another cornerstone of insurance company operations.

The risks or uncertainties associated even with prudent risk-taking suggest that an insurance company needs financial resources (surplus) to assure that it can fulfill its obligations. Surplus provides assurance that those who have participated in the risk-sharing agreement will be compensated when a loss occurs.

An insurer blessed with unlimited surplus could manage any risk-sharing arrangement and need not be concerned about the level of risk-taking borne by the company. In the real world, risk management capacity, that is, surplus, is finite, and to assure their survival, insurers must be mindful of the level of risk-taking they assume.

In practice, an insurer avoids unreasonable levels of risk-taking by appropriately controlling risk. Over the years, the industry has developed widely respected techniques and standards of risk control with respect to C-1 and C-2 risks.

Investment (C-1) risk has been controlled through sound quality standards, effective underwriting and diversification.

Insurance (C-2) risk has been controlled through sound product design, conservative pricing and valuation, effective underwriting, expense control, and reduction or elimination of hazards that could lead to loss. Risk transfer through reinsurance also has been an effective control mechanism for insurance risks.

The C-3 risk involves both insurance and investment operations and so requires coordination of those operations to achieve effective control. The industry has only recently recognized and fully appreciated the C-3 risk, and control mechanisms are not as well developed as those for C-1 and C-2 risks. Research conducted to date clearly reveals that effective control in practical terms requires reasonable matching of asset and liability cash flows. However, there are as yet no accepted, objective standards to measure the degree of matching to assure reasonable control. The potential magnitude of the C-3 risk, relative to C-1 and C-2 risks, suggests that companies must be very concerned about control of this risk.

Based on the understanding of C-4 risk as outlined in Appendix A-2, the Task Force has concluded that effective control of C-4 risk, or at least that portion of the risk within control of management, is closely related to the idea of good management. Good management assures that a company will maintain or enhance its financial strength as a going concern.

By the very nature of C-4 risk, some portion related to action or events beyond the reach of management cannot be controlled. Even in this case, however, good management can be expected to minimize the adverse impact on the company.

While it is clearly essential for management to control the risks assumed, it must be recognized that even with reasonable efforts in this regard, there will always remain residual risks. Thus, the issues of transferring and financing risks must still be addressed.

Transfer of Risk

After having identified each risk involved in its business and developing controls to the extent it wants to do so or to the extent it is able to do so, an insurance company must decide whether it wants to retain the risk or transfer part or all of it.

Reinsurance is a method of risk transfer for an insurance company. It should be clear that risk transfer can have a material bearing on risk exposure in an insurance company. However, risk transfer is not directly relevant to the thrust of the work of the Task Force, so no further mention will be made of it.

Financing Risk

If the company decides to retain the risk, it must be prepared to finance it; that is, the company must be prepared to pay for any losses it may suffer because of materialization of that risk. Fundamentally, this is the area of concern to the Combination of Risks Task Force.

It has been customary to think of financing risk in terms of statutory surplus. Clearly, in an historical context statutory surplus has been a reasonable and objective measure of financial strength and ability to finance risk. This fact prompted the Task Force to undertake an analysis of actual surplus levels within the industry, and this analysis is presented in Appendix A-4. Major findings include the following:

- Capitalization is clearly inversely related to size.
- There is a markedly lower capitalization of companies that have annuities as a primary business.
- Annuity premium is a primary determinant of capitalization based on regression analysis.
- No clear trend in the level of capitalization over the period of study was apparent.
- There is no basis to objectively relate statutory capitalization to underlying C-1, C-2, C-3 and C-4 risks.

A statutory balance sheet represents a *point-in-time* summary of the assets and liabilities of an insurer, with financial strength representing the excess of the value of the assets over the value of the liabilities at that time. In reality, the assets and liabilities of the insurer represent convenient summarizations of *anticipated* cash inflows and outflows.

Positive financial strength in a current statutory balance sheet is a reasonable measure of ability to maintain solvency only under the following assumptions:

(1) The interest rate environment has been and remains relatively stable.
(2) Actual cash inflows and outflows are reasonably consistent with those anticipated.

Under these assumptions, statutory surplus will grow at interest and will increase or decrease with variations between actual and assumed experience in the determination of asset or liability values (no new business is assumed). Of course, shareholder dividends will reduce surplus.

When actual cash inflows and outflows develop differently than anticipated in the balance sheet and there is volatility in the external interest rate environment, the impact (favorable or unfavorable) flows through the income statement (Gain from Operations) and is reflected in a changed assessment of financial strength in the statutory balance sheets prepared as of subsequent dates.

Deviations from anticipated cash flows have become the norm rather than the exception in the operation of an insurance company, and the manifestation of risk can produce dramatic deviations from anticipated cash flows. With interest rates now exhibiting more volatility than heretofore considered possible, the stage has been set for unusual departures from the financial strength presented in a current statutory balance sheet.

The fundamental problem with the statutory balance sheet is that the statutory values of assets and liabilities are not necessarily consistent with their respective economic values. Accordingly, statutory financial strength does not equate to real "economic strength," which can be thought of as the excess of the market value of all the company's assets over the present value of the *actual* cash flows that will arise from all the company's existing insurance contracts. Economic strength may be materially different from the traditional statutory accounting definition of financial strength. To the extent that the cash flows implicit in the statutory valuation differ from what reasonably may be expected under then currently anticipated conditions, and to the extent that the assumed cash flows are discounted at interest rates which are not representative of current market rates, then assuredly the two measures of financial strength will be different.

Much can be learned from the relationship between statutory financial strength and "economic strength." If "economic strength" is significantly less than statutory financial strength, there is reason to be concerned as this indicates that statutory losses will likely develop in future years and that in the absence of an improved economic climate the insurer eventually could fail a statutory solvency test. On the other hand, if economic strength is greater than statutory financial strength, statutory gains can be expected in future years. While economic strength is useful to understand the real financial strength of an insurer, solvency is defined in terms of the statutory balance sheet.

From a practical standpoint, therefore, ability to finance risk requires sufficient financial strength in a current statutory balance sheet to mature all obligations as they fall due, that is, meet all cash outflows as they fall due, and meet a statutory balance sheet test of solvency over the lifetime of the block of business. This assessment must consider both normal and adverse assumptions with due recognition for the risks assumed by an insurer, which can produce deviations from anticipated cash flows.

Understanding ability to finance risk thus ultimately leads to a need to understand cash flows within an insurance company and how these cash flows may shift as the result of manifestation of risk. This is the approach that the Task Force expects to follow in its analysis of combination of risks. It is, in fact, the same approach suggested in the original work of the Trowbridge Committee (*RSA* 5, no. 1 (1979):257). The Conceptual Framework defined in that report reduced the assets and liabilities to cash-flow streams and defined the basic insurance risks as deviations from the expected cash-flow streams.

The notion of risk as cash-flow deviations is clear when there is either a payout of cash, as in the case of a death benefit or annuity payment (C-2 risk), or a failure to receive cash that was expected in the case of an asset default (C-1 risk). The effect in the case of losses associated with the C-3 risk is more subtle, but is still directly related to cash flows: if cash flows are "too high" in periods of low interest or "too low" in periods of high interest, the company will sustain losses. C-4 risk, as explained in Appendix A-2, is usually manifest in terms of C-1, C-2 and C-3 risks and thus also may be viewed in terms of cash flows.

Deviations from expected cash inflows and outflows are the essence of risk in an insurance company, and the Task Force believes that in the final analysis all efforts to quantify risk must be reduced to cash-flow deviations.

APPENDIX A-2

C-4 RISK

JAMES A. GEYER AND MICHAEL E. MATEJA

Any business enterprise is exposed to the risk of failure. While the acknowledged goal of business is to produce profits, the record shows that many businesses instead produce losses and eventually fail. During the last 40 years, the annual rate of industrial and commercial business failures has ranged from 0.2 percent to 0.6 percent. Statistics about failures of insurance companies are not readily available, but it is clear that over the years many insurance companies have indeed failed. It is also clear that many insurance company failures are not attributable to C-1, C-2 or C-3 risks in the conventional sense.

The reasons for business failures are varied and complex, ranging from the classic story of obsolescence associated with the buggy whip maker to the complex problems that led to the recent failures of once powerful and respected companies such as Itel Corporation, Manville Corporation, Revere Copper and Brass, and Wickes. Competition, overexpansion, poor management, poor business conditions generally, and plain bad luck are among the many reasons one can find in a study of business failures. More often than not, there is a complex series of interdependent factors that eventually lead to business failure.

Insurance companies as business enterprises are subject to most, if not all, of the same business risks that threaten and contribute to the failure of other business enterprises. These risks collectively have been called C-4 risks, and conceptually they include all risks not specifically associated with those attributable to C-1, C-2 and C-3 risks.

Little is known about the magnitude of the C-4 risk, but an analysis of actual insolvency data (see Appendix A-3) indicates that C-4 risk was a major contributing factor in many companies that were placed in rehabilitation or insolvency. This fact suggested that it would be necessary for the Combination of Risks Task Force to specifically recognize the C-4 risk in its work.

In order for the C-4 risk to be incorporated into the analysis of combination of risks, it was necessary to develop an understanding of the nature of the C-4 risk comparable to the understanding that the Task Force has with respect to the other risks assumed by an insurance company. Specifically, it was necessary to understand the C-4 risk in terms of cash-flow characteristics.

An example perhaps best illustrates the nature and cash-flow characteristics of C-4 risk. Suppose a financially sound company decides on an aggressive and somewhat speculative investment program. Suppose further that at some future date substantial losses are realized from the program. While the resultant losses are clearly C-1 losses, they really have their origin in the original management decision to pursue the more aggressive investment program. The company actually was "at risk" from the time that the new aggressive investment program was implemented. The original investment decision represented conscious acceptance of more investment risk, which we believe is appropriately characterized as C-4 risk. This C-4 risk ultimately manifests itself in cash-flow terms as C-1 risk.

Most C-4 risks seem to share timing and manifestation characteristics common to those in the above example. First, some action or event occurs that at some subsequent date contributes to or somehow produces a financial loss. Loss in this sense is a material negative deviation from expected cash flow. At the time the loss actually occurs, it can be clearly related to or identified with the basic C-1, C-2 or C-3 risks assumed by an insurer.

The action or event that precedes the reality of a future financial loss in many cases can be recognized as clearly adverse to the company's financial interests. Loss of a key executive, legislation producing increased taxes, new competition, introduction of a new product by a competitor, or new regulation of sales practices are examples of recognizable actions or events that could adversely affect a company's future financial interests. Other actions or events, such as the decision to pursue a more aggressive investment program as in the above example, are more difficult to recognize as adverse to the company's future financial interests. It is only when a loss is realized that it is possible to analyze and understand its origin.

Whether the action or event that could adversely affect a company's future financial interests is recognizable or not, it is unlikely that the future loss associated with the action or event could be quantified on any disciplined basis. Quantification conceptually is possible where the action or event is recognized in advance, but the process would be highly subjective and company specific. How does one put value on the loss of a key executive, new competition, or new legislation or regulation? Quantification becomes virtually impossible in the case of actions or events that are not recognized as potentially adverse to a company's future financial interests. In this latter case, the quantification process would require some means to assess actual company operating policies and procedures relative to pricing, investment, marketing, etc. against some industry norm or average. Companies pursuing more aggressive programs would require a larger C-4 risk component. The difficulty of introducing some discipline into such a process should be apparent.

The only thing that is clear about the cash-flow characteristics of C-4 risk is that the loss potential is real and that it is prospective. These characteristics led the Task Force to define the C-4 risk as ". . . any development which adversely affects the business as a going concern." This is purposefully a very broad definition, but we believe it appropriately characterizes the nature of the C-4 risk as developed above.

The prospective nature of C-4 risk suggests that at any given time an insurer (or the industry at large) probably has several "accruing" losses that have their origins in C-4 type actions or events. The problem of concern to the Task Force is how to treat such accruing losses, particularly those that cannot be clearly recognized as increasing the company's exposure to a future loss. How can such accruing losses be reduced to cash flows?

The Task Force concluded that until more research was conducted on the magnitude of C-4 risk, any choice of cash-flow deviation to reflect this risk would be arbitrary. Practical recognition of this risk, however, would be possible by introducing a degree of conservatism in the choice of cash flows for the recognized insurance risks.

APPENDIX A-3

ANALYSIS OF 1930s INSOLVENCIES
THE ROLE OF C-4 RISK

ANTHONY AMODEO

The purpose of this is to study the relationship between the C-4 risk and life insurance company failures on an historical basis. This study demonstrates that C-4 risk, in particular, management error, is indeed a prime factor in failures. As discussed in Appendix A-2, however, to see this relationship often means looking beyond the particular crisis that caused the insolvency to the ultimate risk-taking decisions.

The degree to which management could have anticipated the final problem is a sensitive point in any particular case, since it will involve personal reputations and possibly legal liability. In general, the following categories may summarize the possibilities:

1. Management could not have anticipated the eventual problem at all, and it resulted despite the normal, prudent operations of a life insurance company.
2. Management took risks beyond those acceptable as normal, prudent operations of a life insurance company, but they were expected to be profitable.
3. Management's prime constituency was outside the life insurance company, and it took risks that may have benefited themselves either directly or through other parts of the organization, but were not in the best interests of the life insurance company itself.
4. Management was incompetent.

The following case histories represent an attempt to discuss the foregoing considerations in a concrete form. The companies examined include all insolvencies from 1930 to 1939 where the loss to policyholders was initially estimated at $1 million or more. The information is drawn from the testimony of Alfred M. Best before the U.S. Senate Temporary National Economic Committee (TNEC). All value judgments are drawn strictly from this testimony and are purely illustrative. Any serious analysis of management decisions, especially of attempting to assign culpability, would certainly require more balanced and insightful investigation.

1. Home Life (Arkansas). The proximate cause of insolvency was asset default, nominally a C-1 risk. However, there was a concentration of risk in stock of a chain of small banks, which were controlled by the same management as that of the

insurance company. If the sole concern of management had been the running of the insurance company, its assets would not have been concentrated in that way. The ultimate cause would, therefore, seem to be a type-3 management position.

2. National Benefit (D.C.). The proximate cause of failure was overinvestment in its home office building (C-1), although bad management was ultimately to blame (type-2), plus lack of knowledge of insurance, which could be type-4.
3. Inter-Southern (Kentucky). Once again, the proximate cause was bad investments (C-1), but these were investments in other insurance affiliates. The investments were made not with the belief that they were valuable securities in themselves, but because of their role in the corporate structure. Once pyramiding started, it became impossible to stop. ". . . believing it to be a purely temporary expedient, . . . it was in there permanently." Mr. Best claimed it was not due to bad faith, but rather "terribly bad judgment." It appears that the ultimate cause was management type-3.
4. Mississippi Valley Life (Missouri). Bad investments and excessive expenses were involved, but "From almost every point of view that was a very badly managed life insurance company." Also, "they used better judgment in their own behalf than they did for the policyholders." (Type-3)
5. Old Colony Life (Illinois). Speculative investments, not related to management interests. (Type-2)
6. Security Life (Illinois). Pyramiding, using the assets to acquire more insurance companies for the holding company (type-3). Mentioned specifically as similar to Home Life and Inter-Southern, which were also type-3.
7. Northern State Life (Indiana). General asset depreciation, seemingly not the fault of management. (Type-1)
8. Missouri State Life (Missouri). Although there was some lack of conservatism in writing business and in investments, the main problem seemed to be the general economic troubles. If the economy had been reasonably healthy, it would seem that this company would have remained solvent. The cause was C-1 and C-3, following a reasonable type-1 management course of action.
9. National Life (Illinois). Excessive concentration in one investment, common stock of the Continental Illinois Bank. This was not a case of interlocking directorates, but rather of a poor investment decision. Type-2 management was the real cause, rather than C-1.
10. Royal Union Life (Iowa). The cause of this failure was bad investments acquired through reinsurance transactions, rather than directly. This could have been anticipated. This is C-1 resulting from type-2 management.
11. Independent Life (Tennessee). Although investment losses precipitated this solvency (C-1), the ultimate cause was felt to be "rather generally incompetent management." (Type-4)
12. Peoria Life (Illinois). Overconcentration in a large hotel and the home office. The investments were not sound to begin with, and the Depression caused policy cash

demands to combine with asset losses so that these deficiencies came to light. Once again, although asset losses precipitated the insolvency, management error was clearly at fault. (Type-4)

13. Register Life (Iowa). This was a case of "honorable management" having "made a bad guess" in mortgage investments and suffering asset losses. C-1, with no management error. (Type-1)
14. Pacific States Life (Colorado). "The company was never a well-managed concern from the time it started." (Type-4)
15. Federal Reserve Life (Missouri). There was considerable discussion of the relationship between management, the insurance department, and other related companies. There seems to be a type-3 management problem here.
16. Continental Life (Missouri). The president owned a bank and used each for his own benefit. Evidently, the interrelationship between these institutions allowed some obfuscation. A pure management problem. (Type-3)
17. Detroit Life (Michigan). Mortgage foreclosures were the immediate cause of loss. Not only were "bad investments" at fault but also "bad management." It's not clear whether this is type-2 or type-4.
18. American Life (Michigan). Overconcentration of assets in the Rio Grande Valley, but not for personal gain of management. Nonetheless, this could have been anticipated as unwise. (Type-2)
19. Pacific Mutual. The cause was underwriting losses on disability policies. Although this seems a pure C-2 event, there was considerable criticism of the judgment shown by management. (Type-2)
20. Illinois Life. Overconcentration of assets in two hotels, one of the which was promoted by the principal officers and stockholders of the life company. The outside constituency implies type-3 management. (Type-3).

The essential understanding that can be derived from this study is that, contrary to most expectations, the severe economic difficulties of the Depression did not cause these failures so much as they brought some fundamental shortcomings to light.

The above represents a fairly large sampling of companies and should be highly representative of the experience during the Depression. One could surmise that experience of smaller companies would show that mismanagement predominated to an even greater extent. Mr. Best's testimony indicates that his rating had been withheld from many or most of these 20 companies. Thus, to an astute outsider, it was evident that mismanagement existed before the final difficulties occurred. This lends great credence to the criticisms of management that ran through these hearings.

If there was ever a period in which one would expect failures resulting from C-1, C-2 and even, to some extent, C-3 risks, it would certainly be the Great Depression. Yet, we find that C-4 predominates. Among the 20 companies cited, in only three cases was management considered prudent and honest, suffering a fate beyond their control. The management of all the other companies was roundly criticized. The majority of the companies failed because the management's risk-taking posture fell somewhere between imprudent and incompetent. There was also a remarkable number of cases in which the life company assets were used for the benefit of persons other than policyholders, either directly for the benefit of management, or indirectly to support related entities.

Regarding this misuse of insurance company assets, it appears that small, closely held stock companies can be under considerable pressure to service the shareholders by shoring up their other businesses, particularly banks. This can occur to the long-term investment detriment of the insurance company. Similar pressures arose to create "monuments," as in home office buildings, hotels, or in one case a "Garden of Eden" in Texas. For these reasons, it became common to overvalue assets and prematurely distribute surplus as shareholder dividends when the risks would have justified retaining earnings. Mutual companies and widely held stock companies appear to have been insulated from these pressures or were at least able to afford any "monuments" that they built.

This record indicates that, even in a severe economic dislocation, well-managed life insurance companies have been able to control losses from C-1, C-2 and C-3 risks to the point at which they do not result in insolvency. The consistent cause of failures is shown to arise in the C-4 risk.

Interestingly, this conclusion regarding life insurance companies is quite consistent with the causes of failure for U.S. companies in general. In *The Business Failure Record* published by Dun & Bradstreet, 16,794 company failures of 1981 are classified by cause of failure, based upon the "opinion of informed creditors and information in Dun & Bradstreet Reports." The results are as follows:

Cause	Percentage of Total Failures
Mismanagement	
Lack of experience in the line	11.1%
Lack of managerial experience	12.5
Unbalanced experience	19.2
Incompetence	45.6
Total	88.4%
Other	
Neglect	0.7%
Fraud	0.3
Disaster	0.5
Reason unknown	10.1

Note that "disaster" caused just 0.5 percent of all failures, while mismanagement was judged responsible for 88.4 percent of all failures.

APPENDIX A-4
INDUSTRY CAPITALIZATION

STEPHEN SMITH AND PAUL TWOROG

This study addresses two major issues in actual insurance industry capitalization. The first is the level of industry capitalization and the second is the trend over the past five years. For the purposes of this study, capitalization will be defined as the ratio of Capital + Surplus + MSVR to Total Liabilities − MSVR. From a "legal" perspective, MSVR is a liability, but from a financial management perspective it can be considered as part of total capital and surplus and is thus included in our capitalization measures. A total of 826 life/health companies were selected based on complete availability of needed data for the years 1979 to 1983. All data are on a statutory basis and reflect individual companies' filings of annual statements rather than aggregation of subsidiaries to a corporate level. Analysis focused on testing the hypothesis that capitalization is dependent on a variety of factors. These included total assets, total premium, Best rating, percentage of total premium attributed to group life and group A&H, percentage of total premium due to annuities, and whether the company was a stock or mutual company. All data pertaining to lines of business come from the appropriate columns of annual statement, page 5, and include row 1 (premium and annuity consideration) and row 1A (annuity and other fund deposits). Results show that all these factors do influence the level of capitalization and that this relationship has been statistically stable over the study period. The primary determinant of capitalization proved to be the percentage of premium accounted for by annuities. No clear trend in the level of capitalization over time was discovered.

Methodology

Two different approaches were used to analyze the data. The first was simply to divide the sample into various groups. Initial divisions were based on total assets, total premium, Best rating, and stock versus mutual classification. Asset and premium groups were further divided by line of business criteria. A company with more than 25 percent of its total premium in a category was defined as having that line as a primary business; otherwise the line was considered secondary. All the grouping divisions are based on 1983 data.

The second approach utilized was a multiple regression. An equation with the capitalization ratio expressed as function of the hypothesized factors was estimated. This method allows us to examine all the companies and factors simultaneously. The equation was specified for all companies for each individual year. Tests were also performed to determined whether the estimates of the parameters have changed during the study period.

Results (Grouping)

Tables A-4-I through A-4-IV display the mean capitalization ratio for the various groups and subgroups. Table A-4-I shows the total premium breakdown with the size class groupings defined at the bottom. Concentrating first on the initial breakdown by total premium, it is clear that there is an inverse relation between company size and level of capitalization in any particular year. For example, in 1983 the 654 small Class I (premiums less than $100 million) companies had an average capitalization ratio of 0.2654, while the 129 Class II (premiums greater that $100 million and less than $500 million) had a mean capitalization ratio of 0.1654. This pattern continues right through to 20 very large Class IV (premium greater than $1 billion) companies whose capitalization ratio averages only 0.0774. Looking at this same breakdown by premium size over time (reading across the columns) does not reveal any clear trend. The same conclusions hold if we examine the similar breakdowns by asset size shown in Table A-4-II.

Looking now at the further subdivisions of premium and assets by line of business, there is a markedly lower capitalization of companies that have annuities as a primary business. Here, capitalization also has trended down over the sample period. This may, however, just reflect the increasing importance of annuity business over the period. The group life and A&H line of business grouping does not reveal any obvious patterns.

Table A-4-III shows us the data based on Best rating and provides little insight into the problem.

Table A-4-IV divides the sample into a stock and nonstock grouping. Stock companies have a significantly higher mean capitalization ratio than nonstock companies. It should be noted again that the sample is based on individual filings rather than aggregations. Many mutual companies may have wholly owned stock subsidiaries that would appear here as stock companies if that is how they file. No trend for either stock or nonstock companies over time is discernible.

TABLE A-4-I

MEAN CAPITALIZATION RATIO
TOTAL PREMIUM BREAKDOWN

	1979	1980	1981	1982	1983
Total Sample N = 826	0.2458	0.2523	0.2472	0.2406	0.2417
Premiums (Class I) N = 654	0.2644	0.2718	0.2702	0.2644	0.2654
Annuity Primary N = 134	0.2459	0.2343	0.2272	0.2107	0.2045
Annuity Secondary N = 520	0.2691	0.2815	0.2812	0.2783	0.2811
Group Life A&H Primary N = 143	0.2629	0.2714	0.2637	0.2565	0.2648
Group Life A&H Secondary N = 511	0.2648	0.2719	0.2720	0.2661	0.2656
Premiums (Class II) N = 129	0.1914	0.1941	0.1750	0.1626	0.1654
Annuity Primary N = 46	0.1607	0.1524	0.1231	0.0931	0.0945
Annuity Secondary N = 83	0.2084	0.2172	0.2037	0.2011	0.2046
Group Life A&H Primary N = 55	0.1824	0.1790	0.1593	0.1657	0.1659
Group Life A&H Secondary N = 74	0.1980	0.2053	0.1867	0.1603	0.1650
Premiums (Class III) N = 23	0.1736	0.1778	0.1504	0.1446	0.1384
Annuity Primary N = 12	0.0928	0.0933	0.0844	0.0730	0.0738
Annuity Secondary N = 11	0.2617	0.2700	0.2223	0.2226	0.2089
Group Life A&H Primary N = 12	0.1887	0.2041	0.1687	0.1714	0.1517
Group Life A&H Secondary N = 11	0.1571	0.1492	0.1304	0.1153	0.1240
Premiums (Class IV) N = 20	0.0740	0.0762	0.0731	0.0763	0.0774
Annuity Primary N = 6	0.0522	0.0545	0.0511	0.0513	0.0513
Annuity Secondary N = 14	0.0834	0.0855	0.0825	0.0870	0.0885
Group Life A&H Primary N = 14	0.0797	0.0818	0.0783	0.0821	0.0839
Group Life A&H Secondary N = 6	0.0608	0.0633	0.0710	0.0627	0.0622

Class I —Companies with total 1983 premium less than $100 million.
Class II —Companies with total 1983 premium greater than $100 million and less than $500 million.
Class III —Companies with total 1983 premium greater than $500 million and less than $1 billion.
Class IV —Companies with total 1983 premium greater than $1 billion.
Primary —More than 25% of total premium.
Secondary—Less than 25% of total premium.

TABLE A-4-II

MEAN CAPITALIZATION RATIO
TOTAL ASSET BREAKDOWN

	1979	1980	1981	1982	1983
Total Sample N=826	0.2458	0.2523	0.2472	0.2406	0.2417
Assets (Class I) N=467	0.3016	0.3123	0.3095	0.3016	0.3022
Annuity Primary N=92	0.2790	0.2680	0.2589	0.2408	0.2371
Annuity Secondary N=375	0.3071	0.3232	0.3219	0.3165	0.3182
Group Life A&H Primary N=99	0.3082	0.3213	0.3102	0.3028	0.3135
Group Life A&H Secondary N=368	0.2998	0.3099	0.3093	0.3012	0.2922
Assets (Class II) N=212	0.1993	0.2010	0.1934	0.1890	0.1877
Annuity Primary N=46	0.2158	0.2087	0.1856	0.1573	0.1452
Annuity Secondary N=166	0.1947	0.1989	0.1956	0.1978	0.1994
Group Life A&H Primary N=66	0.1922	0.1909	0.1761	0.1748	0.1740
Group Life A&H Secondary N=146	0.2025	0.2056	0.2012	0.1954	0.1939
Assets (Class III) N=64	0.1545	0.1508	0.1398	0.1325	0.1375
Annuity Primary N=21	0.1223	0.0970	0.0866	0.0782	0.0800
Annuity Secondary N=43	0.1703	0.1770	0.1657	0.1590	0.1656
Group Life A&H Primary N=25	0.1529	0.1508	0.1385	0.1443	0.1505
Group Life A&H Secondary N=39	0.1556	0.1507	0.1407	0.1250	0.1291
Assets (Class IV) N=67	0.1367	0.1392	0.1312	0.1261	0.1347
Annuity Primary N=33	0.0988	0.0968	0.0881	0.0703	0.0741
Annuity Secondary N=34	0.1734	0.1803	0.1731	0.1803	0.1934
Group Life A&H Primary N=24	0.1443	0.1463	0.1371	0.1403	0.1331
Group Life A&H Secondary N=43	0.1324	0.1353	0.1280	0.1182	0.1355
Assets (Class V) N=16	0.0578	0.0607	0.0565	0.0582	0.0560
Annuity Primary N=6	0.0584	0.0606	0.0567	0.0583	0.0549
Annuity Secondary N=10	0.0574	0.0607	0.0565	0.0681	0.0566
Group Life A&H Primary N=10	0.0522	0.0555	0.0505	0.0513	0.0501
Group Life A&H Secondary N=6	0.0671	0.0694	0.0666	0.0698	0.0658

Class I —Companies with total 1983 assets less than $100 million.
Class II —Companies with total 1983 assets greater than $100 million and less than $500 million.
Class III —Companies with total 1983 assets greater than $500 million and less than $1 billion.
Class IV —Companies with total 1983 assets greater than $1 billion.
Class V —Companies with total 1983 assets greater than $5 billion.
Primary —More than 25% of total assets.
Secondary—Less than 25% of total assets.

TABLE A-4-III

MEAN CAPITALIZATION RATIO
TOTAL COMPANY RATE BREAKDOWN

	1979	1980	1981	1982	1983
Total Sample N=826	0.2458	0.2523	0.2472	0.2406	0.2417
Rated A+ N=232	0.2154	0.2197	0.2118	0.2107	0.2116
Rated A N=206	0.2715	0.2711	0.2679	0.2550	0.2497
Rated B+ N=122	0.2377	0.2521	0.2660	0.2634	0.2714
Rated B N=76	0.2617	0.2732	0.2853	0.2704	0.2730
Rated C+ N=52	0.2489	0.2686	0.2373	0.2454	0.2575
Rated C N=27	0.2519	0.2573	0.2043	0.2010	0.2255
Without Assigned Rating N=11	0.2567	0.2628	0.2511	0.2386	0.2321

TABLE A-4-IV

MEAN CAPITALIZATION RATIO
STOCK VERSUS NONSTOCK BREAKDOWN

	1979	1980	1981	1982	1983
Total Sample N=826	0.2458	0.2523	0.2472	0.2406	0.2417
Stock Companies N=705	0.2638	0.2694	0.2640	0.2553	0.2567
Non-Stock Companies N=121	0.1413	0.1526	0.1493	0.1554	0.1542

Results (Regression)

While the grouping approach helps identify some of the main factors in capitalization and presents a useful summary view of the data, a more complete summarization can be derived if we look at all factors and companies at the same time. Specifically, a regression equation of following form was estimated:

$$\begin{aligned}\text{Capitalization Ratio} = {} & \text{Intercept} + B_1\,(\text{Total Assets}) + B_2\,(\text{Total Premium}) \\ & + B_3\,(\text{Group Life} + \text{A\&H/Premiums}) \\ & + B_4\,(\text{Annuities/Premiums}) + B_5\,(\text{Best Rating}).\end{aligned}$$

Preliminary results showed that when total assets were used in conjunction with total premium, neither was significant. Used separately, the total premium term produces a marginally better fit so that result is reported. The variables, change in premium and change in assets, were also tested but proved insignificant. Table A-4-V shows the estimates of the parameters as well as the significance of each. All the results appear reasonable, with the percentage of total premium accounted for by annuities having the greatest

TABLE A-4-V

MEAN CAPITALIZATION RATIO
REGRESSION RESULTS

Parameter/Significance	1979	1980	1981	1982	1983
Intercept	0.184	0.183	0.179	0.193	0.184
Significance	100%	100%	100%	100%	100%
Total Premium	−2.71E−11	−2.44E−11	−3.02E−11	−3.19E−11	−3.14E−11
Significance	96%	96%	98%	100%	99%
Group Life and A&H/Premium	0.003	−0.021	−0.004	−0.028	−0.055
Significance	10%	68%	15%	100%	100%
Annuities/Premium	−0.229	−0.244	−0.234	−0.236	−0.249
Significance	100%	100%	100%	100%	100%
Rated A+	−0.922	−0.005	−0.008	−0.001	0.018
Significance	74%	21%	30%	6%	62%
Rated A	0.014	0.031	0.027	0.030	0.043
Significance	54%	87%	80%	87%	96%
Rated B+	−0.005	0.021	0.038	0.035	0.043
Significance	19%	63%	89%	88%	94%
Rated B	−0.007	0.015	0.018	0.011	0.030
Significance	24%	45%	52%	36%	75%
Rated C+	−0.023	−0.001	−0.038	−0.045	0.004
Significance	55%	3%	79%	85%	10%
Rated C	−0.094	−0.086	−0.081	−0.047	−0.036
Significance	98%	95%	94%	78%	65%
Stock	0.115	0.116	0.116	0.100	0.100
Significance	100%	100%	100%	100%	100%

Weighted R-Square = 0.15
Significant at 100% level.

explanatory power. It also should be noted that while individual rating classes may not be particularly significant, 1983 results are a good example, the joint significance of all rating terms taken together exceeds the 90 percent level. In other words, while it is difficult to determine precisely how a specific rating like A+ affects capitalization, it can be shown that rating does affect capitalization.

The estimates of the parameters have changed over the study period. The most important change occurred in 1983 when the percentage of business in Group Life and A&H became significant. The standard test for structural change in the parameters was conducted. That is, the hypothesis that each individual term estimate for 1983 equals the individual estimate for 1979 was tested. In no case could the hypothesis be rejected. Thus, while parameter estimates have undergone change, it has not been significant in a statistical sense.

Conclusions

The best summarization of the study is found in the estimated regression equation. This can be used to generate predicted capitalization levels, given the characteristics of a company. For example, in 1983 a nonstock company with no assigned rating would be predicted to have a capitalization ratio of:

$$0.184 - 3.14\text{E}{-11}\ (\text{Premium } \$) - 0.055\ (\text{Group Life \& A\&H/Premium}) - 0.249\ (\text{Annuities/Premium}).$$

A similar stock company would have an additional term, namely, 0.104. An A+ rated company would also have an additional term, 0.018.

In summary, strong conclusions about the level of capitalization have been reached. The estimated equations appear stable and reasonable. Percentage of premium from annuity business was the most important term. The significantly higher capitalization of stock companies was the most surprising result. Less light has been shed on the trend of the capitalization ratio over time. More history is perhaps needed before conclusion can be drawn on that issue.

APPENDIX B
COMBINATION OF RISKS TASK FORCE COMPUTER MODEL

TERRYN J. BOUCHER

Overview

The Combination of Risks computer model is a FORTRAN program written to study the effects of individual risks and their combinations on reserve adequacy and required surplus. The model essentially tracks cash flows and prepares income statements and balance sheets over time. The original model was developed by Jim Geyer and Tim Corbett at the Aetna.

The initial focus of the model was on C-3 risk and its effect on cash flows. Once the C-3 risk had been studied for the SPDA report, the primary focus became the study of combinations of risk. Asset default cash flows were modeled and an expense decrement was added, allowing us to study C-1 and C-2 risk, respectively. By studying all three risks in the model in this way, we were able to begin to understand the effects of a combination of risks acting simultaneously.

The basic purpose of the model is to determine required surplus for a given interest and experience scenario. Required surplus is the amount of current statutory surplus needed to assure solvency at all future durations. Initial assets are set equal to initial liabilities, plus initial surplus, if any. The model then projects forward for 40 years. If at any time in this projection period the surplus becomes negative, then a greater initial surplus is required. The model uses an iterative procedure, running 40-year projections with varying amounts of initial surplus, until the minimum amount of initial surplus required to keep the surplus balance from ever becoming negative is found. This minimum is defined as required surplus. The model then generates two sets of output: one using the original initial surplus, if any, and the other using required surplus as calculated by the model.

At the end of each of the first ten years, Cash-Flow-Based Surplus (CFS) is calculated, equal to the present value of future asset cash flows less the present value of future liability cash flows. The cash flows and discount rates used in this calculation assume continuation of the then current interest rate. Thus these are "market" values of both assets and liabilities.

The model contains options for various situations. There are two asset options, three liability options, plus options for features such as asset calls, asset defaults, varying credited rates, and expense factors. These options, along with input and output, are explained in the paragraphs that follow.

Asset Options

One of the following options must be chosen as an input assumption:

1. The first option assumes that a block of business has been built up over a prior ten-year period, during which interest rates remained constant and there were no asset defaults. Insurance cash flows must be entered for each of these ten years. These cash flows are then invested according to specified assumptions to develop the initial assets.
2. The second option develops initial assets using amounts of principal rolled over read from a special asset file.

Asset Calls

The model contains a provision for asset calls. The input are the call protection period and the break-even spread, which is the amount interest rates must drop before it is economically feasible for the borrower to call the assets. There are no call premiums.

Asset Defaults

The model contains a provision for asset defaults. It is assumed that when an asset defaults, no principal or interest is received for a period of n years. After n years, there is a settlement for a renegotiated asset equal to $x\%$ of the cash flows for the original asset. The number of remaining payments is the same as the number at the time of the default of the original asset.

Liability Options

One of the following liability options must be chosen:

1. In option one, lapse rates are applied to beginning-of-year liabilities to determine liability cash flows. A formula that is a function of the spread between the new money rate and the credited rate is used to determine the lapse rates.
2. In option two, lapse *rates* are read directly from an input file.
3. In option three, actual lapse *amounts* are read from an input file.

Credited Rates

The initial and guaranteed credited rates must be entered into the model. There are three alternatives for specifying credited rates for later years:

- Set all future credited rates equal to the initial rate
- Read in specific credited rates from an input file
- Have the model generate later credited rates based on the average earned rates.

Expense Factors

Expense factors applied to initial liabilities can be used to increase or decrease expenses. One use of this capability is to model C-2 risks. Extra mortality, for example, can be translated into a percentage of liabilities and can thus be entered as an expense factor.

