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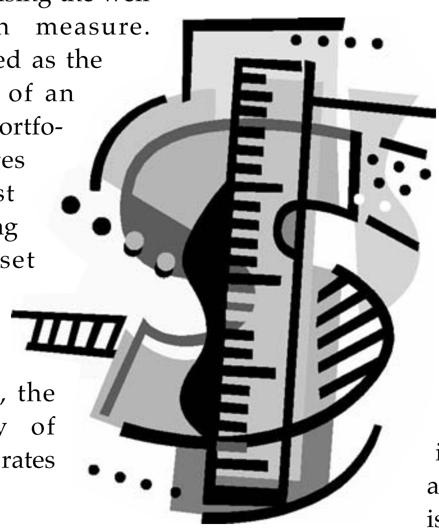
Insurance Liability Duration In a Low-Interest-Rate Environment

by Paul J. Heffernan

The recent low interest rate environment has created a challenging situation for insurers. Products with fund balances which grow at a crediting rate, at least equal to some contractual minimum, face profitability pressures and some tricky asset-liability management issues, as well. Examples of such are deferred annuities, universal life and settlement options.

Actuaries are familiar with asset-liability management (ALM) as a technique to protect surplus against changes in interest rates. The essential idea is to measure the price sensitivity to interest rate changes of both liabilities and the assets supporting them, using the well-known duration measure. Duration is defined as the price sensitivity of an asset or liability portfolio to small changes in current interest rates. By measuring and equating asset and liability durations (or, more properly, duration dollars), the price sensitivity of surplus to interest rates becomes small.



Product actuaries must consider duration in many aspects of their work, including product design and development, pricing, setting new money and old money crediting rates, forecasting and communicating with asset managers. The actuary must understand the drivers of duration and the methodology used to calculate it.

There are many versions of the duration measure, with the two oldest, and perhaps best-known, being Macauley and modified duration. Both versions do a fine job of measuring price sensitivity to interest rates when the cash flows of an asset or liability do not vary with interest rates. However, when a change in rates can alter the cash flows, a more robust measure is needed. This measure is called effective duration, and it is the product of option-adjusted analysis (OAA). The methodology of OAA originally was developed for assets, such as callable bonds, mortgage-backed securities and CMO securities, but it is equally valuable for insurance liabilities. Today's low-interest-rate environment makes this an opportune time for actuaries to learn or brush up on the basics of OAA.

By measuring and equating asset and liability durations (or, more properly, duration dollars), the price sensitivity of surplus to interest rates becomes small.

With this goal in mind, we'll explore OAA in this paper and apply it to a challenge that arises when the supportable crediting rate on a fund accumulation product falls below the minimum, namely, the lengthening of the liability duration. Working with the example of a deferred annuity and using OAA, we will show that the duration of the annuity with a minimum crediting rate can be longer than that of a similar annuity without the minimum. We will show that it is helpful to decompose the product into two parts, an annuity without a crediting rate minimum and an interest rate floor. In this way we will attribute the additional duration to the embedded derivative that is the interest rate floor that results from the crediting rate floor.

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Duration and insurance liabilities

Interest rate changes pose potential risks to insurers, since interest rate movements can change the valuation of insurance liabilities and the fixed-income assets that the insurer holds to support them. The measure that actuaries and asset managers use to quantify this relationship is duration. Duration can be defined in words as the percentage change in value per change in interest rates, and is written symbolically as:

$$D = -(\Delta P/P)/\Delta i$$

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When interest rates are the key driver of the value of both liabilities and the assets that back them, duration matching is a valuable tool for protecting surplus against rate movements. Defining "duration dollars" of a portfolio of assets or liabilities as its duration times value by managing asset duration dollars to equal liability duration dollars, the level of surplus is unaffected by modest rate changes. In order to more effectively protect surplus, a company should review additional measures such as convexity, but in any event duration makes a powerful starting point.

