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Interest Rate Regimes—An Empirical Description

by Joseph Koltisko

What is a "regime" in the interest rate environment? If the regime changed, how would you know? For long-term descriptive interest rate models, it is useful to develop a concise view about this. With it, we can produce internally consistent scenario sets for a given regime. We can investigate the financial consequences of moving from the current regime to alternate ones. Our views about the likelihood of transitioning from one regime to another can also be incorporated into a stochastic stress test. With a view to these applications, this article proposes a way to describe the interest rate regime.

As many have pointed out, a statistical technique called "principal components analysis" (PCA) provides a remarkably stable result. Changes in the yield curve can be decomposed into shifts in "level," "steepness" and "curvature." This is the starting point for investigating changes in the U.S. yield curve. The monthly Constant Maturity Treasury (CMT) series from the U.S. Federal Reserve Web site was used. This site provides market rates for each month since 1953.

Now, by itself, a principal components representation for the yield curve is inadequate for a term structure model. On one hand, any change in the yield curve can be represented with a linear combination of the PCs, for instance, 3 * level vector + 2 * steepness vector. These multiples (3 and 2) are sometimes called changes in the "state variables." A typical attempt would be to assume changes in the state variables are independent identically distributed normal random variables, with mean reversion. Perhaps enough constraints can be found on such a model to make it useful for risk reporting. The usual, unsatisfying result when we attempt to use PCA for a term structure model is that the curve becomes too inverted or humped, while rates may become negative or very high. The constrained mean-reverting multivariate normal model must be missing something critical.

Clearly, the PCs themselves are only building blocks. The interesting part of the model comes when we specify how the state variables change over time. As input to such a model, we should first observe the actual path for the state variables in the CMT data.

Some preparation work is needed first. Given the monthly CMT series, I interpolated the zero coupon bond prices for all points between time zero and 20 years. For this, I used a Lagrange interpolating polynomial with anchors at the beginning, middle and end of the observed data. With the zero coupon bond price at time t for a unit payment at time T, P(t, T), we can calculate the associated continuous spot rate series $r(t, T) = -\ln[P(t, T)] / T$. I used the r(t, T) series as the yield curve for that month.

PCA applies to the change in the yield curve. I used the matrix M of absolute changes (in bp) at each time t, rather than the percentage change or normalized percentage change. The square matrix $A = M^T \times M$ is related to the variance-covariance matrix for the original data series. Any data point (change in the yield curve at time t) can be represented with a column vector $\mathbf{v}^T$, and this in turn can be expressed as A**j** for some **j**.

$$A = M^T \times M$$
$$A' = \text{inverse of } A$$
$$A'\mathbf{v}^T = \mathbf{j}$$

Now we can represent the data point **j** more efficiently. The columns of A form a basis that spans the original data set. We can transform A into a set of orthogonal vectors (eigenvectors) that also spans the data set. The PCs are the largest eigenvectors of $M^T \times M$. Thus any of the observed data in M can be represented with a linear combination of the PCs.

If $A\mathbf{w} = \lambda\mathbf{w}$ then **w** is an eigenvector of A, and λ is the associated eigenvalue. The size of λ tells us how much of the variation in A can be represented with each eigenvector. Table 1 and Figure 1 show the results of this analysis.

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From Page 9

Many statistical packages can calculate principal components. These results are reasonably in line with expectations. Note we can choose the scale and sign for these factors as we like, since the same data point results when we make equal and opposite adjustments to the state variables. In Table 1, they have been scaled so that the monthly change in the state variables has roughly a variance of 1.

The next step was to use these building blocks to describe the original spot rate series r(t, T). Since any change in r(t, T) can be represented with a linear combination of the PCs, so can the original series. For this I used Excel solver to back into the state variables at each point. The full range of data over the 50-year period can be expressed this way. For example, the state variables for three representative dates are shown in Table 2.