Specifics of the Model—Input

The following is a list of the input parameters for the model:

1. Asset Assumptions. The interest rate that initial assets are invested at must be specified. If asset calls are to be used, a call protection period and break-even spread must be specified. The asset option must be chosen (options described above). The type and length (not to exceed 30 years) of the initial asset as well as reinvested assets must be chosen. Also, asset default information must be specified.
2. Liability Assumptions. Initial surplus and the initial liabilities must be specified. A reserve factor setting the reserve equal to a percentage of the fund value also can be specified. The initial and guaranteed minimum credited rate must be entered along with how future credited rates are to be calculated. The liability option for lapse rates must also be chosen (options described above).
3. Other Modeling Assumptions
    (a) FIT Percentage. Federal income tax equals this input percentage of Gain From Operations. If FIT is negative, it can be set equal to zero. A capital gains tax rate is also specified.
    (b) Percentage of Positive GFO Paid to Shareholders as Dividends. This percentage is applied to GFO after FIT. A minimum shareholder dividend may also be specified as a percentage of beginning of year liabilities.

(c) Miscellaneous. The items listed below can vary in each of the first 10 years of the projection period. The rates in years 11 through 40 are set equal to the rate in year 10.
- New money rates for both borrowed and invested money.
- Asset default rates and the percentage of the asset that will be recovered.
- The prior ten-year insurance cash flows for asset option 1.
- Expense factors.

Processing Cash Flows

All input assumptions are used to define a series of cash flows. If the net cash flow at the end of any year is positive, then the cash can be invested. If it is negative, cash must be borrowed. These investments and borrowings generate more cash flows and so on for 40 years. It soon becomes apparent why a computer model is needed. The model keeps track of these cash flows year by year and tracks the net cash flow throughout the 40-year projection period.

Output

There are three sections of output:

1. Input parameters page
2. Forty-year projection period assuming initial surplus entered
3. Forty-year projection period with required surplus as computed by the model

The input parameters page also contains a summary of results giving present values at time zero, duration statistics, and the required initial surplus.

The 40-year projection is shown in four parts:

1. Summary of Operations. Investment income is shown separately for initial assets, later investments, and "ex-defaulted" assets. For subsequent investments, it is also broken down between invested money and borrowed money. An average earned rate is calculated by dividing investment income by the sum of the beginning-of-year asset and non-admitted asset balances. Interest credited equals the credited rate multiplied by beginning-of-year fund value.

$$\text{Earnings} = \text{investment income} + \text{reserve released} - \text{interest credited} - \text{defaulted assets} - \text{net expenses} - \text{FIT}.$$

2. Cash Flows. All cash flows are assumed to occur at year-end. Total rollover is separated into the same components as investment income in the Summary of Operations.

   Asset cash flows = investment income + rollover − assets called

   Liability cash flows = lapse benefits + expenses + FIT

   Shareholder dividends are shown separately. Thus,

   Net cash flow = asset cash flow − liability cash flow − shareholder dividends

3. Balance Sheet. The Balance Sheet has four accounts: assets, liabilities, surplus, and nonadmitted assets. Assets are increased by investment income and decreased by the difference between book value and market value of defaulted assets, and by lapses, expenses, FIT, and shareholder dividends. Liabilities are increased by interest credited and decreased by lapses and reserves released. Surplus is increased by GFO after FIT and decreased by shareholder dividends. Nonadmitted assets are increased by the difference between book value and market value of defaulted assets and decreased at settlement of assets by the amount of principal lost upon settlement.
4. An analysis section containing cash-flow present values is included with the first 10 years of the initial 40-year projection. Duration statistics for projected asset and liability cash flows calculated at the end of each of the first 10 years are also included.

Determination of Required Surplus

Required surplus is found by an interpolation and looping procedure. The procedure stops when the lowest surplus balance is within the tolerance range of 0.0005 percent about zero, or when an initial surplus balance of zero produces no negative surplus balances. Termination automatically occurs if a required surplus amount has not been found after 50 iterations. When required surplus is found, it is added to initial assets and is assumed to be invested in the same bonds and mortgages as the initial assets.

Example

Attached is a portion of output from a sample run. An SPDA is being modeled and this run corresponds to the low crediting strategy discussed in Appendix D-1. The following assumptions were used:

- The FIT rate is 36.8 percent and the capital gains tax rate is 28 percent. Negative FIT is not set equal to zero.
- 50 percent of positive GFO is paid as shareholder dividends, with no minimum dividend.
- Interest rates are 14 percent for initial assets and 20 percent for later investments and borrowing. Lapse rates are 25 percent. The credited rate is a constant 13 percent.
- There are no asset defaults.
- Money is invested in 15-year mortgages and borrowed assuming constant rollover over 10 years.

The initial required surplus calculated is 2.907 percent of initial liabilities. CFS at issue is $ – 14,892.

To facilitate an understanding of the model, the first two years of this example will be discussed at length here. Refer to Table B-1.

Sections I–IV show most of the input assumptions described above. Section V is a summary of the results of this specific run. The present values at time zero of the asset and liability cash flows are shown, both pre- and post-tax, and their difference is Cash-Flow-Based Surplus, or CFS. Note, as derived in Appendix C-1, that CFS equals the present values of shareholder dividends. Duration statistics are shown and then the required surplus as calculated by the model. (The number 21 above the required surplus amount indicates that this amount was determined using 21 iterations of the model. Remember, as noted above, that the model automatically stops looping once the iteration procedure reaches 50 iterations.)

The calculations for years 1 and 2 are described below in detail:

Year 1

Under the Summary of Operations, investment income in the first year is equal to the initial assets of 1,000,000 times the 14 percent initial new money rate (initial assets equal initial liabilities, 1,000,000 in this case, plus initial surplus, 0 in this case). There are no subsequent investments made in the first year as all cash flows are assumed to occur at year-end.

The average earned rate is calculated as:

$$\text{Avg Earned Rate} = \frac{\text{Investment Income}}{\text{BOY Assets} + \text{BOY Nonadmitted Assets}}$$

Beginning-of-year assets and nonadmitted assets are found on the Balance Sheet. (Nonadmitted assets are generated when asset defaults are assumed.)

TABLE B-1

I. Asset Assumptions		
A.	Interest rate initial assets are invested at	0.14000
B.	Asset calls	N
	Call protection period	0
	Break-even spread	0.00000
C.	Option 1 (initial block of assets developed from prior ten years of insurance cash flows (ICFs) and from chosen investment vehicle)	Y
	Asset Type	2
	Length of asset	15
D.	Asset default assumptions	
	Years asset is in default	0
	Fraction of defaulted assets recovered	See IV.C.
	Default future positive reinvestments	M
E.	Option 2 (principle rollover for initial assets is read from asset input file)	N
F.	Reinvestment assumptions	
	Positive net cash flow asset type	2
	Length of asset	15
	Negative net cash flow asset type	3
	Length of asset	10
II. Liability Assumptions		
A.	Initial surplus	0
B.	Initial liability reserve	1,000,000
C.	Reserve factor	1.00000
D.	Expense option (see expense factors IV.D)	
E.	Credited/reserve interest rates	
	Initial rate	0.13000
	Minimum rate	0.4000
	Later rates set equal to initial rate	Y
	Rates read from liability input file	N
F.	Option 1 (lapse formula for lapse rates)	N
	Single pay (S) or installment (I) annuity	S
	Moderate (M) or high (H) lapse rates	M
G.	Option 2 (lapse rates are read from Liability input file)	Y
H.	Option 3 (actual lapse amounts are read from Liability input file)	N
	Initial liability reserve calculated using Credited/reserve interest rates	N
III. Other Modeling Assumptions		
A.	Federal income tax rate	0.36800
B.	Capital gains tax rate	0.28000
C.	Negative fit set equal to zero	N
D.	Positive GFO paid as owner dividends	0.50000
E.	Minimum dividend applied to BOY liabilities	0.00000

TABLE B-1—*Continued*

IV. Miscellaneous

Yr	A. New Money Rates		B. Rates of Default	C. Fraction Recoverable	D. Prior to ICF's	E. Expense Factors	F. Asset Type Input
	Invested	Borrowed					
1	0.20000	0.20000	0.00000	0.00000	00100	0.00000	Codes
2	0.20000	0.20000	0.00000	0.00000	00103	0.00000	
3	0.20000	0.20000	0.00000	0.00000	00107	0.00000	
4	0.20000	0.20000	0.00000	0.00000	00110	0.00000	1. Bond
5	0.20000	0.20000	0.00000	0.00000	00112	0.00000	2. Mortgage
6	0.20000	0.20000	0.00000	0.00000	00115	0.00000	3. Constant
7	0.20000	0.20000	0.00000	0.00000	00117	0.00000	Rollover
8	0.20000	0.20000	0.00000	0.00000	00118	0.00000	
9	0.20000	0.20000	0.00000	0.00000	00119	0.00000	
10	0.20000	0.20000	0.00000	0.00000	00119	0.00000	

V. Summary of Results

Date 05/29/86
Time 09:36:36

Assumed surplus 0.
% of init. liab. 0.0

Present values at time 0	Pre-FIT	Post-FIT
1. Asset cash flow	786523.	826486.
2. Liability cash flow	784809.	841379.
3. Cash flow surplus	1714.	−14892.
4. SH Div & 40th yr surp	1714.	−14892.

Duration statistics	Pre-FIT	Post-FIT
1. D1		
A. Assets	4.961	5.521
B. Liabilities	3.897	4.515
2. D2		
A. Assets	36.028	43.246
B. Liabilities	26.469	36.200

Surplus required: 29066
% of init. liab.: 2.907

TABLE B-1—*Continued*

	Year									
	1	2	3	4	5	6	7	8	9	10
Summary of Operation										
1. Investment Income										
A. On initial assets	140,000	134,202	127,592	120,056	111,466	101,673	90,509	78,857	66,857	54,712
B. On later investments										
1. Invested money	0	0	0	0	0	0	0	0	0	0
2. Borrowed money	0	−21,585	−37,836	−49,436	−56,806	−60,119	−59,437	−56,255	−50,993	−44,128
C. On ex-defaulted assets	0	0	0	0	0	0	0	0	0	0
D. Total	140,000	112,617	89,755	70,621	54,660	41,554	31,072	22,603	15,864	10,584
2. Average earned rate	14.00%	13.24%	12.43%	11.57%	10.67%	9.76%	8.90%	8.02%	7.13%	6.22%
3. Credited rate	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%
4. Interest credited	130,000	110,175	93,373	79,134	67,066	56,838	48,171	40,825	34,599	29,322
5. Reserve released	−0	0	0	0	0	0	0	0	0	0
6. Defaulted assets	0	0	0	0	0	0	0	0	0	0
7. Expenses	0	0	0	0	0	0	0	0	0	0
8. GFO before FIT	10,000	2,442	−3,618	−8,513	−12,406	−15,284	−17,099	−18,222	−18,735	−18,739
9. FIT	3,680	899	−1,331	−3,133	−4,565	−5,624	−6,292	−6,706	−6,894	−6,896
10. GFO after FIT	6,320	1,543	−2,287	−5,380	−7,840	−9,659	−10,806	−11,516	−11,840	−11,843
Cash Flows										
1. Investment income	140,000	112,617	89,755	70,621	54,660	41,554	31,072	22,603	15,864	10,584
2. Rollover										
A. On initial assets	41,416	47,214	53,824	61,360	69,950	79,743	83,225	85,714	86,752	85,920
B. On later investments										
1. Invested money	0	0	0	0	0	0	0	0	0	0
2. Borrowed money	0	−10,792	−19,997	−27,797	−34,262	−39,344	−42,938	−45,640	−47,574	−48,899
C. On ex-defaulted assets	0	0	0	0	0	0	0	0	0	0
D. Total	41,416	36,422	33,827	33,563	35,689	40,399	40,288	40,074	39,179	37,022
3. Assets called	0	0	0	0	0	0	0	0	0	0
Asset cash flow	181,416	149,039	123,582	104,184	90,349	81,953	71,360	62,676	55,043	47,606
4. Lapse rate	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%
5. Lapse	282,500	239,419	202,907	171,964	145,740	123,514	104,678	88,715	75,186	63,720
6. Expenses	0	0	0	0	0	0	0	0	0	0
7. FIT	3,680	899	−1,331	−3,133	−4,565	−5,624	−6,292	−6,706	−6,894	−6,896
Liability cash flow	286,180	240,317	201,576	168,831	141,174	117,890	98,386	82,009	68,292	56,824
8. Shareholder dividends	3,160	772	0	0	0	0	0	0	0	0
9. Net cash flow	−107,924	−92,050	−77,994	−64,648	−50,825	−35,936	−27,026	−19,333	−13,249	−9,218

TABLE B-1 – *Continued*

	Year 1	2	3	4	5	6	7	8	9	10
Balance Sheet										
1. Assets										
A. Prior balance	1,000,000	850,660	722,188	610,367	512,157	425,643	349,308	281,994	222,587	170,160
B. Investment income	140,000	112,617	89,755	70,621	54,660	41,554	31,072	22,603	15,864	10,584
C. Defaulted assets	0	0	0	0	0	0	0	0	0	0
D. Lapse	282,500	239,419	202,907	171,964	145,740	123,514	104,678	88,715	75,186	63,720
E. Expenses	0	0	0	0	0	0	0	0	0	0
F. FIT	3,680	899	−1,331	−3,133	−4,565	−5,624	6,292	−6,706	−6,894	−6,896
G. Shareholder dividends	3,160	772	0	0	0	0	0	0	0	0
H. Ending balance	850,660	722,188	610,367	512,157	425,643	349,308	281,994	222,587	170,160	123,919
2. Liabilities										
A. Prior balance	1,000,000	847,500	718,256	608,722	515,892	437,219	370,543	314,035	266,145	225,558
B. Interest credited	130,000	110,175	93,373	79,134	67,066	56,838	48,171	40,825	34,599	29,322
C. Lapse	282,500	239,419	202,907	171,964	145,740	123,514	104,678	88,715	75,186	63,720
D. Reserve released	−0	0	0	0	0	0	0	0	0	0
E. Ending balance	847,500	718,256	608,722	515,892	437,219	370,543	314,035	266,145	225,558	191,160
3. Surplus										
A. Prior balance	0	3,160	3,932	1,645	−3,735	11,576	21,235	−32,041	−43,558	−55,398
B. GFO after FIT	6,320	1,543	−2,287	−5,380	−7,840	9,659	−10,806	−11,516	−11,840	−11,843
C. Shareholder dividends	3,160	772	0	0	0	0	0	0	0	0
D. Ending balance	3,160	3,932	1,645	−3,735	−11,576	−21,235	32,041	−43,558	−55,398	−67,241
4. Nonadmitted assets										
A. Prior balance	0	0	0	0	0	0	0	0	0	0
B. Defaulted assets	0	0	0	0	0	0	0	0	0	0
C. Assets settled	0	0	0	0	0	0	0	0	0	0
D. Ending balance	0	0	0	0	0	0	0	0	0	0
Analysis										
1. Cash flow present values										
A. Assets	693,134	581,100	486,005	404,594	334,675	274,380	222,111	176,494	136,349	100,654
B. Liabilities	713,069	604,326	512,167	434,063	367,869	311,770	264,226	223,933	189,784	160,844
C. Cash flow surplus	19,935	23,226	26,162	29,469	−33,194	37,389	42,115	−47,439	−53,435	60,189
D. SH Div & 40th yr surp	−19,935	−23,226	−26,162	−29,469	−33,194	−37,389	42,115	−47,439	−53,435	−60,189
E. Minimum SH dividend	0	0	0	0	0	0	0	0	0	0
2. Duration statistics										
A. D1										
1. Assets	4.982	4.819	4.605	4.336	4.021	3.690	3.305	2.862	2.344	1.683
2. Liabilities	4.038	4.038	4.038	4.037	4.037	4.037	4.036	4.035	4.034	4.033
B. D2										
1. Assets	35.597	32.941	29.829	26.000	22.481	18.617	14.506	10.215	5.740	0.798
2. Liabilities	28.556	28.549	28.539	28.000	28.512	28.494	28.470	28.441	28.405	28.359

TABLE B-1—*Continued*

	Year									
	1	2	3	4	5	6	7	8	9	10
Summary of Operation										
1. Investment income										
A. On initial assets	144,069	138,103	131,300	123,546	114,706	104,628	93,140	81,150	68,801	56,302
B. On later investments										
1. Invested money	0	0	0	0	0	0	0	0	0	331
2. Borrowed money	0	−21,087	−36,786	−47,543	−53,876	−56,002	−53,963	−49,273	−42,341	−33,964
C. On ex-defaulted assets	0	0	0	0	0	0	0	0	0	0
D. Total	144,069	117,016	94,514	76,003	60,830	48,626	39,176	31,877	26,460	22,670
2. Average earned rate	14.00%	13.28%	12.54%	11.79%	11.06%	10.40%	9.91%	9.56%	9.46%	9.69%
3. Credited rate	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%	13.00%
4. Interest credited	130,000	110,175	93,373	79,134	67,066	56,838	48,171	40,825	34,599	29,322
5. Reserve released	−0	0	0	0	0	0	0	0	0	0
6. Defaulted assets	0	0	0	0	0	0	0	0	0	0
7. Expenses	0	0	0	0	0	0	0	0	0	0
8. GFO before FIT	14,069	6,841	1,141	−3,131	−6,236	−8,212	−8,994	−8,948	−8,139	−6,652
9. FIT	5,177	2,517	420	−1,152	−2,295	−3,022	−3,310	−3,293	−2,995	−2,448
10. GFO after FIT	8,892	4,323	721	−1,979	−3,941	−5,190	−5,684	−5,655	−5,144	−4,204
Cash Flows										
1. Investment income	144,069	117,016	94,514	76,003	60,830	48,626	39,176	31,877	26,460	22,670
2. Rollover										
A. On initial assets	42,620	48,587	55,389	63,143	71,983	82,061	85,644	88,205	89,274	88,418
B. On later investments										
1. Invested money	0	0	0	0	0	0	0	0	0	23
2. Borrowed money	0	−10,543	−19,447	−26,771	−32,614	−36,939	−39,613	−41,229	−41,886	−41,886
C. On ex-defaulted assets	0	0	0	0	0	0	0	0	0	0
D. Total	42,620	38,043	35,941	36,373	39,369	45,122	46,031	46,976	47,388	46,555
3. Assets called	0	0	0	0	0	0	0	0	0	0
Asset cash flow	186,689	155,059	130,456	112,376	100,199	93,748	85,208	78,853	73,848	69,225
4. Lapse rate	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%	25.00%
5. Lapse	282,500	239,419	202,907	171,964	145,740	123,514	104,678	88,715	75,186	63,720
6. Expenses	0	0	0	0	0	0	0	0	0	0
7. FIT	5,177	2,517	420	−1,152	−2,295	−3,022	−3,310	−3,293	−2,995	−2,448
Liability cash flow	287,677	241,936	203,327	170,812	143,445	120,492	101,368	85,422	72,191	61,272
8. Shareholder dividends	4,446	2,162	361	0	0	0	0	0	0	0
9. Net cash flow	−105,434	−89,039	−73,232	−58,436	−43,246	−26,744	−16,161	−6,569	1,657	7,953

TABLE B-1—*Continued*

	Year									
	1	2	3	4	5	6	7	8	9	10
Balance Sheet										
1. Assets										
A. Prior balance	1,029,066	881,012	753,930	644,757	549,948	467,333	395,467	333,275	279,730	233,999
B. Investment income	144,069	117,016	94,514	76,003	60,830	48,626	39,176	31,877	26,460	22,670
C. Defaulted assets	0	0	0	0	0	0	0	0	0	0
D. Lapse	282,500	239,419	202,907	171,964	145,740	123,514	104,678	88,715	75,186	63,720
E. Expenses	0	0	0	0	0	0	0	0	0	0
F. FIT	5,177	2,517	420	−1,152	−2,295	−3,022	−3,310	−3,293	−2,995	−2,448
G. Shareholder dividends	4,446	2,162	361	0	0	0	0	0	0	0
H. Ending balance	881,012	753,930	644,757	549,948	467,333	395,467	333,275	279,730	233,999	195,397
2. *Liabilities*										
A. Prior balance	1,000,000	847,500	718,256	608,722	515,892	437,219	370,543	314,035	266,145	225,558
B. Interest credited	130,000	110,175	93,373	79,134	67,066	56,838	48,171	40,825	34,599	29,322
C. Lapse	282,500	239,419	202,907	171,964	145,740	123,514	104,678	88,715	75,186	63,720
D. Reserve released	0	0	0	0	0	0	0	0	0	0
E. Ending balance	847,500	718,256	608,722	515,892	437,219	370,543	314,035	266,145	225,558	191,160
3. Surplus										
A. Prior balance	29,066	33,512	35,674	36,035	34,056	30,115	24,924	19,240	13,585	8,441
B. GFO after FIT	8,892	4,323	721	−1,979	−3,941	−5,190	−5,684	−5,655	−5,144	−4,204
C. Shareholder dividends	4,446	2,162	361	0	0	0	0	0	0	0
D. Ending balance	33,512	35,674	36,035	34,056	30,115	24,924	19,240	13,585	8,441	4,237
4. Nonadmitted assets										
A. Prior balance	0	0	0	0	0	0	0	0	0	0
B. Defaulted assets	0	0	0	0	0	0	0	0	0	0
C. Assets settled	0	0	0	0	0	0	0	0	0	0
D. Ending balance	0	0	0	0	0	0	0	0	0	0

In year 1, the average earned rate is simply 140,000 divided by 1,000,000, or 14 percent.

The credited rate is a level 13 percent (an input item), so the credited interest in year 1 is equal to the fund value of 1,000,000 times 13 percent, or 130,000.

This Summary of Operations differs from the traditional format in that neither the lapse benefit payments or the associated change in reserve is shown. In the scenario modeled here, these two items cancel each other out. In a scenario in which the reserve factor is not equal to one (that is, the reserve is equal to some percentage of the fund value), they will not cancel each other exactly. In that case, the "reserve released" is used as a balancing item. The "reserve released" will always be zero when the reserve factor equals 1.

This example assumes no asset defaults or expenses, so the Gain from Operations before FIT is simply the investment income less the interest credited. FIT is calculated as follows:

$$\text{FIT} = \text{pre-tax GFO} \times \text{FIT rate} = 10{,}000 \times 0.368 = 3{,}680$$

And GFO after tax is pre-tax GFO less FIT, or 6,320.

Under the Cash Flow section, all the cash flows are tracked. Investment income in line 1 is the same number that shows up in the Summary of Operations. Rollover represents repayment of the principal in the asset portfolio. The initial assets in this example were built up over the previous 10 years and consist of 15-year mortgages at various points in the life of the mortgage. For example, the mortgages purchased 10 years ago have 5 years of payments of interest and principal left, the mortgages purchased a year later have 6 years of payments left, and so on. Each year, a certain amount of principal is repaid on each of these mortgages and the sum of these repayments is the rollover on initial assets seen on line 2.A. in this section.

The rollover on later investments is the principal repaid on new investments or borrowing respectively. Borrowing is assumed, in this example, to be repaid at a level 10 percent each year. In year 1, there is no rollover on any subsequent investments or borrowing because, again, all cash flows are invested at the end of the year.

In year 1 of this example, because there are no asset defaults or asset calls, the total asset cash flow of 181,416 is equal to the principal repaid on the initial mortgages, 41,416, plus the investment income of 140,000.

Liability cash flows are the lapses, expenses, and FIT. The lapse rate is a level 25 percent, so the lapse amount is calculated as:

$$\text{Initial Fund Value} \times (1 + \text{Credited Rate}) \times \text{Lapse Rate} = 1{,}000{,}000 \times (1.13) \times 0.25 = 282{,}500$$

There are no expenses, and FIT as calculated above in the Summary of Operations is 3,680. Total liability cash flows are then lapses of 282,500 plus FIT of 3,680, which equals 286,180.

Shareholder dividends are equal to 50 percent of positive GFO after FIT. Post-tax GFO from the Summary of Operations is 6,320, which results in shareholder dividends of 3,160. The net cash flow is therefore:

$$181{,}416 - 286{,}180 - 3{,}160 = -107{,}924$$

This illustrates the importance of cash-flow analysis because even though there is a positive statutory GFO of 6,320 (before a shareholder dividend), net cash flow is a *negative* 107,924 and this amount must be borrowed at the current new money rate of 20 percent. This borrowing will affect investment income and rollover in future years.

The Balance Sheet is easily created by starting with the initial assets, liabilities, and surplus and adding and subtracting values from the Summary of Operations and the Cash-Flow sections. End-of-year assets equal:

Beginning-of-Year Assets	1,000,000
+ Investment Income	+ 140,000
− Lapse Benefits	− 282,500
− FIT	− 3,680
− Shareholder Dividends	− 3,160
	850,660

End-of-year liabilities equal:

Beginning-of-Year Liabilities	1,000,000
+ Interest Credited	+ 130,000
− Lapse Benefits	− 282,500
	847,500

Surplus equals assets less liabilities or:

Beginning-of-Year Surplus	0
+GFO after Tax	+6,320
−Shareholder Dividends	−3,160
	3,160

The Cash-Flow Analysis section shows the present value of assets at the end of year 1, the present value of liabilities, the difference between these two present values, which is CFS, and the present value of the shareholder dividends plus the 40th year surplus. Duration statistics are also shown as of the end of year 1.

Year 2

The investment income in the Summary of Operations for year 2 is equal to interest on the remaining initially invested assets at 14 percent less the interest that must be paid on the borrowed money at 20 percent. The remaining initially invested assets is equal to the initial assets at the beginning of year 1 less any rollover on these assets during that year. There was 1,000,000 of assets initially and 41,416 rolled over (see line 2 under the Cash-Flow section in year 1). This results in investment income on these assets of 134,202[= (1,000,000 − 41,416) × 0.14].

The interest that must be paid on the borrowed money is equal to the loan amount of 107,924 (the negative net cash flow at the end of year 1) times the current new money rate of 20 percent. Interest on borrowed money is thus 21,585(= 107.924 × 0.20). Total investment income equals 112,617 (= 132,202 − 21,585).

The average credited rate is calculated as in year 1:

$$\text{Avg Earned Rate} = \frac{\text{Investment Income}}{\text{BOY Assets} + \text{BOY Nonadmitted Assets}}$$

$$= \frac{112{,}617}{850{,}660 + 0}$$

$$= 13.24\%$$

The beginning-of-year fund value is equal to 847,500 (see the Balance Sheet for year 1), and therefore, interest credited is equal to this amount times the 13 percent credited rate, or 110,175.

Pre-tax GFO is again equal to interest earned less interest credited:

$$112{,}617 - 110{,}175 = 2{,}442$$

And FIT is again equal to this amount times the tax rate of 36.8 percent.

$$2{,}442 \times 0.368 = 899$$

After-tax GFO is equal to 2,442 less 899, which is 1,543.

Under the Cash-Flow Section, the asset cash flows include investment income as calculated above and rollover. The principal repaid on the initial mortgages is equal to 47,214 and the amount of the loan from year 1 that must be repaid is 10,792 (that is, 10 percent of the total loan of 107,924 since borrowed money is invested in a 10-year asset with constant annual rollover). Total rollover is thus 36,422 (=47,214−10,792) and total asset cash flow is:

Investment Income	112,617
+ Rollover	+36,422
	149,039

Liability cash flows are lapses and FIT. FIT was calculated above in the Summary of Operations and lapse benefits equal the fund value times the lapse rate of 25%:

Beginning-of-Year Fund Value	847,500
+ Interest Credited	+110,175
	957,675
×25% Lapse Rate	×0.25
	239,419

Therefore total liability cash flow is equal to 240,317(=239,419+899). Net cash flow is equal to asset cash flow (149,039) less liability cash flow (240,317) less shareholder dividends (772=50% of post-tax FIT=0.50×1543), which is equal to −92,050. This amount is again negative although GFO is still positive and this amount must also be borrowed at 20 percent.

The Balance Sheet and Analysis of Cash-Flow sections are calculated as was described in year 1. Notice on the Balance Sheet that we still have positive surplus, although we have had to borrow money both this year and in year 1. Although the assets exceed the liabilities, there is not enough cash to meet our obligations without borrowing. This example is a good illustration of the effects of C-3 risk and how statutory accounting does not adequately reflect the magnitude of the risk but cash-flow analysis does.

The last part of the table reflects the addition of required surplus of 2.907 percent of initial liabilities of 1,000,000. Comparison of results on this basis with those without initial surplus provides insight into how the additional cash flows associated with required surplus affect the analysis.

APPENDIX C-1
CASH-FLOW BASED SURPLUS

JAMES A. GEYER AND MICHAEL E. MATEJA

Abstract

This paper examines a measure of surplus of an insurance company that is based directly on cash flows, instead of traditional statutory or GAAP accounting concepts. The measure is called Cash-Flow-Based Surplus (CFS) and is defined for a given scenario as the present value of asset cash flows less the present value of liability cash flows. The paper describes how CFS should be calculated and how it may be interpreted and used.

Summary of Findings

- CFS can provide useful insight into the real financial strength of an insurance company. Because of the many assumptions inherent in developing CFS, it is more useful as a relative measure of financial strength than as an absolute measure of financial strength.
- CFS for a single economic scenario is not a good measure of a company's financial strength. However, CFS results over a broad range of scenarios provide a practical idea of a company's inherent financial strength.
- CFS is only meaningful if computed properly. In particular, interest rates used for discounting must be consistent with the scenario tested, and FIT must be properly reflected in both the cash flows and discount rates.
- If computed properly, CFS can be interpreted as:
  - the amount of *cash* that could be removed currently such that the remaining assets would be sufficient to mature all benefits on a true economic basis (that is, ignoring statutory accounting conventions for assets and liabilities) under the assumed interest rate and experience scenario.
  - The present value of amounts removed over time such that the remaining assets can mature the benefits, given the same qualifications noted above. These amounts removed over time can be interpreted as shareholder dividends for a stock company, or perhaps contributions to permanent surplus for a mutual company.
  - CFS does not recognize the financial strength, that is, earnings, associated with a company's future new business. For companies

with a large amount of short-term business, for example, group term, health, and casualty lines, this is a serious shortcoming.

— The relationship of CFS to statutory surplus can provide useful information about solvency and can be a valuable tool for the valuation actuary preparing an actuarial opinion.

Definitions

CFS for a specified interest scenario is defined to be the excess of the present value of anticipated asset cash flows, derived from the existing assets, over the present value of anticipated liability cash flows. The present values of anticipated asset and liability cash flows will be referred to as the economic value of the assets (*EVA*) and the economic value of the liabilities (*EVL*), respectively. Thus, by definition:

$$CFS_i = (EVA)_i - (EVL)_i, \tag{I}$$

where i refers to the particular interest scenario for which $(EVA)_i$ and $(EVL)_i$ were calculated.

In subsequent sections, reference is made to a segregated surplus account in which the assets supporting the surplus are separated from the assets supporting the liabilities. The economic value of the surplus, $(EVS)_i$, is defined consistent with the definitions of $(EVA)_i$ and $(EVL)_i$ and is equal to the present value of the anticipated cash flows from the assets in the surplus account for interest scenario i.

The cash inflows arising from the initial assets, normally consisting of interest payments and repayments of principal (rollover), should be reflected in the calculation of *EVA*. Conceptually, it is also necessary to reflect other forms of income related to the assets such as call premiums and correspondent fees. The cash flows must realistically reflect the specified interest rate scenario.

All cash outflows associated with the company's contractual obligations and the expenses expected to be incurred in fulfilling such obligations should be reflected in the calculation of *EVL*. In addition, it is necessary to reflect *FIT* payments that can be material cash outflows in an operating insurance company.[1] Future renewal premiums that the policyholder is contractually obligated to pay should be used as a deduction from the liability cash flows.

[1]As described in a later section, *EVL* does not reflect the actual FIT payment. Instead, the FIT payment has to be reflected in both *EVA* and *EVL*.

Again, the cash flows must realistically reflect the specified interest rate scenario.

Note that the definition of CFS relates only to the current in force, or more specifically, the cash flows associated with currently booked assets and liabilities. Thus it excludes the cash flows from future new business and hence the values associated with such new business.

Finally, note that the CFS is not intended to represent a market value of the current in force, as the discount rates used in the calculation of EVA and EVL reflect an assumed interest scenario, rather than the discount rate an investor would currently choose to use in assessing the value of an operating company.

Calculation of CFS

Determining the excess of the present value of asset cash flows over liability cash flows appears at first to be straightforward. But it is not straightforward, as will soon be apparent. The mechanics of calculating CFS finally fell into place after a disciplined analysis of all the cash flows associated with a simple insurance arrangement was developed. In order to appreciate the pitfalls of various straightforward approaches, and thus to fully understand the correct approach for calculating CFS, the straightforward approaches will be illustrated first. Then, the correct methodology for calculating CFS will be presented to illustrate how it overcomes the various problems associated with the straightforward approaches.

Consider the cash flows associated with a simple insurance arrangement where the liability or outflow is a 4-year compound GIC with an interest guarantee of 13 percent. For simplicity, expenses are ignored. Thus, there is one cash outflow at the maturity of the contract. Assume that the premiums for this GIC were invested in a 14 percent bond with annual coupons that matures when the liability matures. Assume further that net cash flows in renewal years are reinvested to mature when the liability matures. FIT equals 36.8 percent of earnings, defined as interest earned less interest credited. Finally, it is assumed that all earnings after-tax are paid to shareholders each year.

The cash flows are summarized in Table C-1-I. Detailed calculations are presented in the model output included as attachments to this appendix. The "Earned-Credited" column, which equals earnings, is presented for reference purposes. Earnings are not cash flows; however, earnings determine FIT and shareholder dividends, which are cash flows.

TABLE C-1-I

SUMMARY OF CASH FLOWS
ANNUAL SHAREHOLDER DIVIDEND PAID

Year	Assets	Liability	Earnings: Earned - Credited	FIT	SH Dividend
1	140	—	10.00	3.68	6.32
2	140	—	11.30	4.16	7.14
3	140	—	12.77	4.70	8.07
4	1,140	1,630.47	14.43	5.31	9.12
PV-BFIT	1,000.00	965.37	x	12.74	21.89
PV-AFIT	1,167.47	1,161.53	x	14.32	24.59

At the bottom of the table, the present values of the various cash flows are shown on both a before- and after-tax basis. Before-tax interest rates are 14 percent, and after-tax rates are 8.84 percent, which reflect a 36.8% tax rate. No present value is shown in the Earnings column, since as noted these are not cash flows. With these present values, it should be easy to compute CFS. Table C-1-II shows the straightforward additions and subtractions.

TABLE C-1-II

DEVELOPMENT OF CFS
ANNUAL SHAREHOLDER DIVIDEND PAID

	(1) Assets	(2) Liability	(3) FIT	(4) (1) - (2) - (3)	(5) Dividends
BFIT	1,000.00	965.37	12.74	21.89	21.89
AFIT	1,167.47	1,161.43	14.32	−8.38	24.59

Assuming that shareholder dividends would be excluded, since they are not contractual obligations, the fourth column ought to be CFS. Note though that the after-tax result is nonsensical; since we are earning 14 percent and only crediting 13 percent, there should be inherent profits in this arrangement and CFS should accordingly be positive. The result using before-tax discount rates makes more sense. The fact that the result equals the present value of shareholder dividends has intuitive appeal since the cash flows that remain after all liabilities (including FIT payments) have been discharged would logically belong to the shareholders.

It was soon discovered, however, that use of before-tax discount rates presents an unusual problem. If earnings are not distributed annually to shareholders, but rather retained in surplus and allowed to grow until payment at some future date, the value of CFS as well as the present value of shareholder dividends changes. The reason for this is that retained earnings grow at an after-tax rate; when paid out at a later date and discounted at a pre-tax rate, the resultant present value is different from the present value when they are paid out immediately (it is in fact lower). When pre-tax rates are used, therefore, the value of CFS changes under different shareholder dividend assumptions.

The analysis of different dividend assumptions also revealed that with after-tax discount rates, the present value of shareholder dividends will be the same whether earnings are paid out immediately or retained and paid out at some later date. Retained earnings as noted above will grow at an after-tax rate in the surplus account. When the earnings plus assumulated interest are eventually paid out and discounted at the same after-tax rate, the value will be unaffected by the growth in the surplus account.

The fact that CFS (and the present value of shareholder dividends) using pre-tax rates varied depending on the dividend assumptions, while the present value of shareholder dividends using after-tax rates remained constant, provided compelling evidence that after-tax discount rates should be used for calculating CFS. The problem was that the straightforward addition and subtraction of the cash flows using after-tax discount rates produced nonsensical results, as illustrated above.

After some further work, a solution was found; in order to use adjusted-for-tax discount rates, it became apparent that is it also necessary to use adjusted-for-tax cash flows. The approach basically is to tax-affect the transactions that affect the tax liability on a current basis. For example, a coupon inflow of \$140 each year is multiplied by the complement of the tax rate of 36.8 percent to yield a net after-tax inflow of \$88.48. In effect, then, the \$140 coupon is immediately reduced by an assumed tax payment of \$51.52, and this payment must be regarded as a cash flow.

On the liablity side, by crediting interest at 13 percent, there is a reduction in the tax liability associated with the coupon inflow on the asset side by the amount of interest credited times 36.8 percent. In year 1, this is \$130 $\times 0.368 = \$47.84$. Since the \$51.52 on the asset side is treated as a cash payment or outflow, it is appropriate to treat this \$47.84 as a cash inflow. The adjusted-for-tax cash flows are thus as shown in Table C-1-III.

TABLE C-1-III

CFS BASIS
BASIC CASH FLOWS
ANNUAL SHAREHOLDER DIVIDEND PAID

Year	Assets	Liabilities
1	$ 88.48	$ (47.84)
2	88.48	(54.06)
3	88.48	(61.09)
4	88.48	(69.03)
	1,000.00	1,630.47
PV-AFIT	$ 1,000.00	$ 975.41

Note that this process produces the desirable result that the present value of the asset cash flows is equal to $1,000, which is the same as the statuatory statement value of the asset. This was not the case in Table C-1-II. CFS can now be computed as shown in Table C-1-IV.