We should note that when we say "price" or "value" we mean a market value. Since insurance liabilities do not trade frequently, it often makes sense to think in terms of the fair value, a concept which is gaining popularity in the actuarial profession because of its focus on the underlying economics. Consequently, when a company seeks to protect surplus by matching asset and liability durations, it is protecting economic surplus, and because of timing differences, such a strategy may not be enough to ensure a steady pattern of earnings under GAAP or statutory reporting.

Crediting rate floors lengthen duration, because they offset some of the effect of resets.

Now consider two characteristics common in liabilities that shorten duration:

- crediting rate resets,
- withdrawals,

and one that lengthens duration:

- crediting rate floors

To see how crediting rate resets shorten duration, consider the extreme example of a deferred annuity maturing in three years, where the crediting rate is reset annually to equal current interest rates less a 100 bp spread. Furthermore, assume the reset is next week. If we immediately increase interest rates by, say, 10 bp, then upon the reset the crediting rate will also increase 10 bp, offsetting the effect of the higher discount rate in the present value calculation. We can see intuitively that we essentially get a duration of zero. This means that, at least in the week before the rate reset, the annuity's value is not sensitive to interest rates. Of course, after the reset date passes, the duration would lengthen to the amount of time until the next reset (in this case, one year), since cash flows for that period, but not after, would be invariant to interest rate changes.

Withdrawals shorten duration because the average life of the cash flows decreases. The risk of disintermediation under rising market rates poses an additional ALM challenge, one that insurers attempt to lessen through surrender charges and other product design features.

Crediting rate floors lengthen duration because they offset some of the effect of resets. The higher the floor, the more duration is lengthened. We can see this by considering extreme examples of a very low and very high floor. With a low floor, the crediting rate will usually be set off of current rates, so the floor has little effect on crediting rates or duration. With a high floor, the crediting rate essentially is fixed at the floor, and as in the example of a GIC or a zero-coupon bond, duration is close to the time to maturity.

Option-adjusted analysis and interest rate trees

In all our examples, we will use a three-year maturity date, and all interest rates will be stated on an annual basis (usually semi-annual, or bond-equivalent, interest rates are used with bonds, but we'll simplify the math with annual rates). We will use the three-year yield curve shown in Table 1. The table gives the yields converted to spot rates and to forward rates. In our calculations of duration, we

will shock the yield up and down by 10 bp, so Table 2 shows these yields and the resulting spot and forward rates.

We implement OAA through a binomial tree, shown in Figure 1. This tree represents the yield curve of Table 1, but models the uncertainty of future interest rates. The tree starts at time t=0 with a single node, since we know with certainty the current one-year spot rate (which equals the one-year forward rate at time zero). Going forward, however, interest rates are uncertain, so the starting node branches out to two nodes at time t=1, representing a pair of possible one-year rates at that time. In moving forward to the next period, each of the two nodes at t=1 branch out to two nodes at t=2. Our tree is "recombining," however, which means that by moving up in the first period and then down in the second, we reach the same node as moving down then up, so that time t=2 has just three nodes, not four. Each of these nodes represents a possible one-year rate at t=2.

We write HH to represent the path taken by taking the up move at t=0, and another up at t=1. Path HH represents the set of forward rates 3.00%, 4.82%, 6.17%. Likewise, path HL is an up move followed by a down move, and represents the forward rates 3.00%, 4.82%, 4.14%. Paths LH and LL are the other two possibilities in our three-year tree with one-year time periods.

We choose a probabilistic process to move from nodes in earlier to later periods, whereby we assume there is a 0.5 probability of either an up or down move leaving any node. For an assumed standard deviation σ, we require the relationship $r^H = e^{2\sigma} * r_L$ to hold for the pair of forward rates leading from a node. For example, in Figure 1 we use σ = 20%, and for the two rates leading out of the node at t=0, we have 4.82% = $(e^{.4})$*3.23%.