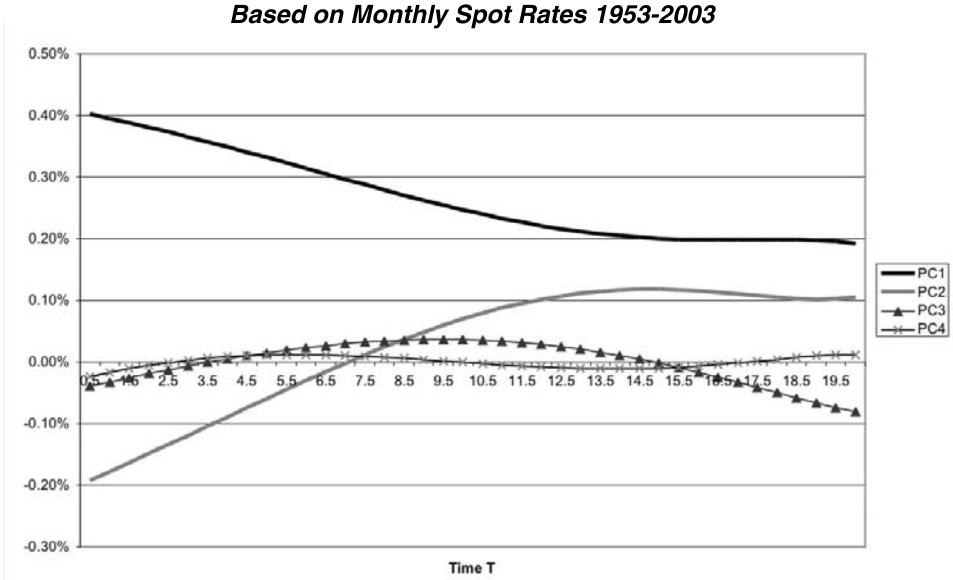
These are (respectively) low/steep, inverted and high/flat. The first two PCs (in this implemenation) explain 98 percent of the estimated spot rate data, so the rest of this analysis only uses the first two PCs. Now, this may compound errors in the smoothing process. The reconstituted result may be off from actual historical data by 30bp or more at intermediate points. However, this forces us to look for patterns in two dimensions.

Most people appreciate visual patterns best. It is insightful to present the state variable series as a graph, with height (y-axis) as a function of steepness (x-axis). This representation is sometimes called the "phase plane." Any point on the plane represents a particular state of the yield curve. We can move up, down or sideways from a given state to the next one. In Figure 2, "up" means greater Height factor, "right" is greater Steepness.

Table 1: Principal Component Vectors

	PC1	*PC2*	*PC3*	*PC4*
0.5	0.40%	-0.19%	-0.04%	-0.02%
1	0.40%	-0.18%	-0.03%	-0.02%
2	0.38%	-0.15%	-0.02%	-0.01%
3	0.37%	-0.12%	-0.01%	0.00%
5	0.33%	-0.06%	0.01%	0.01%
7	0.30%	0.00%	0.03%	0.01%
10	0.25%	0.07%	0.04%	0.00%
12	0.22%	0.10%	0.03%	-0.01%
15	0.20%	0.12%	0.00%	-0.01%
20	0.19%	0.10%	-0.08%	0.01%
Significance	86.6%	12.0%	1.3%	0.1%

Figure 1: Principal Components of U.S. Yield Curve
Based on Monthly Spot Rates 1953-2003



This tells us a lot. Even though changes in the state variables are uncorrelated (by definition of PCs), Figure 2 shows a link between steepness and height. Steepness follows height closely, with much of the variation being around the center line where the two are equal. This makes intuitive sense: When rates are very high, the steepness/inversion factor has the same relative size but may be much higher than normal in terms of the absolute 10yr-2yr yield difference.

Note the areas in the lower left of Figure 2. This represents the period from 1953 to 1963. There seem to be two distinct periods here in which rates stayed about the same "distance" from the origin, but that the "angle" for the data point shifted back and forth.

This suggests we should transform the data series of steepness-height state variables into polar coordinates. Define:

Steepness	S
Height	H
Distance	D = sq root($D^2 + H^2$)
Angle	A = arc tan (H/S) in radians

The resulting time series contains some interesting regularities.

Again, the path of rates is a continuous, connected figure in the D-A phase diagram. The 1953-1963 period as seen in Figure 3, now shows up as relatively vertical lines to the left of the chart. It seems that during the last 50 years, the pattern changes every five years or so. Three such periods are shown in Figure 4. This is what I would like to capture with the concept of a "regime" of interest rates.

