



# RISKS AND REWARDS

THE NEWSLETTER OF THE INVESTMENT SECTION

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## Deflators—The Solution to a Stochastic Conundrum?

by Don Wilson

**S**tochastic modeling of life insurance products has become increasingly important over the last few years. The complex nature of the guarantees that exist in many products has generally required the use of a Monte Carlo approach, involving the calculations being performed repeatedly for each scenario, potentially many hundreds or thousands of times.

As this modeling has evolved, it has divided down two paths—real-world and risk-neutral. In this article, I discuss this division and show how these paths may be re-united through the use of deflators.

### Path 1—real world

In many stochastic applications, the requirement is to test the robustness of product design or business strategy and to quantify the range of possible financial outcomes. For this type of application, the scenarios must represent the real world. By this I mean that the outcomes for each scenario produced by the stochastic economic generator used must represent a path that could occur in the future. The range of the scenarios represents the population of possible future outcomes.

### Path 2—risk neutral

Increasingly, the valuation or pricing of a product option or guarantee, benefit, line of business or company requires a stochastic process for the full financial intricacies to be captured. For many applications, the fundamental requirement is that there is consistency with the techniques used to value or price assets—so that both sides of the balance sheet are consistent. As a consequence, there is a requirement that the economic scenarios be “risk neutral.” When such scenarios are used, discounting the projected cash flows at the risk-free rate appropriate for each scenario and taking the mean gives a value or price that is consistent with a market valuation of the assets.

I do not propose in this article to dwell on the technical differences between these two types of economic scenarios, nor to discuss how to build the stochastic generators. Most practitioners do not need to be experts on such matters and there is much published material already available. What is most important is the ability to

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# Chairperson's Corner

by Mark W. Bursinger

The growth and value of our profession is dependent on the ability of our members to stay current and evolve as the financial services industry and world changes around us. The Investment Section is designed to help this cause. Our mission is to facilitate the professional development of our members in regard to the investment of institutional funds, especially insurance company and pension fund assets and in the measurement and management of those assets in relation to the institution's liabilities. To date, we have worked to accomplish this mission in a number of ways, including:

- Development of investment-related sessions at SOA meetings
- Sponsorship of seminars such as the Investment Actuary Symposium, Beginner and Advanced Risk Management Seminars and the Stochastic Modeling Symposium
- Publication of our newsletter *Risks and Rewards* (articles are always welcome)
- Provision of grants for investment-related research and awards for investment-related publications
- Sponsorship of events to broaden our experience

There are other groups within the SOA that also serve to develop and advance our members, like practice areas and exam committees. The Board of Governors recently approved a motion designed to improve the SOA's responsiveness to its members. A response to this directive is well underway and will likely result in significant changes in the SOA structure of sections and practice areas. A large number of volunteers are working to identify what is important to our members and how best to deliver it.

But are we doing enough? I'm certainly not suggesting that the already dedicated volunteers aren't giving enough of their time. Are we doing the right things to position our profession for growth and prosperity, particularly in the field of risk management and investments? My personal view is that we could do more.

I find it surprising that there is a debate about the inclusion of financial economics in the required material for the new exam syllabus. I think many companies wished their actuaries had known a little more about pricing equity-related options before they developed guaranteed minimum benefits on variable products. Perhaps there would be less confusion on fair value of liabilities if we all used the same valuation paradigm and were only left to debate the assumptions. There are many other examples of where a better understanding of financial theory would have been beneficial to our work.

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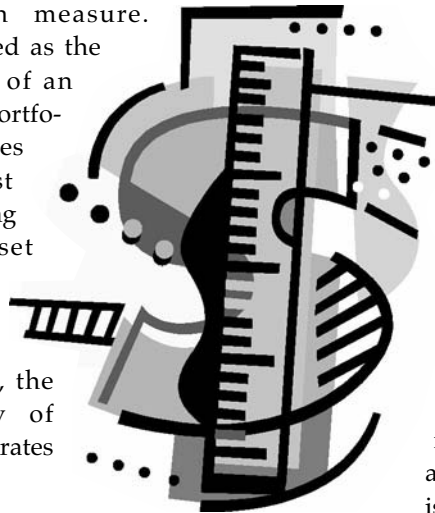
# Insurance Liability Duration In a Low-Interest-Rate Environment

by Paul J. Heffernan

The recent low interest rate environment has created a challenging situation for insurers. Products with fund balances which grow at a crediting rate, at least equal to some contractual minimum, face profitability pressures and some tricky asset-liability management issues, as well. Examples of such are deferred annuities, universal life and settlement options.

Actuaries are familiar with asset-liability management (ALM) as a technique to protect surplus against changes in interest rates. The essential idea is to measure the price sensitivity to interest rate changes of both liabilities and the assets supporting them, using the well-known duration measure.

Duration is defined as the price sensitivity of an asset or liability portfolio to small changes in current interest rates. By measuring and equating asset and liability durations (or, more properly, duration dollars), the price sensitivity of surplus to interest rates becomes small.



Product actuaries must consider duration in many aspects of their work, including product design and development, pricing, setting new money and old money crediting rates, forecasting and communicating with asset managers. The actuary must understand the drivers of duration and the methodology used to calculate it.

There are many versions of the duration measure, with the two oldest, and perhaps best-known, being Macauley and modified duration. Both versions do a fine job of measuring price sensitivity to interest rates when the cash flows of an asset or liability do not vary with interest rates. However, when a change in

rates can alter the cash flows, a more robust measure is needed. This measure is called effective duration, and it is the product of option-adjusted analysis (OAA). The methodology of OAA originally was developed for assets, such as callable bonds, mortgage-backed securities and CMO securities, but it is equally valuable for insurance liabilities.

Today's low-interest-rate environment makes this an opportune time for actuaries to learn or brush up on the basics of OAA.

With this goal in mind, we'll explore OAA in this paper and apply it to a challenge that arises when the supportable crediting rate on a fund accumulation product falls below the minimum, namely, the lengthening of the liability duration. Working with the example of a deferred annuity and using OAA, we will show that the duration of the annuity with a minimum crediting rate can be longer than that of a similar annuity without the minimum. We will show that it is helpful to decompose the product into two parts, an annuity without a crediting rate minimum and an interest rate floor. In this way we will attribute the additional duration to the embedded derivative that is the interest rate floor that results from the crediting rate floor.

## Duration and insurance liabilities

Interest rate changes pose potential risks to insurers, since interest rate movements can change the valuation of insurance liabilities and the fixed-income assets that the insurer holds to support them. The measure that actuaries and asset managers use to quantify this relationship is duration. Duration can be defined in words as the percentage change in value per change in interest rates, and is written symbolically as:

$$D = -(\Delta P/P) / \Delta i$$

**By measuring and equating asset and liability durations (or, more properly, duration dollars), the price sensitivity of surplus to interest rates becomes small.**



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When interest rates are the key driver of the value of both liabilities and the assets that back them, duration matching is a valuable tool for protecting surplus against rate movements. Defining “duration dollars” of a portfolio of assets or liabilities as its duration times value by managing asset duration dollars to equal liability duration dollars, the level of surplus is unaffected by modest rate changes. In order to more effectively protect surplus, a company should review additional measures such as convexity, but in any event duration makes a powerful starting point.

We should note that when we say “price” or “value” we mean a market value. Since insurance liabilities do not trade frequently, it often makes sense to think in terms of the fair value, a concept which is gaining popularity in the actuarial profession because of its focus on the underlying economics. Consequently, when a company seeks to

**Crediting rate floors lengthen duration, because they offset some of the effect of resets.**

protect surplus by matching asset and liability durations, it is protecting economic surplus, and because of timing differences, such a strategy may not be enough to ensure a steady pattern of earnings under GAAP or statutory reporting.

Now consider two characteristics common in liabilities that shorten duration:

- crediting rate resets,
- withdrawals,

and one that lengthens duration:

- crediting rate floors

To see how crediting rate resets shorten duration, consider the extreme example of a deferred annuity maturing in three years, where the crediting rate is reset annually to equal current interest rates less a 100 bp spread. Furthermore, assume the reset is next week. If we immediately increase interest rates by, say, 10 bp, then upon the reset the crediting rate will also increase 10 bp, offsetting the effect of the higher discount rate in the present value calculation. We can see intuitively that we essentially get a duration of zero. This means that, at least in the week before the rate reset, the annuity’s value is not sensitive to interest rates. Of course, after the reset date passes, the

duration would lengthen to the amount of time until the next reset (in this case, one year), since cash flows for that period, but not after, would be invariant to interest rate changes.

Withdrawals shorten duration because the average life of the cash flows decreases. The risk of disintermediation under rising market rates poses an additional ALM challenge, one that insurers attempt to lessen through surrender charges and other product design features.

Crediting rate floors lengthen duration because they offset some of the effect of resets. The higher the floor, the more duration is lengthened. We can see this by considering extreme examples of a very low and very high floor. With a low floor, the crediting rate will usually be set off of current rates, so the floor has little effect on crediting rates or duration. With a high floor, the crediting rate essentially is fixed at the floor, and as in the example of a GIC or a zero-coupon bond, duration is close to the time to maturity.