To calculate the value of an asset or liability using the tree, we determine the cash flows along each possible path of up and down moves and discount the cash flows with the set of forward rates found along that path. Since we assume up and down moves are equally likely at each node, every possible path is equally likely, so the value of an instrument is simply

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Table 1: Example Yield Curve

		Maturity	
	1-year	2-year	3-year
Yield	3.00%	3.50%	3.75%
Spot rate	3.00%	3.51%	3.77%
Forward rate	3.00%	4.02%	4.28%

All rates are annual

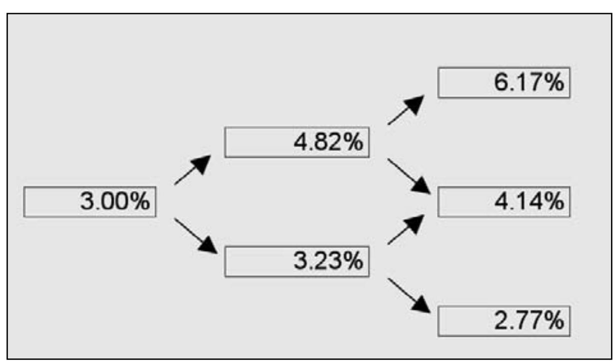
Table 2: Example yield curve with +/- 10 bp shock

		Maturity	
	1-year	2-year	3-year
+10 bp shock			
Yield	3.10%	3.50%	3.75%
Spot rate	3.10%	3.51%	3.77%
Forward rate	3.10%	4.02%	4.28%
+10 bp shock			
Yield	2.90%	3.40%	3.75%
Spot rate	2.90%	3.51%	3.77%
Forward rate	2.90%	4.02%	4.28%

Table 3: Valuing Payment of 100 at t=3

With spot rate:	
P = 100 / (1.03766^3)	= 89.50
With binomial tree:	
P(HH) = 100 / { (1.0300)*(1.0482)*(1.0617) }	= 87.24
P(HL) = 100 / { (1.0300)*(1.0482)*(1.0414) }	= 88.94
P(LH) = 100 / { (1.0300)*(1.0323)*(1.0414) }	= 90.31
P(LL) = 100 / { (1.0300)*(1.0323)*(1.0277) }	= 91.51
P{P(HH) + P(HL) + P(LH) + P(LL) } / 4	= 89.50

Figure 1: Interest Rate Tree



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the average present value of cash flows across all the paths.

An important point is that the set of forward rates on the tree must produce the same present value for a set of cash flows as do the true forward rates. This requirement is commonly called fitting the model to the term structure of the yield curve. The rates in Figure 1 have been chosen with this intent. We verify this in Table 3, where we discount a payment of 100 at t=3 with the three-year spot rate, and by averaging the discounted values along the four possible paths, we see that we get the same price each way.

Figure 2: Deferred annuity with 3.0% crediting rate floor
Crediting rate is greater of 3.0% and current rate less 100 bp

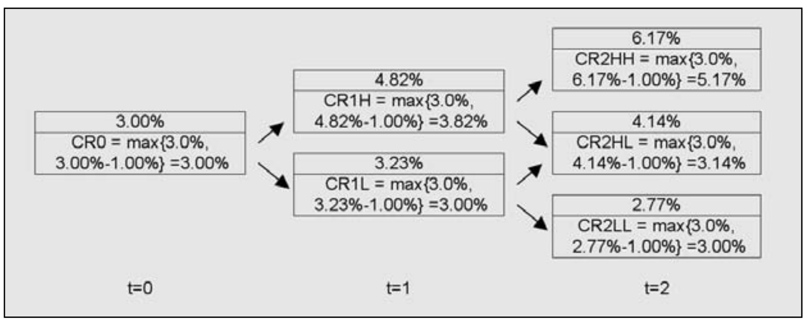

Table 4: 3-year deferred annuity, with 3% crediting rate floor

Premium		=	**100**
Crediting rate current rate less 1%, subject to 3% floor			
P(HH)	= 100 * { (1.0300)*(1.0382)*(1.0517) } / { (1.0300)*(1.0482)*(1.0617) }	=	98.11
P(HL)	= 100 * { (1.0300)*(1.0382)*(1.0314) } / { (1.0300)*(1.0482)*(1.0414) }	=	98.09
P(LH)	= 100 * { (1.0300)*(1.0300)*(1.0314) } / { (1.0300)*(1.0323)*(1.0414) }	=	98.82
P(LL)	= 100 * { (1.0300)*(1.0300)*(1.0300) } / { (1.0300)*(1.0323)*(1.0277) }	=	100.00
P=	{P(HH) + P(HL) + P(LH) + P(LL) } / 4	=	98.76