Here are the "stylized facts" that seem evident from this representation shown in Figure 4.

turn to page 12

Table 2: State Variables for Three Dates

State Variables

	May-54	Aug-69	Dec-00
PC1	7.07	23.47	18.77
PC2	10.65	11.68	13.04
PC3	(2.16)	(3.02)	(10.47)
PC4	5.47	(5.89)	(4.20)

Figure 2: Steepness-Height Phase Diagram for U.S. Yield Curve
Monthly 1953-2003

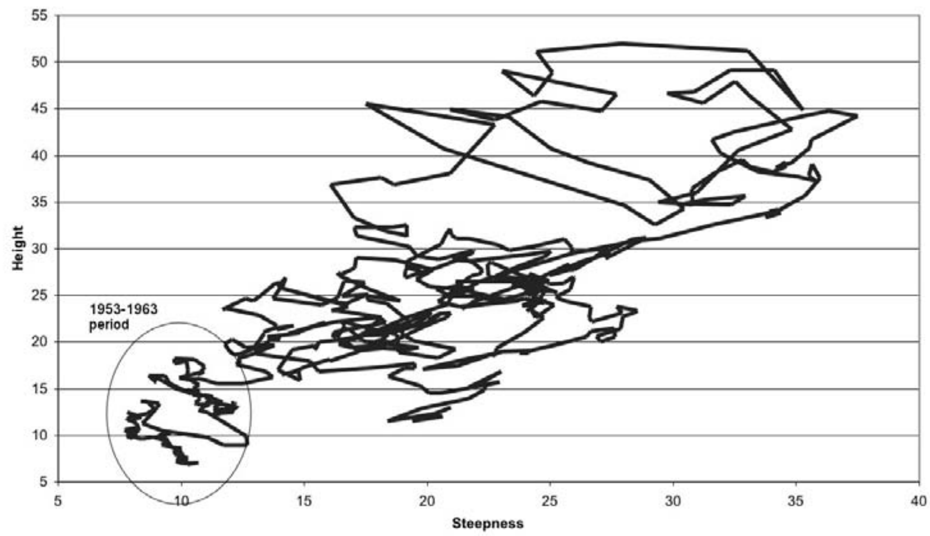
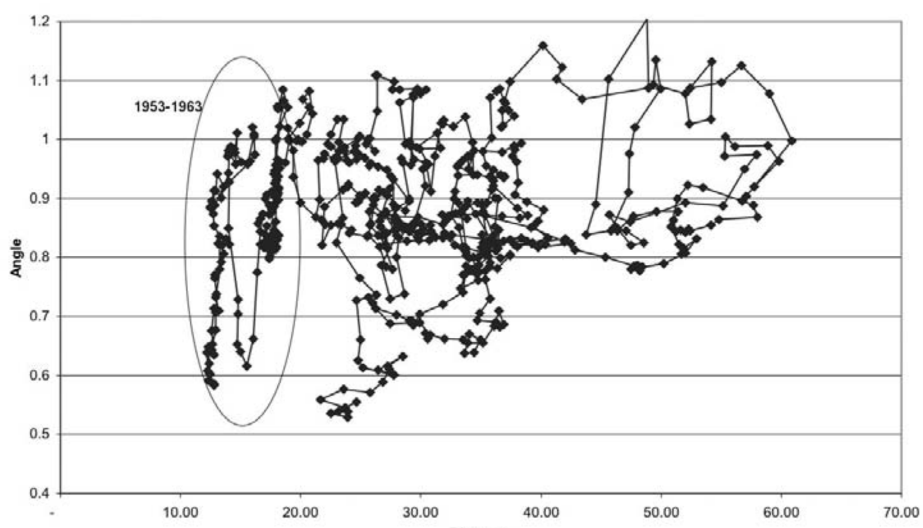