### Option-adjusted analysis and interest rate trees

In all our examples, we will use a three-year maturity date, and all interest rates will be stated on an annual basis (usually semi-annual, or bond-equivalent, interest rates are used with bonds, but we’ll simplify the math with annual rates). We will use the three-year yield curve shown in Table 1. The table gives the yields converted to spot rates and to forward rates. In our calculations of duration, we



will shock the yield up and down by 10 bp, so Table 2 shows these yields and the resulting spot and forward rates.

We implement OAA through a binomial tree, shown in Figure 1. This tree represents the yield curve of Table 1, but models the uncertainty of future interest rates. The tree starts at time  $t=0$  with a single node, since we know with certainty the current one-year spot rate (which equals the one-year forward rate at time zero). Going forward, however, interest rates are uncertain, so the starting node branches out to two nodes at time  $t=1$ , representing a pair of possible one-year rates at that time. In moving forward to the next period, each of the two nodes at  $t=1$  branch out to two nodes at  $t=2$ . Our tree is "recombining," however, which means that by moving up in the first period and then down in the second, we reach the same node as moving down then up, so that time  $t=2$  has just three nodes, not four. Each of these nodes represents a possible one-year rate at  $t=2$ .

We write HH to represent the path taken by taking the up move at  $t=0$ , and another up at  $t=1$ . Path HH represents the set of forward rates 3.00%, 4.82%, 6.17%. Likewise, path HL is an up move followed by a down move, and represents the forward rates 3.00%, 4.82%, 4.14%. Paths LH and LL are the other two possibilities in our three-year tree with one-year time periods.

We choose a probabilistic process to move from nodes in earlier to later periods, whereby we assume there is a 0.5 probability of either an up or down move leaving any node. For an assumed standard deviation  $\sigma$ , we require the relationship  $r^{H} = e^{2\sigma} \cdot r_L$  to hold for the pair of forward rates leading from a node. For example, in Figure 1 we use  $\sigma = 20\%$ , and for the two rates leading out of the node at  $t=0$ , we have  $4.82\% = (e^4) \cdot 3.23\%$ .

To calculate the value of an asset or liability using the tree, we determine the cash flows along each possible path of up and down moves and discount the cash flows with the set of forward rates found along that path. Since we assume up and down moves are equally likely at each node, every possible path is equally likely, so the value of an instrument is simply

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Table 1: Example Yield Curve

	1-year	Maturity 2-year	3-year
Yield	3.00%	3.50%	3.75%
Spot rate	3.00%	3.51%	3.77%
Forward rate	3.00%	4.02%	4.28%
All rates are annual			

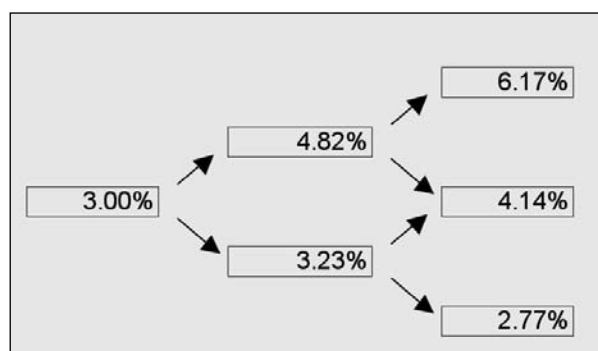
Table 2: Example yield curve with +/- 10 bp shock

	1-year	Maturity 2-year	3-year
+10 bp shock			
Yield	3.10%	3.50%	3.75%
Spot rate	3.10%	3.51%	3.77%
Forward rate	3.10%	4.02%	4.28%
-10 bp shock			
Yield	2.90%	3.40%	3.75%
Spot rate	2.90%	3.51%	3.77%
Forward rate	2.90%	4.02%	4.28%

Table 3: Valuing Payment of 100 at  $t=3$

With spot rate:		
$P = 100 / (1.03766^3)$		= 89.50
With binomial tree:		
$P(HH) = 100 / \{ (1.0300) \cdot (1.0482) \cdot (1.0617) \}$		= 87.24
$P(HL) = 100 / \{ (1.0300) \cdot (1.0482) \cdot (1.0414) \}$		= 88.94
$P(LH) = 100 / \{ (1.0300) \cdot (1.0323) \cdot (1.0414) \}$		= 90.31
$P(LL) = 100 / \{ (1.0300) \cdot (1.0323) \cdot (1.0277) \}$		= 91.51
$P\{P(HH) + P(HL) + P(LH) + P(LL)\} / 4$		= 89.50

Figure 1: Interest Rate Tree

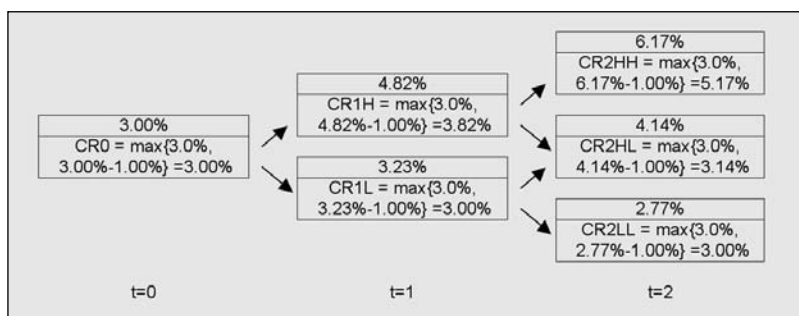


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the average present value of cash flows across all the paths.

An important point is that the set of forward rates on the tree must produce the same present value for a set of cash flows as do the true forward rates. This requirement is commonly called fitting the model to the term structure of the yield curve. The rates in Figure 1 have been chosen with this intent. We verify this in Table 3, where we discount a payment of 100 at  $t=3$  with the three-year spot rate, and by averaging the discounted values along the four possible paths, we see that we get the same price each way.

**Figure 2: Deferred annuity with 3.0% crediting rate floor**  
**Crediting rate is greater of 3.0% and current rate less 100 bp**



**Table 4: 3-year deferred annuity, with 3% crediting rate floor**

<b>Premium</b>	<b>= 100</b>
<b>Crediting rate current rate less 1%, subject to 3% floor</b>	
$P(HH) = 100 * \{ (1.0300)^*(1.0382)^*(1.0517) \} / \{ (1.0300)^*(1.0482)^*(1.0617) \}$	<b>= 98.11</b>
$P(HL) = 100 * \{ (1.0300)^*(1.0382)^*(1.0314) \} / \{ (1.0300)^*(1.0482)^*(1.0414) \}$	<b>= 98.09</b>
$P(LH) = 100 * \{ (1.0300)^*(1.0300)^*(1.0314) \} / \{ (1.0300)^*(1.0323)^*(1.0414) \}$	<b>= 98.82</b>
$P(LL) = 100 * \{ (1.0300)^*(1.0300)^*(1.0300) \} / \{ (1.0300)^*(1.0323)^*(1.0277) \}$	<b>= 100.00</b>
$P = \{ P(HH) + P(HL) + P(LH) + P(LL) \} / 4$	<b>= 98.76</b>

**Table 5: 3-year deferred annuity, with 3% crediting rate floor, +10bp yield curve shock**

<b>Premium</b>	<b>= 100</b>
<b>Crediting rate current rate less 1%, subject to 3% floor</b>	
$P(HH) = 100 * \{ (1.0300)^*(1.0394)^*(1.0532) \} / \{ (1.0310)^*(1.0494)^*(1.0632) \}$	<b>= 98.02</b>
$P(HL) = 100 * \{ (1.0300)^*(1.0394)^*(1.0323) \} / \{ (1.0310)^*(1.0494)^*(1.0423) \}$	<b>= 98.00</b>
$P(LH) = 100 * \{ (1.0300)^*(1.0300)^*(1.0323) \} / \{ (1.0310)^*(1.0331)^*(1.0423) \}$	<b>= 98.65</b>
$P(LL) = 100 * \{ (1.0300)^*(1.0300)^*(1.0300) \} / \{ (1.0310)^*(1.0331)^*(1.0284) \}$	<b>= 99.76</b>
$P = \{ P(HH) + P(HL) + P(LH) + P(LL) \} / 4$	<b>= 98.61</b>

### Evaluating crediting rate floors

An interest rate tree is invaluable when the cash flows of an asset or liability can vary with interest rates. We will explore how to use the tree to analyze a deferred annuity with a crediting rate floor.

Assume we have a three-year deferred annuity that credits each year the one-year market rate less a 100 bp spread, subject to a 3.0% crediting rate floor. To focus our attention on the effect of floors, we will assume there are no withdrawals before the maturity date. At each node of Figure 2, the crediting rate is the greater of 3.0% and the one-year rate associated with the node less 100 bp. For the node at  $t=0$ , this is  $CR0 = \max \{ 3.0\%, 3.00\% - 1.00\% \} = 3.00\%$ , meaning that the floor rate is higher than the supportable crediting rate.