Table 5: 3-year deferred annuity, with 3% crediting rate floor, +10bp yield curve shock

Premium		=	**100**
Crediting rate current rate less 1%, subject to 3% floor			
P(HH)	= 100 * { (1.0300)*(1.0394)*(1.0532) } / { (1.0310)*(1.0494)*(1.0632) }	=	98.02
P(HL)	= 100 * { (1.0300)*(1.0394)*(1.0323) } / { (1.0310)*(1.0494)*(1.0423) }	=	98.00
P(LH)	= 100 * { (1.0300)*(1.0300)*(1.0323) } / { (1.0310)*(1.0331)*(1.0423) }	=	98.65
P(LL)	= 100 * { (1.0300)*(1.0300)*(1.0300) } / { (1.0310)*(1.0331)*(1.0284) }	=	99.76
P	= { P(HH) + P(HL) + P(LH) + P(LL) } / 4	=	98.61

Evaluating crediting rate floors

An interest rate tree is invaluable when the cash flows of an asset or liability can vary with interest rates. We will explore how to use the tree to analyze a deferred annuity with a crediting rate floor.

Assume we have a three-year deferred annuity that credits each year the one-year market rate less a 100 bp spread, subject to a 3.0% crediting rate floor. To focus our attention on the effect of floors, we will assume there are no withdrawals before the maturity date. At each node of Figure 2, the crediting rate is the greater of 3.0% and the one-year rate associated with the node less 100 bp. For the node at t=0, this is CR0 = max { 3.0%, 3.00% - 1.00% } = 3.00%, meaning that the floor rate is higher than the supportable crediting rate.

At t=1, there are two nodes, representing an up move and down move in rates in the first year. Under the up move, we have CR1H = max { 3.0%, 4.82% - 1.00%} = 3.82%, so the current rate determines the crediting rate. However, for the down move, we have CR1L = max { 3.0%, 3.23% - 1.00% } = 3.00%, so the floor rate gives us the crediting rate. In a similar fashion, at t=2 we have CR2HH = 5.17%, CR2HL = CR2LH = 3.14%, and CR2LL = 3.00%.

In Table 4, we show the initial fund of 100 growing to an ending fund under each of the four possible interest rate paths, and the associated present value of each. We average the four present values to obtain the price of the contract, 98.76.

We then value the contract under the upwards shocked tree, shown in Figure 3. The binomial tree again has been calibrated to preserve the term structure of the yield curve. The resulting price is P+ =98.61. We show this in Table 5.

If we shock the tree downward (recalibrating the tree) and revalue, we get P- = 98.91. This gives a duration of 1.5, as shown in Table 6.

Earlier we discussed a three-year deferred annuity which each year credits the current one-year market rate less 100 bp, and argued that immediately before its rate reset it has a duration of zero. This example is an identical annuity with a crediting rate floor added, and we see that adding the floor adds duration. A higher floor lengthens the duration further. For example, a floor of 4.0% results in a duration of 2.6. When cash flows do not vary with interest rates—either because the crediting rate over three years is fixed, or because the floor is extremely high—we get a duration of 2.9, close to the time to maturity. In this manner, we see that the more the floor affects crediting rates, the more sensitive the price of the liability becomes.

The embedded derivative—an interest rate floor

We gain further insight into the effect of a crediting rate floor on price sensitivity by decomposing the contract into components—the underlying contract and the embedded derivative. In our example, the annuity can be decomposed into a three-year deferred annuity without a floor and an interest rate floor derivative. This derivative has a three-year maturity, a notional of 100, a strike rate of 4 percent, and an annual reset. Each year it pays any excess of the strike rate over the current one-year rate, times the notional. Note that 4 percent less the current one-year rate is equivalent to the excess of 3 percent over the current one-year rate less the 100 bp spread. The reason this decomposition works is that the cash flows of the annuity with floor equal the sum of the cash flows of the two components. Not surprisingly, when we value the interest rate floor derivative on the binomial tree (Table 7), we get a value of 1.61, which is the difference between the values of the deferred annuities with (98.76) and without (97.14) a crediting floor.