Figure 3: Distance-Angle Diagram of U.S. Interest Rates



From Page 11

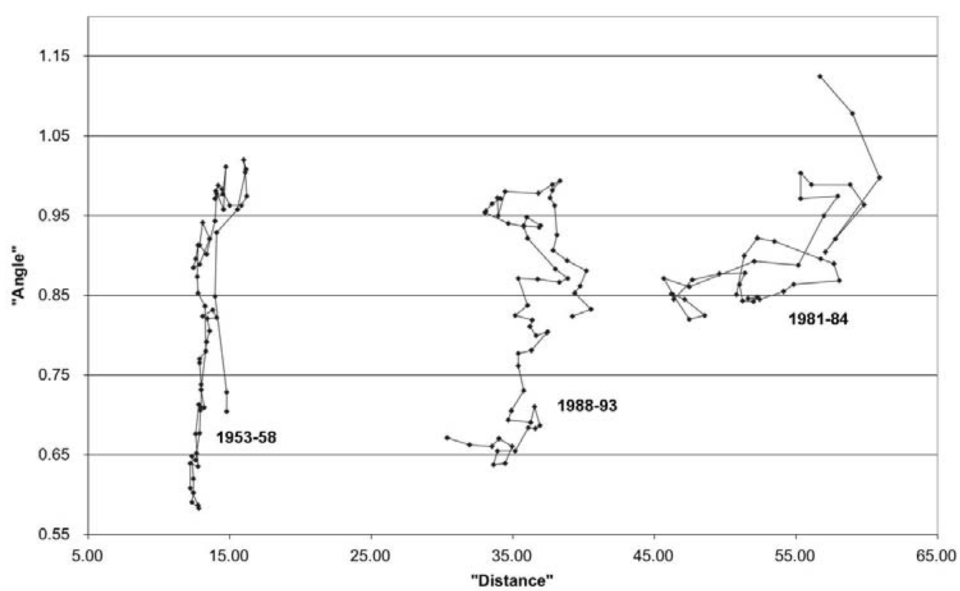
- The state of the yield curve at any time is specified as a linear combination of the two PCs given in Table 1, where H (height) is a multiple of PC1, and S (steepness) is a multiple of PC2.
- We can transform the time series in (S,H) coordinates to a series in polar (D,A) coordinates.
- The path of the yield curve under both coordinate systems can be assumed to be continuous.
- D has ranged from about 12 to 60
- A has ranged from about .5 to 1.2
- During an interest rate "regime," the yield curve stays in a smaller region of the D-A phase plane anchored on a line.
- The slope of this line varies with D, from about .09 when D = 12 toward zero for D > 45.
- The yield curve stays in a given regime for three to six years, then transitions to another similar one.

By "anchored" I mean that the path of a point in the D-A phase plane will differ from the line by a stochastic component. It may speed up or reverse course or move laterally for while and then resume course. All the math used to describe motion in two dimensions can be applied to develop a specific model form. For example, one approach would be to start with stochastic differential equations for oscillating motion in X and Y, such as

$$d^2X/dt^2 = - uX + \sigma_D Z_1$$
$$d^2Y/dt^2 = - vY + \sigma_A Z_2$$

with initial conditions X(0), X'(0), Y(0), Y'(0) and volatility parameters σ_D and $\sigma_A$and then rotate/translate the resulting figure to the desired line in the D-A plane.

Figure 4: D-A Phase Plane for U.S. Interest Rate Regimes



Note that D mainly controls the level of interest rates, while A mainly controls the amount of inversion. A slope of .09 results in most of the movement occurring at the short end, while a slope of zero results in a parallel shift.

Here is an example of the yield curves along an "anchor" line, in which D moves from 20 to 25 in increments of 1, while the angle A follows the line

$$A = -1.05 + .08*D$$

as seen in Figure 5.

We'll stop here with this qualitative description of an interest rate regime. Though convoluted, this derivation of the form of an interest rate model has a number of advantages:

- Scenario output can be easily compared to actual historical levels for D and A.
- It is straightforward to extrapolate to very low and very high interest rates and still preserve reasonable relationships between steepness and height. The same model can apply for Brazil as for Japan.
- We can specify parameters for several regimes and allow the model to transition from one to the next
- We can distinguish "velocity" of movement within a given regime, from transitional movement to a new regime.

Figure 5: Interest Rate Regime Anchor Line