At  $t=1$ , there are two nodes, representing an up move and down move in rates in the first year. Under the up move, we have  $CR1H = \max \{ 3.0\%, 4.82\% - 1.00\% \} = 3.82\%$ , so the current rate determines the crediting rate. However, for the down move, we have  $CR1L = \max \{ 3.0\%, 3.23\% - 1.00\% \} = 3.00\%$ , so the floor rate gives us the crediting rate. In a similar fashion, at  $t=2$  we have  $CR2HH = 5.17\%$ ,  $CR2HL = 3.14\%$ , and  $CR2LL = 3.00\%$ .

In Table 4, we show the initial fund of 100 growing to an ending fund under each of the four possible interest rate paths, and the associated present value of each. We average the four present values to obtain the price of the contract, 98.76.

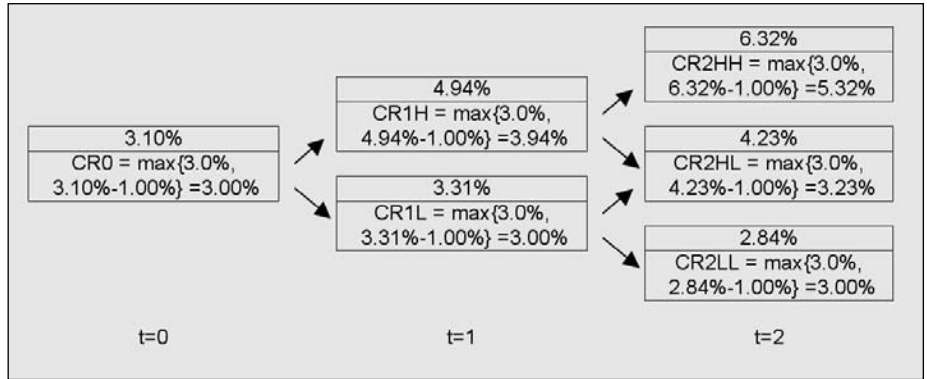
We then value the contract under the upwards shocked tree, shown in Figure 3. The binomial tree again has been calibrated to preserve the term structure of the yield curve. The resulting price is  $P+ = 98.61$ . We show this in Table 5.

If we shock the tree downward (recalibrating the tree) and revalue, we get  $P- = 98.91$ . This gives a duration of 1.5, as shown in Table 6.

Earlier we discussed a three-year deferred annuity which each year credits the current one-year market

rate less 100 bp, and argued that immediately before its rate reset it has a duration of zero. This example is an identical annuity with a crediting rate floor added, and we see that adding the floor adds duration. A higher floor lengthens the duration further. For example, a floor of 4.0% results in a duration of 2.6. When cash flows do not vary with interest rates—either because the crediting rate over three years is fixed, or because the floor is extremely high—we get a duration of 2.9, close to the time to maturity. In this manner, we see that the more the floor affects crediting rates, the more sensitive the price of the liability becomes.

**Figure 3: Deferred annuity with 3.0% crediting rate floor +10bp yield curve shock**  
**Crediting rate is greater of current rate less 100 bp and 3.0%**



**The embedded derivative—  
an interest rate floor**

We gain further insight into the effect of a crediting rate floor on price sensitivity by decomposing the contract into components—the underlying contract and the embedded derivative. In our example, the annuity can be decomposed into a three-year deferred annuity without a floor and an interest rate floor derivative. This derivative has a three-year maturity, a notional of 100, a strike rate of 4 percent, and an annual reset. Each year it pays any excess of the strike rate over the current one-year rate, times the notional. Note that 4 percent less the current one-year rate is equivalent to the excess of 3 percent over the current one-year rate less the 100 bp spread. The reason this decomposition works is that the cash flows of the annuity with floor equal the sum of the cash flows of the two components. Not surprisingly, when we value the interest rate floor derivative on the binomial tree (Table 7), we get a value of 1.61, which is the difference between the values of the deferred annuities with (98.76) and without (97.14) a crediting floor.

**Table 6: Calculating the duration**

$$\begin{aligned}
 D &= - (1 / P) * (P+ - P-) / (2 * \Delta i) \\
 &= - (1 / 98.76) * (98.61 - 98.91) / (0.0020) \\
 &= 1.5
 \end{aligned}$$

**Table 7: Interest rate floor derivative**

Notional	100	
Maturity	3 years	
Strike rate	4%	
$  P(HH) = 100 * \left\{ \begin{aligned}  &\max[0, 0.0400 - 0.0300] / (1.0300) \\  &+ \max[0, 0.0400 - 0.0482] / (1.0300 * 1.0482) \\  &+ \max[0, 0.0400 - 0.0617] / (1.0300 * 1.0482 * 1.0617) \end{aligned} \right\} = 0.97  $		
$  P(LL) = 100 * \left\{ \begin{aligned}  &\max[0, 0.0400 - 0.0300] / (1.0300) \\  &+ \max[0, 0.0400 - 0.0323] / (1.0300 * 1.0323) \\  &+ \max[0, 0.0400 - 0.0277] / (1.0300 * 1.0323 * 1.0277) \end{aligned} \right\} = 2.82  $		
In similar manner, P(HL) = 0.97 and P(LH) = 1.69		
P	= { P(HH) + P(HL) + P(LH) + P(LL) } / 4	= 1.61

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We can take this a step further by considering duration dollars. The duration dollars of an instrument is its price times its duration, and by equating the duration dollars of assets and liabilities, managers can insulate surplus from interest rate movements. Just as the price of the deferred annuity with crediting rate floor equals the sum of the prices of its two components, its duration dollars equal the sum of the duration dollars of the components.

duration when designing and managing products. It is also important to keep in mind that interest rate floors, and therefore liabilities with embedded floors, have high convexity, which adds a further challenge in managing interest rate risk. These challenges are especially important in the low rate environment that we continue to experience.  $\delta$

After computing the duration of the interest rate floor derivative with our OAA approach to be 94.4, we can summarize the decomposition as shown in Table 8.

Actuaries should be aware of the effect of crediting floors on

**Table 8: Decomposition of annuity with crediting rate floor**

Annuity w/ floor	=	Annuity w/o floor	+ interest rate floor
P(Ann w/ floor)	=	P(Ann w/o floor)	+ P(int rate floor)
98.76	=	97.14	+ 1.61
Dur\$(Ann w/ floor)	=	Dur\$(Ann w/o floor)	+ Dur\$(int rate floor)
1.5*98.76	=	0*97.14	+ 94.4*1.61

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# Interest Rate Regimes—An Empirical Description

by Joseph Koltisko

**W**hat is a “regime” in the interest rate environment? If the regime changed, how would you know? For long-term descriptive interest rate models, it is useful to develop a concise view about this. With it, we can produce internally consistent scenario sets for a given regime. We can investigate the financial consequences of moving from the current regime to alternate ones. Our views about the likelihood of transitioning from one regime to another can also be incorporated into a stochastic stress test. With a view to these applications, this article proposes a way to describe the interest rate regime.

As many have pointed out, a statistical technique called “principal components analysis” (PCA) provides a remarkably stable result. Changes in the yield curve can be decomposed into shifts in “level,” “steepness” and “curvature.” This is the starting point for investigating changes in the U.S. yield curve. The monthly Constant Maturity Treasury (CMT) series from the U.S. Federal Reserve Web site was used. This site provides market rates for each month since 1953.

Now, by itself, a principal components representation for the yield curve is inadequate for a term structure model. On one hand, any change in the yield curve can be represented with a linear combination of the PCs, for instance, 3 \* level vector + 2 \* steepness vector. These multiples (3 and 2) are sometimes called changes in the “state variables.” A typical attempt would be to assume changes in the state variables are independent identically distributed normal random variables, with mean reversion. Perhaps enough constraints can be found on such a model to make it useful for risk reporting. The usual, unsatisfying result when we attempt to use PCA for a term structure model is that the curve becomes too inverted or humped, while rates may become negative or very high. The constrained mean-reverting multivariate normal model must be missing something critical.

Clearly, the PCs themselves are only building blocks. The interesting part of the model comes when we specify how the state variables change over time. As input to such a model, we should first observe the actual path for the state variables in the CMT data.

Some preparation work is needed first. Given the monthly CMT series, I interpolated the zero coupon bond prices for all points between time zero and 20 years. For this, I used a Lagrange interpolating polynomial with anchors at the beginning, middle and end of the observed data. With the zero coupon bond price at time  $t$  for a unit payment at time  $T$ ,  $P(t, T)$ , we can calculate the associated continuous spot rate series  $r(t, T) = -\ln[P(t, T)] / T$ . I used the  $r(t, T)$  series as the yield curve for that month.