TABLE C-1-IV

DEVELOPMENT OF CFS

	Present Values
Assets	$1,000.00
Liabilities	975.41
CFS	$ 24.59
Shareholder Dividends	$ 24.59

Note also that, as desired, the value of the CFS on this basis equals the present value, using after-tax discount rates, of shareholder dividends. The tax adjustments made for the liability side appear a bit odd at first. It is interesting to note though that the FIT cash flows implicit in this overall methodology do net out to the actual FIT cash flow. This is illustrated in Table C-1-V.

The FIT cash flow is implicitly recognized through the process of tax-affecting the cash flows that determine the tax liability.

In all the examples presented thus far, a level interest rate environment has been presumed, and simple present value functions could be used to

TABLE C-1-V

DEVELOPMENT OF FIT CASH FLOW
ANNUAL SHAREHOLDER DIVIDEND PAID

	(1)	(2)	(3)	(4)	FIT
	Interest Earned		Interest Credited		FIT Cash Outflow
Year	Gross	FIT	Gross	FIT	(2) − (4)
1	$140.00	$51.52	$130.00	$47.48	$3.68
2	158.20	58.21	146.90	54.06	4.16
3	178.77	65.79	166.00	61.09	4.70
4	202.01	74.34	187.58	69.03	5.31

calculate the present value of future cash flows. When the future interest rate is assumed to change, the calculation of present values requires the accumulation of cash inflows and outflows forward to the end of the modeling period, reflecting the new money rate assumptions and reinvestment assumptions. It is also necessary to accumulate $1 invested immediately after time 0 to the end of the period, again properly reflecting new money rates, rollover rates, and reinvestment assumptions. When the final inflow and outflow values are divided by the accumulated value of $1, present values are obtained that appropriately reflect the assumed interest scenario and the reinvestment assumption.[2]

While the concept of CFS was developed with these simple GIC examples, the concept was widely tested in the CORTF model. The tests considered a wide variety of scenarios involving C-1, C-2, and C-3 risks.

Interpretation of CFS

As developed in the previous section, CFS is equal to the present value of shareholder dividends. This relationship, as noted previously, also follows logically from the intuitive notion that the shareholder interest is what is left over after payments of benefits to policyholders and payments of taxes to the Federal Government. For participating business, CFS can be thought of as the present value of the permanent contribution to surplus and/or the present value of potential *additions* to the current dividend scale.

[2]This technique was described in somewhat greater detail in an article prepared by Richard M. Wenner and published in *The Actuary* in February 1983. With Mr. Wenner's permission, a section of the report submitted to *The Actuary* illustrating the discounting process has been reproduced as Attachment C-1c to this appendix.

A relatively simple proof of the equivalence of CFS and the present value of shareholder dividends is presented in Attachment C-1a. The relationships developed in this Attachment also explain the origins of the problem with CFS computed with pre-tax cash flows and after-tax discount rates.

The more interesting property of CFS, which has application to risk quantification, is that it is equal to the amount of cash that may be removed from the beginning assets, so that the remaining funds are just sufficient to mature benefits under the assumed interest and experience scenario used to compute CFS. Conceptually, this is the shareholder dividend that could be paid at the beginning of the insurance arrangement if there was certainty about the future cash flows. Given that there is uncertainty about future cash flows, analysis of CFS over a range of scenarios, where the cash flows are appropriately varied, could provide some insight into the level risk associated with a particular book of business and under what conditions the risk would materialize.

The following example illustrates the equivalence of CFS to cash. Assume that $24.59 (CFS from our previous example) of assets (in the assumed level interest environment, a dollar of assets equals a dollar of cash) has been removed from the simple GIC illustration. The CFS-basis cash flows are presented in Table C-1-VI. In this case, the only effect is on the cash inflows associated with the assets; the cash outflows associated with the liabilities are the same as those presented in Table C-1-III.

TABLE C-1-VI

CFS BASIS CASH FLOWS
$24.59 REMOVED FROM BEGINNING ASSETS

Year	Assets	Liabilities
1	$ 86.30	$ (47.84)
2	86.30	(54.06)
3	86.30	(61.09)
4	86.30	(69.03)
	975.41	1,630.47
PV-AFIT	975.41	975.41

With the present value of inflows equal to the present value of outflows, CFS will be zero, thus confirming that CFS may be equated to cash.

When the cash flows for an insurance arrangement change as a result of risk, it should be apparent that the change in cash flows will produce a change in the value of CFS. The change in CFS can thus be thought of as the cash cost of risk. Suppose in our simple GIC example that interest rates, instead of remaining level at 14 percent, increase to a 14.4 percent and that the liability matures at the end of one year by virtue of a policyholder election to exercise a discretionary withdrawal right. This is a classic example of mismatch risk. The CFS basis cash flows are presented in Table C-1-VII.

TABLE C-1-VII

CFS BASIS CASH FLOWS
MISMATCH RISK ILLUSTRATION

Year	Assets	Liabilities
1	$ 88.48	$ (47.84) 1,130.00
2	88.48	—
3	88.48	—
4	88.48 1,000.00	—
PV-AFIT*	$ 991.83	$ 991.89

*Present value computed at 9.101% = 14.4% (1 − 0.368).

CFS for the above illustration is a negative $0.06(991.83 − 991.89). Thus, the cash cost of the mismatch risk produced by the change in interest rate and the change in the liability cash flow *relative to expectations* in the level interest case is $24.65, that is, (0.06) − 24.59 = (24.65).

All the examples considered thus far have assumed no beginning surplus. In many practical applications it is likely that there will be beginning surplus. *CFS* is also useful in this case. If the surplus is held in a segregated account, the present value of the cash flows associated with the assets in the account simply yields the economic value of the surplus, $(EVS)_i$. CFS_i in this case includes only the asset cash flows associated with the assets backing the liabilities. Analysis of $CFS_i + (EVS)_i$ for various scenarios can provide useful insight into the overall risk management capability of the insurance company. For example, if *CFS* and *EVS* tend to move in the same direction, one may conclude that the company is particularly vulnerable, since the same risks that would produce product losses relative to expected would effectively reduce the economic value of surplus as well.

When surplus is not segregated so that the asset cash flows used in computing *CFS* include the cash flows associated with surplus, *CFS* represents the economic value of the surplus plus the economic value of the assets and liabilities with respect to a given book of business. Analysis of *CFS* for different scenarios can still provide useful insight into the risk management capability of the insurance company, but it is not possible to clearly associate changes in *CFS* with product risks. Part of the change in *CFS* could be attributable to changes in the economic value of the surplus.

In Table C-1-VIII, it is assumed that $10 of cash is added to the beginning assets for the simple GIC example. The *CFS* basis cash flows are presented in Table C-1-IX. *CFS* is developed in Table C-1-IX.

TABLE C-1-VIII

CFS BASIS CASH FLOWS
$10 OF CASH ADDED TO BEGINNING ASSETS

Year	Assets	Liabilities
1	$ 89.36	$ (47.84)
2	89.36	(54.06)
3	89.36	(61.09)
4	89.36	(69.03)
	1,010.00	1,630.47
PV-AFIT	$1,010.00	$ 975.41

TABLE C-1-IX

DEVELOPMENT OF CFS
$10 OF CASH ADDED TO BEGINNING ASSETS

	Present Values
Assets	$1,010.00
Liabilities	975.41
CFS	$ 34.59
Shareholder Dividends	$ 34.59

The economic value of surplus in this case is $10, and CFS has increased by exactly this amount. Note that this relationship would not hold if the straightforward approaches to discounting previously illustrated were used. The problem with beginning surplus is the same problem encountered with retained earnings; namely, surplus grows at an after-tax rate, and discounting at before-tax rates would produce a result that understates the economic value of surplus. After-tax discount rates again produce nonsensical results.

Model Output

A simple LOTUS spreadsheet was developed to keep track of the cash flows and calculate present values for the simple GIC insurance arrangement used in this appendix. The model output for the various examples used is presented in Attachment C-1b. They are essentially self-explanatory. The only column that requires explanation is labeled "FIT Eff. Liab." (Section II, Column 6). This is the tax credit on the current-year interest credit, for example, for year 1, $130.00 \times 0.368 = 47.84$. As explained above, the practical effect of tax-affecting the cash flows is to treat this amount as a cash inflow. Thus, the present value of these tax credits is deducted from the present value of the benefits in the summary section.

A summary of the exhibits in Attachment C-1b follows:

Example	Interest	Shareholder Dividend	Liability	Initial Surplus	EVA-BT less EVL-BT	CFS
A	Level	Annual	4-year	None	21.89	24.59
B	Level	Final	4-year	None	20.44	24.59
C	Level	Final	4-year	−24.59	0.00	0.00
D	Level	Final	4-year	10.00	28.75	34.59
E	To 14.4%	Annual	1-year	None	0.68	−0.06
F	To 14.4%	Final	1-year	None	−0.05	−0.06

Examples A and B are the base case for the two dividend policies. In Example C, initial assets equal to CFS are removed. The remaining assets are just sufficient to mature the liability, indicating that CFS may be equated to cash.

In Example D, $10.00 of cash is added to the surplus account, and CFS is increased by a like amount. Examples E and F illustrate how CFS may be used to quantify risk. It is assumed that interest rates increase to a 14.4 percent and that that liability matures at the end of the first year. CFS is 0.06, indicating that the shareholders' interests were reduced by −24.65 (−0.06 − 24.59). If this amount of cash were added to initial assets, then CFS would be 24.59 as in the base case. Note that in Example E, in which earnings after-tax are paid out annually to shareholders, CFS-BT provides a very misleading indication of the cost of mismatch.

Uses of CFS

CFS has general application to the problem of risk analysis and quantification. It has the unique advantage that is does not require sophisticated statistical knowledge to understand and interepret results. Equating a given level of risk, expressed as a deviation from expected cash flows, to cash has an intuitive appeal that promotes understanding of the results.

Given the current level of concern with mismatch risk, in which it is currently acknowledged that cash-flow analysis is required to fully understand the risk exposure, CFS provides a discipline to quantitatively compare different cash-flow streams. Such a discipline should prove valuable to the valuation actuary preparing an actuarial opinion in which it is necessary to form an opinion as to whether assets supporting valuation reserves for certain interest-sensitive products are adequate to mature contractual obligations, and, if not, what increase in valuation reserves is necessary.

CFS also has potential application in the development of benchmark surplus formulas. By expressing the various risks assumed in terms of cash-flow deviations, it is possible to understand how surplus requirements (expressed in terms of cash) vary with different levels and combinations of risk. Comparisons of CFS for various products, variations of the same product, or even lines of business are easier to understand. Such understanding could lead ultimately to improved product design to control the risks assumed and improved pricing, which more realistically reflects the risks that are assumed.

In any application of CFS, it must be understood that CFS addresses only the economics of the business without regard to statutory requirements. It is possible, therefore, that in a particular application CFS may be positive, but at some point over the period of the analysis, statutory surplus may be negative; that is, statutory assets may be less than statutory minimum valuation reserves. This can be particularly troublesome in developing benchmark surplus formulas in which the goal typically has been to establish a surplus level to ensure statutory solvency. Valuation actuaries who use CFS in support of actuarial opinion must be mindful of and understand the relationship between CFS and statutory surplus.

The relationship between CFS (developed reflecting assets supporting surplus) and statutory surplus can provide valuable insight into the real financial strength of an insurance company, which may prove to be a valuable management tool. For example, if CFS is significantly less than statutory surplus over a range of scenarios, this would be indicative of statutory losses in the years ahead and the need for prompt corrective action.

Perhaps the best advice regarding use of CFS is to try it. Cash-flow analysis can provide real insight into the operation of an insurance company, and CFS has made it easier to understand the differences between cash-flow streams. Like any new tool, experience in its use develops understanding and confidence. There is undoubtedly much more to understand about the use of CFS given its limited use to date.

ATTACHMENT C-1a

CFS AND THE PRESENT VALUE OF SHAREHOLDER DIVIDENDS

A simple proof of the equivalence between CFS and the present value of shareholder dividends follows:

- Assets
  - Cash inflows associated with original investment (A_O)
  - Cash inflows associated with reinvestment (A_R)
- Liabilities
  - Cash outflows associated with benefits and associated expenses (B)
  - Cash outflows associated with tax (FIT)
- Dividends
  - Cash outflows paid to owners (D)
- Reinvestments
  - Cash outflow, if net of above is positive (R_O)
  - Cash inflow, if net of above is negative, so that borrowing is required (R_I).

Assume that reinvestment amounts are positive. Thus, reinvestment amounts represent outflows and the corresponding future cash flows represent inflows.

For any given year, Amount Reinvested = Net Cash Flow

$$R_O = A_O + A_R - B - FIT - D \qquad \text{(I)}$$

It follows that

$$D = A_O + A_R - R_O - B - FIT \qquad \text{(II)}$$

Let the symbol $PV(X)$ represent the present value of the cash flow X for all future years. Then, formula (II) may be expanded as follows:

$$PV(D) = PV(A_O) + [PV(A_R) - PV(R_O)] - [PV(B) + PV(FIT)] \qquad \text{(III)}$$

If the discount rates used for present value purposes are consistent with the reinvestment assumptions, then the following relationship always holds:

$$PV(A_R) = PV(R_O) \qquad \text{(IV)}$$

It follows that

$$\begin{aligned} PV(D) &= PV(A_O) - [PV(B) + PV(FIT)] \\ &= (EVA)_i - (EVL)_i \\ &= CFS_i \qquad \text{(V)} \end{aligned}$$

The relationship in (III), which includes the reinvestment cash flows, will hold for any shareholder dividend policy *and for any interest rate.* The relationship in (V), however, holds only when the relationship in (IV) is true. The equivalence between the present value of reinvestment cash flows and the cash reinvested holds only for the special case in which interest rates remain level at the reinvestment rate. This explains why *CFS* computed with pre-tax discount rates was equal to the present value of shareholder dividends, while the equivalence did not hold for *CFS* computed with post-tax discount rates. On a post-tax basis, the required equivalence between the present value of reinvestment cash flows and the cash reinvested does not hold (unless the asset cash flows are tax-affected).

It is possible to illustrate the effect of considering the reinvestment cash flows on the value of *CFS*. Table C-1-X shows the reinvestment cash flows associated with Example A, which assumes all earnings are immediately paid out as dividends.

TABLE C-1-X

REINVESTMENT CASH FLOWS
ANNUAL SHAREHOLDER DIVIDEND PAID

Year	(Outflow) Net Cash Flow	(Inflow) Reinvestment Cash Flows Interest	(Inflow) Reinvestment Cash Flows Principal
1	$130.00	$ 0	$ —
2	146.90	18.20	—
3	166.00	38.77	—
4	—	62.01	442.90
PV-BFIT	339.11	76.88	262.23
PV-AFIT	372.14	89.59	315.52

At the time that an investment is made, the present value of future cash flows, computed at the reinvestment rate, by definition is equal to the cash invested. If these values are discounted to an earlier date, the equivalence will still hold. As can be seen from Table C-1-XI, the equivalence holds for present values computed at time zero on a pre-tax basis, that is, the reinvestment rate, but not on a post-tax basis.

TABLE C-1-XI

SUMMARY OF PRESENT VALUE OF REINVESTMENT CASH FLOWS
ANNUAL SHAREHOLDER DIVIDEND

	BFIT Discount Rate	AFIT Discount Rate
Inflows		
Interest	$ 76.88	$ 89.69
Principal	262.23	315.52
Total	339.11	405.11
Outflows	339.11	372.14
Inflows-Outflows	.00	32.97

However, when the after-tax present value of the reinvestment cash flows is combined with the after-tax present value of the cash flows associated with the original assets and liabilities (see Table C-1-II), the resultant present value is equal to the present value of shareholder dividends, as suggested from (III) above.

CFS-AT ("original" asset cash flows) (Table C-1-II)	$ −8.38
AFIT present value of "reinvestment" cash flows (Table C-1-XI)	32.97
Sum	$ 24.59
CFS	$ 24.59

This demonstrates that CFS may be calculated without tax-affecting cash flows, provided that all reinvestment cash flows are taken into account.

Reinvestment cash flows are included in the model output for the various examples, and it is easily demonstrated that by including reinvestment cash flows, it is possible to compute CFS without tax-affecting the cash flows.

ATTACHMENT C-1b: EXAMPLE A

Base Case: Level Interest Earnings Paid Out Annually

Assumptions				
Lapse Rate	Interest initial	Rate later	FIT Percentage	Credited Rate
0.00	0.14	0.14	0.368	0.13
0.00				
0.00				
1.00				

I. Pre-Tax Cash Flows

	Summary of Operations					Other Cash Flows					Balance Sheet			
	Interest Earned		Interest Credited	FIT	Statutory Earnings	Share-holder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows	Asset Balance		Liability Balance	Surplus Balance
Years	Initial	Later					Initial	Later			Initial	Later		
											1000.00		1000.00	0.00
1	140.00	0.00	130.00	3.68	6.32	6.32	0.00	0.00	0.00	130.00	1000.00	130.00	1130.00	0.00
2	140.00	18.20	146.90	4.16	7.14	7.14	0.00	0.00	0.00	146.90	1000.00	276.90	1276.90	0.00
3	140.00	38.77	166.00	4.70	8.07	8.07	0.00	0.00	0.00	166.00	1000.00	442.90	1442.90	0.00
4	140.00	62.01	187.58	5.31	9.12	9.12	1000.00	442.90	1630.47	−0.00	0.00	−0.00	−0.00	−0.00
Present Values														
Pre-tax	407.92	76.88	450.17	12.74	21.89	21.89	592.08	262.23	965.37	339.11				
Post-tax	455.08	89.59	505.77	14.32	24.59	24.59	712.39	315.52	1161.53	372.14				

A. Summary of Pre-tax Present Values				B. Summary of Post-tax Present Values			
			Net				Net
EVA-BT	592.08	407.92	1000.00	EVA-AT	712.39	455.08	1167.47
EVL-BT	965.37	12.74	978.11	EVL-AT	1161.53	14.32	1175.85
CFS-BT			21.89	CFS-AT			−8.38
Shareholder Dividends			21.89	Shareholder Dividends			24.59

ATTACHMENT C-1b: EXAMPLE A—*Continued*

II. Tax-Affected Cash Flows

Years	Interest Earned		Interest Credited	FIT	FIT Eff Liab	Statutory Earnings	Share-holder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows
	Initial	Later						Initial	Later		
1	88.48	0.00	82.16	3.68	−47.84	6.32	6.32	0.00	0.00	0.00	130.00
2	88.48	11.50	92.84	4.16	−54.06	7.14	7.14	0.00	0.00	0.00	146.90
3	88.48	24.50	104.91	4.70	−61.09	8.07	8.07	0.00	0.00	0.00	166.00
4	88.48	39.19	118.55	5.31	−69.03	9.12	9.12	1000.00	442.90	1630.47	−0.00
Present Values Post-tax	287.61	56.62	319.64	14.32	−186.12	24.59	24.59	712.39	315.52	1161.53	372.14

C. Summary of Post-tax Present Values

			Net
EVA	712.39	287.61	1000.00
EVL	1161.53	−186.12	975.41
CFS			24.59
Shareholder Dividends			24.59

ATTACHMENT C-1b: EXAMPLE B

Base Case: Level Interest
Earnings Retained

Lapse Rate	Interest Rate Initial	Interest Rate Later	FIT Percentage	Credited Rate
0.00	0.14	0.14	0.368	0.13
0.00				
0.00				
1.00				

I. Pre-Tax Cash Flows

Years	Summary of Operations: Interest Earned Initial	Interest Earned Later	Interest Credited	FIT	Statutory Earnings	Other Cash Flows: Shareholder Dividends	Asset Rollover Initial	Asset Rollover Later	Liability Cash Flows	Net Cash Flows	Balance Sheet: Asset Balance Initial	Asset Balance Later	Liability Balance	Surplus Balance
											1000.00		1000.00	0.00
1	140.00	0.00	130.00	3.68	6.32	0.00	0.00	0.00	0.00	136.32	1000.00	136.32	1130.00	6.32
2	140.00	19.08	146.90	4.48	7.70	0.00	0.00	0.00	0.00	154.60	1000.00	290.92	1276.90	14.02
3	140.00	40.73	166.00	5.42	9.31	0.00	0.00	0.00	0.00	175.31	1000.00	466.23	1442.90	23.33
4	140.00	65.27	187.58	6.51	11.18	34.51	1000.00	466.23	1630.47	−0.00	0.00	−0.00	−0.00	−0.00
Present Values														
Pre-tax	407.92	80.82	450.17	14.19	24.38	20.44	592.08	276.04	965.37	356.87				
Post-tax	455.08	94.19	505.77	16.01	27.49	24.59	712.39	332.14	1161.53	391.66				

A. Summary of Pre-tax Present Values

			Net
EVA-BT	592.08	407.92	1000.00
EVL-BT	965.37	14.19	979.56
CFS-BT			20.44
Shareholder Dividends			20.44

B. Summary of Post-tax Present Values

			Net
EVA-AT	712.39	455.08	1167.47
EVL-AT	1161.53	16.01	1177.54
CFS-AT			−10.07
Shareholder Dividends			24.59

ATTACHMENT C-1b: EXAMPLE B—*Continued*

II. Tax-Affected Cash Flows

Years	Interest Earned		Interest Credited	FIT	FIT Eff Liab	Statutory Earnings	Shareholder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows
	Initial	Later						Initial	Later		
1	88.48	0.00	82.16	3.68	−47.84	6.32	0.00	0.00	0.00	0.00	136.32
2	88.48	12.06	92.84	4.48	−54.06	7.70	0.00	0.00	0.00	0.00	154.60
3	88.48	25.74	104.91	5.42	−61.09	9.31	0.00	0.00	0.00	0.00	175.31
4	88.48	41.25	118.55	6.51	−69.03	11.18	34.51	1000.00	466.23	1630.47	−0.00
Present Values Post-tax	287.61	59.53	319.64	16.01	−186.12	27.49	24.59	712.39	332.14	1161.53	391.66

C. Summary of Post-tax Present Values

			Net
EVA	712.39	287.61	1000.00
EVL	1161.53	−186.12	975.41
CFS			24.59
Shareholder Dividends			24.59

ATTACHMENT C-1b: EXAMPLE C

Base Case: Level Interest
Earnings Retained
CFS from Ex B Removed

Lapse Rate	Interest Rate Initial	Interest Rate Later	FIT Percentage	Credited Rate
0.00	0.14	0.14	0.368	0.13
0.00				
0.00				
1.00				

I. Pre-Tax Cash Flows

Years	Summary of Operations: Interest Earned Initial	Interest Earned Later	Interest Credited	FIT	Statutory Earnings	Other Cash Flows: Shareholder Dividends	Asset Rollover Initial	Asset Rollover Later	Liability Cash Flows	Net Cash Flows	Balance Sheet: Asset Balance Initial	Asset Balance Later	Liability Balance	Surplus Balance
											975.41		1000.00	−24.59
1	136.56	0.00	130.00	2.41	4.14	0.00	0.00	0.00	0.00	134.14	975.41	134.14	1130.00	−20.44
2	136.56	18.78	146.90	3.11	5.33	0.00	0.00	0.00	0.00	152.23	975.41	286.38	1276.90	−15.11
3	136.56	40.09	166.00	3.92	6.73	0.00	0.00	0.00	0.00	172.73	975.41	459.11	1142.90	−8.38
4	136.56	64.28	187.58	4.88	8.38	0.00	975.41	459.11	1630.47	−0.00	0.00	−0.00	−0.00	−0.00
Present Values:														
Pre-Tax	397.89	79.57	450.17	10.04	17.24	0.00	577.52	271.83	965.37	351.40				
Post-Tax	443.89	92.73	505.77	11.35	19.50	0.00	694.87	327.06	1161.53	385.67				

A. Summary of Pre-tax Present Values

			Net
EVA-BT	577.52	397.89	975.41
EVL-BT	965.37	10.04	975.41
CFS-BT			0.00
Shareholder Dividends			0.00

B. Summary of Post-tax Present Values

			Net
EVA-AT	694.87	443.89	1138.76
EVL-AT	1161.53	11.35	1172.89
CFS-AT			−34.12
Shareholder Dividends			0.00

II. Tax-Affected Cash Flows

Years	Interest Earned		Interested Credited	FIT	FIT Eff Liab	Statutory Earnings	Share-holder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows
	Initial	Later						Initial	Later		
1	86.30	0.00	82.16	2.41	−47.84	4.14	0.00	0.00	0.00	0.00	134.14
2	86.30	11.87	92.84	3.11	−54.06	5.33	0.00	0.00	0.00	0.00	152.23
3	86.30	25.34	104.91	3.92	−61.09	6.73	0.00	0.00	0.00	0.00	172.73
4	86.30	40.62	118.55	4.88	−69.03	8.38	0.00	975.41	459.11	1630.47	−0.00
Present Values: Post-Tax	280.54	58.60	319.64	11.35	−186.12	19.50	0.00	694.87	327.06	1161.53	385.67

C. Summary of Post-tax Present Values

			Net
EVA	694.87	280.54	975.41
EVL	1161.53	−186.12	975.41
CFS			0.00
Shareholder Dividends			0.00

ATTACHMENT C-1b: EXAMPLE D

Base Case: Level Interest
Earnings Retained
CFS from Ex B Removed

Lapse Rate	Interest Rate Initial	Interest Rate Later	FIT Percentage	Credited Rate
0.00	0.14	0.14	0.368	0.13
0.00				
0.00				
1.00				

I. Pre-Tax Cash Flows

Years	Interest Earned Initial	Interest Earned Later	Interest Credited	FIT	Statutory Earnings	Shareholder Dividends	Asset Rollover Initial	Asset Rollover Later	Liability Cash Flows	Net Cash Flows	Asset Balance Initial	Asset Balance Later	Liability Balance	Surplus Balance
											1010.00		1000.00	10.00
1	141.40	0.00	130.00	4.20	7.20	0.00	0.00	0.00	0.00	137.20	1010.00	137.20	1130.00	17.20
2	141.40	19.21	146.90	5.04	8.66	0.00	0.00	0.00	0.00	155.56	1010.00	292.77	1276.90	25.87
3	141.40	40.99	166.00	6.03	10.36	0.00	0.00	0.00	0.00	176.36	1010.00	469.12	1442.90	36.23
4	141.40	65.68	187.58	7.18	12.32	48.55	1010.00	469.12	1630.47	−0.00	0.00	−0.00	−0.00	−0.00
Present Values:														
Pre-Tax	412.00	81.33	450.17	15.88	27.28	28.75	598.00	277.76	965.37	359.09				
Post-Tax	459.63	94.78	505.77	17.90	30.74	34.59	719.51	334.20	1161.53	394.10				

A. Summary of Pre-tax Present Values

			Net
EVA-BT	598.00	412.00	1010.00
EVL-BT	965.37	15.88	981.25
CFS-BT			28.75
Shareholder Dividends			28.75

B. Summary of Post-tax Present Values

			Net
EVA-AT	719.51	459.63	1179.14
EVL-AT	1161.53	17.90	1179.44
CFS-AT			0.29
Shareholder Dividends			34.59

ATTACHMENT C-1b: EXAMPLE D—*Continued*

II. Tax-Affected Cash Flows

Years	Interest Earned		Interest Credited	FIT	FIT Eff Liab	Statutory Earnings	Share-holder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows
	Initial	Later						Initial	Later		
1	89.36	0.00	82.16	4.20	−47.84	7.20	0.00	0.00	0.00	0.00	137.20
2	89.36	12.14	92.84	5.04	−54.06	8.66	0.00	0.00	0.00	0.00	155.56
3	89.36	25.90	104.91	6.03	−61.09	10.36	0.00	0.00	0.00	0.00	176.36
4	89.36	41.51	118.55	7.18	−69.03	12.32	48.55	1010.00	469.12	1630.47	−0.00
Present Values: Post-Tax	290.49	59.90	319.64	17.90	−186.12	30.74	34.59	719.51	334.20	1161.53	394.10

C. Summary of Post-tax Present Values

			Net
EVA	719.51	290.49	1010.00
EVL	1161.53	−186.12	975.41
CFS			34.59
Shareholder Dividends			34.59

ATTACHMENT C-1b: EXAMPLE E

Base Case: Level Interest
Earnings Retained
CFS from Ex B Removed

Assumptions

Lapse Rate	Interest Rate: Initial	Interest Rate: Later	FIT Percentage	Credited Rate
1.00	0.14	0.144	0.368	0.13
0.00				
0.00				
0.00				

I. Pre-Tax Cash Flows

	Summary of Operations					Other Cash Flows					Balance Sheet			
	Interest Earned		Interest Credited	FIT	Statutory Earnings	Shareholder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows	Asset Balance		Liability Balance	Surplus Balance
Years	Initial	Later					Initial	Later			Initial	Later		
											1000.00		1000.00	0.00
1	140.00	0.00	130.00	3.68	6.32	6.32	0.00	0.00	1130.00	-1000.00	1000.00	-1000.00	0.00	0.00
2	140.00	-144.00	0.00	-1.47	-2.53	0.00	0.00	0.00	0.00	-2.53	1000.00	-1002.53	0.00	-2.53
3	140.00	-144.36	0.00	-1.61	-2.76	0.00	0.00	0.00	0.00	-2.76	1000.00	-1005.29	0.00	-5.29
4	140.00	-144.76	0.00	-1.75	-3.01	-8.30	1000.00	-1005.29	0.00	0.00	0.00	-0.00	0.00	-0.00
Present Values:														
Pre-Tax	404.60	-290.97	113.64	-0.00	-0.01	0.68	583.84	-586.93	987.76	-877.90				
Post-Tax	452.56	-334.32	119.16	-0.34	-0.58	-0.06	705.81	-709.54	1035.74	-920.83				

A. Summary of Pre-tax Present Values

			Net
EVA-BT	583.84	404.60	988.44
EVL-BT	987.76	-0.00	987.76
CFS-BT			0.68
Shareholder Dividends			0.68

B. Summary of Post-tax Present Values

			Net
EVA-AT	705.81	452.56	1158.37
EVL-AT	1035.74	-0.34	1035.40
CFS-AT			122.97
Shareholder Dividends			-0.06

ATTACHMENT C-1b: EXAMPLE E—*Continued*

II. Tax-Affected Cash Flows

Years	Interest Earned		Interest Credited	FIT	FIT Eff Liab	Statutory Earnings	Share-holder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows
	Initial	Later						Initial	Later		
1	88.48	0.00	82.16	3.68	−47.84	6.32	6.32	0.00	0.00	1130.00	−1000.00
2	88.48	−91.01	0.00	−1.47	0.00	−2.53	0.00	0.00	0.00	0.00	−2.53
3	88.48	−91.24	0.00	−1.61	0.00	−2.76	0.00	0.00	0.00	0.00	−2.76
4	88.48	−91.49	0.00	−1.75	0.00	−3.01	−8.30	1000.00	−1005.29	0.00	−0.00
Present Values:											
Post-Tax	286.02	−211.29	75.31	−0.34	−43.85	−0.58	−0.06	705.81	−709.54	1035.74	−920.83

C. Summary of Post-tax Present Values

			Net
EVA	705.81	286.02	991.83
EVL	1035.74	−43.85	991.89
CFS			−0.06
Shareholder Dividends			−0.06

ATTACHMENT C-1b: EXAMPLE F

Base Case: Level Interest
Earnings Retained
CFS from Ex B Removed

Assumptions				
Lapse Rate	Interest Rate Initial	Interest Rate Later	FIT Percentage	Credited Rate
1.00	0.14	0.144	0.368	0.13
0.00				
0.00				
0.00				

I. Pre-Tax Cash Flows

	Summary of Operations					Other Cash Flows					Balance Sheet			
	Interest Earned						Asset Rollover				Asset Balance			
Years	Initial	Later	Interest Credited	FIT	Statutory Earnings	Share-holders Dividends	Initial	Later	Liability Cash Flows	Net Cash Flows	Initial	Later	Liability Balance	Surplus Balance
											1000.00		1000.00	0.00
1	140.00	0.00	130.00	3.68	6.32	0.00	0.00	0.00	1130.00	−993.68	1000.00	−993.68	0.00	6.32
2	140.00	−143.09	0.00	−1.14	−1.95	0.00	0.00	0.00	0.00	−1.95	1000.00	−995.63	0.00	4.37
3	140.00	−143.37	0.00	−1.24	−2.13	0.00	0.00	0.00	0.00	−2.13	1000.00	−997.76	0.00	2.24
4	140.00	−143.68	0.00	−1.35	−2.32	−0.09	1000.00	−997.76	0.00	−0.00	0.00	−0.00	0.00	−0.00
Present Values:														
Pre-Tax	404.60	−288.98	113.64	0.73	1.25	−0.05	583.84	−582.54	987.76	−871.52				
Post-Tax	452.56	−332.03	119.16	0.51	0.87	−0.06	705.81	−704.23	1035.74	914.07				

A. Summary of Pre-tax Present Values

			Net
EVA-BT	583.84	404.60	998.44
EVL-BT	987.76	0.73	988.49
CFS-BT			−0.05
Shareholder Dividends			−0.05

B. Summary of Post-tax Present Values

			Net
EVA-AT	705.81	452.56	1158.37
EVL-AT	1035.74	0.51	1036.25
CFS-AT			122.12
Shareholder Dividends			−0.06

ATTACHMENT C-1b: EXAMPLE F—*Continued*

II. Tax-Affected Cash Flows

Years	Interest Earned		Interest Credited	FIT	FIT Eff Liab	Statutory Earnings	Share-holder Dividends	Asset Rollover		Liability Cash Flows	Net Cash Flows
	Initial	Later						Initial	Later		
1	88.48	0.00	82.16	3.68	−47.84	6.32	0.00	0.00	0.00	1130.00	−993.68
2	88.48	−90.43	0.00	−1.14	0.00	−1.95	0.00	0.00	0.00	0.00	−1.95
3	88.48	−90.61	0.00	−1.24	0.00	−2.13	0.00	0.00	0.00	0.00	−2.13
4	88.48	−90.80	0.00	−1.35	0.00	−2.32	−0.09	1000.00	−997.76	0.00	−0.00
Present Values:											
Post-Tax	286.02	−209.84	75.31	0.51	−43.85	0.87	−0.06	705.81	−704.23	1035.74	−914.07

C. Summary of Post-tax Present Values

			Net
EVA	705.81	286.02	991.83
EVL	1035.74	−43.85	991.89
CFS			−0.06
Shareholder Dividends			−0.06

ATTACHMENT C-1c

EXCERPTS FROM "SOME THOUGHTS ON DISCOUNTING"*

RICHARD M. WENNER

As actuaries we have frequently used discounting and present values to the point where they have become second nature to us. Recent valuation developments have forced me to step back and examine discounting and the meaning of its results.

The valuation developments I refer to are those emanating from the New York Life Insurance Department in connection with GICs and annuities under the Dynamic Valuation Law. New York's version of the Dynamic Valuation Law permits the more favorable (higher) valuation interest rate if the actuary can demonstrate through a good and sufficient test (which takes into account the relationship between assets and liabilities) that the resulting reserves are adequate.

In order to demonstrate such adequacy, one approach would be to project along several possible future interest rate paths the cash flow of both the contract liabilities of the book of business in question and the assets which support the reserves being tested. The net cash-flow streams for a given interest rate path could then be converted to a single value through discounting or accumulating. A positive value would demonstrate adequacy for that path; a negative value, inadequacy. (Let me duck the issue of the net book value of the business in question going negative somewhere in the path even though the net cash-flow single value is positive.)

Under a level interest rate path assumption, traditional discounting and accumulating at the assumed, single-interest rate are valid and produce meaningful results. But what about nonlevel interest rate paths? One has to question the meaning of the results in these cases if a single interest rate is used for discounting or accumulating. One can also question the meaning of the results of an approach whereby the accumulation or discount factors are derived by simply "stringing together" the applicable new money interest rates of the interest rate path in question.

My thinking on discounting and accumulating under a nonlevel future interest rate assumption has led me to conclude that what is theoretically called for is a type of investment-year method that takes into account an assumed reinvestment strategy. (I am using the word "reinvestment" to

*An abbreviated version of Mr. Wenner's paper appeared in the February 1983 edition of *The Actuary*. The excerpts reproduced here are from the original paper.

apply to cash flow from "insurance operations" as well as "investment operations.") The assumed reinvestment strategy needs to be defined for each future year of a given interest rate path and represents the strategy that would actually be employed to handle the cash flow that would emerge each year, if in fact that interest rate path were to materialize. The reinvestment strategy applicable to any given year's cash flow could be a function of what has transpired to date but, of course, cannot peek ahead to get a glimpse of the future interest rates the path has in store. Thus, two interest rate paths that run identically through year m and diverge thereafter would have to have the same assumed reinvestment strategies through year m.

An assumed disinvestment strategy (for example, selling certain assets, borrowing short term) is also needed to deal with the situation of negative cash flow. For the remainder of this discussion, the disinvestment strategy is considered merely a part of the overall reinvestment strategy.

An example best illustrates how the investment year approach might operate in the accumulation or discounting process. Assume a simple book of business consisting of (i) a $1,000 deposit at time 0 that is guaranteed to be repaid with interest at the rate of 9 percent per annum at the end of 3 years and (ii) a $1,000 bond, bought at par at time 0, paying annual coupons of $90 and maturing at the end of year 4.

Assume we are holding a reserve of $1,090 (equal to the accrued contract liability and supported by the $1,000 bond and first-year coupon) at the end of year 1 when interest rates are 10 percent and that we want to perform with respect to that reserve a good and sufficient test that takes both asset and liability cash flows into account. Further assume one future interest rate path to be tested has a 12 percent prevailing rate at the end of year 2, 14 percent at the end of year 3, and 16 percent at the end of year 4.

