Designing Exams to Test Higher Levels of Learning

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Outline

- Learning Taxonomies
- Assessment is Curriculum
- Higher Level Questions
- Examples
- How you can use these ideas
Learning Taxonomies – Bloom’s

- Remember: recall facts and basic concepts
  - define, duplicate, list, memorize, repeat, state

- Understand: explain ideas or concepts
  - classify, describe, discuss, explain, identify, locate, recognize, report, select, translate

- Apply: use information in new situations
  - execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch

- Analyze: draw connections among ideas
  - differentiate, organize, relate, compare, contrast, distinguish, examine, experiment, question, test

- Evaluate: justify a stand or decision
  - appraise, argue, defend, judge, select, support, value, critique, weigh

- Create: produce new or original work
  - design, assemble, construct, conjecture, develop, formulate, author, investigate
<table>
<thead>
<tr>
<th>Retrieval</th>
<th>Comprehension</th>
<th>Analysis</th>
<th>Knowledge Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recognize</strong> • recognize (from a list) • select (from a list) • identify (from a list) • determine (true / false)</td>
<td><strong>Symbolizing</strong> • use models • symbolize • depict • represent • draw • illustrate • show • diagram • chart</td>
<td><strong>Classifying</strong> • classify • organize • sort • identify different types or categories • Identify a broader category</td>
<td><strong>Problem-Solving</strong> • solve • adapt • develop a strategy • figure out a way • how would you overcome • how will you reach your goal under these conditions</td>
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<td><strong>Recall</strong> • name • list • describe • state • identify who, where, or when • describe what</td>
<td><strong>Matching</strong> • compare and contrast • categorize • sort • differentiate • discriminate • distinguish • create an analogy or metaphor</td>
<td><strong>Analyzing Errors</strong> • edit • revise • identify errors or problems • evaluate • identify issues or misunderstandings • assess • critique • diagnose</td>
<td><strong>Experimenting</strong> • experiment • generate &amp; test • test the idea that • what would happen if • how would you test that • how can this be explained • how would you determine if • based on the experiment, what can be predicted</td>
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<tr>
<td><strong>Executing</strong> • use • demonstrate • show • make • draft • complete</td>
<td></td>
<td><strong>Generalizing</strong> • form conclusions • Create a principle, generalization, or rule • trace the development of • generalize • what conclusions can drawn • what inferences can be made</td>
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<td><strong>Integrating</strong> • summarize • paraphrase • describe the key parts of • describe the relationship between • Explain the ways in which • describe how or why • describe the effects</td>
<td></td>
<td><strong>Specifying</strong> • make &amp; defend • predict • what would have to happen • develop an argument for • judge • under what conditions • deduce</td>
<td><strong>Investigating</strong> • investigate • research • find out about • take a position on • how &amp; why did this happen • what would happen if • what are differing features of</td>
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Assessment is Curriculum

• What is important that your students know?

What skills do you want them to have?

• Test them on that!

• But how…
How to Think of High Level Qs

- Look for inspiration in Bloom/Marzano verb lists
- Keep a list as you teach of interesting ideas
- When creating your tests, insert where appropriate
- Practice: the more you do it, the easier it gets!
How to Include High Level Qs

• Start with a simple calculation
  • Can give the answer to less accuracy – why?
• Then some more complex calculations
  • Using the first part or similar techniques
• Finally a conceptual question
  • Extending the material
How to Include High Level Qs

• Prove/Disprove instead of True/False
• Ask to graph something
• Apply models to a completely new situation
• Translate between symbols and words
• Similarities and differences
How to Grade High Level Qs

• Look for key words
• Scan through several responses before starting, to determine quality benchmarks
• Use a rubric
• Design questions that can be easier to mark
  • Matching/checkbox vs listing
  • Graphing vs describing
  • Prove/disprove
Example 1 - Probability

3. (10) Major data security breaches (where customer data or passwords are made public, either by accident or by hackers) can occur at any time. The probability that any month has no major data security breaches is 0.135.

(a) (3) Do you think that the three conditions for a Poisson process would hold in this situation in real life? Discuss each one in 1-2 complete sentences.