PCA applies to the change in the yield curve. I used the matrix  $M$  of absolute changes (in bp) at each time  $t$ , rather than the percentage change or normalized percentage change. The square matrix  $A = M^T \times M$  is related to the variance-covariance matrix for the original data series. Any data point (change in the yield curve at time  $t$ ) can be represented with a column vector  $\mathbf{v}^T$ , and this in turn can be expressed as  $A\mathbf{j}$  for some  $\mathbf{j}$ .

$$\begin{aligned} A &= M^T \times M \\ A' &= \text{inverse of } A \\ A'\mathbf{v}^T &= \mathbf{j} \end{aligned}$$

Now we can represent the data point  $\mathbf{j}$  more efficiently. The columns of  $A$  form a basis that spans the original data set. We can transform  $A$  into a set of orthogonal vectors (eigenvectors) that also spans the data set. The PCs are the largest eigenvectors of  $M^T \times M$ . Thus any of the observed data in  $M$  can be represented with a linear combination of the PCs.

If  $A\mathbf{w} = \lambda\mathbf{w}$  then  $\mathbf{w}$  is an eigenvector of  $A$ , and  $\lambda$  is the associated eigenvalue. The size of  $\lambda$  tells us how much of the variation in  $A$  can be represented with each eigenvector. Table 1 and Figure 1 show the results of this analysis.

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Many statistical packages can calculate principal components. These results are reasonably in line with expectations. Note we can choose the scale and sign for these factors as we like, since the same data point results when we make equal and opposite adjustments to the state variables. In Table 1, they have been scaled so that the monthly change in the state variables has roughly a variance of 1.

The next step was to use these building blocks to describe the original spot rate series  $r(t, T)$ . Since any change in  $r(t, T)$  can be represented with a linear

combination of the PCs, so can the original series. For this I used Excel solver to back into the state variables at each point. The full range of data over the 50-year period can be expressed this way. For example, the state variables for three representative dates are shown in Table 2.

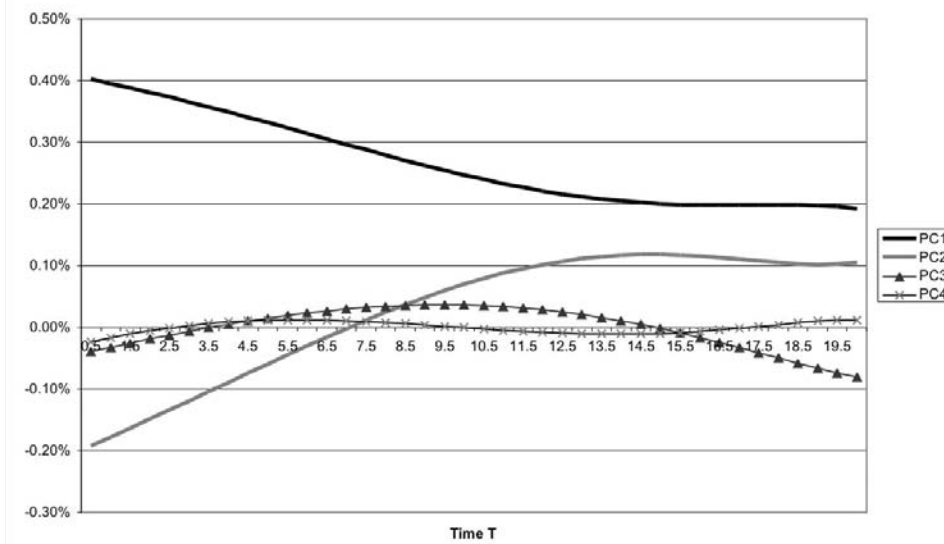
These are (respectively) low/steep, inverted and high/flat. The first two PCs (in this implementation) explain 98 percent of the estimated spot rate data, so the rest of this analysis only uses the first two PCs. Now, this may compound errors in the smoothing process. The reconstituted result may be off from actual historical data by 30bp or more at intermediate points. However, this forces us to look for patterns in two dimensions.

Most people appreciate visual patterns best. It is insightful to present the state variable series as a graph, with height (y-axis) as a function of steepness (x-axis). This representation is sometimes called the “phase plane.” Any point on the plane represents a particular state of the yield curve. We can move up, down or sideways from a given state to the next one. In Figure 2, “up” means greater Height factor, “right” is greater Steepness.

**Table 1: Principal Component Vectors**

	PC1	PC2	PC3	PC4
0.5	0.40%	-0.19%	-0.04%	-0.02%
1	0.40%	-0.18%	-0.03%	-0.02%
2	0.38%	-0.15%	-0.02%	-0.01%
3	0.37%	-0.12%	-0.01%	0.00%
5	0.33%	-0.06%	0.01%	0.01%
7	0.30%	0.00%	0.03%	0.01%
10	0.25%	0.07%	0.04%	0.00%
12	0.22%	0.10%	0.03%	-0.01%
15	0.20%	0.12%	0.00%	-0.01%
20	0.19%	0.10%	-0.08%	0.01%
Significance	86.6%	12.0%	1.3%	0.1%

**Figure 1: Principal Components of U.S. Yield Curve  
Based on Monthly Spot Rates 1953-2003**



This tells us a lot. Even though changes in the state variables are uncorrelated (by definition of PCs), Figure 2 shows a link between steepness and height. Steepness follows height closely, with much of the variation being around the center line where the two are equal. This makes intuitive sense: When rates are very high, the steepness/inversion factor has the same relative size but may be much higher than normal in terms of the absolute 10yr-2yr yield difference.

Note the areas in the lower left of Figure 2. This represents the period from 1953 to 1963. There seem to be two distinct periods here in which rates stayed about the same “distance” from the origin, but that the “angle” for the data point shifted back and forth.

This suggests we should transform the data series of steepness-height state variables into polar coordinates. Define:

- Steepness S
- Height H
- Distance  $D = \text{sq root}(D^2 + H^2)$
- Angle  $A = \text{arc tan}(H/S)$  in radians

The resulting time series contains some interesting regularities.

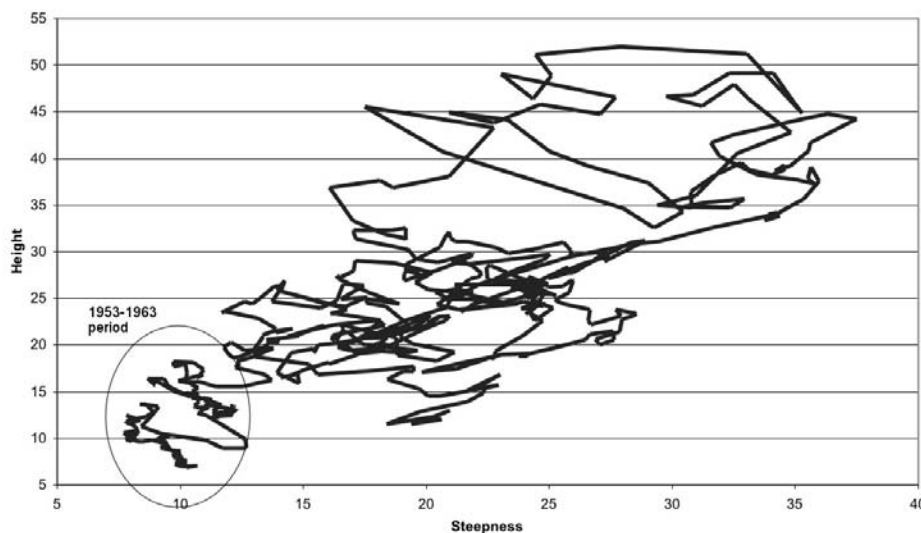
Again, the path of rates is a continuous, connected figure in the D-A phase diagram. The 1953-1963 period as seen in Figure 3, now shows up as relatively vertical lines to the left of the chart. It seems that during the last 50 years, the pattern changes every five years or so. Three such periods are shown in Figure 4. This is what I would like to capture with the concept of a “regime” of interest rates.

Here are the “stylized facts” that seem evident from this representation shown in Figure 4.

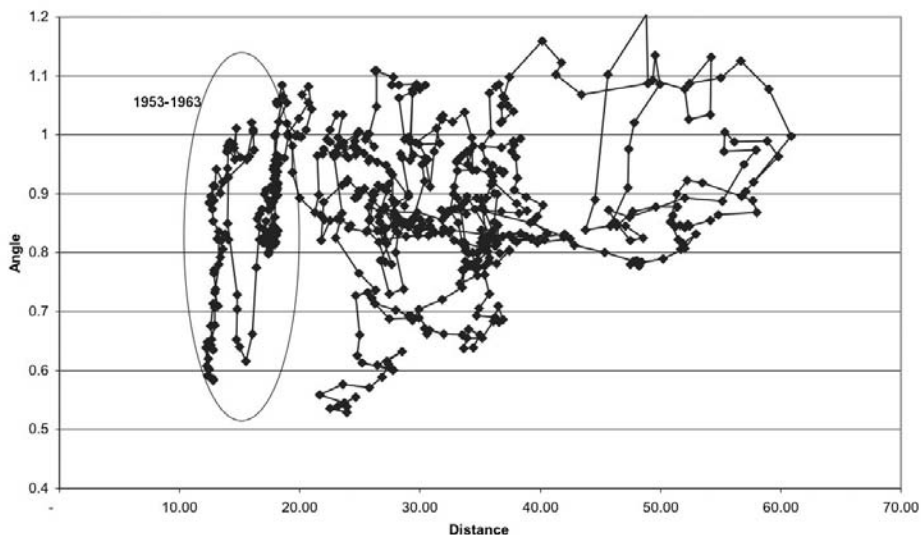
**Table 2: State Variables for Three Dates**