The assumed reinvestment/disinvestment strategy for this interest rate path is as follows. Each year any net positive cash flow is to be reinvested in annual coupon bonds at the prevailing rate to mature at the end of year 4 (assume a flat yield curve). Borrowing on mirror-image terms will be assumed to cover any negative cash flow, except that any investments existing at the end of year 4 will be sold at market value to minimize any borrowing at that point. The projected cash flow from assets existing as of year 1 (before reinvestment of the first-year $90 coupon) and from contract liabilities and their net sum is as follows:

YEAR-END CASH FLOW

	Year			
	1	2	3	4
Assets	$90	$90	$ 90.00	$1,090.00
Liabilities	0	0	(1,295.03)	—
Net (Before Reinvestment)	$90	$90	$(1,205.03)	$1,090.00

The question now is how to discount or accumulate these net cash flows.

Discount Factor Approach

Development of a discounted present value that has meaning in relation to the purpose of the testing requires some careful deliberation. I have concluded that the definition of present value that is needed must involve future accumulation values. The essence of the definition is that two things will be *deemed* equivalent today if they ultimately end up equivalent in the future. More specifically, the definition is as follows:

> The cash-equivalent present value of specified cash flow emerging in some future year is the amount of the current cash needed to ultimately produce over the interest rate path tested the same accumulated value that the specified cash flow in question would ultimately produce, provided both the current cash and the specified cash flow are reinvested in accordance with the assumed reinvestment strategy for that interest rate path.

The cash-equivalent present value of an investment is simply the sum of the cash-equivalent present values of the cash flow expected to be generated by the investment over the given interest rate path.

Note that this definition of present value is a function of both future interest rates *and* the assumed reinvestment strategy. Note also that future interest rates can additionally affect the present value by affecting the expected cash flow, itself, of the investment (and of the liability, for that matter).

By this definition, the discount factors can be derived as shown below. (The "ultimate" value of $1 of cash flow emerging at each year end, including that at the end of year 1, was calculated above.)

Year-End	Ultimate Value of $1 CF Invested at Year-End	Ultimate Value of $1 Current* Cash	Discount Factor	Net Cash Flow Before Reinvestment	Present Values
1	1.3397	÷ 1.3397	= 1.0000	× $ 90	= $ 90.00
2	1.2568	÷ 1.3397	= 0.9381	× 90	= 90.00
3	1.1400	÷ 1.3397	= 0.8509	× (1,205.03)	= (1,025.36)
4	1.0000	÷ 1.3397	= 0.7464	× 1,090	= 813.58
					(37.35)

* That is, at the end of year 1.

Note that, (i) as appropriate, (\$37.35)×1.3397=(50.05), the accumulated value previously calculated,† and (ii) the following discount factors would be inappropriate.

Year-End	Discount Factor		Net CF Before Reinvestment	Present Value
1	1.00	or 1.0000	× $ 90	= $ 90.00
2	1/(1.10)	or 0.9091	× 90	= 81.81
3	1/(1.10)(1.12)	or 0.8117	× (1,205.03)	= (978.12)
4	1/(1.10)(1.12)(1.14)	or 0.7120	× 1,090	= 776.08
				(30.22)

†\$50.05 was the accumulated value of the cash flows based on the progressive accumulation methodology described by Mr. Wenner, which tracks all the cash flows associated with the example.

APPENDIX C-2

VALUATION OF A BUSINESS BASED ON ITS CASH FLOW

OAKLEY E. VAN SLYKE

I. Summary

The committee on Valuation and Related Problems of the Society of Actuaries began studying measures of valuing life insurance companies in 1977. Mateja and Geyer [1] describe the history of work by this committee and its members. As they explain:

> This committee published its landmark paper in 1979* which established a conceptual framework for the balance sheet of an insurance enterprise. Assets and liabilities were viewed as cash-flow streams. The value assigned to the assets was the present value of all cash flows generated by the assets, and the value assigned to the liabilities was the present value of all cash flows generated by the liabilities. Surplus then was defined in accordance with traditional accounting conventions as the difference between the asset value and the liability value.

In valuation work we must be careful to distinguish between current statutory concepts, GAAP concepts, and cash-flow concepts. The same *approach* should be good enough to be applied to all three value systems, but we must be careful to distinguish our terms carefully.

Also, the same *approach* should be good enough to be applied to all the components of an ongoing business, for example:

A. Runoff of in-force
B. Renewals attributable to in-force
C. New sales from existing sales force, product mix, etc. (going-concern values).

A correct approach could be applied to the in-force only (component A) for solvency regulation. It could be applied to renewals (component B) in capitalizing a company's worth for tax purposes during acquisition, or in valuing a quota-share transfer of a block of policies. It could be applied to component C as well when determining the buy or sell price of a going concern. The only difference should be in what is included in the cash flows being considered. The approach to evaluating the cash flows should be the same.

The material that follows is based on well-established principles for evaluating risk by a decision-maker who is averse to risk. Some of the notation

*"Valuation, Surplus and Related Problems," *RSA* 5, no. 1 (1979): 256–84.

is new, in hopes that the new notation will make the fundamental concepts clear. The development of this material can be found in Raiffa [3], Cozzolino [4], Van Slyke [5] and Van Slyke [6], among others. The approach requires that a utility be associated with each cash flow, but plays down the role of any particular utility function. The result is a straightforward (although tedious) analysis of the financial strength of a business as reflected in its anticipated future cash flows.

II. Brief Overview of Interest and Discounting

The reader is trained and experienced in the theory and practice of investment calculations. These few comments are intended to highlight the concepts that will be used later rather than to present any new material.

Interest rates vary from time to time. The same investment, such as the purchase of a one-year U.S. Treasury bill, may yield 5 percent one year and 10 percent another.

Interest rates also vary from one investment to another. Bonds issued by companies with marginal financial histories have higher yields than bonds with the same term issued by the U.S. government. Writers have discussed many reasons for these variations, but for our purposes we can consider this variation simply as a reflection of the risk that the principle or anticipated interest will not be fully realized.

Whatever the reason for the interest level, an anticipated cash flow of \$1.00 in the future is worth less than \$1.00 in hand. We can say that an income of \$1.00 deferred some length of time is less useful than \$1.00 in hand, or that it has lower utility.

The extent to which a cash flow of \$1.00 at time t is worth less than \$1.00 is usually expressed as $\$1v^t$, where t is in years and $v=(1+i)^{-1}$. More generally, we may write $\$1v(t)$, which allows the interest rate i to vary over time and avoids questions about the meaning of i over a fraction of a year. For any cash flow of x at time t, then, the value today is $xv(t)$.

In today's financial marketplace we can generally purchase nearly risk-free investments maturing at nearly any future date. For practical purposes this allows us to distinguish that part of $v(t)$ that is due to the time deferred without risk from that part that is due to risk. For example, a return of 10.24 percent on a one-year investment when risk-free investments are earning 6 percent suggests that the additional return for risk is 4 percent. We shall return to this with some notation after introducing utilities in Section IV.

III. Brief Review of Probability

The reader is also familiar with probability, and we shall keep this section brief. The concept of the *event* plays such a crucial role, however, that we must be careful to define events and probabilities at this point.

We often say that an event i has a probability p_i. (The subscript i should be easily distinguished from the interest rate i. Both bits of nomenclature are firmly established in actuarial literature, and it seems unnecessary to change notation here.) We define the events to be mutually exclusive events, so that

$$\sum_{\text{all } i} P_i = 1. \tag{III-1}$$

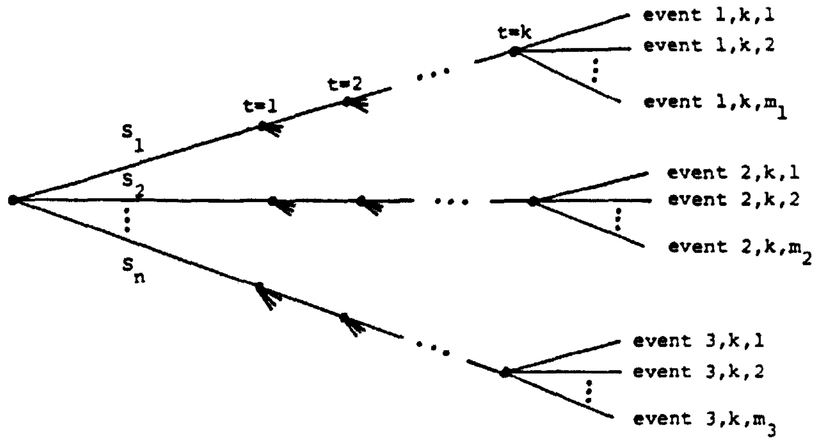
Events at different points in time may be correlated with one another. Income in year 10 may be positively correlated with loss (investment) in year 1, for example. By this we mean that a large investment in year 1 is expected to increase the chance of a large income in year 10. As another example, income in year 10 may be correlated with income in other years. A low lapse rate would tend to increase income in all later years; a high lapse rate would tend to depress income in all later years. Events that are correlated with one another must be carefully distinguished from events that are not correlated with any other.

In financial planning we refer to a set of closely related possible outcomes as a scenario. An "inflationary scenario," for example, refers to a description of the outcomes in which the probabilities of possible outcomes at each point in time are correlated because of the effects of inflation. In valuation work it might be important to consider inflation, interest rates, sales success, lapse rates and other factors to identify the ways in which outcomes in some years are correlated with outcomes in other years.

With these definitions, any particular scenario can be analyzed into many scenarios, each with a description within the more general framework. As a scenario is analyzed into mutually exclusive sets of outcomes (such as "slightly inflationary, with tight credit") it becomes, in the extreme, a list of possible events, each with a specific probability. Any framework for evaluating insurance products or insurance companies should give results that are consistent for various ways of identifying the scenarios and the events that may result under those scenarios.

The concept of event then has the usual meaning in probability work: the set of possible events defines an exhaustive and mutually exclusive set of independent results, and a probability is associated with each result. (See Figure C-2-1.) The probability of occurrence i at time t under scenario j is determined by j and t, but whether event i actually occurs is independent of the results for other values of j and t.

FIGURE C-2-1



In general for a set of scenarios denoted by the subscript j, with $j = 1, \ldots, m$, each with n_j possible events, we can say:

$$\sum_{\text{all } j} \sum_{\text{all } i_j} p_j p(i,j|j) = 1 \tag{III-2}$$

In this expression $p_j p(i,j|j)$ is the probability of occurrence of event i under scenario j.

IV. Brief Review of Decision Theory

This section introduces material that may be new to the reader, and notation that is almost certainly new. There are only three concepts, however, and we have included several examples of each to help the reader gain an intuitive feel for the concepts. Of course, the expected value of a set of cash flows x_i with probability p_i at time 0 is:

$$EV = \sum_{\text{all } i} p_i x_i \tag{IV-1}$$

If cash flows may occur in the future, we have familiar formulas for the expected value and the present value:

$$EV = \sum_{\text{all } i} \sum_{t} p_{i,t} x_{i,t} \qquad \text{(IV-2)}$$

$$PV = \sum_{\text{all } i} \sum_{\text{all } t} p_{i,t} v_{i,t} x_{i,t} \qquad \text{(IV-3)}$$

where

$p_{i,t}$ = probability of event i leading to cash flow $x_{i,t}$ at time t

$v_{i,t}$ = value of $1 cash flow at time t given occurrence of event i.

We can extend this to a problem in which the decision-maker has defined many scenarios. The formulas are:

$$EV = \sum_{\text{all } j} \sum_{\text{all } i} \sum_{\text{all } t} p_j p(i,j,t|j,t) x_{i,j,t} \qquad \text{(IV-4)}$$

$$PV = \sum_{\text{all } j} \sum_{\text{all } i} \sum_{\text{all } t} p_j p(i,j,t|j,t) v_{j,t} x_{i,j,t} \qquad \text{(IV-5)}$$

where

p_j = probability of scenario j

$p(i,j,t|j,t)$ = probability of cash flow $x_{i,j,t}$ at time t in scenario j, given that scenario j has been realized.

$v_{j,t}$ = value of cash flow at time t in scenario j

The third concept (after expected value and present value) is utility. The utility of a particular cash flow is the value that a particular decision-maker attaches to that cash flow. We have mentioned that 1v(t)$ is a measure of the worth (today) of a cash flow of $1 at time t. If the decision-maker uses present-value calculations,

$$U(x) = \sum_{\text{all } t} x_t v(t) \qquad \text{(IV-6)}$$

In general, we can simply write the utility of a cash flow of x as $U(x)$, the utility of a cash flow of x at time t as $U(x, t)$, and the utility of a cash flow x at time t under scenario j as $U(x,t,j)$.

The thrust of this paper is that the utility of a cash flow should be considered because any responsible decision or evaluation should weigh bad outcomes more severely than favorable outcomes. Expected value calculations are merely a special case of utility calculations in which the evaluator's aversion to risk is negligible. Expected value calculations fall short of the needs of regulators, investors, product-developers and others because it is appropriate for these decision-makers to be averse to risk.

As the pioneers of decision theory (for example, Raiffa [3]) showed, consistent decisions can be developed from a given set of estimates of probabilities only when the probabilities and utilities are combined in the following way:

$$U. = \sum_{\text{all } i} p_i u(x_i) \qquad \text{(IV-7)}$$

Note that the utility of an outcome must be evaluated independently of its probability. The utilities of the possible outcomes must be determined, then averaged using their probabilities as weights. The decision-maker will not be able to reach a consistent set of decisions if the utility of the possible events is measured by the sum of the $U(p_i x_i)$, unless the decision-maker has no aversion to risk.

The particular utility functions to be used will not play an important role in this article. Often we will write simply $U(x)$. But simple examples will employ an exponential utility function because the exponential function is reasonably easy to understand and is probably a reasonable approximation to most functions that will be used in practice.

Exponential Utility

The exponential utility function may be defined as follows: the utility of a cash flow (*in*) of x_i with probability p_i is

$$-c \ln[p_i e^{-x_i/c} + (1 - p_i)] \qquad \text{(IV-8)}$$

In this utility calculation a gain of x_i is related to some scale c, then decreased by the exponentiation, and then weighted by its probability, p_i. The value of a gain of zero (which is unity) is given a weight of $1-p_i$. Then the exponentiation and scale shift in c are undone in order to develop a result in the same scale as x_i.

Example 1

$p_1 = 0.1$
$x_1 = \$10$ million
$c = \$150$ million
$EV_1 = 0.1 \times \$10$ million $= \$1.0$ million
Utility $= -\$150$ million $\ln\,[0.1e^{-10/150} + (1-0.1)]$
$= \$0.97$ million

Example 2

$p_2 = 0.01$
$x_2 = -\$100$ million (loss)
$c = \$150$ million
$EV_2 = 0.01 \times (-\$100 \text{ million}) = -\$1$ million
Utility $= -\$150$ million ln $([0.01e^{100/150} + (1-0.01)]$
$= -\$1.41$ million

This definition allows the outcomes to be defined in any degree of detail. In particular, if we associate with every possible outcome a value x_i and a probability p_i, then the utility of the set of outcomes is

$$U. = -c \ln \sum_{\text{all } i} p_i e^{-x_i/c} \quad \text{(IV-9)}$$

This definition can be expanded easily to include sums over several scenarios:

$$U. = -c \ln \sum_{\text{all } j} \sum_{\text{all } i|j} p_j p(i,j|j) e^{-x_{i,j}/c} \quad \text{(IV-10)}$$

where p_j is the probability of the j-th scenario.

Risk-Adjusted Value (RAV)

Whenever a utility measure is in the same units as the cash flow, it makes intuitive sense to call it the risk-adjusted value, or *RAV*, of the set of outcomes.

A feel for utility calculations in general, and exponential utility in particular, can be gained from considering the value of *RAV* at extreme values of p_i and c. The reader can verify the following results for the exponential utility calculations:

Result	Implications
A. $\lim_{p_1 \to 1} RAV = x_1$	As the probaiblity of event 1 increases toward certainty, the risk-adjusted value of all outcomes approaches the value x_1.
B. The addition of a certain cash flow x (one with probability one) to a set of cash flows increases their *RAV* by x.	Amounts that are absolutely certain can be handled outside the *RAV* calculation.
C. $\lim_{c \to \infty} RAV = EV$	As one's scale factor increases toward infinity, the decision or evaluation approaches a simple expected-value calculation.
D. $\lim_{c \to 0} RAV = \min (x_i)$	As the scale factor decreases toward zero, the risk-adjusted value approaches the cost of the worst possible cash outflow (the *minimum* cash inflow).

We call the scale factor c the evaluator's risk capacity.

The last two observations can be restated in common sense ways. If an evaluator uses exponential utility calculations, then:

- An evaluator with very large risk capacity (in relation to the outcomes) is an expected-value decision-maker.
- An evaluator with a very limited risk capacity (in relation to the outcomes) will behave as if the worst possible outcome were certain to occur.

Of course in most practical situations those who take risks have a risk capacity that is significant in relation to the assumed risks, yet assume risks that are significant in relation to this risk capacity.

Determining Risk Capacity

From the point of view of one who must carry out the *RAV* calculations, the risk capacity is no more than the scale factor, c, that must be chosen to calculate a numerical value for *RAV*. In an important sense, however, a firm's risk capacity is a measure of its financial size and ability to withstand an unexpected loss. For example, in the presence of a competitive but not overzealous reinsurance market, a company might be expected to buy reinsurance to protect against a financial loss greater than its risk capacity.

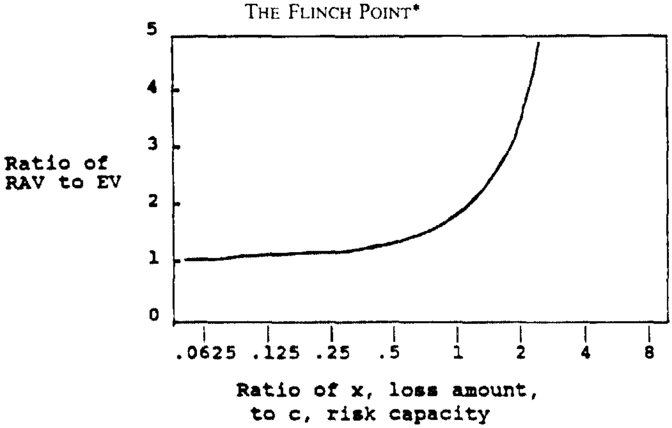
Note that the value, x, the risk capacity, c, and the risk-adjusted value, *RAV*, are all in the same units. If the units of x are cash in U.S. dollars, c and *RAV* are cash in U.S. dollars. If the units of x are statutory earnings, c and *RAV* are statutory earnings.

One corollary to the concept of risk capacity that has an intuitive feel is that a "flinch point" exists at which even a small possibility of a loss causes us to move to an attitude of risk aversion. Figure C-2-2 illustrates the "flinch point" by considering the utility of a small chance of a cash outflow of x for various values of risk capacity c. One's flinch point is at roughly twice one's risk capacity.

Practical experience suggests that a flinch point or a value of c can be identified for any particular valuation problem. For example, in valuing a company with assets of \$10 million, it might be appropriate to use a flinch point of \$200,000, corresponding to a risk capacity of about \$100,000. In valuing a company with assets of \$1 billion, it might be appropriate to use a flinch point of \$10 million, corresponding to a risk capacity of about \$5 million. These examples reflect an assumption that the large company is

FIGURE C-2-2

THE FLINCH POINT*



*The risk-adjusted value of a small chance of losing x increases dramatically when the loss x exceeds about twice the evaluator's risk capacity, c. This is the evaluator's flinch point.

more conservative in percentage terms than the small company, yet has greater risk capacity in absolute terms.

Discussion of utility and risk capacity should not give the impression that risk capacity is objective. Neither should we leave the impression that risk capacity is arbitrary. In practice, the selection of an appropriate value for c requires judgment, but c is confined within a certain range by the uncertainties being considered. (For example, "I'd like to think the ABC company is risk averse, but when I review their underwriting commitments, their risk capacity is clearly between \$10 million and \$50 million.") The value of risk capacity, c, can be selected with the same degree of confidence that the probabilities can be estimated. Most important for insurance company valuation, a consistent set of valuations for a number of companies can be calculated by consistently setting c to be a simple fraction of assets such as 1 percent.

Other Utility Functions

The specific results above depend on the exponential utility function. Other utility functions might be considered as well. Our impression is that other utility functions that provide substantially all the desirable properties offered by the exponential will generally produce results similar to those of the exponential.

V. The Importance of Cash Flow

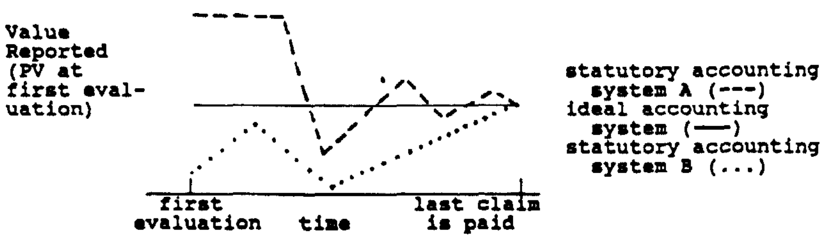
A business is said to be insolvent when it cannot meet its financial obligations. In the absence of any accounting conventions, this occurs when the company is unable to honor its current obligations.

Insurance companies make commitments to pay claims many years in the future. Elaborate accounting conventions have evolved to promote early recognition of financial situations that may lead to later inability to pay claims. These accounting conventions, including both GAAP and statutory, are important, but ultimately the purpose of insurance company valuation is to provide assurance that funds will be available to pay claims.

Advocates of cash-flow measures of financial condition have sometimes been criticized because forecasts of future cash flows are subject to error. Accounts based on historical measures of sales commitment are in some sense more subject to objective audit. This is true, but when the last claim is paid, both cash-flow-based measures and "objective" measures must converge to the same result. The track records of three valuations systems are illustrated in Figure C-2-3.

FIGURE C-2-3

ACCOUNTING SYSTEMS OVER TIME*



*Accounting systems have several goals. One is to reflect at any point in time the value of the company in light of its future obligations. If the business takes on no new obligations, all accounting systems will eventually produce the same answer. At any time before the last claim is closed, an accounting system can be judged by how close its estimate of the value of the company is to the value that is ultimately known (after all estimates are stated in terms of present or accumulated value).

In short, estimates of cash flow are subjective but subject to rational analysis. When the last claim is paid, cash flow is objective, and all other accounting systems converge to accumulated cash flow.

Mateja and Geyer [2] demonstrate two important points that we will take as given. These are:

1. Cash flows must be evaluated after-tax and discounted at after-tax rates of return if they are to provide consistent results.
2. The present value of a company's cash flow is the sum of its current cash, the present value of future dividends, and the present value of the increase in its cash.

In these points, present value includes expected value (as it does in the section on expected value above), and all calculations run until the last claim is closed.

Insurance management cannot ignore statutory and GAAP accounting, of course. Both statutory and GAAP accounting are widely used measures of financial condition, and changes in value measured by statutory and GAAP accounting will affect the range of alternatives company management can take advantage of. A good system of company valuation should be robust enough to be applied to statutory and GAAP profits and losses as well as to cash flow.

VI. Synthesis of Cash Flow and RAV

The final step in company valuation measuring cash flows in light of uncertain future payments is to synthesize the focus on cash flows into the risk-adjusted value calculations outlined above.

Scenarios As Related Sets of Events

As we discussed in Section III, a scenario is generally a set of related possible outcomes. We can associate a particular probability with each scenario. For each scenario, we can define a set of possible cash flows, associating with each cash flow a particular dollar amount, a particular time, and a particular probability.

The probabilities of the various outcomes $x_{i,j,t}$ are not generally independent, however. For example, scenario 1 might be high interest rates. In a high-interest-rate scenario, early cash flows might reflect high levels of cash outflow due to policyholder borrowing and turnover, while later cash flows might reflect substantial cash inflows because of greater margins between interest earned and interest paid to policyholders. These offsetting profits and losses are linked together by the scenario.

Each $RAV_{j,t}(c)$ is the risk-adjusted value of all events that might occur at time t under scenario j. This value is:

$$RAV_{j,t}(c) = -c \ln \sum_{\text{all } i} p(i,j,t|j,t)\, e^{-x_{i,j,t}/c} \quad \text{(VI-1)}$$

Consider, for example, the risk-adjusted value of the possible cash flows at time 0 under scenario 1. This is $RAV_{1,0}(c)$. $RAV_{1,0}(c)$ comprises

$$\begin{array}{l} x_{1,1,0};\ p_{1,1,0} \\ x_{2,1,0};\ p_{2,1,0} \\ \cdot \\ \cdot \\ \cdot \\ x_{n,1,0};\ p_{n,1,0} \end{array}$$

In the general case, $RAV_{j,t}(c)$ comprises the following elements:

$$\begin{array}{l} x_{1,j,t};\ p_{1,j,t} \\ x_{2,j,t};\ p_{2,j,t} \\ \cdot \\ \cdot \\ \cdot \\ x_{n,j,t};\ p_{n,j,t} \end{array}$$

The amounts and probabilities set forth above must reflect all risk if risk is to be treated consistently regardless of its source. A statement like, "Under scenario 1 at time 10, there is a 1 percent chance of a cash inflow of $100," should reflect all sources of uncertainty about the amount of the cash flow. Some of the cash flow may be investment income, some may be renewal premium, some (negative) amount may be renewal premium, some (negative) amount may be administrative expenses, and so on. Each source presumably has an expected value and an uncertainty about it. The pairs of x_i, p_i for a particular set of j and t should reflect all sources of cash flow and the uncertainty about that total cash flow.

Within a given scenario, early investments may be offset by later gains, or early high premium revenues may be offset by later high mortality. The present-value calculation should bring together these investment-return situations.

	$t = 0$	$t = 1$	...	t
Scenario 1:	$RAV_{1,0}(c)$	$RAV_{1,1}(c)$	...	$RAV_{1,t}(c)$
Scenario 2:	$RAV_{2,0}(c)$	$RAV_{2,1}(c)$	...	$RAV_{2,t}(c)$
	.	.		.
	.	.		.
	.	.		.
Scenario j:	$RAV_{j,0}(c)$	$RAV_{j,1}(c)$	...	$RAV_{j,t}(c)$

where $RAV_{j,t}(c)$ is a single number reflecting all possible cash flows at time t under scenario j. These $RAV_{j,t}$'s are all certainty equivalents for given scenarios because all investment risk and all underwriting risk is reflected in the p_i's.

The next step is to take the present value of each scenario at a risk-free rate of return. A risk-free rate of return is appropriate because all investment risk has been reflected in the p_i's.

The correct summary is therefore:

$$RAV_j(c) = \sum_t v_t RAV_{j,t}(c)$$
$$= -c \sum_t v_t \ln \sum_{\substack{\text{all } i \\ \text{within } j}} p(i,j,t|j,t) e^{-x_{i,j,t}/c} \qquad \text{(VI-2)}$$

The overall risk-adjusted value is determined from these RAV_j's. The risk-adjusted value for each scenario must be given proper adjustment for risk and probability. The formula is

$$RAV(c) = -c \ln \sum_j p_j e^{-RAV_j(c)/c} \qquad \text{(VI-3)}$$

In summary, there are three steps to the calculations. First, any uncertainty about the value associated with each time under each scenario is resolved by replacing a probability distribution (the set of $p(i,\ j,\ t|j,t)$, $x_{i,j,t}$) with its utility, $RAV_{j,t}(c)$. Second, the present value of each scenario is taken so that the results that are correlated with one another over time are allowed to offset or compound one another. Third, the present value of the utility of each scenario is treated as a single element of a traditional utility calculation, one that gives greater weight to adverse results (adverse scenarios) than to favorable scenarios.

These calculations allow early investments to be offset by later rewards, while preserving the flexibility to envision scenarios in which the anticipated rewards do not materialize.

General Form

The intermediate steps shown above are intended to make the rationale more clear. They are not essential. The formulas for $RAV_j(c)$ and $RAV_{j,t}(c)$ can be replaced with their equivalent expressions in terms of the p_i's and $x_{i,j,t}$'s. The results are:

$$RAV(c) = -c \ln \sum_j p_j \exp\left(\sum_t v_t \ln \sum p(i,j,t|j,t) e^{-x_{i,j,t}/c} \right) \quad \text{(VI-4)}$$

Every atom of cash flow under any scenario is weighted by the utility function, which in this case is $\exp(-x/c)$, and by its probability, which is $p_j p(i,j,t|j,t)$.

Large negative values of $(x_{i,j,t})/c$ will play a key role in the actual value of $RAV(c)$ unless they are offset by positive values at other times within the scenario. The calculations can be inspected for such values to see the sensitivity of the value of $RAV(c)$ to specific assumptions about the cash flows. Recall that an evaluation with a limited risk capacity in relation to outcomes will give results that correspond to the outcome of the worst possible result.

Large negative values of $(x_{i,j,t})/c$ may be due to poor assumptions, but they may be due to real risks the insurer faces. In the latter case, the company may want to adjust its underwriting or investment portfolio to reduce the magnitude of the possible loss.

Investment Risk

These calculations reflect a particular way to handle investment risk. As we discussed in Section II, investment risk is an integral part of any present value factor whenever the investment anticipates a return greater than the risk-free rate of return. The discussion of utility in Section IV pointed out that risk aversion implies the evaluator should give greater weight to significant adverse cash flows than to modest adverse cash flows or to favorable cash flows. Each discount factor (the $v(t)$'s of Section II) is divided into a risk-free present value factor and a utility adjustment for risk.

This treatment of interest is important for two reasons:

1. It supports the important concept of immunization. Specifically, to the extent the company has cash outflows equal to cash inflows, the net cash flow is zero and the company has removed the element of risk from both the investment portfolio and the underwriting portfolio. This

support comes directly from the careful distinction between risk-free return and uncertainty in the dollar amount of return.

2. It provides to the final result the important associative property of algebra. Specifically, a risk-adjusted value can be associated with any set of risky outcomes. The *RAV* for the entire set of outcomes does not depend on how finely the evaluator enumerates the possible outcomes.

The Relationship of RAV to Risk-Free Rate of Return

It is often useful to ignore changes in the risk-free rate of return over time and represent v_t by $(1+r)^{-t}$, where r is an estimate of the risk-free rate of return.

With this selection of v_t, the total risk-adjusted value of the cash flows is

$$RAV(c,r) = -c \ln \sum_j p_j \exp\left(\sum_t -\frac{RAV_{j,t}(c)}{c(1+r)^t} \right) \qquad \text{(VI-5)}$$

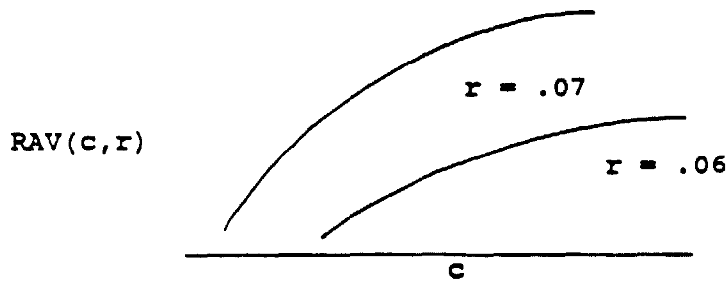
That is, for a particular set of possible cash flows, the *RAV* is a function of the firm's risk capacity and the assumed risk-free rate of return. This can be viewed as a series of curves reflecting the risk-adjusted value of the possible cash flows under various alternative assumptions about c and r.

The curves in Figure C-2-4 show why consistent results cannot be obtained over all types of assumptions when one presumes surplus is invested for later return. That presumption leads to apparently better results for a company when risk-free rates of return are low than when they are high. The correct treatment, as shown by *RAV*, leads to the appropriate result that an insurance company is more valuable when risk-free rates of return are high and future claim payments are more discounted. This important result is not dependent on the particular utility function chosen.

The curves in Figure C-2-4 would allow a valuation actuary to rank a set of insurance companies without selecting for any selected values of r. For example, the valuation actuary would be able to make statements such as, "If the risk-free rate of return stays at over 6 percent, all ten companies have positive risk-adjusted values. If risk-free rates of return are only 4 percent, companies 4 and 7 fail to have positive risk-adjusted values. Companies 1, 2, and 3 are well enough immunized that they have positive risk-adjusted values for all risk-free rates of return."

FIGURE C-2-4

RAV AS A FUNCTION OF RISK-FREE RATE OF RETURN (r) AND RISK CAPACITY (c)*



*The risk-adjusted value increases asymptotically toward the present value as c increases. Assuming initial cash outflows and later cash inflows, an investment has a lower *RAV* if the risk-free rate of return is greater. A growing life insurance company, on the other hand, might be characterized as a set of early cash inflows followed by a set of later cash outflows. The risk-adjusted value at time zero for a growing insurance company would be greater if the risk-free rate of return were high than if the risk-free rate of return were low.

Based on our experience in other fields, if $RAV(c,r)$ were measured with exponential utility, and c were set using a rule of thumb such as 1 percent of assets, the curves of $RAV(c,r)$ would not be much affected by modest changes in c. For example, the curves would be about the same if c were taken at 0.6 percent of assets or 1.8 percent of assets, a 3-to-1 range. Also, if a company is reasonably well-immunized, the value of $RAV(c,r)$ will not be much affected by the choice of r. Poorly funded companies will be clearly distinguished from well funded companies over a wide range of c and r.

VII. Comparison of RAV(c,r) *to Statutory Surplus*

Statutory surplus will continue as a widely recognized method of company valuation even if a cash-flow-based approach becomes generally popular. There are important differences between the two.

First, if c is very large, risk considerations are small. Several observations emerge:

(a) If $RAV(c,r)$ is negative, while current statutory surplus is positive, then the company is headed for insolvency, although it may be saved by good luck. There is a real risk that future statutory losses will ultimately bring statutory surplus to a negative position.

(b) If $RAV(c,r)$ is positive and greater than statutory surplus, the company is headed for increases in statutory surplus.
(c) If $RAV(c,r)$ is positive, but less than statutory surplus, then either:
   (1) There is an expectation that statutory losses will be greater than statutory gains, or, at best,
   (2) There is an expectation that statutory surplus will grow at a rate less than r.
(d) If current statutory surplus is negative, but $RAV(c,r)$ is positive, then future statutory gains are expected to be sufficient, *if retained*, to bring statutory surplus to a positive position at some future time. An insurer with negative statutory surplus and positive $RAV(c,r)$ might be an excellent prospect for sale to an insurer with positive statutory surplus.

Second, if c is not very large in relation to the risks of the cash flows, then the value of $RAV(c,r)$ may be much smaller than the expected present value. In this case,
(a) If $RAV(c,r)$ is negative, the company can be said to have undertaken risks inappropriate for its risk-bearing ability or to have failed to reduce its risks to prudent levels by diversification and immunization.
(b) If $RAV(c,r)$ is positive and greater than statutory surplus, then the company can be expected to realize gains in statutory surplus.
(c) If $RAV(c,r)$ is positive but less than statutory surplus, the company runs a substantial risk of loss of statutory surplus.

Solvency regulation might be improved by changing the computation of statutory surplus to better measure risk-adjusted value. This might be possible without a major departure from current recordkeeping practices.

VIII. Are RAV Calculations Worth the Effort?

RAV calculations will be worth the effort only if the advantages of the *RAV* results are important. These advantages are:
(1) The results of the risk-adjusted value calculations are objective in an important sense: different valuation actuaries working with the same set of assumptions will reach substantially the same results. As a corollary, although different *RAV*'s may be developed by several actuaries, the differences can be traced to specific differences in assumptions. This advantage is held by statutory and GAAP calculations, but not by the judgmental comparisons of surplus to underwriting and investment risks that regulators must make in order to use statutory and GAAP results.

(2) The *RAV* calculations in all their detail lead to better understanding of a company's financial strength and the sources of that strength. Those details also help pinpoint events of high possible loss that can be reduced by changes in underwriting and investment strategies. The *RAV* effort may lead to better management.
(3) The *RAV* approach can be generalized. For example, a regulator reviewing a merger of two life insurance companies can consider the *RAV* of the two companies and of the new consolidated company. A wise merger would have a consolidated *RAV* greater than the sum of the two separate *RAV*'s.
(4) The *RAV* calculations not only reflect the extent of immunization but also show the sensitivity of the valuation to changes in assumptions about underwriting losses and investment returns.

Considering the enormous effort currently spent on solvency regulation and the potentially harmful effects of poor solvency regulation, and the knowledge that would come from the *RAV* calculations themselves, the *RAV* calculations do seem worth the effort.

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APPENDIX D-1

C-3 RISK ASSOCIATED WITH THE SPDA PRODUCT

JAMES A. GEYER AND MICHAEL E. MATEJA

The purpose of this report is to present the results of an analysis of C-3 risk associated with a typical single-premium deferred-annuity (SPDA) product and an assessment of these results.

Background

When the Combination of Risks Task Force decided to use cash-flow analysis in its effort to understand and quantify combinations of risks, it became necessary to develop a computer model to support the necessary analysis. The process of constructing such a model quickly established that while modeling future cash flows associated with an insurance product subject to different risks was relatively straightforward, understanding and interpreting the resultant cash flows was not nearly as simple and straightforward. A great deal of effort was eventually devoted to the subject of discounting cash flows and the end result was the concept of CFS.

Once the model took shape, it became obvious that the model offered the opportunity to more thoroughly understand mismatch risk. When a relationship between "required surplus" and "duration" was developed, it was concluded that it would be worthwhile to complete a more disciplined analysis of C-3 risk that would serve as a point of departure when this risk was combined with other risks in subsequent phases of the research effort.

Attachment D-1a contains a detailed description of the assumptions used to model the SPDA product. The mechanics of the computer model are described in Appendix B-1.