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Figure 3: Deferred annuity with 3.0% crediting rate floor
+10bp yield curve shock
Crediting rate is greater of current rate less 100 bp and 3.0%

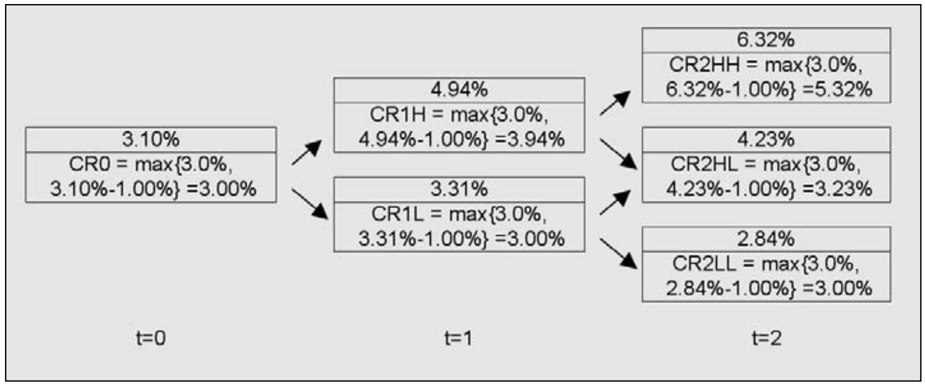

Table 6: Calculating the duration

$$
\begin{aligned}
D &= -(1/P)*(P+ - P-)/(2*\Delta i)\\
&= -(1/98.76)*(98.61 - 98.91)/(0.0020)\\
&= 1.5
\end{aligned}
$$

Table 7: Interest rate floor derivative

Notional	100
Maturity	3 years
Strike rate	4%

P(HH) = 100 * { max[0,0.0400 – 0.0300] / (1.0300)
+ max[0,0.0400 – 0.0482] / (1.0300*1.0482)
+ max[0,0.0400 – 0.0617] / (1.0300*1.0482*1.0617) } = 0.97

P(LL) = 100 * { max[0,0.0400 – 0.0300] / (1.0300)
+ max[0,0.0400 – 0.0323] / (1.0300*1.0323)
+ max[0,0.0400 – 0.0277] / (1.0300*1.0323*1.0277) } = 2.82

In similar manner, P(HL) = 0.97 and P(LH) = 1.69

P = { P(HH) + P(HL) + P(LH) + P(LL) } / 4 = 1.61

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We can take this a step further by considering duration dollars. The duration dollars of an instrument is its price times its duration, and by equating the duration dollars of assets and liabilities, managers can insulate surplus from interest rate movements. Just as the price of the deferred annuity with crediting rate floor equals the sum of the prices of its two components, its duration dollars equal the sum of the duration dollars of the components.

After computing the duration of the interest rate floor derivative with our OAA approach to be 94.4, we can summarize the decomposition as shown in Table 8.

Table 8: Decomposition of annuity with crediting rate floor

Annuity w/ floor	=	Annuity w/o floor	+ interest rate floor
P(Ann w/ floor)	=	P(Ann w/o floor)	+ P(int rate floor)
98.76	=	97.14	+ 1.61
Dur$(Ann w/ floor)	=	Dur$(Ann w/o floor)	+ Dur$(int rate floor)
1.5*98.76	=	0*97.14	+ 94.4*1.61

Actuaries should be aware of the effect of crediting floors on duration when designing and managing products. It is also important to keep in mind that interest rate floors, and therefore liabilities with embedded floors, have high convexity, which adds a further challenge in managing interest rate risk. These challenges are especially important in the low rate environment that we continue to experience.

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