State Variables			
	May-54	Aug-69	Dec-00
PC1	7.07	23.47	18.77
PC2	10.65	11.68	13.04
PC3	(2.16)	(3.02)	(10.47)
PC4	5.47	(5.89)	(4.20)

**Figure 2: Steepness-Height Phase Diagram for U.S. Yield Curve Monthly 1953-2003**



**Figure 3: Distance-Angle Diagram of U.S. Interest Rates**



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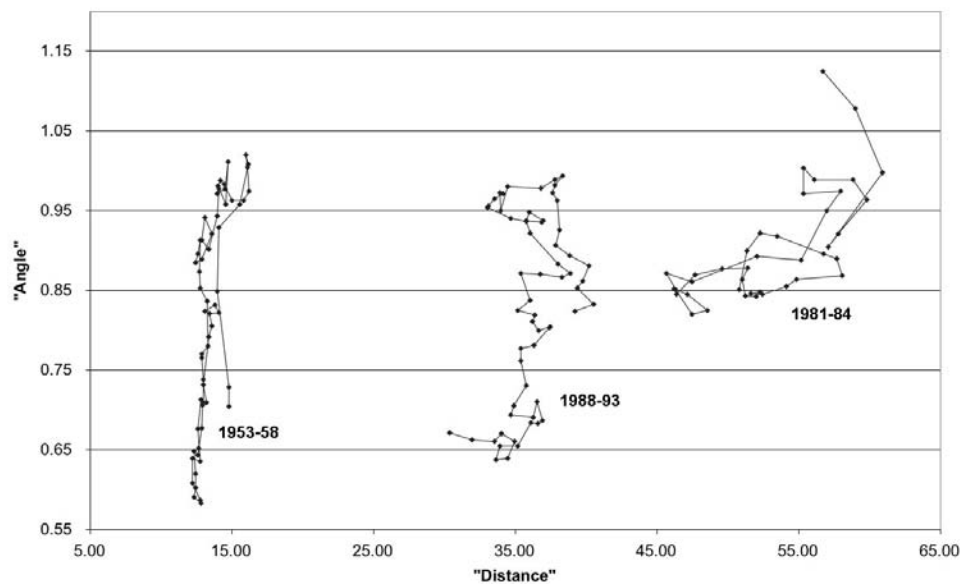
- The state of the yield curve at any time is specified as a linear combination of the two PCs given in Table 1, where H (height) is a multiple of PC1, and S (steepness) is a multiple of PC2.
- We can transform the time series in (S,H) coordinates to a series in polar (D,A) coordinates.
- The path of the yield curve under both coordinate systems can be assumed to be continuous.
- D has ranged from about 12 to 60
- A has ranged from about .5 to 1.2
- During an interest rate “regime,” the yield curve stays in a smaller region of the D-A phase plane anchored on a line.
- The slope of this line varies with D, from about .09 when D = 12 toward zero for D > 45.
- The yield curve stays in a given regime for three to six years, then transitions to another similar one.

By “anchored” I mean that the path of a point in the D-A phase plane will differ from the line by a stochastic component. It may speed up or reverse course or move laterally for while and then resume course. All the math used to describe motion in two dimensions can be applied to develop a specific model form. For example, one approach would be to start with stochastic differential equations for oscillating motion in X and Y, such as

$$\begin{aligned} d^2X/dt^2 &= -uX + \sigma_D Z_1 \\ d^2Y/dt^2 &= -vY + \sigma_A Z_2 \end{aligned}$$

with initial conditions  $X(0)$ ,  $X'(0)$ ,  $Y(0)$ ,  $Y'(0)$  and volatility parameters  $\sigma_D$  and  $\sigma_A$  and then rotate/translate the resulting figure to the desired line in the D-A plane.

**Figure 4: D-A Phase Plane for U.S. Interest Rate Regimes**



Note that D mainly controls the level of interest rates, while A mainly controls the amount of inversion. A slope of .09 results in most of the movement occurring at the short end, while a slope of zero results in a parallel shift.

Here is an example of the yield curves along an “anchor” line, in which D moves from 20 to 25 in increments of 1, while the angle A follows the line

$$A = -1.05 + .08 * D$$

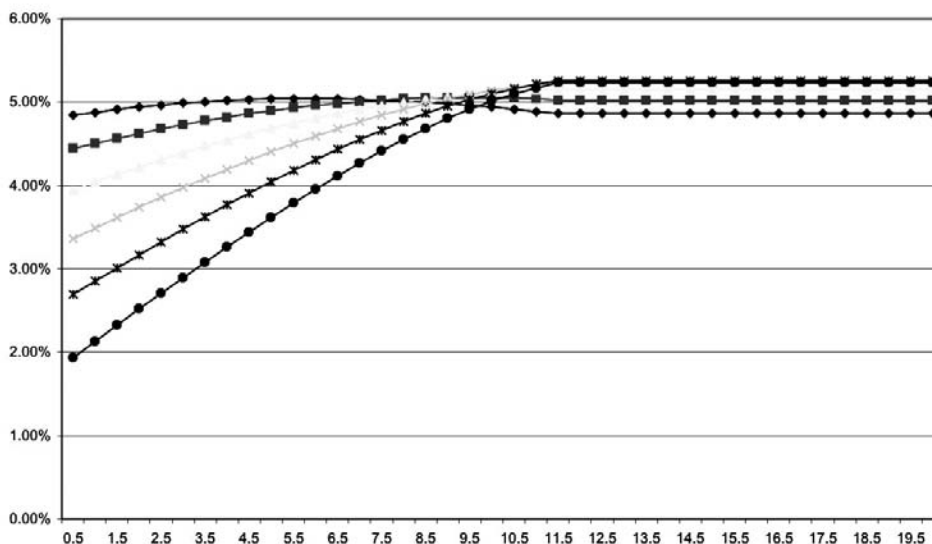
as seen in Figure 5.

We’ll stop here with this qualitative description of an interest rate regime. Though convoluted, this derivation of the form of an interest rate model has a number of advantages:

- Scenario output can be easily compared to actual historical levels for D and A.

- It is straightforward to extrapolate to very low and very high interest rates and still preserve reasonable relationships between steepness and height. The same model can apply for Brazil as for Japan.
- We can specify parameters for several regimes and allow the model to transition from one to the next
- We can distinguish “velocity” of movement within a given regime, from transitional movement to a new regime.  $\delta$

Figure 5: Interest Rate Regime Anchor Line



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understand that there are differences and to make sure that the right type of generator is used in each application.

Discounting the projection results from real-world scenarios does not give a “market-consistent” valuation—a valuation which measures riskiness in the same

**Deflators, or more precisely state-price deflators, bridge the divide between real world and risk-neutral scenarios.**

way as the capital markets. Such a valuation is needed for contracts with guaranteed death or living

benefits; for instance, if you want an indication of the potential cost of hedging. Some sort of value may be obtained by taking, say, the 75% conditional tail expectation (CTE) where the x% CTE is defined as the average of the worst (1-x)% scenario outcomes. This gives a single numeric value that reflects what is happening in the tail of the distribution. However, the choice of appropriate CTE level is not obvious and the result obtained is not necessarily consistent with asset valuations. Moreover, such scenarios are generally not arbitrage free and there is no consensus for the discount rate to be used.

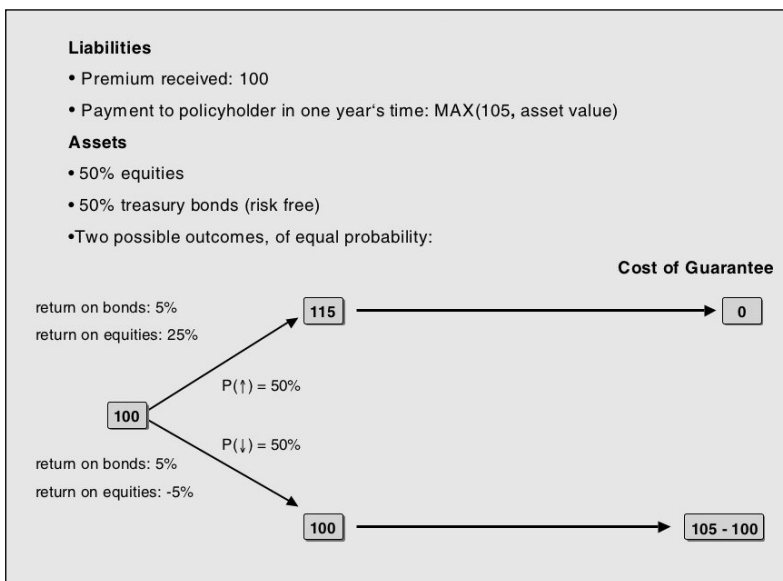
Inspection of the individual scenarios in a risk-neutral valuation gives no insight into the financial

dynamics of the model. Percentile distributions of the outcomes are meaningless and it is impossible to work through the calculations for individual scenarios to satisfy yourself as to their reasonability. They are an artificial construct intended purely to produce a market valuation.