(b) (4) Assuming a Poisson process is appropriate, find the probability of observing at least 2 breaches in 1 month.

(c) (3) Find the probability that you must wait 6 (non-overlapping) months until you observe 2 months with no breaches.
Example 2 – Financial Math

5. [10 marks] The number of fruit flies in a lab grows at a constant daily force $\delta = 0.2$. There are 10,000 fruit flies in the lab at 9:00 am today.

(a) [3 marks] What will be the increase in the number of flies between 9:00 pm and 10:00 pm tonight?

(b) [3 marks] At what day and time (to the nearest hour) will there be 20,000 flies in the lab?

(c) [4 marks] If at exactly 9:00 am each day, starting tomorrow, 1% of the normal fly population mutates, and mutated flies do not reproduce, how many mutated flies will there be in the lab just after 9:00 am in one week (7 mutations)?
Example 3 – Life Cons 1

1. [16 marks] The survival function for a life age 0 is given by $S_0(x) = \left(\frac{\lambda}{\lambda + \beta x}\right)^\alpha$, where $\alpha$, $\beta$, and $\lambda > 0$.

(a) [6 marks] Verify that $S_0(x)$ satisfies the three conditions to be a valid survival function, and briefly justify why each condition makes sense.

(b) [3 marks] Show that the force of mortality $\mu_x = \frac{\alpha \beta}{\lambda + \beta x}$.

(c) [3 marks] Show that for $x \geq 0$, $\dot{p}_x = \left(\frac{\lambda^*}{\lambda^* + \beta t}\right)^\alpha$, for some new $\lambda^*$ that you should identify.

(d) [2 marks] For $\alpha = 50$, $\beta = 0.001$, and $\lambda = 2$, calculate $\hat{e}_0$, the average lifetime of an individual.

(e) [2 marks] Roughly sketch a graph of the force of mortality for typical human lives.
Example 4 – Life Cons 2

2. A life age 50 buys a fully discrete 20 year term insurance policy with sum insured 100,000. The interest assumption for both premiums and policy values is 5% per year effective, and mortality follows the attached table.

(a) [2] Show that the premium is $313 to the nearest $1.
(b) [3] Calculate the policy value 15 years after the policy is issued.
(c) [3] Using the recursive relationship, calculate the policy value 14 years after the policy is issued.
(d) [4] Your boss needs the policy value at time 14.5. He suggests simply taking the average of the policy values at time 14 and 15. Explain briefly why this would give a poor approximation, and determine a more accurate estimate.
Example 5 – Probability Models

4. (a) [7] Check off whether each of the following statements hold for Markov Chains, Poisson Process, and/or Standard Brownian Motion.

<table>
<thead>
<tr>
<th>State space is discrete</th>
<th>MC</th>
<th>PP</th>
<th>SBM</th>
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<tbody>
<tr>
<td>Process starts at 0 at time 0</td>
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<tr>
<td>Process has the Markov property</td>
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<td>Equilibrium distribution exists</td>
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<td>Process is nowhere differentiable</td>
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<td>Process is stationary</td>
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<tr>
<td>Process is non-decreasing</td>
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(b) [6] Sketch a possible trajectory (sample path) for each of the following continuous-time stochastic processes, illustrating the differences between them:

i. Poisson Process
ii. Standard Brownian Motion
iii. Geometric Brownian Motion with $\mu > 0$
Example 6 – Time Series

6. ARCH/GARCH Models

(a) [6] How can we tell that an ARCH/GARCH model might be appropriate for a set of data? Discuss 3 features that ARCH/GARCH models help capture, and how you would identify these features in your data from plots.

(b) [2] Suppose you have fitted an ARCH(1) model to some log stock returns, and the fitted parameters are \( \mu = -0.0017, \alpha_0 = 0.462, \) and \( \alpha_1 = 0.085. \) The most recent stock price is $62.48. Predict what the stock price will be 3 business days later. No prediction interval is needed.
How You Can Use These Ideas

• Universally applicable
• Test what you teach
• Brainstorm questions throughout the term
• Give students consistent assessments
• Explain the importance of understanding
• You can do it!
Thank You

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Teaching blog: http://www.math.uwaterloo.ca/~dkchisho/teachingblog.html