Major Findings

1. Management of the interest-crediting strategy can influence the timing of reported losses associated with the C-3 risk, but perhaps not the ultimate economic cost.
2. The degree of mismatch or C-3 risk as measured by required surplus is extremely sensitive to the investment strategy, the level to which future interest rates rise, and withdrawal rates. Other factors, not commonly associated with C-3 risk, have a significant bearing on required surplus including the treatment of negative FIT, the shareholder dividend policy, and the margin for adverse deviation contained in the reserves.

3. Assuming a level future interest rate environment, there appears to be a relationship between the difference between asset and liability durations, the basic earnings margin, and required surplus. Specifically, required surplus can be approximated by the following formula:

$$\text{Required Surplus} = (i_{\text{actual}} - i_{\text{base}})(D^A - D^L) - C$$

where

i_{actual} = the assumed level future new money rate
i_{base} = the interest rate at which the initial assets are invested
D^A = the Macaulay duration of the assets
D^L = the Macaulay duration of liabilities
C = a constant that is a function of the earnings margin present prior to an adverse interest scenario.

This approximation formula was derived from regression analysis performed on the model results. It appears to hold on both a pre- and post-tax basis, where on a post-tax basis, i_{actual} and i_{base} are after-tax rates, and the durations are calculated using after-tax interest rates.

These three major findings are discussed in more detail below.

I. Management of Interest-Crediting Strategy

The nature of the C-3 risk for a typical SPDA product as measured by required surplus is dependent on the interest-crediting strategy adopted by management when interest rates rise dramatically. Testing has concentrated on the effects of two basic crediting strategies:

(1) Low Crediting Strategy. Keep credited rates low and risk a great outflow of cash, which, if it produces negative cash flow, requires liquidation of assets at a loss or borrowing at high rates, which produces an equivalent loss.
(2) High Crediting Strategy. Increase credited rates to prevent the cash outflow, thus producing a *certain* immediate loss due to crediting more interest than the assets are actually generating.

Results for a CORTF computer model run when the low crediting strategy was followed are summarized below in Table D-1. The major assumptions are as follows: initial assets earn 14 percent with a duration of 5.5 percent; interest rates immediately rise to and remain at 20 percent; the 13 percent credited rate is maintained indefinitely; policyholders lapse at the rate of 25 percent each year, and initial liabilities equal $1,000,000.

TABLE D-1

CORTF MODEL RESULTS
LOW CREDITING STRATEGY

Year	New Money Interest Rate	Credited Rate	Lapse Rate	Liability Balance EOY (000)	Net Cash Flow (000)	Average Earned Rate	GFO AFIT (000)
1	20.0%	13.0%	25.0%	$847.5	$ −107.9	14.0%	$ 6.3
2	20.0	13.0	25.0	718.3	−92.1	13.2	1.5
3	20.0	13.0	25.0	608.7	−78.0	12.4	−2.3
4	20.0	13.0	25.0	515.9	−64.6	11.6	−5.4
5	20.0	13.0	25.0	437.2	−50.8	10.7	−7.8
10	20.0	13.0	25.0	191.2	−9.2	6.2	−11.8
15	20.0	13.0	25.0	83.6	−13.0	−33.5	−10.8
20	20.0	13.0	25.0	36.5	−46.1	−20.0	−21.2

Assumed high lapses due to low credited rates immediately cause net cash flow to turn negative. In the CORTF computer model, negative net cash flow is covered by borrowing at the then current interest rate, that is, 20 percent. If assets are liquidated, the losses would show up immediately. By borrowing, the losses are deferred, but not avoided.

The average earned rate on the net assets (invested assets less borrowing) falls steadily as more money is borrowed at 20 percent. Since the assumed asset is a 15-year mortgage, the last of the initial assets roll over at the end of year 15, and the average earned rate stabilizes at −20 percent.[1] In year 12, the asset balance becomes negative, indicating a net borrowed position.

Since lapses are assumed to occur at the end of the year, first-year earnings are unaffected by the interest rate increase. Earnings quickly become negative, however, as the average earned rate drops below the credited rate.

Consider next how the C-3 risk is manifested when the high crediting strategy is chosen, that is, management increases the credited rate ("chases" the new money rate) in an attempt to minimize cash outflow. Assume it is decided to credit what the average earned rate would have been if there were no lapses. Assume further that lapse rates are still high initially, but considerably reduced from the prior case. Table D-2 presents the specific credited rate and lapse rate assumptions and the results:

[1]A negative sign is used to denote the situation in which both interest and assets are negative.

TABLE D-2

CORTF MODEL RESULTS
HIGH CREDITING STRATEGY

Year	Interest Rate	Credited Rate	Lapse Rate	Liability Balance EOY (000)	Net Cash Flow (000)	Average Earned Rate	GFO AFIT (000)
1	20.0%	14.0%	22.0%	$ 888.9	$ −62.9	14.0%	$ 7.0
2	20.0	15.0	19.2	825.9	−19.1	13.6	−0.7
3	20.0	15.8	16.9	794.6	13.5	13.5	−5.0
4	20.0	16.6	14.6	791.2	46.5	13.6	−8.3
5	20.0	17.3	12.6	811.6	78.1	14.0	−10.4
10	20.0	19.0	7.5	1,212.8	203.1	17.7	−7.5
15	20.0	19.0	7.5	1,959.9	277.8	19.9	10.7
20	20.0	19.0	7.5	3,167.2	474.9	20.0	23.9

Note how different the manifestation of the risk now appears. Except for the first two years, the net cash flows are all positive. Lapses still create loss in that they reduce the cash flow otherwise available for investment at the high interest rates. But little borrowing at high rates is required because of lapse, which produced the losses in the low crediting strategy. Losses in this second example can be directly related to the decision to credit at a rate greater than supported by the assets. What makes the high crediting strategy potentially attractive is that ultimately the normal profit margin is achieved, and there is a substantial book of business in force.

To absorb these statutory losses, a certain amount of initial statutory surplus would be required. We have defined "required surplus" to denote the least amount of statutory surplus that is required to maintain statutory solvency in each future year. Furthermore, we assume such required surplus is backed by assets having the same characteristics as those assets backing the liabilities.

Required surplus for these two crediting strategies is as follows:

Crediting Strategy	Required Surplus
Low	2.9% of initial liabilities
High	7.9

The higher required surplus for the high crediting strategy implies that this is a far riskier strategy. As developed below, this conclusion is not necessarily accurate. Another means to compare these two crediting strategies is to determine the cash-flow-based surplus (CFS), which equals the net present value of asset less liability cash flows. CFS is useful here to demonstrate the "economic" as opposed to the statutory impact of different crediting strategies.

Under the low crediting strategy, required surplus is 2.9 percent and CFS is −1.5 percent, where both are percentages of initial liabilities. Under the high "chase the rate" crediting strategy, required surplus is 7.9 percent, and CFS is −2.3 percent. The CFS results are sufficiently close for us to conclude that the two strategies are current economic equivalents. The material difference between the two strategies is that with the high "chase the rate" strategy, a large, profitable[2] block of in-force business remains after the losses due to the C-3 risk disappear. However, the high crediting strategy may be difficult to implement since management must be prepared to accept immediate losses and surplus requirements to fund the losses are considerably higher.

A more moderate "chase the rate" strategy was investigated to see what effect variations in the lapse rate and credited rate would have on required surplus and CFS. Consider the following "intermediate" crediting strategy, which is assumed to produce lapses somewhere in between the two presented above. Table D-3 summarizes the high and intermediate "chase the rate" assumptions. Remember that in the low crediting strategy, the credited rate was held at 13 percent with lapses at 25 percent.

Under the intermediate strategy, the credited rates are pegged at a lower level relative to those assumed in the high strategy, so that we would expect greater lapse rates. With the assumptions of the intermediate strategy, required surplus is 4.1 percent and CFS is −1.8 percent. These results fall within the range established by the high and low crediting strategy results. The CFS results suggest that for a given mismatch situation, management of the interest credits will not appreciably alter the underlying economic loss.

Mismatch risk is dependent on asset cash flows as well as liability cash flows, that is, lapse rates. Thus, the effects of the crediting strategy could be influenced by the assets backing the liabilities. Table D-4 illustrates the effects on required surplus and CFS of the above three strategies for four

[2]Assuming no further C-3 risk manifestation!

TABLE D-3

"CHASE THE RATE" CREDITING STRATEGIES

	High		Intermediate	
Year	Credited Rate	Assumed Lapse Rate	Credited Rate	Assumed Lapse Rate
1	14.0%	22.0%	13.5%	23.5%
2	15.0	19.2	14.0	22.0
3	15.8	16.9	14.5	20.6
4	16.6	14.6	14.8	19.8
5	17.3	12.6	15.2	18.6
6	17.9	10.8	15.5	17.8
7	18.4	9.3	15.7	17.2
8	18.8	8.1	15.9	16.6
9	19.0	7.5	16.0	16.3
10	19.0	7.5	16.2	15.8
11	19.0	7.5	16.6	14.6
12	19.0	7.5	17.2	12.9
13	19.0	7.5	17.9	10.8
14	19.0	7.5	18.4	9.3
15	19.0	7.5	18.8	8.1
16+	19.0	7.5	19.0	7.5

TABLE D-4

EFFECT OF INTEREST CREDITING STRATEGIES ON REQUIRED SURPLUS AND CFS FOR VARIOUS ASSETS

	CFS			Required Surplus		
Asset	Low	Intermediate	High	Low	Intermediate	High
10-yr. Bond	0.0%	−0.1%	−0.6%	1.5%	3.0%	6.3%
15-yr. Mortgage	−1.5	−1.8	−2.3	2.9	4.1	7.9
20-yr. Bond	−9.8	−10.1	−10.6	16.0	16.7	19.6
30-yr. Mortgage	−10.4	−10.7	−11.2	17.5	17.5	19.5

asset types. Required surplus and CFS are expressed as a percentage of initial liabilities.

These results clearly indicate increasing risk with increasing asset length, but interestingly in all cases, required surplus materially overstates the economic loss as measured by CFS. It is also interesting to note that for a particular investment, the economic loss as measured by CFS is remarkably stable for the various crediting strategies. The low crediting strategy produces the lowest level of loss on a present-value basis, primarily because the losses are deferred into the future, whereas under the intermediate and high crediting strategies, most of the loss is recognized upfront.

Through testing of various crediting strategies, it was found that these types of evaluations are highly dependent on the lapse assumptions. Thus, the "best" strategy is highly dependent on one's perceptions of how policyholders will react to the different levels of credited rates.

One additional interest-crediting strategy was tested where the credited rate was immediately reduced to 4 percent. It was assumed that policyholders would lapse in droves: 35 percent in year 1, then 50 percent in later years. Using the 15-year mortgages, required surplus was 6.4 percent, and CFS was −2.6 percent. These results suggest that there is a lower limit to losses associated with low crediting strategies. Results, however, are extremely sensitive to lapse assumptions. For instance, if lapses remain at 30 percent in this example, no additional surplus would be required.

II. Risk Factors

The effect on required surplus of combination of the various risks assumed by an insurance company has been the focus of the CORTF research effort. As development of the CORTF model progressed, it became clear that required surplus is dependent on many other factors not immediately associated with the basic insurance risks. The Task Force felt it was necessary to develop a better understanding of these factors. The purpose of this section is to examine those factors that have been found to affect required surplus in a C-3 risk environment. Some of these factors have been previously addressed by the C-3 Risk Task Force. When a relationship between required surplus for C-3 risk and duration was discovered (this relationship is examined in the following section), it was decided that it was necessary to reexamine some of the work of the C-3 Risk Task Force.

The following factors in particular were found to have a material effect on the degree of risk as measured by required surplus.

(1) Investment strategy
(2) Future interest rate assumption
(3) Withdrawal assumptions
(4) Owner dividend policy
(5) Treatment of FIT
(6) Reserve conservatism/earnings margin
(7) Interest crediting strategy.

All tests presented in this report assume interest rates increase immediately to some high level and remain there. Thus, the future reinvestment strategy has no effect on required surplus; with variable future interest rates, the reinvestment strategy would have a significant effect.

The above risk factors were examined one at a time by holding all other risk factors constant and appropriately varying the relevant risk factor. Except when "chasing the rate," the credited rate is set equal to the average earned rate of the investments less the earnings margin, but never less than the initial credited rate.

(1) Investment Strategy

In the CORTF computer model, the characteristics of the initial assets are defined by two variables: the type of asset invested in, and the distribution of the insurance cash flows invested over the prior ten-year period. Aggregate rollover rates of the initial block of assets can then be developed.

Table D-5 summarizes the asset and liability durations[3] and required surplus for the various assets tested. It is assumed that interest rates immediately rise to and remain at 20 percent and that lapse rates are 25 percent each year.

The durations shown in this and subsequent tables are *post-tax* durations, calculated by using an after-tax interest rate. Although it is still not clear whether pre- or post-tax durations are "better," the latter produces regression formulas for required surplus as a function of duration that appear to have a better "fit" (this is covered in greater detail in a later section).

Note that as the asset length increases, and the spread between D^A and D^L increases, the required surplus increases dramatically. It is interesting to note though that there are exceptions (the 7-year bond), and some surplus is required even when the asset duration is less than the liability duration.

The reason for these exceptions is that duration is not a perfect indicator of risk potential. As discussed earlier, losses develop when negative net cash

[3]The duration index used is Macaulay Duration. It is equal to the weighted average number of years to each future cash flow where the weights are the present value of each cash flow. In symbols:

$$\frac{\Sigma t v^t CF_t}{\Sigma v^t CF_t}$$

where $v = 1/(1+i)$. For the liability duration, the cash flows equal the amount of liabilities lapsed each year, using the *projected* lapse rates. For the asset duration, the cash flows equal the sum of investment income and principal rollover on the initial assets.

TABLE D-5

EFFECT OF INVESTMENT STRATEGY ON REQUIRED SURPLUS DURATIONS AS OF BEGINNING OF PROJECTION PERIOD
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Asset Type	Duration			Required Surplus
	Asset	Liability*	$D^A - D^L$	
5-yr. Bond	2.8	5.3	(2.5)	0.0%
7-yr. Bond	3.9	5.1	(1.3)	0.3
10-yr. Mortgage	3.9	5.0	(1.1)	0.0
12-yr. Mortgage	4.6	4.6	0.0	0.9
10-yr. Bond	4.9	4.5	0.3	2.3
15-yr. Mortgage	5.5	4.5	1.0	3.7
20-yr. Mortgage	7.0	4.5	2.5	9.1
15-yr. Bond	7.0	4.5	2.5	10.4
20-yr. Bond	8.5	4.5	4.0	16.6
30-yr. Mortgage	9.2	4.5	4.7	17.9
30-yr. Bond	10.4	4.5	5.8	23.5

*The liability duration is influenced by the asset length through the effect on credited rates. Since the short assets roll over quicker, losses are not as large, and the average earned rate ultimately rises to the new money rate. These lead to higher credited rates, and lower lapse amounts in later years.

flows lower the average earned rate below the credited rate. In general, assets with shorter durations produce greater cash flows, thereby reducing, and sometimes eliminating, the negative net cash flows. If all net cash flows are positive (where the credited rate is kept below the earned rate), statutory losses do not develop.

However, assets with similar durations can have very different cash flow patterns. In general, the mortgages produce more uniform cash flows by year than the bonds. With the 7-year bond, for example, although the duration is quite short, there is relatively little cash flow generated in the first few years, so that negative cash flows and statutory losses do develop.

(2) Future Interest Rate Assumptions

Table D-6 illustrates the effect on required surplus of different future interest levels for various assets. The scenario is the same as that described for Table D-5, where initial assets are invested at 14 percent, and lapse rates are 25 percent each year.

For a given asset type, required surplus increases dramatically with higher interest rate levels.

TABLE D-6

EFFECT OF FUTURE INTEREST ASSUMPTION
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Future Interest Rate	7-Year Bond	10-Year Bond	15-Year Mortgage	20-Year Mortgage	30-Year Mortgage	30-Year Bond
17%	0.0%	0.5%	1.2%	3.9%	8.3%	11.1%
20	0.3	2.3	3.7	9.1	17.9	23.5
25	1.3	5.2	7.8	17.5	33.2	43.3
30	2.3	8.0	11.9	25.6	47.4	61.6

In reality, the sensitivity of required surplus to different interest rate levels would be even greater, since withdrawal rates could be expected to increase as the spread between new money rates and credited rates increases.

For example, consider the results for a 15-year mortgage where lapse rates are determined using a formula relating lapse rates to this spread (see Attachment D-1b) for the four interest scenarios above.

The lapse rates shown for the two formulas are the highest lapse rates experienced in the given scenario according to the formula followed (Table D-7). Clearly, withdrawal assumptions together with the future interest rate assumptions can have a very material effect on required surplus.

TABLE D-7

REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Future Interest Rate	Flat 25% Lapse		Intermediate Lapse Formula		High Lapse Formula	
	Lapse Rate	Required Rate	Lapse Rate	Required Rate	Lapse Rate	Required Rate
17%	25%	1.2%	16.3%	0.0%	20.4%	0.0%
20	25	3.7	24.8	3.6	39.4	10.2
25	25	7.8	37.6	18.3	75.0	29.6
30	25	11.8	48.4	33.8	75.0	43.5

(3) Withdrawal Assumptions

For a given assumption, interest environment, and interest-crediting strategy, the surplus required is very much a function of lapse rates. Table D-8 illustrates the surplus required under a variety of withdrawal assumptions. A 15-year mortgage is assumed to be the underlying asset and interest rates are assumed to go to 20 percent. A "low" crediting strategy is assumed where the credited rate is set equal to the average earned rate less 1.00 percent subject to a minimum of 13 percent, which is the initial credited rate. ("Low" crediting strategy will be used in this context hereafter.)

TABLE D-8

EFFECT OF WITHDRAWAL ASSUMPTIONS
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

	A(mod)	B(ext)	C	D	E	F	G	H	I
Year	Lapse Rates								
1	24.8%	39.4%	20%	25%	15%	20%	30%	25%	40%
2	24.8	39.4	20	25	20	20	30	30	40
3	24.8	39.4	20	25	25	30	30	35	40
4	24.8	39.4	20	25	30	35	30	40	40
5	24.8	39.4	20	25	35	40	30	40	40
6 -	24.8*	39.4	20	25	40	40	30	40	40
	Durations (Post-Tax)								
Liability	4.6	2.7	6.6	4.5	4.2	3.7	3.7	3.3	2.7
Asset-Liab.	1.0	2.8	−1.0	1.0	1.3	1.8	1.8	2.2	2.9
	Required Surplus								
	3.6%	10.2%	0.4%	3.7%	4.0%	6.0%	6.0%	7.7%	10.4%

*In year 9, cash flow turns positive (thanks to scheduled rollover from initial assets relative to a smaller in force) and earned rates begin to rise; in year 14, credited rates increase and lapses decrease, reaching an ultimate level of 7.5 percent in year 16.

Tests A and B above have lapse rates developed by the lapse formulas relating lapse rates to the spread between new money and credited rates. A has "intermediate" rates and B has "high" rates. Attachment D-1b discusses the formulas and other considerations due to the withdrawal assumptions used throughout these tests. The intermediate lapse formula has generally been used unless otherwise indicated.

From Table D-8 it is evident that the surplus required increases substantially as the liability duration shortens, or as seen earlier, as the spread between asset and liability duration increases.

(4) Owner Dividend Policy

In the testing to this point, it has been assumed, somewhat arbitrarily, that 50 percent of GFO after FIT would be paid out as owner dividends, and the remaining 50 percent would be retained in surplus.[4] Thus, a portion

[4]The owner dividend requirement can also be viewed partially or fully as the requirement to fund new business.

of earnings in early years accumulates in surplus, which is available to cover losses in later years.

In reality, even the retained portion of GFO after FIT might not be available for risk management. For an ongoing company, earnings from in-force blocks are used to fund new business. Amounts left over, if not needed to maintain surplus at a target level, are paid out in the form of dividends to owners.

Tests to this point have also assumed that a "minimum owner dividend" is paid even when there are statutory losses, or depressed earnings. The minimum owner dividend is equal to 32 basis points of the beginning-of-year liability balance.[5] This assumption recognizes that an ongoing company may wish to maintain a certain level of dividends (or support new business), even if it is necessary to borrow money.

Table D-9 illustrates the effect of these dividend assumptions. It is assumed that interest rates rise to and remain at 20 percent, withdrawal rates are at 25 percent, and the underlying asset is a 15-year mortgage invested at 14 percent. Further, the "low" crediting strategy is assumed.

TABLE D-9

EFFECT OF DIVIDEND POLICY
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Scenario	Minimum Owner Dividends	Percentage of Positive GFO as Dividends	Required Surplus
A	No	0%	2.2%
B	No	50	2.9
C	No	100	3.7
D*	Yes†	50	3.7
E	Yes†	100	4.5

*This is the assumed dividend policy used in earlier tests.
†32 basis points times beginning of year liability balance.

The first three scenarios illustrate the effect of the assumption as to how much of the statutory earnings will be paid as dividends. Note that this assumption only matters in years of positive GFO. Under this particular scenario, GFO is positive in the first two years. Assuming all earnings in this period are retained versus all earnings are paid out is worth 150 basis

[5]The minimum dividend of 32 basis points was derived by multiplying the earnings margin of 100 basis points by 50% times (1 − FIT rate of 36.8%).

points of required surplus. If the assumptions were changed to defer the emergence of the loss to later years, it should be apparent that earnings accumulated during this period would provide substantial risk management capacity. Clearly, the ability to retain earnings materially reduces the amount of initial surplus required to manage risk.

Scenarios B versus D and C versus E illustrate the effect that management's need to pay dividends (or fund new business) even during periods of statutory losses may have on required surplus. In these examples, the decision to borrow funds at high interest rates to pay dividends is worth 80 basis points of required surplus.

Unless otherwise noted, the minimum dividend requirement of 32 basis points is used in all tests. The only exception is with the "chasing the rate" scenarios. Minimum dividend requirements when "chasing the rate" are large, since minimum dividends are defined as a function of the liability balance. The liabilities grow large quickly in the "chasing the rate" scenarios, since credited rates are high and lapse rates are low. Consequently, the results presented earlier assumed no minimum dividends for all "chasing the rate" strategies presented, nor for the other strategies that these were compared to.

Table D-10 shows the effect of minimum dividend strategies on required surplus for various interest crediting strategies. The assumptions are consistent with those of Table D-9.

TABLE D-10

EFFECT OF DIVIDEND POLICY
REQUIRED SURPLUS AS PERCENTAGE
OF INITIAL LIABILITIES

	Minimum Owner Dividends*	
Interest-Crediting Strategy	No	Yes
Low	2.9%	3.7%
Intermediate	2.0	2.3
High	5.6	8.0

*It is also assumed that 50% of positive GFO is paid as dividend.

Again, these results are influenced by the assumptions as to the timing and duration of the loss that produces negative earnings, that is, beginning of interest rate increase and assumed level of rates. If the loss extends over a longer period of time, required surplus would be higher and vice versa.

These results are sufficient to establish that the overall dividend policy is a material determinant of required surplus levels. In interpreting C-3 risk surplus requirements, it is essential to understand what dividend policy applies.

(5) Treatment of Negative FIT

All the above results assume negative FIT is available as a current-year tax credit when GFO is negative. In other words, it is assumed there are other lines of business generating positive FIT. The practical effect is to reduce negative cash flows and temper losses that otherwise would materialize. In periods of adversity, this may not be a valid assumption. There also may not be positive FIT available from other lines if the company is growing rapidly.

Table D-11 illustrates the effect that the negative FIT credit assumption has on required surplus. Interest rates are assumed to rise to and remain at 20 percent, withdrawal rates are 25 percent, the assets are 15-year mortgages, and there is no minimum dividend requirement. Further, a "low" crediting strategy is assumed.

TABLE D-11

EFFECT OF NEGATIVE FIT CREDIT
REQUIRED SURPLUS AS PERCENTAGE
OF INITIAL LIABILITIES

Percentage of Positive GFO as Dividends	Required Surplus	
	FIT Credit	No FIT Credit
0%	2.2%	3.0%
50	2.9	3.9
100	3.7	4.9

As would be expected, the availability of the FIT credit reduces required surplus, since the credit effectively reduces the size of the losses that would otherwise be covered by borrowing in the model.

Note that the required surplus associated with the "no FIT credit" scenario is somewhat overstated, in that the possibility of loss carry-forwards and carry-backs is ignored.

Again, in assessing C-3 surplus requirements, it is essential to understand the effect of the tax credit assumption on the results.

(6) Reserve Conservatism/Earnings Margin

The above results all assume that the statutory reserve equals the cash surrender value (CSV). We have tested more conservative reserves to examine the effect on required surplus. These results are presented in Table D-12 for various assets. Interest rates are assumed to immediately rise to and remain at 20 percent, lapse rates are 25 percent, and there is no minimum dividend requirement. Further, a "low" crediting strategy is assumed.

TABLE D-12

EFFECT OF LIABILITY VALUATION ON REQUIRED SURPLUS
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Asset	Statutory Reserve Equals CSV Plus		
	0% of CSV	2% of CSV	4% of CSV
10-yr. Bond	1.5%	0.4%	0%
15-yr. Mortgage	2.9	1.7	0.7
20-yr. Bond	16.1	14.8	13.6
30-yr. Mortgage	17.5	16.3	15.2

Note that surplus is expressed as a percentage of initial liabilities, which includes the extra x percent of CSV. Also, note that an increase in the statutory reserve of x percent does not translate into a corresponding decrease in required surplus. In these examples, an extra 2 percent of reserves decreases surplus by only 1.0–1.3 percent. This relationship arises because the additional reserve "requirement" must be maintained even when adversity strikes. In contrast, required surplus is available in full to cover losses as required.

Table D-13 shows how much of a margin must be introduced into the reserve so that no initial surplus is required. The low crediting strategy and the high "chasing the rate" strategy discussed in Part I are used as examples here.

In both cases, reserves must be increased well in excess of the required surplus when based on reserves equal to cash surrender values. The necessary increase in reserves to get required surplus to zero is especially dramatic with the high "chase the rate" crediting strategy.

TABLE D-13

RELATIONSHIP BETWEEN RESERVE CONSERVATISM AND REQUIRED SURPLUS
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Interest-Crediting Strategy	Statutory Reserve Increased by x% of CSV	Required Surplus	Extra Reserve Plus Surplus†
Low	0.0%	2.9%	2.9%
	2.9	1.2	4.1
	5.3	0.0	5.3
High	0.0*	7.9*	7.9
	7.9	3.8	11.7
	17.9	0.0	17.9

*These results relate to Tables D-1 and D-2, respectively.
†Note that the increase in statutory reserves applies in all policy years, whereas the required surplus is expressed as a percentage of initial liabilities. Thus this total must be used with caution.

Under both strategies, the conservatism in the reserves is released upon lapse and then is available to cover losses. In the low crediting strategy, the high level of lapses in the early years results in a high level of reserve conservatism released. In the high crediting strategy, which produces a lower level of lapse, relatively more reserve conservatism is necessary to cover the same level of loss.

Another interesting difference between the two strategies is the relationship between interest earned on the additional reserve less interest credited to support the increase in additional reserve. Under the low crediting strategy, the extra reserve provides additional interest margin, which helps cover the losses and reduces the level of additional reserve required. However, under the high crediting strategy, the interest margin on the incremental reserve piece is actually negative, that is, the extra reserve is increased by the credited rate, which is greater than the interest earned on the assets. Thus, required reserve conservatism is further increased.

These results suggest that reserve conservatism, at least in the form of a fixed percentage increase, is not necessarily an effective means to assure solvency. Clearly, management would opt for the lowest combination of surplus and reserve conservatism that would assure that obligations could be matured, and these results indicate that this is achieved when all risk management capacity is held in surplus.

Another form of "reserve conservatism" that is relied on for statutory purposes is to use a low discount rate in the present value calculations. Once a reserve with a conservative discount rate is established, the conservatism in later years emerges as a larger statutory earnings margin. In the model, this form of reserve conservatism is simulated by varying the margin between earned and credited interest.

For most of the runs, a pre-tax earnings margin of 100 basis points, net of expense, has been assumed. Table D-14 illustrates how various earnings margins impact required surplus. Interest rates are assumed to go to 20 percent, withdrawal rates are 25 percent, and there is no minimum dividend requirement.

TABLE D-14

EFFECT OF EARNINGS MARGIN ON REQUIRED SURPLUS
REQUIRED SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

Asset	Earnings Margin		
	50 Basis Points	100 Basis Points	150 Basis Points
10-yr. Bond	2.3%	1.5%	0.7%
15-yr. Mortgage	3.8	2.9	2.1
20-yr. Bond	16.9	16.1	15.2
30-yr. Mortgage	18.3	17.5	16.7

Since earnings are the first line of defense against risk, it would be expected that higher margins would decrease required surplus. This expectation is confirmed by the above results, though the magnitude of the effect is smaller than was anticipated. The reasons for this are twofold: first, FIT reduces the 50 basis points differential to 32 basis points. Second, lapses are so high that the liability balance to which the 32 basis points applies shrinks rapidly.

This approach of just varying the earnings margin does not necessarily simulate the reserve conservatism present when low discount rates are used, depending on the reserve/CSV relationship. Where low discount rates are used for both, such that reserves and CSVs are approximately equal, this should be an adequate representation. However, where lower discount rates are used for reserves, such that reserves exceed cash values, the impact on required surplus would be a combination of Tables D-12 and D-14. This combination has not been investigated.

(7) Interest-Crediting Strategy

The effect of the interest crediting strategy on required surplus was discussed in detail in Part I.

III. Relationship between Duration and Required Surplus

In the testing, a very interesting relationship was discovered between required surplus and the difference between the asset and liability durations, where the liability duration reflects the projected lapse rates. Regression formulas for required surplus, based on the bulk of the data presented in the preceding tables, exhibit the following general relationship:

$$\text{Required Surplus} = (i_{actual} - i_{base})\,(D^A - D^L) - C$$

where

i_{actual} = the assumed level future new money rate
i_{base} = the interest rate at which the initial assets are invested
D^A = the Macaulay duration of the assets
D^L = the Macaulay duration of the liabilities
C = a constant that is a function of the earnings margin present prior to an adverse interest scenario.

We came upon this formula by applying regression analysis to our data, where we defined the independent variable to be the asset/liability duration difference (denoted by x) and the dependent variable to be the required surplus percentage (denoted by y).

Illustrations of our analysis follow. Only those results where the asset duration exceeds the liability duration and initial surplus is required were included in the calculation of the regression formulas unless otherwise noted.

The first set of regression formulas was derived from the data in Table D-6, where the effect of different future interest assumptions was examined. (Attachment D-1c contains all the raw data used to develop the regression formulas that follow in Table D-15.) Both pre-tax and post-tax data were tested.*

Note that the coefficient of x, the duration difference, determined by the regression analysis, is close to $(i_a - i_b)$ under both the pre-tax and post-tax basis. Note also the very high correlation coefficients, indicating a very good fit. However, this may be somewhat misleading as relatively few points, 5 to 11, were used.

*For "post-tax," durations are computed as the post-tax interest rate, and $(i_a - i_b)$ is post-tax.

TABLE D-15

REGRESSION ANALYSIS FOR VARIOUS FUTURE INTEREST RATE LEVELS

Future Interest Rate	$i_a - i_b$	Regression Formula	Correlation Coefficient
Pre-tax Interest Rates			
17%	3%	$y = 3.02\%x - 1.93\%$	0.9855
20	6	$y = 5.85\%x - 1.89$	0.9880
25	11	$y = 10.69\%x - 2.56$	0.9910
30	16	$y = 15.18\%x - 2.80$	0.9925
Post-tax Interest Rates			
17%	1.9%	$y = 1.92\%x - 0.43\%$	0.9965
20	3.8	$y = 3.91\%x - 0.29$	0.9968
25	7.0	$y = 7.01\%x - 1.93$	0.9965
30	10.1	$y = 9.92\%x - 3.66$	0.9948

The post-tax results appear slightly "better," in that the correlation coefficients are closer to 1, and the coefficient of x remains closer to $(i_a - i_b)$ over the interest rate range.

Table D-16 compares surplus versus regression formula surplus for the 20 percent post-tax regression formula.

TABLE D-16

ACTUAL SURPLUS VERSUS REGRESSION SURPLUS
SURPLUS AS PERCENTAGE OF INITIAL LIABILITIES

	10-yr. Bond	15-yr. Mortg.	20-yr. Mortg.	15-yr. Bond	20-yr. Bond	30-yr. Mortg.	30-yr. Bond
Duration Difference (x)	0.33	1.01	2.45	2.51	4.01	4.66	5.85
$3.91\%x + 0.29\%$	1.6%	4.2%	9.9%	10.1%	16.0%	18.5%	23.2%
Act.Req.Surp.	2.3%	3.7%	9.1%	10.4%	16.6%	17.9%	23.5%

Thus, the formula does remarkably well at matching the actual required surplus determined by the model.

The regression analysis was repeated, but using the data in Table D-14, where the effect of different earnings margins was examined.

Interestingly, the coefficient of the duration difference, x, remains approximately equal to the difference between the interest rates. But the constant C seems to vary at some constant rate depending on the earnings margin. On a pre-tax basis, it appears to be an offset to required surplus, an offset that becomes larger as the earnings margin grows. This makes sense since the earnings margin is the first line of defense in the management of risk. Thus, the larger the earnings margin, the less surplus that is necessary to manage risk.

The same general reasoning applies to the post-tax formulas, even though the constant C is positive in the first two cases. Thus, the constant C is more negative (or less positive) as the initial earnings margin increases.

The fact that this constant is sometimes positive for the particular formula is somewhat troubling, however. For example, it suggests that surplus of 1.77 percent is required when asset and liability durations are equal, and no earnings margin is present. This may well be true in some situations, if cash flows are not well matched, but the pre-tax formula implies required surplus of negative 0.55 percent for the same case. All this reinforces the observation that duration is not a perfect measure of risk, and these formulas are not necessarily precise.

Note that the correlation coefficients in Table D-17 suggest the post-tax formulas consistently provide the better fit. There is also less variation of $(i_{actual} - i_{base})$ on a post-tax basis than on a pre-tax one. These results led to the use of durations computed at post-tax interest rates in earlier sections of this appendix.

TABLE D-17

REGRESSION FORMULAS FOR VARIOUS EARNINGS MARGIN LEVELS

Earnings Margin	$i_a - i_b$	Regression Surplus Formula	Correlation Coefficient
Pre-tax Interest Rates			
0 bp	6%	$y = 6.02x - 0.55$	0.9887
50	6	$y = 5.98x - 1.62$	0.9873
100	6	$y = 5.87x - 2.44$	0.9848
150	6	$y = 6.15x - 4.38$	0.9837
Post-tax Interest Rates			
0 bp	3.8%	$y = 3.99x + 1.77$	0.9969
50	3.8	$y = 4.01x + 0.49$	0.9969
100	3.8	$y = 3.99x - 0.50$	0.9963
150	3.8	$y = 3.91x - 1.27$	0.9946

The next area of interest was the effect of various lapse assumptions, which influence the duration of the liabilities (see Table D-18). The following regression formulas were derived from the following data:

(1) Table 5, various assets, interest to 20%, 25% lapse
(2) Table 8, 15-year mortgage, interest to 20%, various lapses
(3) Various assets, interest to 20%, intermediate lapse formula (results not shown in prior tables).

TABLE D-18

REGRESSION FORMULAS FOR VARIOUS LAPSE ASSUMPTIONS

Durations	Required Surplus Formula	Correlation Coefficient
Pre-tax		
(1) Table 5	$y = 5.85x - 1.89$	0.9880
(2) Table 8	$y = 4.60x - 1.05$	0.9981
(3) Intermediate Lapse Formula	$y = 6.60x - 4.37$	0.9909
(4) Data Set Combined	$y = 6.06x - 2.78$	0.9863
Post-tax		
(1) Table 5	$y = 3.91x - 0.29$	0.9968
(2) Table 8	$y = 3.62x - 0.23$	0.9912
(3) Intermediate Lapse Formula	$y = 4.03x - 0.25$	0.9970
(4) Data Sets Combined	$y = 4.06x - 0.52$	0.9946

Interestingly, there is remarkable consistency in the formulas derived from the different data sources, especially with the post-tax formulas. We expected some variation in the magnitude of the "C" factor, since the rate of lapse influences the distribution and amount of earnings over the period during which losses due to the C-3 risk are occurring. Although "C" does vary, the variation is not severe, a result we find encouraging.

The regression formulas above were based only on tests where the asset duration exceeds the liability duration and initial surplus is required. Now consider the case where D^A is less than D^L, which is encountered when we "chase the rate." The following set of regression formulas (see Table D-19) is based on the different interest-crediting strategies found in Table D-4.

Interestingly, the coefficient of x is reasonably stable, yet "x" in these high crediting runs is generally negative. However, the correlation coefficient, and thus the fit of the regression formula, for the intermediate and high scenarios are not nearly as good as in previous regressions. But again, there seems to be slightly better results among the post-tax formulas.

TABLE D-19

REGRESSION FORMULAS FOR DIFFERENT INTEREST CREDITING SCENARIOS

Scenario	Required Surplus Formula	Correlation Coefficient
	Pre-tax Durations	
Low	$y = 4.87x - 2.44$	0.9848
Intermediate	$y = 4.77x + 15.38$	0.9545
High	$y = 4.73x + 38.17$	0.9792
	Post-tax Durations	
Low	$y = 3.99x - 0.50$	0.9963
Intermediate	$y = 4.77x + 24.64$	0.9836
High	$y = 3.41x + 51.46$	0.9834

The large positive "*C*" factor with the intermediate and high crediting strategies again appears to be related to the "earnings margin." In the intermediate and high strategies, more interest is credited than is earned, thus producing negative earnings margins. This logically leads to the need for an addition to required surplus, which would not be a direct function of the duration difference. Instead, the large losses occur in the first few years and are roughly of the same magnitude with different assets.