This divide into two paths would not matter greatly if the modeling requirements were always similarly divided so that the appropriate technique could easily be chosen and applied. However, this is not the case. For instance, some companies are now seeking to use an economic capital methodology to determine capital allocation, calculating the liabilities using a “fair value” (i.e., market consistent) measure. This method of valuation of the liabilities requires a risk-neutral type approach but the capital requirement is usually determined to achieve an x% probability of insolvency in y years (where x and y are determined by the company to reflect their desired position in the market) and, therefore, requires the use of real-world scenarios.

Luckily, a solution is at hand—deflators. Deflators, or more precisely state-price deflators, bridge the divide between real-world and risk-neutral scenarios. In short, they may be used to calculate market consistent valuations of any cash flow stream using real-world scenarios. In the next section, I describe in more detail what they are and show how they work. Then in the rest of this article, I give a practical example of their application.

Figure 1: Simple Model of a Guarantee



**What are deflators?**

To define deflators and to contrast them with a risk-neutral valuation, let us consider a very simple model—see Figure 1. This simple model provides a minimum guaranteed return to the policyholder at the end of one year. The premium is invested in assets assumed to have two equally likely outcomes. One pays out more than the minimum required, but the other leaves a shortfall. The question we wish to address is, given these assets, what is the value of the policyholder guarantee?

We start with the risk-neutral approach. Figure 2 outlines the construction of the risk-neutral probabili-

ties that need to be assumed in this model to avoid the possibility of arbitrage. These probabilities are then applied in Figure 3 to derive the risk-neutral value of guarantee.

With this under our belt, I set out a definition of deflators in Figure 4. Technically speaking, deflators are path-dependent stochastic risk discount factors. Separate factors are associated with each real-world scenario. Their effect is to put a greater emphasis on those scenarios in which risky assets perform badly. The riskiness and downside aversion that is experienced in the market valuation of assets is absorbed within the deflator values. This contrasts with risk-neutral valuations, where it is absorbed within the economic scenarios themselves.

We can apply the definition of deflators in Figure 4 to construct the deflator values for our simple model. Applying them (see Figure 5) leads to a value of guarantee that, as we would expect, is the same as that calculated using a risk-neutral valuation. The value of the guarantee is the expected value of the deflated cash flows. You can think of this value as being equal to the value of the hedging portfolio that you would need, assuming that such a hedging portfolio is available to close out the risk completely.

Unfortunately, the construction of deflators is not normally this simple. They cannot just be derived on top of existing sets of scenarios as additional streams of values. You need a stochastic economic generator that has been purpose-built to generate the deflator values alongside its other simulated economic outcomes (interest rates, equity returns, inflation indexes, etc.). Given this, you're all set!

One hugely important property of deflators is that the values are dependent only on scenario and time. The values are independent of the assets and liabilities to which they are applied. This means that they can be used to put a value on any stream of cash flows that varies according to the economic assumptions used. The market-consistent valuation of these cash flows is always the mean value of the deflated cash flows.

Figure 2: Risk-Neutral Probabilities

In order to obtain no arbitrage opportunities, two conditions for risk-neutral probabilities must be preserved:

- Normalization:  

$$P_{RN}(↑) + P_{RN}(↓) = 1$$
- Conservation of market efficiency:  

$$\text{initial portfolio value} * P_{RN}(↑) * (1 + \text{return}(↑)) + \text{initial portfolio value} * P_{RN}(↓) * (1 + \text{return}(↓)) = \text{initial portfolio value} * (1 + \text{risk-free rate})$$

Yielding the risk-neutral probabilities :

$$P_{RN}(↑) = \frac{\text{risk-free rate} - \text{return}(↓)}{\text{return}(↑) - \text{return}(↓)} \quad P_{RN}(↓) = \frac{\text{return}(↑) - \text{risk-free rate}}{\text{return}(↑) - \text{return}(↓)}$$

Figure 3: Risk-Neutral Valuation

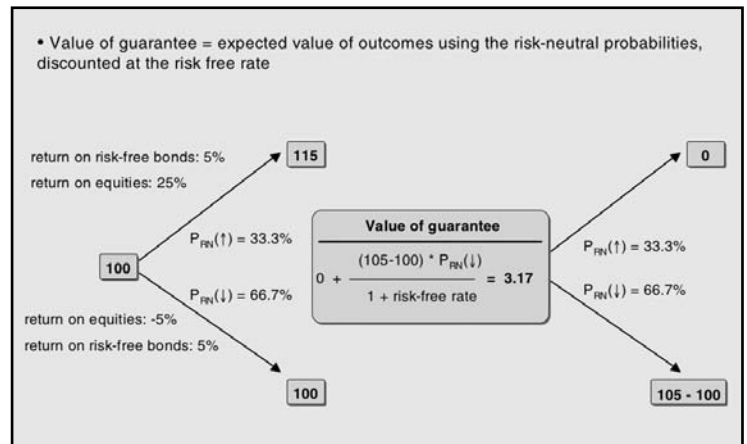


Figure 4: Definition of Deflators

Using risk-neutral probabilities  $P_{RN}$  and discounting with risk-free rates corresponds to using real-world probabilities  $P$  and discounting with deflators  $D$ :

$$D(↑) * P(↑) = \frac{1}{1 + \text{risk-free rate}} * P_{RN}(↑)$$

$$D(↓) * P(↓) = \frac{1}{1 + \text{risk-free rate}} * P_{RN}(↓)$$

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More background on deflators and the theory behind them may be found in “Modern Valuation Techniques,” by Stuart Jarvis, Frances Southall and Elliot Varnell. This paper was presented to the Staple Inn Actuarial Society in the UK and copies of it may be downloaded from The Smith Model Web site at [www.thesmithmodel.com](http://www.thesmithmodel.com). This award-winning paper is highly recommended.

**With the market downturn in 2000, the benefits which had been offered for little or no additional cost have moved significantly into the money and threaten to cause measurable financial pain to an industry coming out of several years of record sales.**

I now move on to describe a practical application of deflators—the valuation of variable annuity guaranteed income and death benefits. This is based on a real project (the values have been changed) performed very rapidly by my colleagues, and I am indebted to Jason Grosse for his help in building the model.

**Background to practical example**

During the market boom of the late 1990s, the issuance of variable annuity contracts with rich guaranteed benefits thrived. With the market downturn in

2000, the benefits which had been offered for little or no additional cost have moved significantly into the money and threaten to cause measurable financial pain to an industry coming out of several years of record sales.

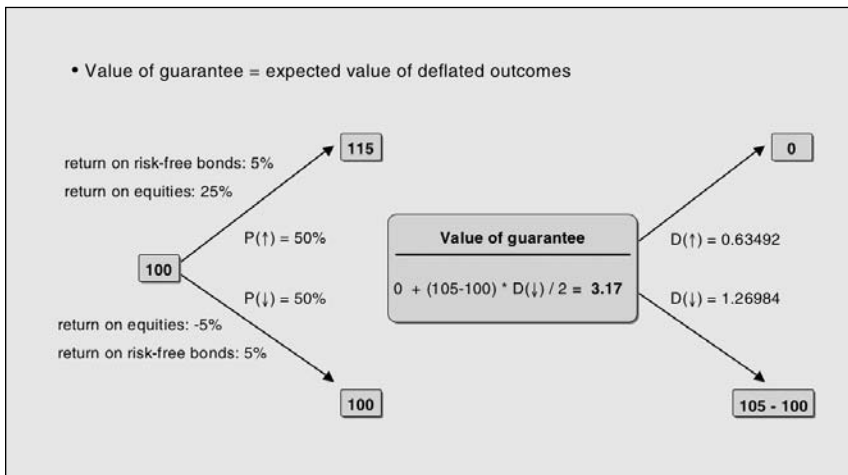
As a consequence, many companies moved the valuation of these guaranteed benefits to the top of their to-do list and focused on the accurate valuation of these benefits. While a Monte Carlo approach is necessary in this exercise, questions remain as to the choice of scenarios and discount rates.

Some companies use real-world scenarios, discounting either at a risk-free rate plus a spread (e.g. the 90-day Treasury plus X bps), or at the spot rate, or at a level rate, representing the company’s cost of capital for all years. The results from these scenarios are then analyzed to come up with a distribution of potential guaranteed benefit costs. This is reasonable for strategic planning and capital allocation. However, as discussed earlier in this article, it does not provide an accurate market consistent value.

Other companies use risk-neutral scenarios and discount at the risk-free rate. This approach gives a market-consistent valuation but has the disadvantage that it gives no strategic insight into the future. This is because the individual scenarios do not represent possible paths through the future, unlike the real-world scenarios.

Our approach was to use deflators. The guaranteed benefits can be thought of as policyholder options, which are valued much like equity put options. A market-consistent valuation is the expected value of the deflated excess of the guaranteed benefit cash flows over the funded account values and represents the current cost of hedging all of the market risk associated with these guaranteed benefits. To achieve this valuation, we used The Smith Model (TSM) stochastic economic generator. This generator produces market-consistent, arbitrage-free scenario sets that include deflators. More information is available at The Smith Model Web site.

**Figure 5: Deflator Valuation**





### Comparative scenarios

We also used an alternative set of scenarios in this project. The second set was a subset of the scenarios recently published by the American Academy of Actuaries (AAA) and made available at [www.actuary.org/life/phase2.htm](http://www.actuary.org/life/phase2.htm). They were generated using a regime-switching lognormal generator and intended primarily to meet the recently published C-3 Phase 2 RBC requirements. The AAA scenarios are not arbitrage-free, and as stated in the documentation supplied by the AAA, should strictly not be used to price securities or derivatives, or in this case, liability cash flows. They were included in the project to demonstrate how the valuation result could differ based on the source of the scenarios. The cash flows produced using these scenarios were discounted at a flat rate of 8 percent, representing an assumed cost of capital.

The observed mean return and volatility assumptions for the equity fund modeled in each set of scenarios are shown in Table 1.

The mean values in this table reflect the geometric average annual rate over 30 years for all scenarios in each set. You can see that volatility of the TSM scenarios is much higher than that of the Academy scenarios. This is because the Academy scenarios were calibrated using historical volatilities whilst the TSM calibration used an implied volatility consistent with current market conditions at the valuation date. The observed value is slightly higher than the input assumptions due to the effect of convexity. We could have calibrated TSM using historical volatilities, though this would have been inappropriate for this project.

### Results

The results of the calculations, using in-force policy data similar to that used in the real project, are displayed in Table 2. To determine a valuation from

Table 1

Measure	Academy	TSM
Mean	10.62%	10.41%
Volatility	16.18%	23.60%

Table 2

Scenario Set: Discount Rate:	Academy 8%	TSM 8%	TSM Deflators
<b>Measure</b>	<b>Results</b>		
Mean	66.0	100.6	234.6
Min	24.3	23.3	1.4
Max	295.7	430.8	7374.4
Standard Deviation	46.3	66.0	642.1
<b>CTE</b>	<b>CTE of Combined GMDB &amp; GMIB Benefit Costs</b>		
95 CTE	295.7	362.4	
90 CTE	215.1	287.3	
75 CTE	181.7	251.7	
50 CTE	129.8	196.8	
25 CTE	92.7	148.6	
5 CTE	76.4	120.4	
1 CTE	68.0	104.2	
	66.4	101.3	
Amount in \$ millions			

the Academy scenarios, we used CTEs. We also included a “middle” set of calculations to quantify the extent to which the results are driven by the scenarios themselves or by the discount rates. The higher CTEs calculated for the TSM scenarios discounted at 8 percent are primarily a result of the higher volatility.

One advantage of the deflator approach is immediately apparent. It gives a suitable value directly, without any need first to decide on an appropriate CTE level. It condenses the results to a single number, the mean value in this table.

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# Expensing Employee Stock Options

by Mark D. J. Evans

**F**ASB 123 requires the disclosure of Black-Scholes based valuation of stock options given to employees. FASB is considering requiring income to be based on a Black-Scholes calculation or binary tree method. Black-Scholes assumptions imply independence between the grantor and receiver of an option and the underlying security. In the case of employee stock options, the incorporation of Black-Scholes must be modified to reflect that the stock underlying the option is equity in the grantor. While the binary tree methods discussed are effective in recognizing the impact of various exercise restrictions and contingencies unique to employee stock options, they do not address the impact of the relationship between the underlying and grantor discussed in this article.

The February 2004 issue of the *Venture Capital Journal* contains an article entitled "FASB's New Method to Value Options is Flawed" by Kim Marie Boylan. While this article focuses on other issues related to expensing employee stock options, it does question whether "FASB should take a step back and look at the fundamental question of *whether* employee stock options are in fact a corporate expense or, rather, a cost to the other shareholders in the form of potential dilution." This is similar to the question addressed by this article.

Consider an illustrative example. Company XYZ is a small company with a volatile stock price and limited net worth. XYZ pays no dividends. It offers generous stock options to its highly skilled employees. Let us assume a strike price equal to the current stock price of 100. XYZ grants one million options in addition to one million shares previously outstanding. The options are struck at the money and are 10-year Europeans. XYZ can issue additional stock at any time. XYZ has net equity of 50 million. At 50 percent volatility and 5 percent risk-free interest, the value of one call is 67.32, according to Black-Scholes. On this basis, the value of the call option exceeds the company's net worth. In actuality, Company XYZ is a viable corporation. The employee options in this

case redefine how the company's future earnings may be split among equity stakeholders, but do not impair the total amount of those future earnings. If XYZ performs well over the next 10 years, then most likely its net equity and stock price will grow. The options will become valuable, but so will the company's fortunes and therefore ability to support the options. On the other hand, if the company does poorly, the options are likely to expire with little or no value.

In issuing employee stock options, company XYZ is essentially creating a contingent liability whereby a claim is placed against equity if XYZ does well, but there is no assessment if XYZ performs poorly or mediocre enough that the stock price at the end of 10 years does not exceed 100. There is a significant difference between XYZ issuing employee stock options and a third party issuing options on XYZ stock. The critical element is the inherent link between success and option value and the ability of XYZ to issue more stock.

For example, assume XYZ's net equity increases to 100 million and the stock price increases to 150 at the end of 10 years. Then XYZ issues one million shares of stock in exchange for 100 million in cash to honor the options. This leaves net equity of 200 million, two million shares and market capitalization of 300 million.

Now assume XYZ's net equity and share price remain flat. The options expire worthless. Net equity is 50 million, we have one million shares and market capitalization of 100 million.

So we see options on XYZ stock issued by XYZ represent a share of the upside potential of XYZ, but not a claim on the economic viability of XYZ. Rather than arbitrarily assigning a cost to employee options, ignoring the relationship between the underlying and the issuer of the derivative, let us consider an approach which recognizes that employee stock options affect future divisions of the pie but do not completely consume the shareholder's equity.

A simple approach is available to address these issues. Define the following variables:

T = time to maturity of employee stock option  
 MV(t) = the market value of company at time t  
 S(t) = stock price at time t  
 C(T) = value of a call option on the stock as of time zero when option expires at time T  
 Shares = number of shares outstanding  
 Options = number of options granted  
 r = risk-free rate of return  
 E = stock holder equity ignoring any claim of option holders to such equity

From risk-neutral assumptions, we can say that the expected value of MV(T) just prior to option expiry is equal to:

$$E[MV(T)] = \text{Shares} * S(0) * \exp(rT) + \text{Options} * C(T) * \exp(rT)$$

Also,

$$MV(0) = E[MV(T)] * \exp(-rT)$$

A portion of MV(0) is associated with stock, but a portion is associated with options. Clearly the portion associated with stock is Shares\*S(0) with the remainder being associated with the options. Simple algebra shows this to be equal to Options\*C(T).

This approach gives us a convenient means to reflect the impact of options on the company. At the end of each accounting period, a portion of the company's equity should be allocated to the optionholders. Algebraically, this equals:

$$E * \text{Options} * C(T) / (\text{Options} * C(T) + \text{Shares} * S(0))$$

This amount would then be set up as a liability. The change in the liability would flow through earnings in each accounting period. If E is negative, then the liability is zero since the presence of options cannot increase the net worth of a company.

In the previous example, the option liability for XYZ is equal to:

$$50,000,000 * 1,000,000 * 67.32 / (1,000,000 * 67.32 + 1,000,000 * 100) = 20,117,140$$

On the one hand, the liability is sensitive to a variety of factors, including stock level and earnings. It can change dramatically from period to period. On the other hand, it will automatically adjust to changing factors. It will always bear a logical relationship to the value of the employee options.

If the stock price rises, then the value of the option, C(T), will increase more than proportionally, meaning that the option liability will be larger in proportion to remaining stockholder equity. Note this is more likely to occur when total equity has increased due to correlation between company success, equity and stock price. If the stock price falls, then all these relationships operate in reverse.

The analysis becomes more tedious due to multiple option grants, exercising rights prior to maturity, restrictions on exercise and the existence of stockholder dividends, but the principles remaining the same.

Hull<sup>1</sup> discusses a company issuing warrants (options on its own stock). While it recognizes these should not be valued as options issued by a third party, their approach assumes market capitalization equals book equity, which is rarely the case. ❖

<sup>1</sup>Hull, John C., *Options, Futures, and Other Derivatives*, 5th edition. Upper Saddle River, New Jersey. Prentice Hall, 2002.



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# SOA Annual Meeting

This year, the SOA Annual meeting will be held October 24-27 among the bright lights of New York City. In addition to the section breakfast and a Tuesday evening reception, the Investment Section is once again sponsoring a number of timely and practical sessions on investment-related topics. The following is a brief description of the sessions and activities planned for the meeting. We look forward to seeing you in October.