The relationship between duration difference and cash-flow surplus (CFS) was also examined. The relationships found were not as strong as those associated with required surplus, although correlations are still very high. This was surprising since CFS is a present value of cash-flow measure and certainly appears more closely related to the duration difference in the regression formula. Results of regression analysis with CFS are presented in Attachment D-1c.

In summary, these formulas are at best approximations. However, the closeness of fit is intriguing, and the formulas have been quite useful for gaining appreciation for the magnitude and sensitivity of risk to the various factors that affect C-3 risk.

Conclusions

As demonstrated by the tests presented herein, initial surplus required to manage manifestations of C-3 risk is dependent upon a variety of variables.

Some variables, such as interest rate, lapse, and investment assumptions, have a very strong effect on required surplus, and they have commonly been associated with C-3 risk.

Other variables, which are to a large extent controlled by management of an insurance company, have been shown to materially affect required surplus in a C-3 risk situation. These variables include the interest-crediting strategy, owner dividend policy, treatment of FIT credits, and earnings margin as determined by pricing policy. The effect of these variables greatly complicates the problem of developing appropriate surplus guidelines for management of C-3 risk.

Finally, it has been shown tnat valuation standards materially affect required surplus. In general, any conservatism in valuation reserves produces a net increase in the total surplus, that is, reserve conservatism plus required surplus, needed to assure solvency. This suggests that conservatism in valuation reserves may not be the best means to assure solvency.

Regression analysis has revealed a strong relationship between durations, earnings, and required surplus. This relationship can be useful for developing an intuitive sense for the effect on risk levels of alternate assumptions.

ATTACHMENT D-1a

SPDA AND MODEL ASSUMPTIONS

Following is a list of the parameters used to develop the "base case" of SPDA business:

- Initial assets invested at 14%
- FIT = 36.8%
- 50% of positive GFO paid out as dividends
- A minimum owner dividend of 32 basis points applied to the beginning of year liability balance
- Utilization of the negative FIT credit
- Underlying asset is a 15-year mortgage
- Lapse rates from the intermediate annuity cash-out formula (see Attachment D-1b)
- Insurance cash flows of the prior ten years in the proportion 100, 103, 107, 110, 112, 115, 117, 119
- Statutory reserve equals cash value, that is, no margin for adverse deviation.

The minimum owner dividend factor was based on the base case, where 50 percent of the after-tax earnings margin is assumed to be paid to shareholders. Thus,

$$50\% \times (1 - 0.368) \times 0.01 = 31.6 \text{ bp} = 32 \text{ bp}$$

The insurance cash flows for the 10 years prior to issue that we used to develop the initial block of assets were derived assuming the SPDA product was first sold ten years ago, sales grew by 10 percent each year, and annual lapses were at 5 percent.

Since interest rates are assumed to immediately rise to some high level and remain there, the future reinvestment or borrowing assumptions have no effect on required surplus. Thus, the debt instrument was arbitrarily chosen to allow 10 percent of the debt to roll over each year for 10 years.

In presenting the test results, an effort has been made to note all deviations in assumptions from those of the base case.

ATTACHMENT D-1b
LAPSE ASSUMPTIONS

Lapse Formula

We initially considered using a lapse formula based upon formulas presented in the C-3 study by Feldman and Kolkman [*RSA* 8, no. 4 (1982): 1557]. Their formula for single pay annuities, "moderate" lapse rates, is as follows:

$$\begin{aligned}\text{Lapse Rate} &= 0.05 + (0.01)(100 \times d)^{1.5}, \text{ for } \% \le d \le 17\%,\\ &= 0.75, \text{ for } d > 17,\end{aligned}$$

where $d =$ (new money rate) − (credited rate) − 0.01.

Feldman and Kolkman also tested "extreme" withdrawal rates, defined as the moderate rates times 2, but not higher than 75 percent.

After extensive testing, and further thought, we decided the moderate formula was unsatisfactory. In particular:

- The formula implies a greater increase in lapse rates for a given increase in the NMR/credited rate spread as this spread reaches higher levels. Our intuition and limited experience suggest an opposite result, that is, that at some point, further increases in the interest spread will lead to only relatively small increases in lapse rates.
- The lapse rate of 5 percent for a 0 percent spread appeared low; we felt 7.5 percent was more appropriate.
- The moderate formula implied lapse rates of 19.8 percent for our base case of 20 percent new money rates, 13 percent credited, versus our usual assumption of 25 percent.

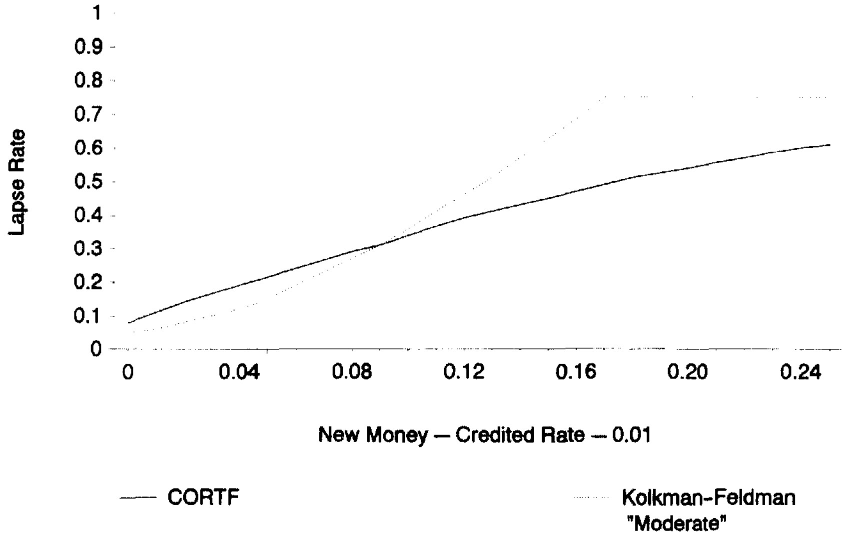
We developed the following formula, which we believe produces more consistent and more reasonable results for different interest and crediting assumptions.

$$\begin{aligned}\text{Lapse Rate} &= 0.075 + 3.0d - 1.5d^2 - 8d^3, \text{ for } \% \le d \le 25\%\\ &= 0.60, \text{ for } d > 0.25\end{aligned}$$

Figure D-1b-1 compares the two formulas. For some tests of "extreme" lapse rates, we continued with the Feldman/Kolkman extreme formula.

FIGURE D-1b-1

LAPSE FORMULAS



ATTACHMENT D-1c

RAW DATA FOR REGRESSION FORMULAS

TABLE D-1c-1

REGRESSION ANALYSIS FOR VARIOUS FUTURE INTEREST RATE LEVELS

	17%			20%			25%			30%		
Asset	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.
5-yr. bond	—	−1.375	−2.028	—	−1.655	−2.475	—	−2.272	−3.687	—	−3.716	−5.093
7-yr. bond	—	−0.390	−0.927	0.261	−0.575	−1.252	1.292	−0.739	−1.478	2.341	−0.853	−1.648
10-yr. bond	0.525	0.549	0.257	2.270	0.593	0.334	5.179	0.596	0.339	8.072	0.596	0.339
15-yr. bond	4.481	2.350	2.514	10.442	2.350	2.514	20.611	2.350	2.514	30.903	2.350	2.514
20-yr. bond	7.486	3.261	4.012	16.545	3.261	4.012	31.726	3.261	4.012	46.720	3.261	4.012
30-yr. bond	11.054	3.980	5.847	23.450	3.980	5.847	43.256	3.980	5.847	61.615	3.980	5.847
10-yr. mort	—	−0.345	−0.790	—	−0.489	−1.055	0.110	−0.805	−1.672	0.604	−1.132	−2.385
12-yr. mort	—	0.233	−0.083	0.851	0.249	−0.042	2.536	0.304	0.055	4.214	0.304	0.055
15-yr. mort	1.218	1.064	1.006	3.708	1.064	1.006	7.828	1.064	1.006	11.869	1.064	1.006
20-yr. mort	3.864	2.113	2.454	9.063	2.113	2.454	17.530	2.113	2.454	25.638	2.113	2.454
30-yr. mort	8.315	3.384	4.662	17.893	3.384	4.662	33.183	3.384	4.662	47.408	3.384	4.662

	17%	20%	25%	30%
Regression Equations:				
Pre-tax:	$y = 3.019x - 1.926$	$y = 5.847x - 1.894$	$y = 10.694x - 2.563$	$y = 15.179x - 2.800$
Post-tax:	$y = 1.924x - 0.427$	$y = 3.906x + 0.288$	$y = 7.009x + 1.930$	$y = 9.916x + 3.663$
Correlation Coefficients:				
Pre-tax:	0.9855	0.9880	0.9910	0.9925
Post-tax:	0.9965	0.9968	0.9965	0.9948
Points Excluded*:				
Pre-tax:	Bond 5,7 / Mort 10,12	Bond 5,7 / Mort 10	Bond 5,7 / Mort 10	Bond 5,7 / Mort 10
Post-tax:	Bond 5,7 / Mort 10,12	Bond 5,7 / Mort 10,12	Bond 5,7 / Mort 10	Bond 5,7 / Mort 10

*Points excluded because of negative duration differences or zero required surplus.

TABLE D-1c-2

REGRESSION ANALYSIS FOR VARIOUS EARNINGS MARGIN LEVELS

Asset	0 Basis Points			50 Basis Points			100 Basis Points			150 Basis Points		
	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.
10-yr. bond	3.235	0.493	0.195	2.322	0.541	0.262	1.464	0.555	0.271	0.689	0.522	0.187
15-yr. bond	11.572	2.247	2.370	10.637	2.299	2.512	9.735	2.350	2.514	8.890	2.399	2.583
20-yr. bond	17.789	3.158	3.868	16.901	3.210	3.941	16.048	3.261	4.012	15.228	3.310	4.081
30-yr. bond	24.886	5.703	5.703	24.043	3.929	5.776	23.234	3.980	5.847	22.456	4.029	5.916
12-yr. mort	1.794	0.201	−0.089	0.895	0.189	−0.130	0.092	0.149	−0.256	—	0.206	−0.171
15-yr. mort	4.714	0.961	0.862	3.779	1.013	0.935	2.907	1.064	1.006	2.099	1.113	1.075
20-yr. mort	10.168	2.010	2.310	9.240	2.062	2.383	8.384	2.113	2.454	7.578	2.162	2.523
30-yr. mort	19.193	3.281	4.518	18.330	3.333	4.591	17.502	3.384	4.662	16.728	3.433	4.731

	0 Basis Points	50 Basis Points	100 Basis Points	150 Basis Points
Regression Equations:				
Pre-tax:	$y = 6.024x - 0.551$	$y = 5.977x - 1.617$	$y = 5.867x - 2.441$	$y = 6.147x - 4.375$
Post-tax:	$y = 3.992x + 1.772$	$y = 4.011x + 0.489$	$y = 3.968x - 0.499$	$y = 3.913x - 1.267$
Correlation Coefficients:				
Pre-tax:	0.9887	0.9873	0.9848	0.9837
Post-tax:	0.9969	0.9969	0.9963	0.9946
Points Excluded*:				
Pre-tax:	12 yr. mort	12 yr. mort	12 yr. mort	12 yr. mort
Post-tax:	12 yr. mort	12 yr. mort	12 yr. mort	12 yr. mort

*Points excluded because of negative duration differences or zero required surplus.

TABLE D-1c-3

Regression Formulas for Various Lapse Assumptions

Asset	Table 5 Data: Req. Surp.	Table 5 Data: Pre-tax Dur.	Table 5 Data: Post-tax Dur.	Intermediate Lapse Formula: Req. Surp.	Intermediate Lapse Formula: Pre-tax Dur.	Intermediate Lapse Formula: Post-tax Dur.	Withdrawal Assumption	Table 8 Data: Req. Surp.	Table 8 Data: Pre-tax Dur.	Table 8 Data: Post-tax Dur.
5-yr. bond	—	−1.655	−2.475				A	3.569	1.032	9.610
7-yr. bond	0.261	−0.575	−1.252				B	0.363	−0.167	−1.034
10-yr. bond	2.270	0.593	0.334				C	3.708	1.064	1.006
15-yr. bond	10.442	2.350	2.514	10.281	2.318	2.469	D	6.593	1.695	1.862
20-yr. bond	16.545	3.261	4.012	16.363	3.229	3.967	E	10.416	2.493	2.869
30-yr. bond	23.450	3.980	5.847	23.264	3.948	5.802	F	3.958	1.035	1.303
							G	5.991	1.504	1.809
10-yr. mort	—	−0.489	−1.055				H	7.657	1.873	2.208
12-yr. mort	0.851	0.249	−0.042							
15-yr. mort	3.708	1.064	1.006	3.569	1.032	0.961				
20-yr. mort	9.063	2.113	2.454	8.894	2.081	2.409				
30-yr. mort	17.893	3.384	4.662	17.709	3.352	4.617				

	Table 5 Data	Intermediate Lapse Formula	Table 8 Data
Regression Equations:			
Pre-tax:	$y = 5.847x - 1.894$	$y = 6.659x - 4.367$	$y = 4.604x - 1.049$
Post-tax:	$y = 3.906x + 0.288$	$y = 4.032x - 0.246$	$y = 3.618x - 0.227$
Correlation Coefficients:			
Pre-tax:	0.9880	0.9981	0.9909
Post-tax:	0.9968	0.9912	0.9970
Points Excluded*:			
Pre-tax:	Bonds 5,7 / Mort 10		(B)
Post-tax:	Bonds 5,7 / Mort 10,12		(B)

*Points excluded because of negative duration differences or zero required surplus.

TABLE D-1c-4

Regression Formulas for Different Interest-Crediting Strategies

Asset	Low			Intermediate			High		
	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.	Req. Surp.	Pre-tax Dur.	Post-tax Dur.
5-yr. bond				0.298	−3.862	−8.190	1.692	−8.129	−14.880
7-yr. bond				1.999	−2.955	−7.188	4.599	−7.222	−13.878
10-yr. bond	1.464	0.555	0.271	2.981	−2.122	−6.180	6.329	−6.389	−12.870
15-yr. bond	9.735	2.350	2.514	12.222	−0.368	−4.005	16.683	−4.635	−10.695
20-yr. bond	16.048	3.261	4.012	16.733	−0.543	−2.507	19.578	−3.724	−9.917
30-yr. bond	23.234	3.980	5.847	24.183	1.262	−0.672	27.423	−3.005	−7.362
10-yr. mort				1.344	−2.944	−6.833	4.300	−7.211	−13.788
12-yr. mort	0.092	0.149	−0.256	2.746	−2.414	−6.464	6.430	−6.681	−13.154
15-yr. mort	2.907	1.064	1.006	4.144	−1.654	−5.513	7.894	−5.921	−12.203
20-yr. mort	8.384	2.113	2.454	10.843	−0.605	−4.065	15.059	−4.872	−10.755
30-yr. mort	17.502	3.384	4.662	17.481	0.666	−1.857	19.471	−3.601	8.547

	Low	Intermediate	High
Regression Equations:			
Pre-tax:	$y = 5.867x - 2.441$	$y = 4.773x + 15.377$	$y = 4.731x + 38.174$
Post-tax:	$y = 3.986x - 0.499$	$y = 3.293x + 24.644$	$y = 3.409x + 51.456$
Correlation Coefficients:			
Pre-tax:	0.9848	0.9545	0.9792
Post-tax:	0.9963	0.9836	0.9834
Points Excluded*:			
Pre-tax:	12 yr. mort		
Post-tax:	12 yr. mort		

*Points excluded because of negative duration differences.

ATTACHMENT D-1d

CASH FLOW SURPLUS AS FUNCTION OF ASSET/LIABILITY DURATIONS

A similar regression analysis was applied to CFS. For example, for various assets for different interest rates, we derived the following formulas:

Interest	CFS Formula*	Corr. Coef.	Required Surplus Formula	Corr. Coef.
17%	$-1.44\%x + 1.54\%$	0.9970	$1.92\%x - 0.43\%$	0.9965
20	$-2.40\%x + 0.60\%$	0.9932	$3.91\%x + 0.29\%$	0.9968
25	$-3.16\%x - 1.39\%$	0.9848	$7.01\%x + 1.93\%$	0.9965
30	$-3.40\%x - 3.19\%$	0.9745	$9.92\%x + 3.66\%$	0.9948

*x is difference between asset and liability durations computed with post-tax interest rates.

Note that the coefficients of the duration difference in the CFS formulas do not increase with increasing interest nearly to the extent that they do with the required surplus formulas. Also, the correlation coefficients for the CFS formulas imply a somewhat worse fit than the required surplus formulas. However, the difference in fit is not very great, as can be seen in the following table, which compares the formula to actual values for the two variables.

INTEREST RATES TO 25%

	Asset						
	10-Yr. Bond	15-Yr. Mortg.	20-Yr. Mortg.	15-Yr. Bond	20-Yr. Bond	30-Yr. Mortg.	30-Yr. Bond
$D^A - D^L$	0.339	1.006	2.454	2.514	4.012	4.662	5.847
CFS							
Formula	−2.5%	−4.6%	−9.1%	−9.3%	−14.1%	−16.1%	−19.9%
Actual	−1.5	−4.0	−9.5	−10.9	−15.4	−15.8	−18.6
Difference	1.0	0.6	0.4	1.6	1.3	0.3	1.3
% Difference	67%	15%	4.2%	14.7%	8.4%	1.9%	7.0%
Req. Surplus							
Formula	4.3%	9.0%	19.1%	19.6%	30.1%	34.6%	42.9%
Actual	5.2	7.8	17.5	20.6	31.7	33.2	43.3
Difference	0.9	1.2	1.6	1.0	1.6	1.4	0.4
% Difference	17.3%	15.4%	9.1%	4.9%	5.0%	4.2%	0.1%

The average absolute difference for CFS is 0.9 versus 1.2 for required surplus. However, as a percentage of average CFS and average required surplus, respectively, the comparison is 8 percent for CFS and 5 percent for required surplus.

APPENDIX E-1
COMBINATION OF RISKS

LINDA C. DINIUS

This appendix provides the details of the methodology developed at Aetna in support of the work of the CORTF for quantifying combinations of two or more risks. Jim Geyer made a significant contribution to the work presented here.

As noted in the main body of this report, the work of the Task Force in combining risks proved to be a logical extension of the work of the C-3 Task Force. But the work of the C-3 Risk Task Force never addressed the issue of defining how much surplus is required to manage the C-3 risk. In our work here at Aetna, we tried to start solving this problem for a combination of risks, and our efforts showed us that a deterministic, probabilistic scenario quantifying a level of surplus was definable, but the process quickly became challenging and complex.

The remainder of this appendix describes our efforts to extend initial cash-flow analysis techniques to a probabilistic level. Admittedly, this effort is only in its initial stages, but the results achieved establish this as an avenue of potential fruitful research.

Previous efforts to quantify combinations of risk have focused on statistical approaches, and Appendix E-2, which was authored by Donald Cody, represents the latest effort in this direction. The efforts of Mr. Cody have provided an opportunity to compare the results of statistical and cash flow approaches to the problem of quantification of risk. We explored the results of the two methodologies in this appendix and as explained below, there are some interesting differences. The relationship between results based on cash flow and statistical methodologies represents another area for further research.

Steps Involved in the Methodology

The steps involved in combining risks and determining levels of surplus using the methodology discussed in this appendix are listed below:

(1) Determine the surplus needed for various combinations of risk using the COR model and choose sample combinations that all require the same level of surplus. These combinations define a "line of constant surplus."

(2) Assign probability distributions to the various risks, and combine these distributions to form the joint probability distribution for several risks

acting simultaneously. Alternatively, the joint probability distribution can be defined directly.

(3) Determine the volume defined by the joint probability function and bounded by the "line of constant surplus." This volume is the probability level associated with a given amount of surplus. For example, a given amount of surplus might be adequate to cover losses from combinations of risk 95 percent of the time as defined by the chosen joint probability distribution.

As explained, these steps lead to a level of surplus that is adequate for x percent of all loss scenarios, where x is the probability calculated in step 3. By developing several "lines of constant surplus," one can determine exactly how much surplus is needed at any probability level.

This process can best be understood by looking first at some simple examples. After developing the methodology on a very elementary level, we will show results using COR model results and probability distributions for the C-1, C-2 and C-3 risks themselves.

Simple Examples

The first example, and the easiest to visualize, is the following:

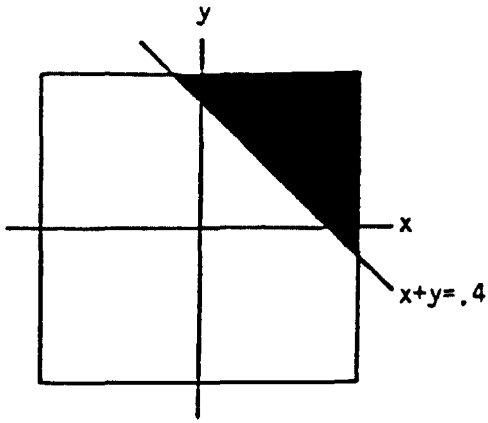
Let X be the random variable for risk 1, $-0.5 \leq X \leq 0.5$, and
Y be the random variable for risk 2, $-0.5 \leq Y \leq 0.5$.

X is defined as the loss (positive number) or gain (negative number) from risk 1 during the period under consideration. Similarly, Y is defined as the loss or gain from risk 2. Assume that the loss from both risks acting simultaneously is the simple sum of the individual losses, $X+Y$. This is pictured in Figure E-1-1.

This square represents all possible values of $X+Y$. The line $X+Y=0.4$ represents all possible values of X and Y where their combined losses equal 0.4. Note that all points in the square below and to the left of the line represent losses less than 0.4 (or gains) and all points above and to the right of the line (the shaded area) represent losses greater than 0.4. Therefore, a surplus amount of 0.4 will be sufficient to cover all losses less than 0.4, that is, all combinations of X and Y below and to the left of this line.

The next step is to determine what percentage of loss situations are covered by this surplus amount of 0.4. To do this, we need to assume a joint probability distribution for X and Y. First, we will assume a distribution for X

FIGURE E-1-1



and Y separately and then we will combine these two distributions into a single joint distribution. To keep this example as simple as possible, we will assume that all possible values of X or Y are equally likely. This is shown below:

$f(x)$ = probability density function (pdf) for the random variable X
$= 1$, for $-0.5 \leq x \leq 0.5$
$f(y)$ = pdf for the random variable Y
$= 1$, for $-0.5 \leq x \leq 0.5$

These are both pdf's as each satisfies the following conditions:

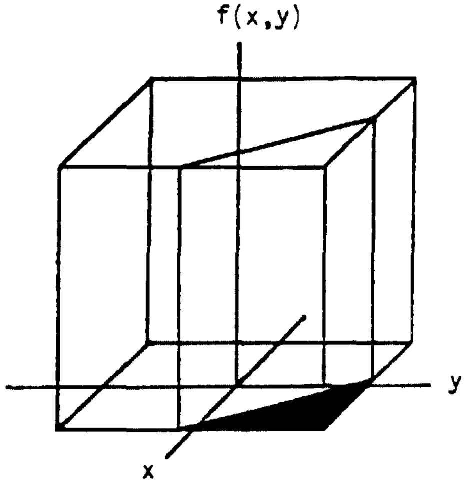
(i) $f(t) > 0$
(ii) $f(t)dt = 1$
(iii) $f(x)dx = P\{a < X < b\}$

If we assume that the random variables X and Y are independent, then the joint pdf for X and Y would be:

$$f(x,y) = f(x)f(y) = 1, \qquad -0.5 \leq x \leq 0.5, \qquad -0.5 \leq y \leq 0.5$$

This joint distribution is shown in Figure E-1-2.

FIGURE E-1-2



The volume of the cube is obviously 1. The line $X+Y=0.4$ is shown as a line across the bottom surface of the cube. The shaded area shown corresponds to the shaded area in Figure E-1-1. The volume of the cube above the shaded area represents the total probability of all the combinations of X and Y that produce losses greater than 0.4. The volume above the unshaded area, on the other hand, represents the total probability of all combinations of X and Y that produce losses that are less than 0.4. In this example, this latter volume is equal to 0.820. Therefore, a surplus of 0.4 will be sufficient to cover losses from risks 1 and 2 acting simultaneously, 82.0 percent of the time.

The line $X+Y=0.4$ in the above example is called a "line of constant surplus" because all points along this line require the same surplus amount. There are a whole family of these "lines," and any surplus can be chosen so that once the "line of constant surplus" is determined and a joint probability distribution is assumed, a probability level can be associated with that surplus amount. The following table shows sample surplus levels and the associated probability adequacy using the above example:

Probability Adequacy	Surplus Level
50%	0.000
75	0.293
82	0.400
90	0.553
95	0.684
98	0.800
99	0.859
100	1.000

Keep in mind that these results are a direct consequence of our assumptions regarding the random variables X and Y and their joint probability density function. Other assumptions would naturally produce completely different results. To illustrate this, let us look at another simple example.

We again define the random variables X and Y as follows:

Let X be the random variable for risk 1, $-0.5 \leq X \leq 0.5$, and
Y be the random variable for risk 2, $-0.5 \leq Y \leq 0.5$.

Again let the losses from the two risks occurring simultaneously be equal to the simple sum $X+Y$. Note that the definition of the lines of constant surplus is completely independent of the assumption we make for the probability density function of X or Y.

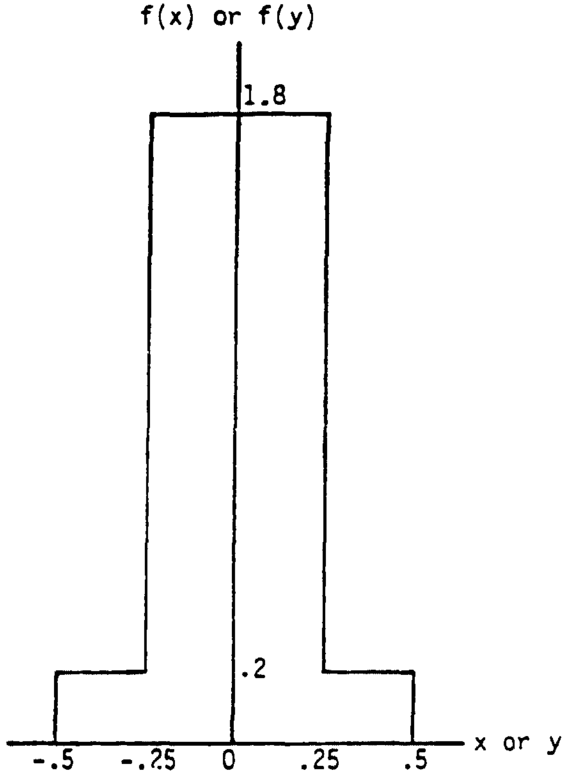
Next we need to define the pdf's for X and Y. They are given below, and Figure E-1-3 is a picture of how they look.

$$f(x) = \begin{cases} 0.2, & -0.50 \leq x \leq -0.25 \\ 1.8, & -0.25 \leq x \leq 0.25 \\ 0.2, & 0.25 \leq x \leq 0.50 \end{cases}$$

$$f(y) = \begin{cases} 0.2, & -0.50 \leq y \leq -0.25 \\ 1.8, & -0.25 \leq y \leq 0.25 \\ 0.2, & 0.25 \leq y \leq 0.50 \end{cases}$$

Here again all the conditions of a pdf are satisfied: all values of the random variables are greater than zero, the area under the pdf is equal to 1, and the area under the pdf between a and b is equal to the probability that X occurs between a and b. The joint pdf, again assuming independence between X and Y, is:

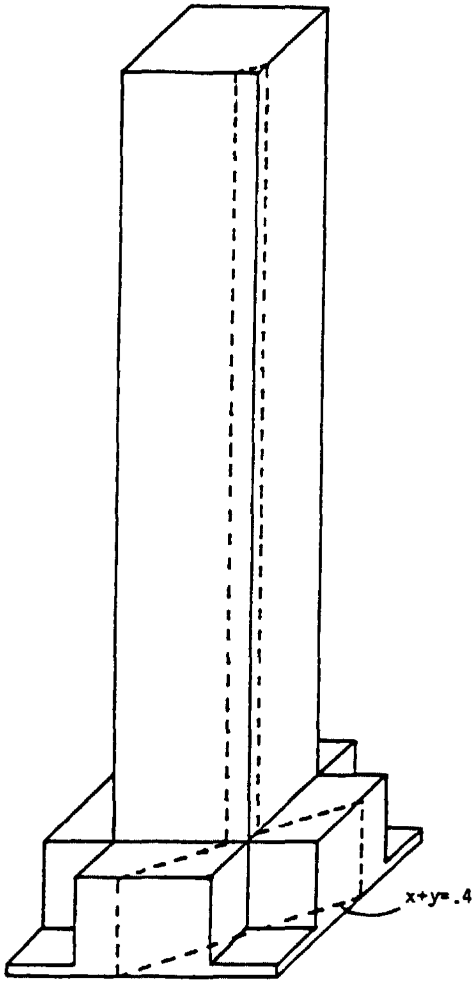
FIGURE E-1-3



$$f(x,y) = f(x)f(y) = \begin{cases} 0.04, & -0.50 \le x \le -0.25, & -0.50 \le y \le -0.25 \\ 0.36, & -0.50 \le x \le -0.25, & -0.25 \le y \le 0.25 \\ 0.04, & -0.50 \le x \le -0.25, & 0.25 \le y \le 0.50 \\ 0.36, & -0.25 \le x \le 0.25, & -0.50 \le y \le -0.25 \\ 3.24, & -0.25 \le x \le 0.25, & -0.25 \le y \le 0.25 \\ 0.36, & -0.25 \le x \le 0.25, & 0.25 \le y \le 0.50 \\ 0.04, & 0.25 \le x \le 0.50, & -0.50 \le y \le -0.25 \\ 0.36, & 0.25 \le x \le 0.50, & -0.25 \le y \le 0.25 \\ 0.04, & 0.25 \le x \le 0.50, & 0.25 \le y \le 0.50 \end{cases}$$

This solid looks like Figure E-1-4. The line on the bottom horizontal plane in Figure E-1-4 is the line $X+Y=0.4$. The area in front of and to the right

FIGURE E-1-4



of this line corresponds to the shaded area in Figure E-1-1. The volume in back of and to the left of the vertical plane cutting through the solid represents the total probability of all combinations of X and Y with losses less than 0.4. This volume is equal to 0.941 as compared to 0.820 for the line $X+Y=0.4$ in our previous example. Thus the probability density function assumed has a significant effect on the ultimate probabilities for the levels of surplus chosen. The table below compares results from examples 1 and 2 just described:

Probability Adequacy	Surplus Level: Example 1 (Figure E-1-2)	Surplus Level: Example 2 (Figure E-1-4)
50%	0.000	0.000
75	0.293	0.177
90	0.553	0.333
95	0.684	0.420
98	0.800	0.529
99	0.859	0.604
100	1.000	1.000

The second example (Figure E-1-4) is more similar to a normal distribution than the first example in that it has "tails" extending out past the central area around the mean where the greatest probability of values of X and Y occur. This concept is illustrated in the above table where in example 2, the surplus level is lower than the level in example 1 at the same probability adequacy.

In order to compare the results of several different probability density assumptions and to compare all of these results to the method described in Appendix E-2, several other joint probability distributions assumptions are described briefly below.

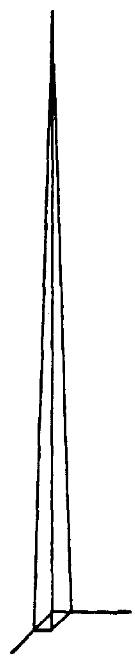
First, we look at a situation where there are no negative values (that is, gains) from a given risk. (Asset defaults are a good example of this assumption as there is no gain from any level of asset defaults. Both better-than-expected mortality and capital gains from falling interest rates represent situations where there are possible gains from C-2 and C-3 risk, respectively.) The individual distributions and the resulting joint probability distribution are given below. Again we assume the two risks are independent.

$$f(x) = 4(1 - 2x), \qquad 0 \le x \le 0.5$$
$$f(y) = 4(1 - 2y), \qquad 0 \le y \le 0.5$$

$$f(x,y) = f(x)f(y) = 16(1 - 2x - 2y + 4xy), 0 \le x \le 0.5, 0 \le y \le 0.5$$

This joint distribution is shown in Figure E-1-5.

FIGURE E-1-5



Note the asymmetry of the above probability distribution; also, the non-zero mean and the fact that only non-negative values of X and Y can occur. These are all characteristics which differ from the distributions assumed in the first two examples and they will materially affect the probability adequacy of the various levels of surplus. Levels of surplus with the associated probability adequacy for this example are shown below along with the results from the prior two examples for comparison purposes:

Probability Adequacy	Surplus Level Example 1 (Figure E-1-2)	Example 2 (Figure E-1-4)	Example 3 (Figure E-1-5)
50%	0.000	0.000	0.321
75	0.293	0.177	0.446
90	0.553	0.333	0.560
95	0.684	0.420	0.630
98	0.800	0.529	0.706
99	0.859	0.604	0.753
100	1.000	1.000	1.000

Note here, in this example, that there is a loss at every combination of X and Y; that is, there are no gains possible, unlike examples 1 and 2.

Another interesting combination occurs when we assume that X has the characteristics from example 2 and Y those from example 3. Example 2 could be thought of as very roughly modeling mortality rates and example 3 as roughly modeling asset defaults. Thus looking at their combination is similar to examining the combination of C-1 and C-2 risks.

This combination is shown below and in Figure E-1-6:

$$f(x) = \begin{cases} 0.2, & -0.50 \le x \le -0.25 \\ 1.8, & -0.25 \le x \le 0.25 \\ 0.2, & 0.25 \le x \le 0.50 \end{cases}$$

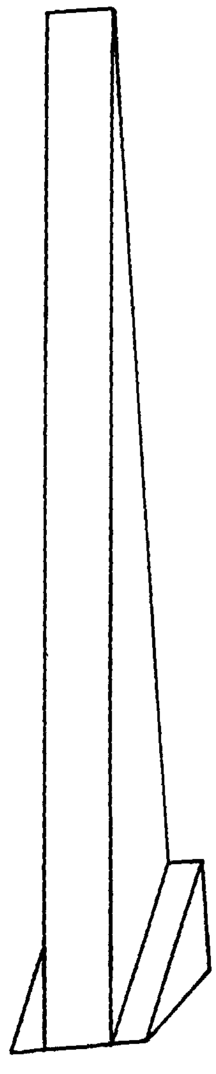
$$f(y) = 4(1 - 2y), \qquad 0 \le y \le 0.50$$

$$f(x,y) = \begin{cases} 0.8(1 - 2y), & -0.50 \le x \le -0.25, \quad 0 \le y \le 0.50 \\ 7.2(1 - 2y), & -0.25 \le x \le 0.25, \quad 0 \le y \le 0.50 \\ 0.8(1 - 2y), & 0.25 \le x \le 0.50, \quad 0 \le y \le 0.50 \end{cases}$$

The probabilities associated with various surplus levels are shown below for this example and the three previous examples as well:

Probability Adequacy	Surplus Level Example 1 (Figure E-1-2)	Example 2 (Figure E-1-4)	Example 3 (Figure E-1-5)	Example 4 (Figure E-1-6)
50%	0.000	0.000	0.321	0.166
75	0.293	0.177	0.446	0.312
90	0.553	0.333	0.560	0.451
95	0.684	0.420	0.630	0.531
98	0.800	0.529	0.706	0.617
99	0.859	0.604	0.753	0.675
100	1.000	1.000	1.000	1.000

FIGURE E-1-6



Note that of the above examples, the one requiring the highest level of surplus to assure adequacy of 95 percent or higher is the cube of example 1, which has no "normal-like" tails. In other words, losses are spread evenly over the entire range of possible values, and therefore more surplus is required to control the losses at the higher probability levels. The lowest surplus at all probability levels is found in example 2.

The above methodology can be extended to combinations of more than two risks. The joint probability distributions with the "lines" of constant surplus are more difficult to visualize, but the underlying methodology remains the same and current computer technology certainly has the ability to evaluate the resulting integrals and equations numerically.

As an illustration, one simple example of the combination of three risks will be shown here.

Define three random variables, X, Y and Z, as in example 1:

Let X be the random variable for risk 1, $-0.5 \leq X \leq 0.5$, and
Y be the random variable for risk 2, $-0.5 \leq Y \leq 0.5$, and
Z be the random variable for risk 3, $-0.5 \leq Z \leq 0.5$.

This time, if we assume the losses from the three risks are additive, we define a *plane* of constant surplus cutting through the solid formed by all the possible combinations of X, Y and Z. This solid is shown in Figure E-1-7 along with the plane of constant surplus $X+Y+Z=0.4$.

Next, we need to define the probability distributions for X, Y and Z in order to determine the probability adequacy level for a given surplus amount. This is done below:

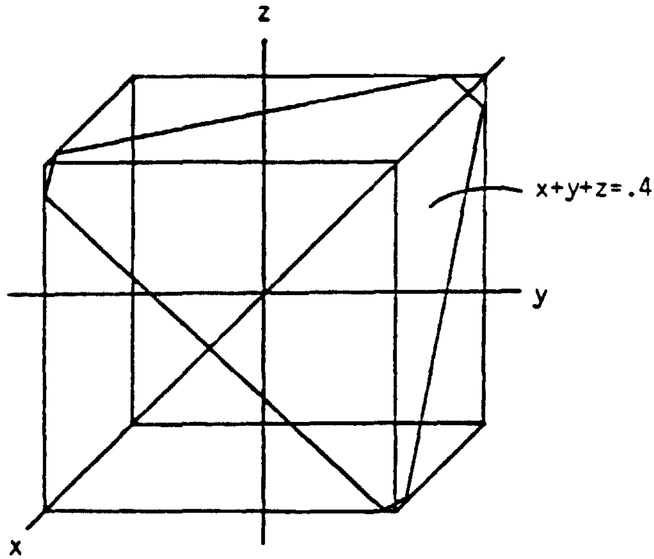
$$f(x) = 1, \qquad -0.5 \leq x \leq 0.5$$
$$f(y) = 1, \qquad -0.5 \leq y \leq 0.5$$
$$f(z) = 1, \qquad -0.5 \leq z \leq 0.5$$

Assuming X, Y and Z are independent, these distributions combine into a single trivariate distribution as follows:

$$f(x,y,z) = f(x)f(y)f(z) = 1, \quad -0.5 \leq x \leq 0.5, \; -0.5 \leq y \leq 0.5, \; -0.5 \leq z \leq 0.5.$$

We now have a four-dimensional "solid" composed of a three-dimensional cube (Figure E-1-7) extending one unit into the fourth dimension.