## **Investment and Financial Reporting Sections' Joint Reception to Welcome the Risk Management Section** (Tuesday 10/26 5:30 pm)

The Investment/Financial Reporting Section's joint reception has been expanded to include the new Risk Management Section. Be sure to take advantage of this opportunity to mingle with your friends and colleagues while enjoying the hors d'oeuvres and drinks that will be served.

## **Forecasting Economic Variables Using the Delphi Method** (Monday 10/25 10:30 am, jointly sponsored with the Futurism Section)

Delphi studies are a futurism technique for extracting a consensus view of the future from a group of subject matter experts. The Investment Section is one of the joint sponsors of the SOA's Delphi study on economic variables, expected to be completed later this year. In this session panelists describe how Delphi studies are conducted and update the progress of the SOA study.

## **How do the Analysts View Your Insurance Company?** (Monday 10/25 2:00 pm)

Rating agency and equity analysts describe the quantitative and qualitative factors that drive insurance company ratings and stock market valuation levels. The analysts identify current issues and concerns for the companies they cover.

## **Quantitative Methods Used in Managing Credit Risk** (Monday 10/25 2:00 pm, jointly sponsored with the Risk Management Section)

What are the main quantitative methods that companies are using to measure and manage individual and portfolio credit risk? Panelists speak about their experiences in credit risk management, describe some of the methods that they are using and compare the relative strengths of the techniques being used.

## **Investment Section Hot Breakfast: Personality Types of Investment Professionals and Actuaries** (Tuesday 10/26 7:00 am, jointly sponsored with the Management & Personal Development Section)

The highlight of the meeting will be the Investment Section breakfast. Once again we have an excellent speaker, Jim Ware, the author of *Investment Leadership: Building a Winning Culture for Long-term Success*. Jim discusses the different personality types of investment professionals and actuaries, and provides techniques for developing and leveraging the talent within investment organizations.



## **Where Has All the Capital Gone, and Where Will We Find it Next?** (Tuesday 10/26 8:30 am, jointly sponsored with the Reinsurance Section)

What's causing a scarcity of capital in the life insurance and life reinsurance industries? What are possible responses to it from ceding companies, reinsurers, regulators and rating agencies? Panelists discuss the issues surrounding capital in the life insurance and life reinsurance industries.

**Life Insurance Securitization**  
**(Tuesday 10/26 10:30 am, jointly sponsored**  
**with the Reinsurance Section)**

With several deals closing in 2003 and more on the way in 2004, life insurance securitization is finally starting to live up to its promise. Three panelists, each with extensive experience on prior securitizations, share their experiences with these deals and discuss their expectations for the future.

**Hedge Fund Strategies**  
**(Tuesday 10/26 2:30 pm)**

What is a hedge fund really and when is a hedge fund an appropriate investment for an insurance company's portfolio? Experts from the hedge fund and insurance industries discuss some of the unique strategies used by hedge funds. The focus of the session will be on funds and strategies particularly germane to life insurance companies.

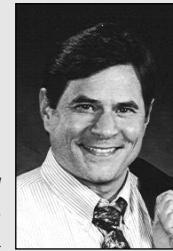
**Investment Issues Facing International Companies**  
**(Wednesday 10/27 8:00 am, jointly sponsored**  
**with the International Section)**

Whether it's a limited universe of assets or a different culture, the international investment actuary encounters some unique investment situations. In this session panelists discuss the issues that international investment actuaries face and provide insights into how companies deal with them.

**Hedging Variable Annuity Guarantees**  
**(Wednesday 10/27 10:00 am)**

The stock market's rise over the past year has lessened the size of the losses stemming from variable annuity guarantees, but not the risks. Panelists describe some of the recent product designs of these guarantees and explore some of the methods companies are using to hedge their exposure. The discussion will include some of the practical day-to-day issues faced by these companies.

**FEATURED  
SPEAKER**



*Jim Ware is a principal at Focus Consulting Group, a firm that helps financial companies plan for sustainable growth. He is a chartered financial analyst with 20 years experience as a research analyst, portfolio manager and director of buy-side investment operations. He has taught investments at the Keller Graduate School of Management and written articles for various trade publications including the Financial Analysts Journal. He is a frequent presenter at trade conferences and has appeared on Fox News and other business channels. He is also a Meyers-Briggs Type Indicator qualified instructor.*



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You can also see from the values in this table that the result obtained from using deflators places a far higher value on the guaranteed benefits than taking the mean value after discounting using a constant discount rate. It is equivalent to a CTE in excess of 95 percent using the Academy scenarios.

### Conclusions

The use of deflators made the job of accurately estimating the cost of the guaranteed benefits very easy. The information we obtained by attempting a comparative valuation using an alternative methodology suggests that valuations based on a CTE approach may differ greatly from market-consistent valuations. This exercise also demonstrated that the cost of hedging the benefit guarantees may well be significantly higher than was previously thought. For reference, see Richard Q. Wendt's article, "An Actuary Looks at Financial Insurance" in the May, 1999 issue of *Risks and Rewards*.

### Summary

In this article I have attempted to explain deflators in a simple, nontechnical way. Along the way I have:

- Shown how a stochastic valuation made using deflators differs from a risk-neutral valuation;
- Explained how deflators enable stochastic valuations to be made using real-world scenarios;
- Indicated why this is useful and may indeed be necessary in some circumstances;
- Highlighted a number of potential pitfalls that may arise if a CTE approach to valuation is used without due care;
- Demonstrated that the application of deflators is straightforward;
- Suggested that, if or when you start to adopt market-consistent valuation techniques for life insurance liabilities, you may find the results disturbing. ☹



## Coming in September!

The **SOA e-mail newsletter** will debut this fall, bringing you news you can use!

Get the latest details about:

- SOA activities & initiatives
- Educational opportunities
- Exam information
- National and global issues for actuaries
- Business news
- And much, much more!

**Stay tuned ... more details to come this summer!**

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While we are busy debating the necessity of studying some material most of us didn't know existed when we took exams, some other risk management organizations have emerged recently with rapidly increasing membership—namely, the Global Association of Risk Professionals (GARP) and Professional Risk Management International Association (PRMIA).

I doubt these organizations will create professionals to compete with our traditional actuarial roles, but they may compete for new opportunities that actuaries could be well qualified to fill. (Hint: include financial economics in the syllabus). Given the rapid growth of these other professional organizations, we risk missing out on opportunities that could be very beneficial to our profession. It takes years to establish a profession, but once established, the barriers to entry are large. We need to make sure we aren't watching from the sidelines as someone else scores the points.

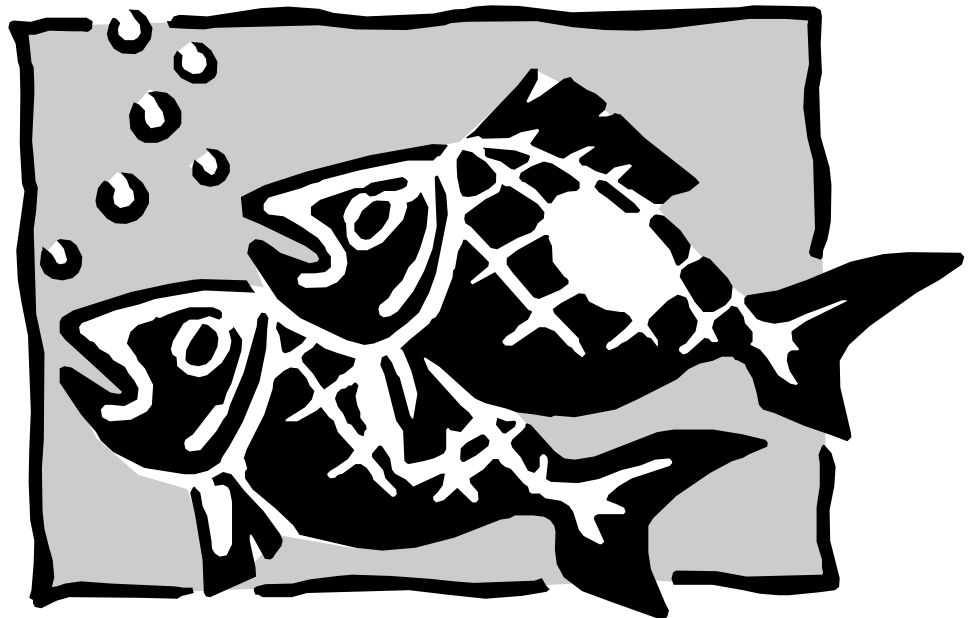
There is one positive event worth mentioning. The Risk Management Section was formed recently. This new section will increase the SOA's ability to develop our members in broader areas of risk management that the Investment Section hasn't historically addressed. The Investment and Risk Management

Sections will coordinate their activities to make sure we are not duplicating efforts. I'd encourage our members to join the Risk Management Section, if you haven't already done so.

Developing and growing the actuarial profession is no small task. I'm reminded of a scene from the movie, "Finding Nemo." It's difficult for an individual or even a few to change the fortunes of our members. But if we all believe in the same cause and swim in the same direction, we can break out of our traditional net releasing us to explore new opportunities. 🐟



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