FIGURE E-1-7



We need to determine the "volume" of probability associated with a given surplus level. To do this, we determine the volume below and to the left of the plane of constant surplus pictured in Figure E-1-7 and then multiply this volume by the "height" of the cube into the fourth dimension. (This is analogous to calculating the volume in the first example by determining the area of the bounded region on the bottom surface of the cube in Figure E-1-2 and then multiplying by the height of the cube, which is the extension of the plane into the third dimension.)

Probability levels associated with various surplus amounts for the combination of both two and three risks are shown below:

	Surplus Level	
Probability Adequacy	Example 1 (Figure E-1-2)	Example 5 (Figure E-1-7)
50%	0.000	0.000
75	0.293	0.353
90	0.553	0.657
95	0.684	0.831
98	0.800	1.007
99	0.859	1.109
100	1.000	1.500

Note that, as would seem intuitively correct, by adding an additional risk, a higher level of surplus is required to achieve the same adequacy. For example, in the first example (which is identical to this example except only two risks are considered) to achieve 99 percent adequacy, one must hold surplus of 0.859. In this example where we are now considering three risks, to achieve 99 percent adequacy, one must hold surplus of 1.109, an increase of approximately 30 percent.

Now that we have looked at several simple examples and compared them to each other, we will next compare the results for these examples from this methodology with the methodology described in Appendix E-2. The approach in Appendix E-2 makes several simplifying assumptions to arrive at a formula based on the losses from the risks individually. The formula then combines the risks and arrives at a surplus amount. Below is the formula modified slightly to take into account that not all our examples have a zero mean. Remember that we have assumed all risks to be independent in our examples. Refer to Appendix E-2 for a better understanding of how this formula is derived.

$$u = (m_1 + m_2 + m_3) + [(u_1 - m_1)^2 + (u_2 - m_2)^2 + (u_3 - m_3)^2]^{1/2}$$

This derivation in Appendix E-2 refers to "an unknown level of error," and this error can be seen by comparing results of the two methodologies. The examples below refer to the five examples described and compared above. A brief summary is given first:

$$\text{Distribution A: } f(x) = 1, \quad -0.5 \le x \le 0.5$$

$$\text{Distribution B: } f(x) = \begin{cases} 0.2, & -0.50 \le x \le -0.25 \\ 1.8, & -0.25 \le x \le 0.25 \\ 0.2, & 0.25 \le x \le 0.50 \end{cases}$$

$$\text{Distribution C: } f(x) = 4(1 - 2x), \quad 0 \le x \le 0.50$$

Example	Distribution of Random Variable		
	X	*Y*	*Z*
1	A	A	—
2	B	B	—
3	C	C	—
4	B	C	—
5	A	A	A

The m_i and u_i in the formula given above represent the mean of the probability distribution for risk i and the surplus needed for risk i at ruin probability level p, respectively, ignoring all other risks. The variable u is the surplus needed for all risks combined at ruin probability p. Below is shown the mean and surplus needed at various probability levels for individual risks assuming the probability distributions above.

Probability Level	Surplus Needed Given the Probability Distribution for Risk i		
	A	*B*	*C*
50%	0.000	0.000	0.146
75	0.250	0.139	0.250
90	0.400	0.222	0.342
95	0.450	0.250	0.388
98	0.480	0.400	0.429
99	0.490	0.450	0.450
100	0.500	0.500	0.500
Mean	0.000	0.000	0.167

Given the above information, surplus needed for a combination of risks can be calculated using the formula, and this surplus can then be compared to the surplus calculated in each of the examples previously. This is shown in Attachment E-1a.

Several things should be noted from these results:

- The formula assumes the underlying distributions of the risks are normal, which is an *infinite* distribution as opposed to the simple *finite* distributions assumed in our examples. Thus, the results at 100 percent probability (the boundaries of our finite examples) will always contain a significant error.
- The three examples that assume distributions more "normal-like" (examples 2, 3 and 4) show a lower percentage error overall.
- Examples 2 and 4 contain the step-function, which has two discontinuities. This results in errors, when compared to the "smooth" formula, which vary significantly in both directions.
- Example 5, which is identical to example 1 except that there are three risks instead of two, results in larger percentage errors when compared to the formula. This would seem to indicate that the "unknown level of error" increases as the number of risks under consideration increases.

Keep in mind that these are very simple probability assumptions, and much more realistic assumptions would be made to model risks in a real-world situation. The comparison of these results to the formula approach merely gives us an indication about the error involved in the simplifying assumptions but this error will be significantly different under different assumptions.

Now that we have explained and compared the basic methodology with several simple examples, we will start applying these principles to real-world combinations of C-1, C-2 and C-3 risk.

Real-World Examples

Combining C-2 and C-3 Risk

The first combinations that we will work with are C-2, pricing risk, and C-3, mismatch risk. We chose our random variables, X and Y, as follows:

Let X be the % change in mortality rates from the expected level at issue, $-\infty \leq X \leq \infty$, and
Y be the % change in interest rates from the expected level at issue, $-\infty \leq Y \leq \infty$.

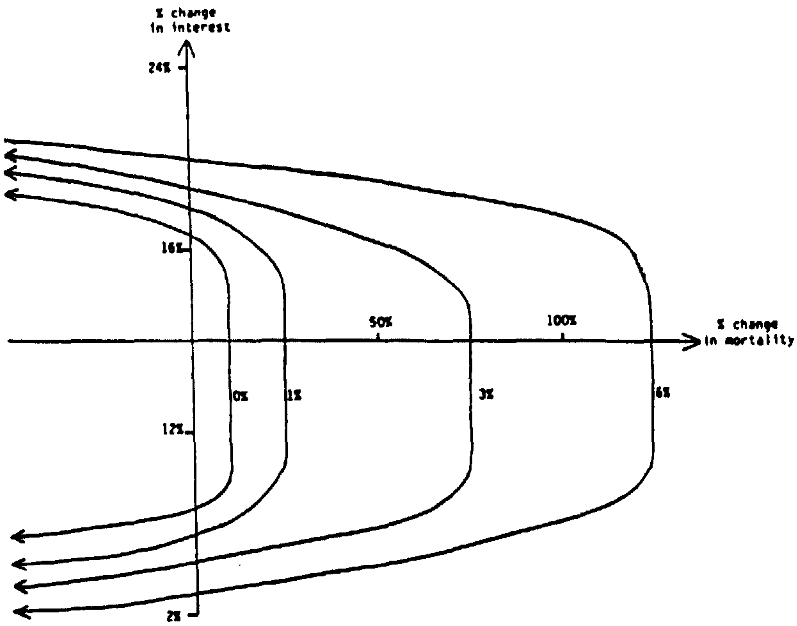
We used the COR model to determine the surplus required given various combinations of the above risks. Our model assumptions were basically the same as with the SPDA product described in Appendix D-1, although by introducing mortality risk, we essentially shifted from an SPDA contract to a universal life contract.

Interest rates could be input into the model directly as new money rates greater than, less than, or equal to the initial assumed rate. Mortality, however, had to be calculated as a change from expected and then converted to either an extra expense charge if it was greater than expected or an extra credit if it was less than expected.

From the model, lines of constant surplus as a percentage of initial liabilities were determined, and sample lines are shown in Figure E-1-8. These lines represent *surplus required* over and above the assumed level of statutory reserves. In these examples, a line of zero surplus indicates the losses for which the bare reserves (with no additional surplus) are adequate.

Note how different these lines of constant surplus are, compared with the lines developed in our simple examples. As with our simple examples, however, the surplus designated by a given line of constant surplus is enough

FIGURE E-1-8



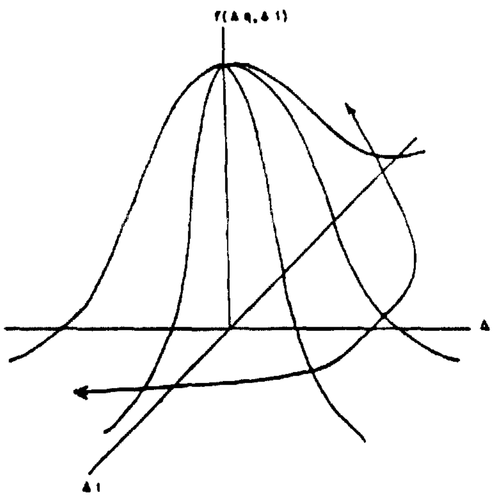
to cover all combinations of risk inside and to the left and inadequate for all combinations outside and to the right.

The shape of the lines above reflects the special characteristics of the C-3 risk. In particular, when interest rates change by 300 to 400 basis points above the starting point, there are no statutory losses. This results in the vertical lines at the right side of Figure E-1-8. The same level of greater-than-expected mortality can be combined with a range of interest rate changes and still require the same level of statutory surplus. Once interest rates increase 300 to 400 basis points, however, a given level of surplus is adequate only if the excess mortality drops substantially. At very high interest rates, we have practically a horizontal line, since a slight increase in interest rates requires big improvements in mortality. A similar result occurs for very low interest rates since we assumed a guaranteed minimum credited rate.

These lines of constant surplus are totally independent of the probability distributions assumed for changes in interest rates and excess mortality.

Instead, they depend solely on the assumptions underlying our computer modeling. But in order to quantify the adequacy level of a given amount of surplus, we must assume a probability distribution for each of the risks. A possible joint probability distribution assumption might result in the three-dimensional picture shown in Figure E-1-9.

FIGURE E-1-9



To determine the probability adequacy of a given surplus amount, we must measure the volume, as shown in Figure E-1-9, that lies above the area on the Δi, Δq_x plane that is within the line of constant surplus. If, for example, 90 percent of the total volume lies inside this line, then this surplus level provides 90 percent adequacy.

Our initial probability distribution assumption for the random variable Y (change in interest rates) was a normal distribution with a mean at 14 percent and a standard deviation of 0.027. These parameters were chosen so that a 450-basis-point increase had a 1-in-20 probability of occurring. This resulted in a 650-basis-point increase having a 1-in-100 probability. We felt this was consistent with economic conditions in the late 1970s and early 1980s.

The probability distribution initially chosen for X (change in mortality) was slightly more complex. For less-than-expected mortality, a normal was again chosen with a zero mean and a standard deviation of 0.081. This resulted in a 30 percent decrease in expected mortality having a 1-in-10,000 probability. This is felt to be an extremely unlikely occurrence, and this probability level reflects this.

For greater-than-expected mortality, our initial assumption was that 50 percent extra mortality is a 1-in-100 occurrence. This corresponds to the level of mortality experienced during the 1918 influenza epidemic. However, a normal distribution with a zero mean and a standard deviation reflecting this assumption did not model other levels of mortality (such as 10 percent or 25 percent) very well. We found that a student's t-distribution with two degrees of freedom and transformed so that a 1-in-100 probability is equal to 50 percent results in 14 percent or higher extra mortality having a 1-in-10 probability and 21 percent or higher extra mortality having a 1-in-20 probability. We felt comfortable with these assumptions.

Other distributions were assumed also for comparison purposes, and these will be discussed below. For a more detailed discussion of all the probability distributions assumed and how they were derived, see Attachment E-1c.

We also assumed that interest rates and mortality are independent, so the joint probability distribution is simply the product of the two marginal distributions. Once a joint distribution was determined, the volume was calculated above and inside the lines of constant surplus such as shown in Figure E-1-9 above. The results corresponding to our initial probability assumptions are shown below:

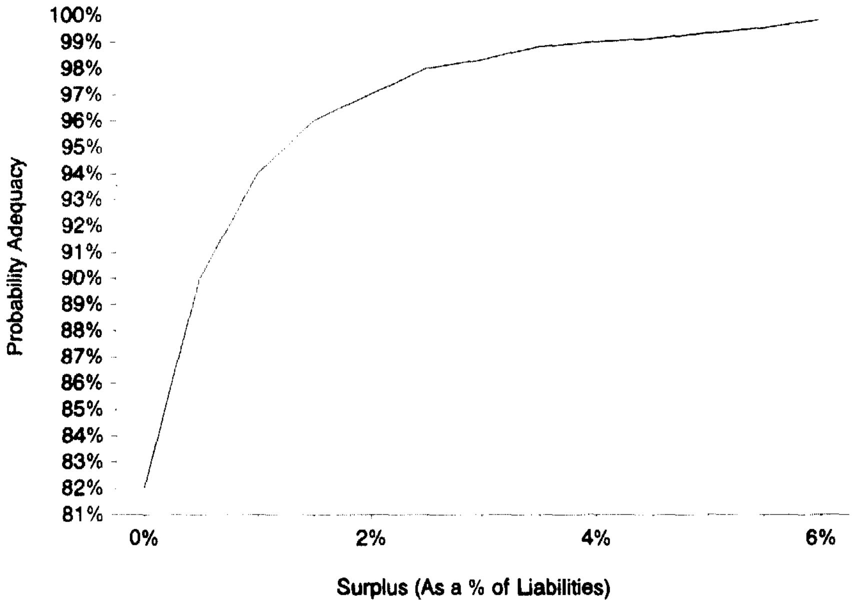
Probability Adequacy	Surplus Level
82%	0.000
90	0.455
95	1.108
97	1.875
98	2.620
99	4.000

Surplus is shown as a percentage of initial liabilities.

Note that 0 percent surplus, that is, the bare reserves, provides adequacy 82 percent of the time. As surplus increases from 0 percent, the probability adequacy improves dramatically at first, but then the improvement tails off as the surplus is raised further. This effect is shown in Figure E-1-10, showing probability adequacy as a function of surplus. Note the steep slope at the lower probabilities and the almost horizontal line at the high probability levels.

FIGURE E-1-10

PROBABILITY ADEQUACY AS A FUNCTION OF SURPLUS
COMBINATIONS OF C-2 AND C-3



To measure the effect that the probability distribution formulas assumed have on the results, we calculated the volumes for the same probability levels assuming slightly different distributions. The four distributions examined are summarized below:

Example 1	Mortality (Less than expected)	Normal
	Mortality (Greater than expected)	Student's t
	Interest	Normal
	Correlation Coefficient	0%
Example 2	Mortality	Normal
	Interest	Normal
	Correlation Coefficient	0%
Example 3	Mortality	Normal
	Interest	Normal
	Correlation Coefficient	50%
Example 4	Mortality (Less than expected)	Normal
	Mortality (Greater than expected)	Student's t
	Interest	Student's t
	Correlation Coefficient	0%

The probability adequacv levels for all four probability situations are shown below:

Probability Adequacy	Surplus Level			
	Example 1	Example 2	Example 3	Example 4
90	0.455	1.099	1.101	0.311
95	1.108	1.643	1.741	0.837
97	1.875	2.055	2.326	1.428
98	2.620	2.439	3.000	2.400
99	4.000	3.450	4.344	5.000

The following observations can be made about the above results:

- From 90 percent to 97 percent adequacy, the two examples with normal distributions exclusively (examples 2 and 3) require more surplus than the other examples, which at least partially incorporate the Student's t.
- Of the two exclusively normal distribution examples (examples 2 and 3), example 3 with a 50 percent correlation coefficient requires more surplus at all probability levels examined.
- Example 4, which is almost exclusively Student's t, requires the least amount of surplus at all probability levels from 90 percent to 98 percent but then requires the most surplus at 99 percent.

The main observation one can make from all this is that the probability distributions assumed significantly affect the surplus results. Also, this effect is different closer to the mean than it is out in the tails of the distributions. Figures E-1-11 and E-1-12 show the difference at the mean and then out in the tails between the normal and the Student's t-distribution assumed for mortality rates:

FIGURE E-1-11

NORMAL VS. STUDENT'S T

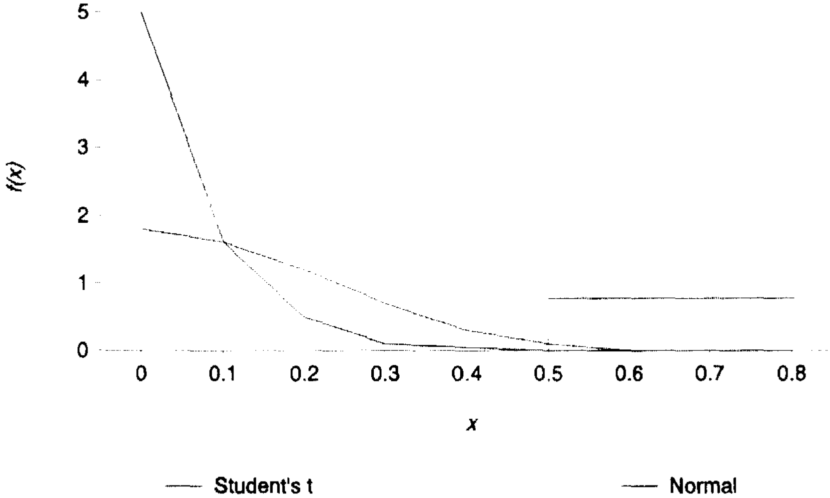
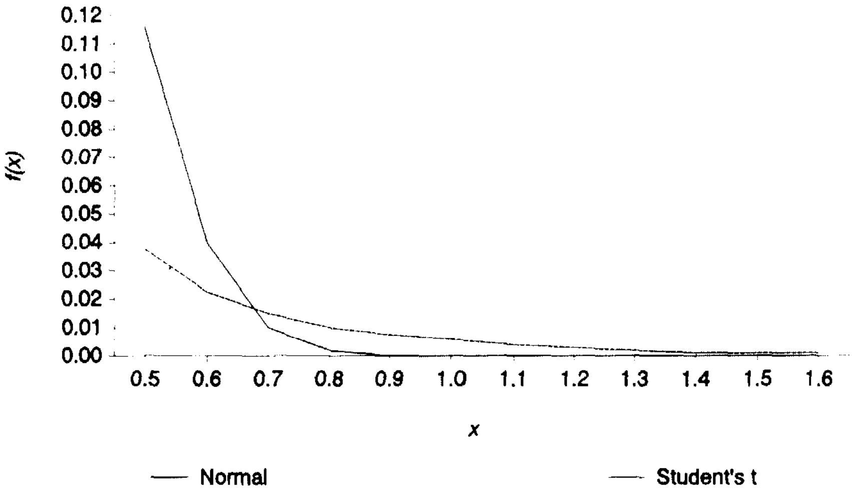


FIGURE E-1-12

BLOWUP OF BOXED AREA



As with the simple examples, the results from these more real-world examples can be compared to results from applying the methodology described in Appendix E-2. The applicable formula from Appendix E-2 is as follows:

$$u = (u_1^2 + u_2^2 + 2r_{12}u_1u_2)^{1/2}$$

As before, u_i is the surplus needed for risk i at ruin probability level p, ignoring all other risks. To determine u_i for our various examples, first the interest rate or mortality rate at a given probability level was calculated assuming the applicable probability distribution. For example, given the normal assumption for interest rates in examples 1, 2 and 3, 17.5 percent interest or higher has a 90 percent probability, 18.5 percent or higher has a 95 percent probability, and so on.

Once the level of individual risk was known at the various probability levels, the COR model was run to calculate the surplus needed for that risk at that ruin probability level keeping all other risks at zero. This is u_i.

The results of comparing the two methodologies using realistic assumptions for C-2 and C-3 risks is shown in Attachment E-1b. Several observations can be made:

- The two examples that are assumed to be normally distributed exclusively (examples 2 and 3) result in the lowest percentage errors. (The formula in Appendix E-2 is derived assuming normal distributions for the *losses* from each risk.)
- Example 4, which is almost exclusive Student's t, has the highest percentage errors.
- The presence of correlation does not seem to particularly affect the closeness of the results of the two methodologies.
- Some of the errors, particularly at the 99 percent level, are low enough to assume that the formula does provide the correct answer within an order of magnitude. Errors at other levels of probability, however, are significant. For instance, in example 1, at the 95 percent probability level, the difference between holding 1.1 percent surplus or 0.6 percent surplus is substantial.

In order to understand why the results from the two methodologies are at times significantly different, we must examine the assumptions made in Appendix E-2 versus the true characteristics of the risks, in particular the C-3 risk.

One assumption made in the methodology in Appendix E-2 is that the random variable representing the losses from a combination of risks is the sum of the random variables of the individual risks. For example, if X is the loss or surplus required for risk 1 at a given probability level and Y is the loss or surplus required for risk 2 at the same probability level, then the required surplus at the same probability level for risk 1 combined with risk 2 is assumed to be $X+Y$.

The table below presents some data from the COR model, which demonstrates that this assumption does not, in reality, hold true:

C-2 Risk		C-3 Risk		Combined		
Extra Mortality	Required Surplus	Interest Rate	Required Surplus	Required Sum	Surplus Actual	Percentage Error
10%	0.000%	18%	0.000%	0.000%	0.070%	100.0%
10	0.000	20	2.160	2.160	2.622	17.6
20	0.531	20	2.160	2.691	3.181	15.4
30	1.0771	20	2.160	3.237	3.740	13.5

These data demonstrate that the required surplus for a given combination of C-2 and C-3 risk is different from the sum of the surplus required for the two risks taken one at a time. This results primarily from two factors: (a) with increasing interest rates under the C-3 risk, the assets are worth less, so the manifestation of another risk is more painful than when the C-3 risk is not present; and (b) the "credit" for the earnings margin (recall the regression formula presented in Appendix D-1) is reflected in each of the individual results, but can only be used once in the combined results.

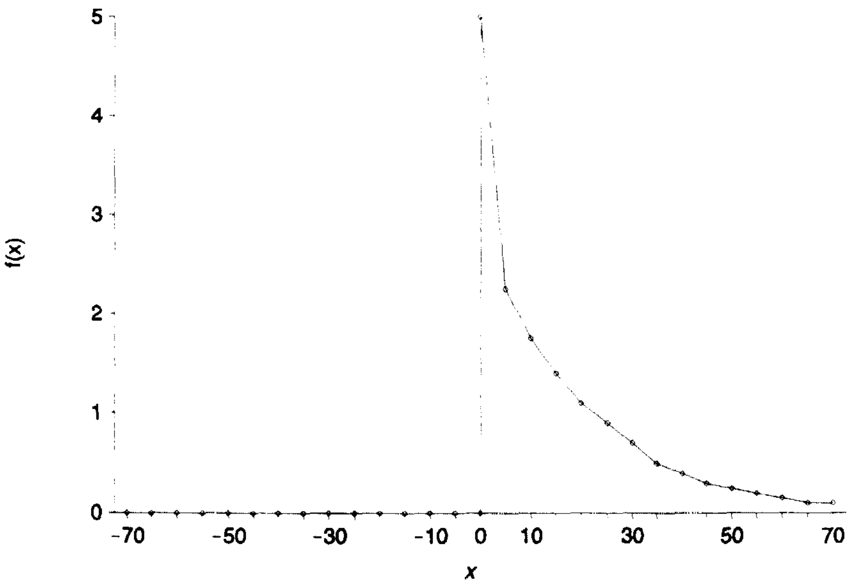
It is exactly these types of interactions that distort the results of statistical approaches that work by combining surplus results for the individual risks.

A second assumption made in the derivation in Appendix E-2 is that the losses from the various risks are normally distributed. This assumes among other things that the losses and gains are distributed symmetrically about the mean. In Appendix E-2, the mean is assumed to be zero with the result that gains and losses are equally likely. In reality this is not necessarily true: losses from changing interest rates and mortality rates are probably much more likely than gains from these risks and there are no gains possible at all from asset defaults. The formula in Appendix E-2 can be adjusted to allow for a non-zero mean, but the normal assumption always implies symmetry and at least some finite probability of gains from the risks involved.

The methodology in this appendix assumes a normal distribution for interest rates, but that is quite different from assuming the *losses from interest rate risk* are normally distributed as is done in Appendix E-2. Assuming the normal distribution for interest rates described in detail in Attachment E-1c and determining the required surplus from the COR model, we were able to construct the distribution of the losses from C-3 risk. This is shown in Figure E-1-13.

FIGURE E-1-13

PROBABILITY DENSITY FUNCTION FOR LOSSES
C-3 RISK



Notice, in particular, under the assumptions of our example, there is no probability associated with negative surplus values (that is, there are no gains). There is very high probability associated with zero surplus, since zero surplus results from the scenarios with decreasing rates down to the guaranteed rate and also from moderate increases in rates. At the right, the tail of the distribution falls much more slowly than the normal, which reflects the property that small increases in interest rates produces large increases in required surplus.

Thus, these are some of the ways the COR model has demonstrated that the true characteristics of the risks are somewhat different from the assumptions made in deriving the simplified formula in Appendix E-2. In this way, we can begin to understand the origin and magnitude of the "unknown level of error" inherent in the formula-type methodology.

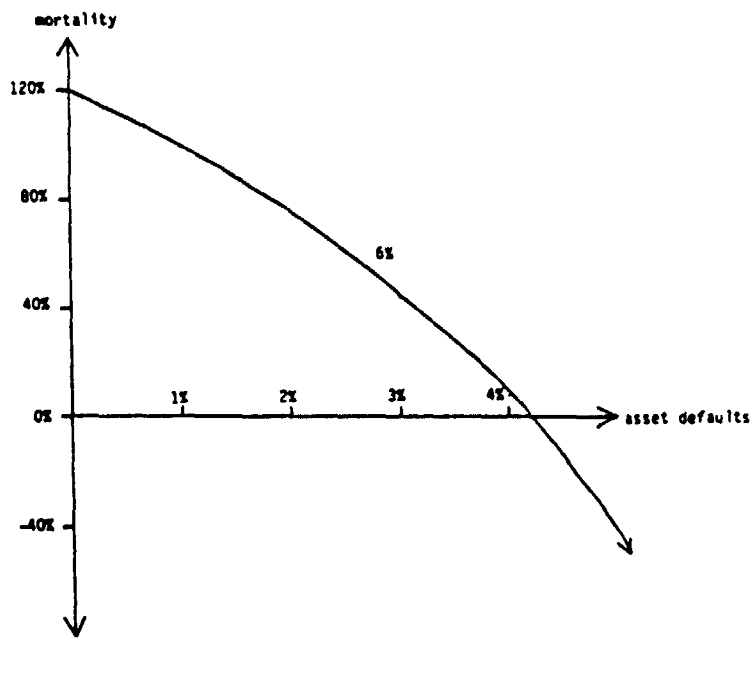
Combining C-1 and C-2 Risk

We have also done some very preliminary testing of the combinations of C-1, asset default risk, and C-2, pricing risk. We chose our random variables, X and Y, as follows:

Let X be the asset default rate, $0 < X < \infty$, and
Y be the percent change in mortality rates from the expected level at issue, $-\infty < Y < \infty$.

Again we used the COR model to determine lines of constant surplus and initial results show that the lines of constant surplus are much closer to straight lines (as was assumed in the simple examples) than was true when C-3 risk was involved. This reflects the more additive nature of these risks. A sample line of constant surplus is shown in Figure E-1-14.

FIGURE E-1-14



The next step is to determine probability distributions for the two risks. For mortality risk, we used the distribution described in the first example above where we were combining C-2 and C-3 risks. For asset default rates, determining a distribution was slightly more complicated than the work we have done so far. Keeping in mind the particular characteristics of asset default rates, we needed a probability distribution with the following aspects:

- No negative values (that is, there do not exist negative rates of asset default)
- A mean close to zero (that is, on average, there is a small finite number of asset defaults at all times)
- A long tail off to the right, which gives reasonable asset default rate values at specific probability levels (for example, 1 percent asset defaults with a 1-in-5 probability)
- The ability to transform the resulting distribution into a normal distribution via a functional transformation so that C-1, asset default risk, and C-3, interest rate risk, when combined can have a non-zero correlation coefficient.

The possible probability distributions that meet the above criteria are discussed in greater detail in Attachment E-1c. For our initial work, we chose a chi-square distribution with three degrees of freedom adjusted to scale so that a 1.5 percent asset default rate has a 1-in-10 probability.

This distribution met some of the above criteria, notably no negative values, a mean close to zero, and a long tail with at least somewhat reasonable surplus values at various probability levels. It cannot, however, be transformed via a functional transformation into a normal distribution although a chi-square with one degree of freedom can be transformed very easily. Additional work on this would have to be done in the future to arrive at the most realistic and most useful distribution for asset default rates.

It was because of our initial inability to work with two normals (via a functional transformation) that our preliminary work with C-1 risk concentrated on the combination of C-1 and C-2, where we can assume the two risks are independent. Of course, it makes intuitive sense that in reality, the correlation between C-1 and C-3 risks is non-zero and more accurate work with this combination would have to take this correlation into account.

Very preliminary results of combining C-1 and C-2 risks are given below. Much more work needs to be done in this area.

Surplus Level	Probability Adequacy
2%	67.76%
4	71.00
6	71.62
9	71.77

From the above probability adequacy levels, we can see that this joint probability distribution has a very long, very shallow tail. Increasing the surplus level a fairly significant amount, from 6 percent to 9 percent, increases the probability adequacy by just a fraction of 1 percent. Obviously, very high surplus levels will have to be assumed to achieve adequacy at the 90 percent to 99 percent level. These results, of course, are a function of the individual probability distributions assumed, and we have already mentioned several inaccuracies in the distribution assumed for asset default rates. This distribution obviously needs more work.

Modified Formula Approach

Very preliminary work has been done at Aetna to devise empirically a formula similar to the one derived by Mr. Cody but adjusted for the errors brought out by our work with the COR model. The proposed form is given below:

$$S_c = [(S_1K_1)^2 + (S_2K_2)^2 + (S_3K_3)^2]^{1/2} + \text{Corr Adj} - PVE$$

where

S_c = surplus required for the combination of all three risks at probability level p

S_i = surplus required for risk i at probability level p, assuming to earnings margin and no required shareholder dividends

K_i = increase in surplus required for risk i caused by the manifestation of other risks

i = 1,2,3

Corr Adj = correlation adjustments

PVE = present value of earnings margin less present value of required shareholder dividends.

Note that probability distribution functions would have to be assumed for the individual risks, which as always will introduce error but that a joint distribution is not required nor is the assumption that the individual distributions are normal. The S_i's and the *PVE* can be determined directly from the model and presumably the K_i's and the correlation adjustments can be determined empirically from multiple runs of various combinations of the risks.

Very little work has been done on this approach and there are no results at this time.

Limitations of Methodologies

It should be apparent from the examples and comparisons discussed above that there are limitations and drawbacks to the methodologies presented in this report. Of course, there are many possible approaches to combining risks, and these are but a few of them. Any methodology would, however, have its own strengths and shortcomings.

The results of all methodologies are sensitive to the input assumptions. Because all types of risks are inherently impossible to predict with complete accuracy, any methodology must make certain guesses and estimates and in this way introduce a certain level of error. The more simple the resulting approach (for example, a single formula versus complete calculations from basis principles), the more numerous are the initial assumptions and estimates and the greater is the potential for gross error.

There is also a certain trade-off between complexity and accuracy. The methodology that uses basic principles described in this appendix is very complex and requires extensive work modeling cash flows, determining lines of constant surplus, developing appropriate probability distributions, and determining the volumes under the joint probability curves. The methodology in Appendix E-2 is extremely easy to apply but, as has been shown, can contain significant errors due to its simplifying assumptions. The modified formula approach is mere conjecture at this point.

Prior to the widespread use of computers, it was often necessary to make simplifying assumptions (assuming a normal curve was extremely common). Today the need for such assumptions is greatly reduced. Whether a methodology as complex as the first one presented here is feasible, however, is open to debate.

In short, a thorough understanding of the risks themselves, the cash flows produced by these risks, and how a true cash-flow analysis of all the cash flows of an insurance concern is accomplished, is an inescapable necessity if one is to truly understand the combinations of risks. Once this basic knowledge is established, the methodology that best fits the appropriate situation then becomes a much easier problem to solve.

ATTACHMENT E-1a

COMPARISON OF METHODOLOGIES USING SIMPLE EXAMPLES

Prob Level	ui (A)	ui (B)	ui (C)	Surplus Level: Example 1 Actual	Example 1 Formula	Example 1 % Diff	Example 2 Actual	Example 2 Formula	Example 2 % Diff	Example 3 Actual	Example 3 Formula	Example 3 % Diff
50%	0.000	0.000	0.146	0.000	0.000	—	0.000	0.000	—	0.321	0.362	−12.7%
75%	0.250	0.139	0.250	0.293	0.354	−20.7%	0.177	0.196	−11.0%	0.446	0.451	−1.2%
90%	0.400	0.222	0.342	0.553	0.566	−2.3%	0.333	0.314	5.6%	0.560	0.581	−3.8%
95%	0.450	0.250	0.388	0.684	0.636	7.0%	0.420	0.354	15.8%	0.630	0.647	−2.6%
98%	0.480	0.400	0.429	0.800	0.679	15.1%	0.529	0.566	−6.9%	0.706	0.705	0.2%
99%	0.490	0.450	0.450	0.859	0.693	19.3%	0.604	0.636	−5.4%	0.753	0.734	2.5%
100%	0.500	0.500	0.500	1.000	0.707	29.3%	1.000	0.707	29.3%	1.000	0.805	19.5%
				Combination: (A),(A)			Combination: (B),(B)			Combination: (C),(C)		

Prob Level	ui (A)	ui (B)	ui (C)	Surplus Level: Example 4 Actual	Example 4 Formula	Example 4 % Diff	Example 5 Actual	Example 5 Formula	Example 5 % Diff
50%	0.000	0.000	0.146	0.166	0.187	−12.6%	0.000	0.000	—
75%	0.250	0.139	0.250	0.312	0.329	−5.3%	0.353	0.433	−22.7%
90%	0.400	0.222	0.342	0.451	0.450	0.3%	0.657	0.693	−5.5%
95%	0.450	0.250	0.388	0.531	0.501	5.7%	0.831	0.779	6.2%
98%	0.480	0.400	0.429	0.617	0.645	−4.6%	1.007	0.831	17.4%
99%	0.490	0.450	0.450	0.675	0.698	−3.5%	1.109	0.849	23.5%
100%	0.500	0.500	0.500	1.000	0.768	23.2%	1.500	0.866	42.3%
				Combination: (B),(C)			Combination: (A),(A),(A)		

(A) $F(X) = 1, \quad -0.5 < X < 0.5$
Mean = 0.00000

(B) $F(X) = 0.2, \quad -0.50 < X < -0.25$
$F(X) = 1.8, \quad -0.25 < X < 0.25$
$F(X) = 0.2, \quad 0.25 < X < 0.50$
Mean = 0.00000

(C) $F(X) = 4(1 - 2X), \quad 0 < X < 0.5$
Mean = 0.16667

Formula: $u = (m_1 + m_2 + m_3) + (u_1 - m_1)^2 + (u_2 - m_2)^2 + (u_3 - m_3)^2]^{1/2}$

mi = Mean of risk i
ui = Surplus required for risk i, individually
u = Surplus required for all risks combined

ATTACHMENT E-1b

COMPARISON OF METHODOLOGIES USING REAL-WORLD EXAMPLES

Prob Level	u_i Int	u_i Mort	r_{12}	Surplus Level - U Actual	Surplus Level - U Formula	% Diff	Assumptions Risk	Assumptions Distribution
Example 1								
90%	0.000	0.164	0%	0.455	0.164	63.9%	Mortality	Normal/
95%	0.000	0.574	0%	1.108	0.574	48.2%		Student's t
97%	0.722	0.956	0%	1.875	1.198	36.1%	Interest	Normal
98%	1.393	1.335	0%	2.620	1.929	26.3%		
99%	2.969	2.170	0%	4.000	3.677	8.1%	Correlation	0%
Example 2								
90%	0.000	0.940	0%	1.099	0.940	14.5%	Mortality	Normal
95%	0.000	1.367	0%	1.643	1.367	16.8%	Interest	Normal
97%	0.722	1.640	0%	2.055	1.792	12.8%		
98%	1.393	1.848	0%	2.439	2.314	5.1%	Correlation	0%
99%	2.969	2.170	0%	3.450	3.677	−6.6%		
Example 3								
90%	0.000	0.940	50%	1.101	0.940	14.6%	Mortality	Normal
95%	0.000	1.367	50%	1.741	1.367	21.5%	Interest	Normal
97%	0.722	1.640	50%	2.326	2.096	9.9%		
98%	1.393	1.848	50%	3.000	2.816	6.1%	Correlation	50%
99%	2.969	2.170	50%	4.344	4.468	−2.9%		
Example 4								
90%	0.000	0.164	0%	0.311	0.164	47.3%	Mortality	Normal/
95%	0.000	0.574	0%	0.837	0.574	31.4%		Student's t
97%	0.000	0.956	0%	1.428	0.956	33.1%	Interest	Student's t
98%	0.000	1.335	0%	2.400	1.335	44.4%		
99%	2.969	2.170	0%	5.000	3.677	26.5%	Correlation	0%

Formula: $u = [u_1^2 + u_2^2 + 2(r_{12})(u_1)(u_2)]^{1/2}$

ui = Surplus required for risk i, individually

u = Surplus required for both risks combined

r_{12} = Correlation between risks 1 and 2

ATTACHMENT E-1c

PROBABILITY DISTRIBUTIONS FOR C-1, C-2, C-3 RISKS

This attachment discusses in detail the probability distributions assumed for the various risks in applying the methodology described in Appendix E-1.

In determining the appropriate probability distribution for a specific risk, both objective and subjective measures were used. Some characteristics, such as the fact that there are never negative asset default rates, can be taken as a given but some characteristics are more elusive. For example, we felt comfortable with 50 percent extra mortality having a 1-in-100 probability, but when a normal curve was defined to produce this assumption, the resulting distribution produced such values as approximately 30 percent extra mortality having a 1-in-10 probability. This just did not *feel* right. An epidemic, such as the influenza epidemic of 1918–19, could reasonably produce 50 percent extra mortality once every 100 years, we felt, but something happening to produce 30 percent extra mortality every 10 years seemed highly unrealistic.

With this in mind, our assumptions, the distributions we used, and how they were developed are discussed below for each of the three types of risk. Keep in mind that we are only doing very preliminary work here, and these are only a few of literally hundreds of possible choices. This discussion should, however, present the ideas necessary to do an in-depth analysis of the risks and their underlying distributions.

Interest Rates, C-3 Risk

Initially, to determine an interest rate distribution, we developed a simple simulation model to randomly calculate interest rate scenarios. The characteristics of the model are given below:

- 1,000 trials of 20 years each were run.
- The interest rates in a given year were assumed to be normally distributed with μ = prior year's interest rate, and $\sigma = 9\% \times \mu$
- The 9 percent used in the formula, above, for the standard deviation was determined from a historical study done of interest rates over the past 60 years. It was determined from this study that the standard deviation of interest rates over time was approximately 9 percent of the prior year's rate.
- The interest rate at year zero was set at 14 percent.

After the model was run, 5-year running averages were taken of each 20-year scenario and the highest 5-year average was chosen. These formed a random sample of interest rates which would at some point in a 5-year period, rise to this chosen rate and stay there for 5 years.

The mean and standard deviation were calculated from this sample of running averages, and a normal distribution was then assumed with this sample mean and standard deviation. Interest rate levels associated with various probabilities were calculated based on this distribution, and the standard deviation was adjusted slightly to produce what we felt were more realistic values. (Of course, one guess as to what are "realistic" interest rates is as good as another.)

The probability density function of the final distribution chosen is shown below with the mean and standard deviation and various sample values:

$$f(x) = \frac{\exp(-0.5(x - 0.14/0.027468)^2}{0.027468\sqrt{2\pi}}, \quad -\infty < x < \infty$$

$$\mu = 0.14$$

$$\sigma = 0.027468$$

$P\{x > X\}$	X
1/10	17.5%
1/20	18.5
1/40	19.4
1/50	19.6
1/80	20.2
1/100	20.4

Given a starting interest rate of 14 percent, we felt these were reasonable values, taking into account the interest rate activity of the past 5–10 years.

In order to compare the results of different probability distributions when analyzing the combination of C-2 and C-3 risk, we also chose a Student's t-distribution for interest rates. This distribution was not as realistic for interest rates as the normal, but it did provide results for comparison purposes.

We chose two different Student's t-distributions, one for rates above 14 percent and one for rates below 14 percent. A distinct distribution was chosen for rates below 14 percent because, even though both the normal and Student's t are infinite distributions and therefore would allow interest rate values below 0 percent, the probability of these negative interest rates should be so low as to be practically nonexistent. To achieve this goal, a Student's t with 5 degrees of freedom was chosen for rates below 14 percent. A Student's t with 2 degress of freedom was chosen for rates above 14 percent and the following adjustments were made to the two distributions:

(1) Both distributions were adjusted for a change in origin from zero to 14 percent.
(2) The distribution above 14 percent was adjusted with a change in scale so that 20.4 percent had a 1/100 probability. This interest rate and probability were chosen from the results from the normal distribution shown above.
(3) The distribution below 14 percent was adjusted with a change in scale, so that the value of the density function at 14 percent was the same for both distributions. This ensured that the final total distribution was continuous without a discontinuity at 14 percent where the two distinct distributions met.

Each of the above calculations are shown below along with the resulting distribution and sample values.

A Student's t-distribution with 5 degrees of freedom is shown below:

$$f(x) = \frac{8}{3\pi\sqrt{5}}\left(1 + \frac{x^2}{5}\right)^{-3}, \quad -\infty < x < \infty$$

By definition, this distribution is symmetric about a mean of zero. Therefore, we know that:

$$\int_{-\infty}^{0} \frac{8}{3\pi\sqrt{5}}\left(1 + \frac{x^2}{5}\right)^{-3} dx = 0.5$$

We can adjust this distribution for a change in origin to 14 percent as follows:

$$\int_{-\infty}^{0.14} \frac{8}{3\pi\sqrt{5}}\left(1 + \frac{(x - 0.14)^2}{5}\right)^{-3} dx =$$

$$u = x - 0.14$$

$$du = dx$$

$$u(0.14) = 0$$

$$u(-\infty) = -\infty$$

$$\int_{-\infty}^{0} \frac{8}{3\pi\sqrt{5}}\left(1 + \frac{u^2}{5}\right)^{-3} du = 0.5$$

Therefore our new distribution with an origin at 14 percent is defined by the density function:

$$f(x) = \frac{8}{3\pi\sqrt{5}}\left(1 + \frac{(x - 0.14)^2}{5}\right)^{-3}, \; -\infty < x < \infty$$

The same steps were followed to change the origin of a Student's t with 2 degrees of freedom. The density function for such a distribution is as follows:

$$f(x) = \frac{1}{2\sqrt{2}}\left(1 + \frac{x^2}{2}\right)^{-3/2}, \; -\infty < x < \infty$$

Again, we know that:

$$\int_0^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{x^2}{2}\right)^{-3/2} dx = 0.5$$

And we can make the following adjustment:

$$\int_{0.14}^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{(x - 0.14)^2}{2}\right)^{-3/2} dx =$$

$$u = x - 0.14$$

$$du = dx$$

$$u(0.14) = 0$$

$$u(\infty) = \infty$$

$$\int_0^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{u^2}{2}\right)^{-3/2} du = 0.5$$

And the resulting density function is:

$$f(x) = \frac{1}{2\sqrt{2}}\left(1 + \frac{(x - 0.14)^2}{2}\right)^{-3/2}, \; -\infty < x < \infty$$

Next, we want to change the scale of this last density function, so that 20.4 percent interest rates or higher have a 1/100 probability. If we again look at an unadjusted Student's t density function with 2 degrees of freedom, we know:

$$\int_{6.952}^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{x^2}{2}\right)^{-3/2} dx = 0.01$$

Looking at the function below, we can evaluate it as follows:

$$\int_{0.204}^{\infty} \frac{108.625}{2\sqrt{2}} \left(1 + \frac{108.625(x - 0.14)^2}{2}\right)^{-3/2} dx =$$

$$u = 108.625(x - 0.14)$$

$$du = 108.625dx$$

$$u(\infty) = \infty$$

$$u(0.204) = 6.952$$

$$\int_{6.952}^{\infty} \frac{1}{2\sqrt{2}} \left(1 + \frac{u^2}{2}\right)^{-3/2} du = 0.01$$

Therefore, the density function with both a change of scale and a change of origin is:

$$f(x) = \frac{108.625}{2\sqrt{2}} \left(1 + \frac{108.625(x - 0.14)^2}{2}\right)^{-3/2}, \quad -\infty < x < \infty$$

Now, both density functions that we have developed so far must equal the same value at 14 percent, so that there is no discontinuity where they meet. The function just developed above gives us the following value at 0.14:

$$f(x) = \frac{108.625}{2\sqrt{2}} \left(1 + \frac{108.625(0.14 - 0.14)^2}{2}\right)^{-3/2}$$

$$= \frac{108.625}{2\sqrt{2}} = 38.405$$

If we add a factor to our adjusted function with five degrees of freedom and evaluate it at 14 percent, we have the following:

$$f(0.14) = \frac{8k}{3\pi\sqrt{5}} \left(1 + \frac{[k(0.14 - 0.14)]^2}{5}\right)^{-3}$$

$$= \frac{8k}{3\pi\sqrt{5}}$$

Setting this expression equal to the 38.405 derived above and solving for k, results in a value of k of 101.170. Therefore, our second density function is the following:

$$f(x) = \frac{101.170(8)}{3\pi\sqrt{5}}\left(1 + \frac{101.170(x - 0.14)^2}{5}\right)^{-3}, \quad -\infty < x < \infty$$

Now, we take the half of the density function just derived below 14 percent and the half of the other density function above 14 percent to arrive at our final probability distribution. We have just proven that the two density functions meet at 14 percent with no discontinuity and as a final step, we must prove that the entire distribution over its entire range is equal to 1. This is shown below:

$$\int_{-\infty}^{0.14} \frac{101.170(8)}{3\pi\sqrt{5}}\left(1 + \frac{101.170(x - 0.14)^2}{5}\right)^{-3} dx$$

$$+ \int_{0.14}^{\infty} \frac{108.625}{2\sqrt{2}}\left(1 + \frac{108.625(x - 0.14)^2}{2}\right)^{-3/2} dx$$

$$u = 101.170(x - 0.14) \qquad v = 108.625(x - 0.14)$$

$$du = 101.170dx \qquad dv = 108.625dx$$

$$u(0.14) = 0 \qquad v(\infty) = \infty$$

$$u(-\infty) = -\infty \qquad v(0.14) = 0$$

$$= \int_{-\infty}^{0} \frac{8}{3\pi\sqrt{5}}\left(1 + \frac{u^2}{5}\right)^{-3} du$$

$$+ \int_{0}^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{v^2}{2}\right)^{-3/2} dv$$

$$= 0.5 + 0.5 = 1$$

The final density function is as follows:

$$f(x) = \begin{cases} \dfrac{101.170(8)}{3\pi\sqrt{5}}\left(1 + \dfrac{101.170(x - 0.14)^2}{5}\right)^{-3}, & -\infty < x < 0.14 \\ \dfrac{108.625}{2\sqrt{2}}\left(1 + \dfrac{108.625(x - 0.14)^2}{2}\right)^{-3/2}, & 0.14 < x < \infty \end{cases}$$

$P\{x > X\}$	X
1/10	15.7%
1/20	16.7
1/40	18.0
1/50	18.5
1/80	19.7
1/100	20.4

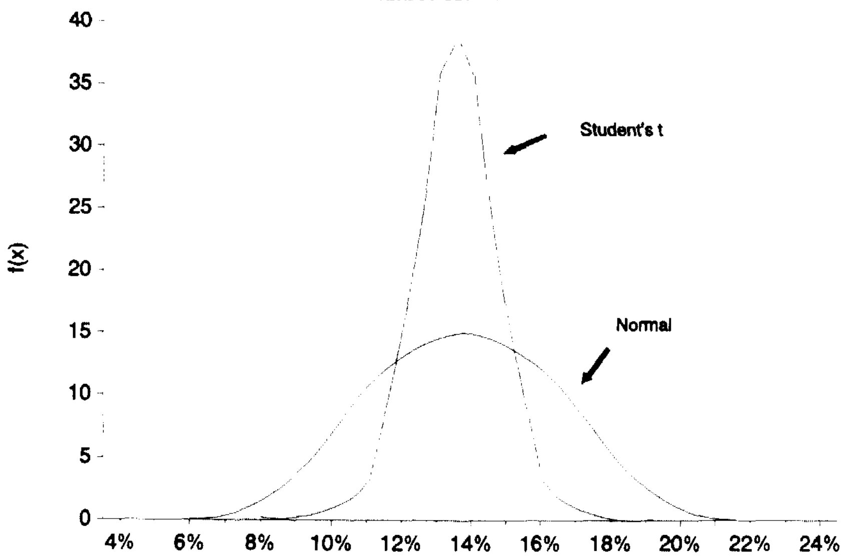
As can be seen, by comparing these sample values with the ones from the normal distribution, the Student's t-distribution results in lower interest rates having a higher probability. We were more comfortable with the normal values, but which one is closer to reality is anyone's guess.

One additional argument for a normal distribution for interest rates is that we need to work with normal distributions when there are non-zero correlation coefficients. When combining C-1 risk, asset defaults, and C-3 risk, interest rates, there surely is a level of correlation that must be modeled.

A graph showing both the normal and the Student's t-distributions just derived for interest rates is shown in Figure E-1c-1.

FIGURE E-1c-1

INTEREST RATES



Mortality Rates, C-2 Risk

As mentioned briefly in the opening paragraphs of this attachment, we initially tried a normal distribution for greater-than or less-than-expected mortality rates but were dissatisfied with the results. Our driving assumption was that 50 percent or greater extra mortality be a 1/100 probability. This was based, as mentioned before, on the influenza epidemic of 1918–19.

Given a zero mean and the probability constraint mentioned above, the resulting normal distribution has a standard deviation of 0.214592 and the density function shown below:

$$f(x) = \frac{\exp(-0.5(x/0.214592)^2}{0.214592\sqrt{2\pi}}, \quad -\infty < x < \infty$$

Sample values from this distribution are as follows:

$P\{x > X\}$	X
1/10	27.5%
1/20	35.3
1/40	42.1
1/50	44.1
1/80	48.1
1/100	50.0

As can be seen, the resulting excess mortality rates at probabilities less than 1/100 just aren't realistic. This distribution was used for our examples where we wanted to look at results from C-2, C-3 combinations of two normals, but we wanted to do most of our work with a more realistic distribution.

There is no compelling reason to choose a normal distribution for excess mortality rates, since we can assume there is no correlation beween C-2 risk and C-1 and C-3 risks. Therefore, we can choose any distribution, and when it is combined with the distribution for another risk, the two are simply multiplied together.

After the normal, the next distribution we looked at was a Student's t and for *excess* mortality rates (that is, those above zero) it worked very well but for *better-than-expected* mortality rates (those below zero), we found the normal actually did a better job. Because we are using two distinct distributions for the two halves of our distribution, we have to make adjustments similar to the ones we made for interest rates.

First, we worked with the Student's t for the half of the distributiion greater than zero. There was no need for a change in origin since our mean is zero (or expected mortality), as is a standard Student's t, but we do need to make a change in scale. A Student's t density function with 2 degrees of freedom is as follows:

$$f(x) = \frac{1}{2\sqrt{2}}\left(1 + \frac{x^2}{2}\right)^{-3/2}, \; -\infty < x < \infty$$

We know that:

$$\int_{6.952}^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{x^2}{2}\right)^{-3/2} dx = 0.01$$

Therefore, we can evaluate the integral below as follows:

$$\int_{0.50}^{\infty} \frac{13.93}{2\sqrt{2}}\left(1 + \frac{(13.93x)^2}{2}\right)^{-3/2} dx =$$

$$u = 13.93x$$

$$du = 13.93dx$$

$$u(\infty) = \infty$$

$$u(0.50) = 6.952$$

$$\int_{6.952}^{\infty} \frac{1}{2\sqrt{2}}\left(1 + \frac{u^2}{2}\right)^{-3/2} du = 0.01$$

Thus, our density function adjusted for a change in scale is:

$$f(x) = \frac{13.93}{2\sqrt{2}}\left(1 + \frac{(13.93x)^2}{2}\right)^{-3/2}, \; -\infty < x < \infty$$

Our two distributions, the normal for values less than zero and the Student's t for values greater than zero, must have the same value at zero, so there is no discontinuity where the two meet. The value of the above density function at zero is:

$$f(0) = \frac{13.93}{2\sqrt{2}}\left(1 + \frac{[13.93(0)]^2}{2}\right)^{-3/2}$$

$$= \frac{13.93}{2\sqrt{2}} = 4.925$$

The density function for a normal distribution is given below and solved for when x equals zero:

$$f(x) = \frac{\exp(-0.5(x/\sigma)^2)}{\sigma\sqrt{2\pi}}, \quad -\infty < x < \infty$$

$$f(0) = \frac{\exp(-0.5(0/\sigma)^2)}{\sigma\sqrt{2\pi}}$$

$$= \frac{1}{\sigma\sqrt{2\pi}}$$

When this expression is set equal to 4.925, as derived above, and solved for, the standard deviation becomes 0.018004. Thus, this density function is:

$$f(x) = \frac{\exp(-0.5(x/.081004)^2)}{0.081004\sqrt{2\pi}}, \quad -\infty < x < \infty$$

As before, we need to show that putting the two halves together results in the area under the entire probability density function being 1. This is shown below:

$$\int_{-\infty}^{0} \frac{\exp(-0.5(x/0.081004)^2)}{.081004\sqrt{2\pi}} dx$$

$$+ \int_{0}^{\infty} \frac{13.93}{2\sqrt{2}} \left(1 + \frac{(13.935x)^2}{2}\right)^{-3/2} dx$$

$$u = 13.93x$$

$$du = 13.93dx$$

$$u(0) = 0$$

$$u(\infty) = \infty$$

$$= \int_{-\infty}^{0} \frac{\exp(-0.5(x/0.081004)^2)}{0.081004\sqrt{2\pi}} dx$$

$$+ \int_{0}^{\infty} \frac{1}{2\sqrt{2}} \left(1 + \frac{u^2}{2}\right)^{-3/2} du$$

$$= 0.5 + 0.5 = 1$$

Therefore, our final density function for greater-than or less-than-expected mortality rates assuming a normal distribution below zero and an adjusted Student's t above zero is given below along with sample values:

$$f(x) = \begin{cases} \dfrac{\exp(-0.5(x/0.081004)^2)}{0.081004\sqrt{2\pi}}, & -\infty < x < 0 \\ \dfrac{13.93}{2\sqrt{2}}\left(1 + \dfrac{13.93x)^2}{2}\right)^{-3/2}, & 0 < x < \infty \end{cases}$$

$P\{x > X\}$	X
1/10	13.3%
1/20	20.8
1/40	31.0
1/50	34.7
1/80	44.5
1/100	50.0

These excess mortality rates are much more reasonable than those produced by the normal distribution that was tried first. The normal distribution used for the half of the distribution below zero results in 30 percent or greater improvement in mortality as having a 1/10,000 probability and this was felt to be realistic as well.

Figure E-1c-2 is a graph showing both the initial normal distribution and the subsequent Student's t/normal combination distribution.

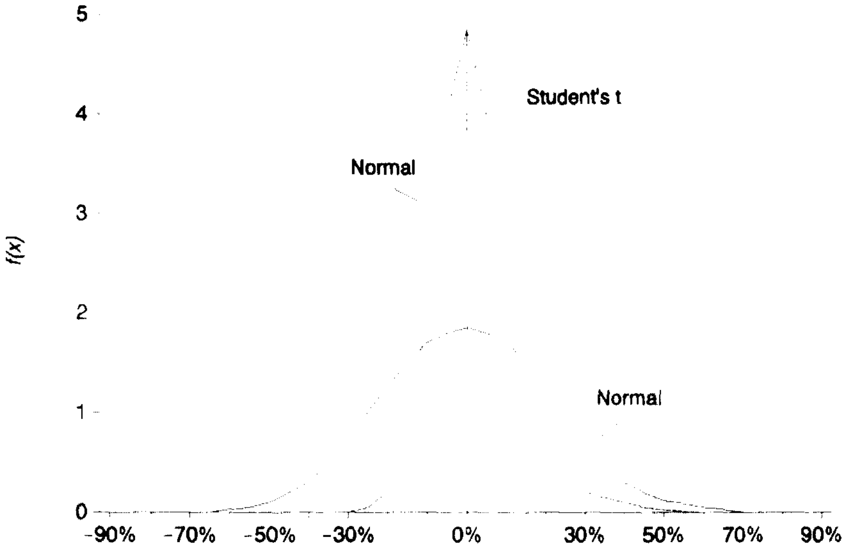
Asset Defaults Rates, C-1 Risk

The work we did on combinations of risk involving C-1 risk is extremely preliminary, so not much has been done to determine an accurate distribution for asset default rates. Asset defaults have several characteristics that must be reflected in order to have a realistic distribution. Some of these which we considered are as follows:

- No negative values (that is, there do not exist negative rates of asset default)
- A mean close to zero (that is, on average, there is a small finite number of asset defaults at all times)

FIGURE E-1c-2

MORTALITY RATES



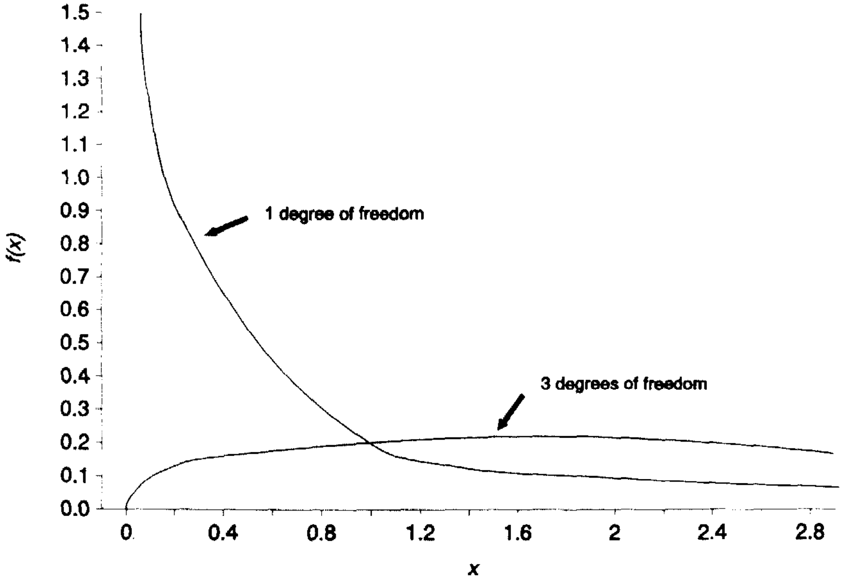
- As values approach zero, the density function should also approach zero (that is, zero asset defaults and those rates near zero have a very small, finite probability of occurring)
- A long tail off to the right, which gives reasonable asset default rate values at specific probability levels (for example, 1.5 percent asset defaults with a 1-in-10 probability)
- The ability to transform the resulting distribution into a normal distribution via a functional transformation so that C-1 and C-3 risk can have a non-zero correlation coefficient.

In the little bit of work that we did with asset default rates, not all these criteria were met. We started with a chi-square distribution with three degrees of freedom. A chi-square does not allow any values less than zero, it has a mean relatively close to zero and a long tail stretching out to the right. All these were desirable characteristics. A chi-square with *one* degree of freedom can very easily be transformed into a normal via a functional transformation, but unfortunately, the limit of the density function as x approaches zero is infinity instead of zero. The lowest degree of freedom where this limit does approach zero is three, and that is the distribution we chose to work with. The combination of C-1 and C-2 risk that we looked at could be assumed to have a zero correlation coefficient but any work that is done with C-1 and C-3 must take into account their correlation.

Figure E-1c-3 is a graph showing the difference in shape between a chi-square with one degree of freedom and a chi-square with three degrees of freedom:

FIGURE E-1c-3

CHI-SQUARE DISTRIBUTION



The asset default rate that we chose to reflect in our final distribution was 1.5 percent or greater having a 1/10 probability of occurring. Thus, the initial chi-square distribution had to be adjusted for a change in scale. The density function for a chi-square with three degrees of freedom is as follows:

$$f(x) = \frac{x^{1/2}e^{-x/2}}{\sqrt{2\pi}}, \ 0 < x < \infty$$

We know that the following equation is true:

$$\int_{6.251}^{\infty} \frac{x^{1/2}e^{-x/2}}{\sqrt{2\pi}} dx = 0.01$$

Therefore, we can evaluate the following expression:

$$\int_{0.015}^{\infty} \frac{416.733(416.733x)^{1/2} \exp(-(416.733x)/2)}{\sqrt{2\pi}} dx =$$

$$u = 416.733x$$

$$du = 416.733dx$$

$$u(\infty) = \infty$$

$$u(0.015) = 6.251$$

$$\int_{6.251}^{\infty} \frac{u^{1/2}e^{-u/2}}{\sqrt{2\pi}} du = 0.01$$

The final density function and sample values are given below:

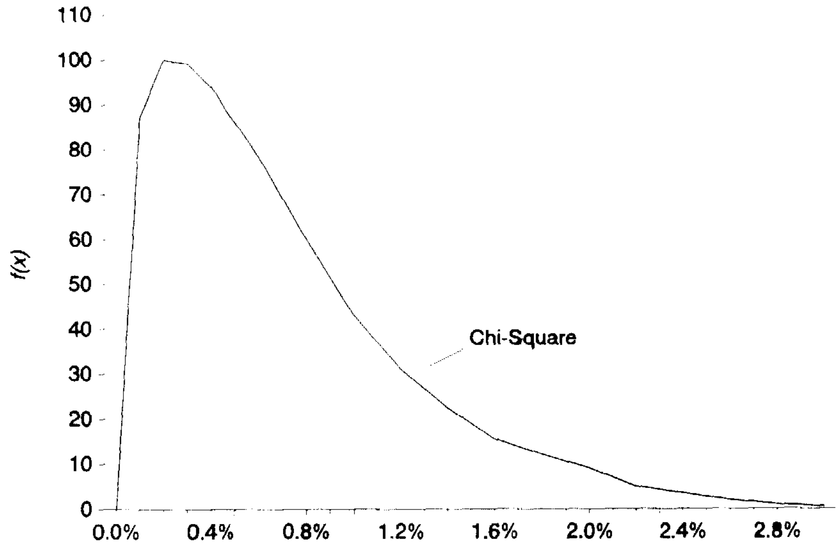
$$f(x) = \frac{416.733(416.733x)^{1/2} \exp(-(416.733x)/2)}{\sqrt{2\pi}}, \ 0 < x < \infty$$

$P\{x > X\}$	X
1/5	1.1%
1/10	1.5
1/20	1.9
1/50	2.4
1/100	2.7

This was all the work that was done on asset default rates. Obviously, there is much left to do. Figure E-1c-4 is a graph showing this final distribution.

FIGURE E-1c-4

ASSET DEFAULT RATES



APPENDIX E-2
MATHEMATICAL CONCEPTS UNDERLYING C-1, C-2 AND C-3 RISKS AND THEIR COMBINATION*

DONALD D. CODY

Abstract

The surplus needed at a chosen ruin probability level p on a block of in-force business against loss caused by each separate risk, ignoring losses from all other risks, can be estimated by procedures set forth in recent actuarial literature. For a few types of risk, surplus needed can be determined from explicit probability distribution functions. However, the procedures for most types of risk utilize "worst" scenarios of plausible deviations, which have a probability p of not being the worst ever to occur in the future. Combination of all such surpluses needed for separate risks at ruin probability p into an overall surplus needed at ruin probability p can be determined by a simple formula based on multivariate mathematical probability theory. The formula derived involves an unknown level of error, which is hoped to be immaterial relative to errors of estimation otherwise present in the procedures and relative to the vagueness and subjectivity inherent in the choice of p.

1. Introduction

Surplus needed for C-1, C-2 and C-3 risks in combination has been studied by the Society of Actuaries Task Force on Combination of Risks, which presented its findings in several impressive reports in the Panel Discussion "Benchmark Surplus Formulas" at the October 1985 Society Meeting. The Aetna members of the Task Force constructed distribution functions of C-1 risk, C-2 mortality risk and C-3 risk and ran myriad scenarios of cash flows, resulting in a distribution of losses from the combined risks. Because even for three risks the process is so elaborate that it may not be practicable except for the few large companies having the necessary resources and firm commitment, there appears to be a need for a simpler approach.

There is another approach based on mathematical probability theory that is both more simple and more comprehensive. The theory involves the general multivariate probability distribution; in the final state the multivariate

*See also CODY, D.D. "Probabilistic Concepts in Measurement of Asset Adequacy," *TSA* XI (1988): 149–72.

normal distribution is introduced with a resultant unknown error from neglect of skewness and higher-order moments known to exist in real-life distributions. This error is hoped to be immaterial relative to errors of estimate otherwise present and relative to the vagueness and subjectivity of the choice of level of probability of ruin p.

2. Listing of Risks

The number of risks in the C-1, C-2 and C-3 risk categories is large, well beyond those treated specifically by the Task Force on Combination of Risks. Here is a partial list:

- *C-1 Risks*. A long serious deflationary depression or stagflation causing cash-flow and capital losses as functions of the quality and type of assets: bonds, preferred stocks, mortgages (commercial and residential), common stocks (market value), real estate, subsidiaries and ventures, futures and options, etc.
- *C-2 Risks*. Losses from increases in claims, asset values, or expenses from mortality on life insurance and annuities recognizing retention levels, antiselection and trends; morbidity and continuance on disability income insurance; medical care insurance; earthquakes (for example, Richter 8 in Los Angeles); epidemics (influenza, AIDS); expense inflation; premium insufficiencies, other than C-1 and C-3 risks; etc.
- *C-3 Risks*. Losses from decreases in interest spreads involving upside and downside and mixed interest environments; call, withdrawal, loan and termination functions; investment policy and contract credit policy; contract design and markets; interest guarantees; etc.

It is evident that total surplus protects against any and all risks and that a straight addition of the surpluses needed for each risk separately will be in excess of surplus needed for the combination. Moreover, correlations, such as between C-1 and C-3 risks and between C-1 risk and C-2 disability income risk, must be recognized, since such combinations arise from the same causes. In addition, surplus needed depends upon the level of reserves.

The following mathematical derivations are not fully detailed; missing details are available in Cramér "Mathematical Methods of Statistics," Princeton University Press, 1946, to which reference is made at several points.

3. Definitions and Concepts

N = Number of risks, denoted as risk i, where $i=1, 2, 3, \ldots, N$

U_i = Present value of losses from risk i for a specific scenario in multivariate N-dimensional probability space, ignoring all other risks (a random variable)

r_{ij} = Correlation coefficient between U_i and U_j ($|r_{ij}| \leqq 1$)

$F(.)$ = Probability distribution function

$f(.)$ = Corresponding probability density function, with moments existing over the N-dimensional multivariate probability space (Cramér Chapter 22)

U = Aggregate present value of losses from risks 1, 2, ..., N for a specific omnibus scenario

p = Ruin probability (suggested as 0.01 for assets equal to sum of reserves and surplus needed; and as 0.10 to 0.25 for assets equal to reserves, with surplus needed zero or negative)

u_i = Surplus needed against risk i for ruin probability p, ignoring all other risks

u = Surplus needed against all risks combined for ruin probability p

Prob $\{U_i \leqq u_i\} = 1-p$ with losses from all other risks ignored

Prob $\{U \leqq u\} = 1-p$ for all risks combined

$S(U_i)$ = Standard deviation of marginal $F(U_i)$

$S(U)$ = Standard deviation of $F(U_1+U_2+\ldots+U_N)$

U_i and U are losses, if positive, and gains, if negative, and have mean value of zero. u_i and u are measured from a "gross premium" reserve, which is calculated on the basis of expected values, that is, with all losses assumed to be zero.

4. Determination of u_i

4.1 Risks Amenable to Explicit Probability Distributions of Losses

In life insurance, the only risk is deviations in the sum of death claims, for which a large literature exists, primarily involving the compound Poisson distribution with convolutions of the distribution of amounts of individual claims and recognition of retention limits; derivation is from first principles. In property casualty insurance, many loss distributions have been derived or fitted successfully. In such cases Prob $\{U_i \leqq u_i\} = 1-p$ can be solved explicitly.

4.2 Risks Not Amenable to Explicit Probability Distributions of Losses

Here, the only available procedures involve constructing "worst" scenarios, ignoring all other risks, defined as those where U_i lies at the hyperpoint $U_i = u_i$. These worst scenarios have the property that the probability is p that even worse scenarios will occur in the future, where p is the ruin probability. The actuary can construct these worst scenarios on a heuristic basis and calculate u_i by use of appropriate models of cash flow under the scenarios. For instance, the scenarios for $p = 0.01$ might be as follows for selected risks:

- A C-1 risk worst scenario based on the Great Depression of the 1930s or an equally serious stagflation worse than the close encounter of the 1970s
- A C-2 risk worst scenario like the 1918 influenza epidemic or a current AIDS epidemic
- A C-3 risk episode much worse than that of the 1970s.

Of course, worst scenarios for $p = 0.10$ to 0.25 applicable to reserve testing would be noncatastrophic and much less severe than for $p = 0.01$, which is a pretty stringent level since it applies to the whole future.

In the literature, scenarios corresponding to $p = 0.01$ are referred to as scenarios of plausible deviations from expected and apply to required assets equal to the sum of reserves and surplus needed. Scenarios corresponding to $p = 0.10$ to 0.25 are referred to as scenarios of reasonable deviations from expected and apply to required assets equal to reserves.

4.3 Credits Against Gross Losses

In the classical literature on ruin probability, income from so-called risk loading on premiums is credited to the cash flow. In our more complex situation, there is a similar credit from operating margins. It appears explicitly in the C-3 risk cash-flow models. It must be introduced into the models used for the other risks as well. However it is introduced, it is equivalent to the tolerable reductions in policyholder dividend and credits, in stockholder dividends, and in level of retained earnings, as seems appropriate in the scenario.

5. Formula for S(U)*, the Standard Deviation of* $F(U_1 + U_2 + \ldots + U_N)$

The basic formula is this:

$$S^2(U) = E(U_1 + U_2 + \ldots + U_N)^2$$
$$= \sum_{i=1}^{N} S^2(U_i) + 2 \sum_{\substack{i,j=1 \\ j<i}}^{N} r_{ij} S(U_i) S(U_j) \qquad \text{(Formula A)}$$

where $S(U_i)$ is the standard deviation of the marginal distribution $F(U_i)$ with all other risks ignored. This formula applies whether $F(U_1, U_2, \ldots, U_n)$ is nonsingular, that is, all $|r_{ij}|<1$, or singular, that is, some $|r_{ij}|=1$. (Cramér 22.3.3.)

6. Formula for u*, the Surplus Needed for All Risks Combined at Ruin Probability* p

So far it has been unnecessary to specify the functional form of $F(U_1, U_2, \ldots, U_N)$, other than that moments exist, or the functional form of the marginal distributions, $F(U_1)$. Now, however, it is necessary to introduce a normalcy assumption to enable statement of the relationship between the standard deviations in Formula A and surpluses needed. We now assume that $F(U_1, U_2, \ldots, U_N)$ is a nonsingular multivariate normal distribution in N-dimensional probability space. As shown in Cramér Chapter 24, it follows that the marginal distributions $F(U_1)$ and the distribution for the combined risks $F(U_1 + U_2 + \ldots + U_N)$ are all normal. Let K be defined as follows:

$$\frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{K} \exp\left(-\frac{t^2}{2}\right) dt = 1 - p$$

Surplus needed, u_i and u, in the marginal and combined distributions, at ruin probability p is equal to K multiplied by the respective standard deviations. By multiplying Formula A through by K^2, we thus obtain the following, which is the result desired:

$$u^2 = \sum_{i=1}^{N} u_i^2 + 2 \sum_{\substack{i,j=1 \\ j<i}}^{N} r_{ij} u_i u_j \qquad \text{with } |r_{ij}| \leqq 1 \qquad \text{(Formula B)}$$

where all u_x are determined with losses from all other risks ignored, estimated by taking losses from all other risks at their mean values zero. Note that even though the multivariate normal distribution has been assumed to be nonsingular with no $|r_{jk}|=1$, Formula B turns out to apply also where some $|r_{jk}|=1$.

There is, of course, no skewness in normal distributions, but it is notable that normalcy is not introduced until this Section 6. The determination of the u_i does not involve normalcy and does reflect skewness, and for different levels of p, the variance parameters of the normal distributions introduced here will be different, which would not have been true if normalcy had been assumed from the start.

Risks are assigned to categories of degree of correlation with other risks. Some correlated risk pairs (l,m) can be assumed to have correlation coefficients of $r_{lm}=1$; for example, C-2 risk disability income losses and C-1 risk losses. Some pairs (j,k) have $0<r_{jk}<1$, for example, C-1 risk losses and C-3 risk losses, where r_{jk} might be set at 1/2. Many pairs (i, x) are completely independent with $r_{ix}=0$; for example, C-2 risk sum of death claims losses and C-1 risk losses.

7. *Statutory Financials*

The surplus needed discussed above is the cash-flow-based surplus, where the excess of the present value of liability cash flows over the present value of asset cash flows on a block of in-force on the valuation date equals zero, and the assets on the valuation date equal the sum of reserves and surplus needed. The reserves are "gross premium" reserves calculated on the basis of expected values with losses assumed to be zero. Thus, the surplus needed in the formulas relates to no financials. On statutory financials, the statutory reserves are larger than such gross premium reserves and, as a consequence, contain some portion of the u_i and u of the formulas. In addition, there is a desideratum that surplus needed in statutory financials be calculated so that assets along scenarios be no less than reserves at future durations of the scenarios. Thus, when Formula B is applied to surplus needed on statutory financials, with u_i and u measured from a variety of statutory reserves, there is an additional error of unknown size.

8. A Final Comment

One may ask why such complex mathematics is applied to derive a formula for combination of risk with an unknown error level. The mathematics enables the derivation of an imaginative design not otherwise available for application in a very complex and only partially understood real world. The estimates entering into the procedures for determination of surpluses needed for each risk, ignoring other risks, are imperfect and involve an unknown level of error. The choice and meaningfulness of the level of ruin probability are subjective and vague. Hopefully, the additional errors introduced by the multivariate probability theory are not material relative to the estimate errors already existing. The determination of surplus needed has its own rewards for actuaries and management in an improved understanding of our risk business. The idealized procedures recited in this paper constitute a structure for sharpening such judgment.

9. Acknowledgments

Any shortcomings in this paper are, of course, my own responsibility. However, I do wish to say that the comments by Jim Hickman in response to earlier work-ups guided and educated me in the intricacies of multivariate distributions. Equally important has been the critique of an earlier draft by Linda Dinius, whose knowledge of combinatorial theory and experience with the models used by the Combination of Risks Task Force exposed omissions and weaknesses in my reasoning, which I have tried to remove.