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Deflators—The Solution to a Stochastic Conundrum?

by Don Wilson

Stochastic modeling of life insurance products has become increasingly important over the last few years. The complex nature of the guarantees that exist in many products has generally required the use of a Monte Carlo approach, involving the calculations being performed repeatedly for each scenario, potentially many hundreds or thousands of times.

As this modeling has evolved, it has divided down two paths—real-world and risk-neutral. In this article, I discuss this division and show how these paths may be re-united through the use of deflators.

Path 1—real world

In many stochastic applications, the requirement is to test the robustness of product design or business strategy and to quantify the range of possible financial outcomes. For this type of application, the scenarios must represent the real world. By this I mean that the outcomes for each scenario produced by the stochastic economic generator used must represent a path that could occur in the future. The range of the scenarios represents the population of possible future outcomes.

Path 2—risk neutral

Increasingly, the valuation or pricing of a product option or guarantee, benefit, line of business or company requires a stochastic process for the full financial intricacies to be captured. For many applications, the fundamental requirement is that there is consistency with the techniques used to value or price assets—so that both sides of the balance sheet are consistent. As a consequence, there is a requirement that the economic scenarios be "risk neutral." When such scenarios are used, discounting the projected cash flows at the risk-free rate appropriate for each scenario and taking the mean gives a value or price that is consistent with a market valuation of the assets.

I do not propose in this article to dwell on the technical differences between these two types of economic scenarios, nor to discuss how to build the stochastic generators. Most practitioners do not need to be experts on such matters and there is much published material already available. What is most important is the ability to

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understand that there are differences and to make sure that the right type of generator is used in each application.

Discounting the projection results from real-world scenarios does not give a "market-consistent" valuation—a valuation which measures riskiness in the same way as the capital markets. Such a valuation is needed for contracts with guaranteed death or living benefits; for instance, if you want an indication of the potential cost of hedging. Some sort of value may be obtained by taking, say, the 75% conditional tail expectation (CTE) where the x% CTE is defined as the average of the worst (1-x)% scenario outcomes. This gives a single numeric value that reflects what is happening in the tail of the distribution. However, the choice of appropriate CTE level is not obvious and the result obtained is not necessarily consistent with asset valuations. Moreover, such scenarios are generally not arbitrage free and there is no consensus for the discount rate to be used.

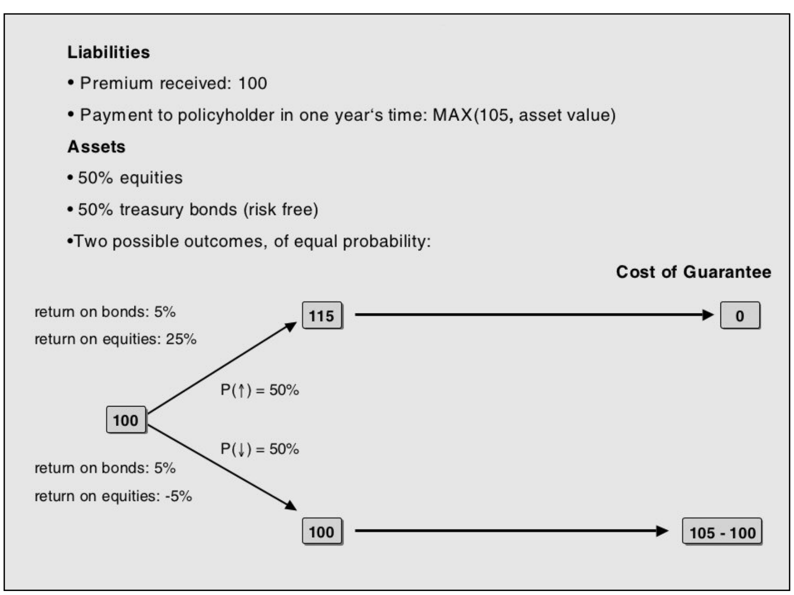
Deflators, or more precisely state-price deflators, bridge the divide between real world and risk-neutral scenarios.

Inspection of the individual scenarios in a risk-neutral valuation gives no insight into the financial dynamics of the model. Percentile distributions of the outcomes are meaningless and it is impossible to work through the calculations for individual scenarios to satisfy yourself as to their reasonability. They are an artificial construct intended purely to produce a market valuation.

This divide into two paths would not matter greatly if the modeling requirements were always similarly divided so that the appropriate technique could easily be chosen and applied. However, this is not the case. For instance, some companies are now seeking to use an economic capital methodology to determine capital allocation, calculating the liabilities using a "fair value" (i.e., market consistent) measure. This method of valuation of the liabilities requires a risk-neutral type approach but the capital requirement is usually determined to achieve an x% probability of insolvency in y years (where x and y are determined by the company to reflect their desired position in the market) and, therefore, requires the use of real-world scenarios.

Luckily, a solution is at hand—deflators. Deflators, or more precisely state-price deflators, bridge the divide between real-world and risk-neutral scenarios. In short, they may be used to calculate market consistent valuations of any cash flow stream using real-world scenarios. In the next section, I describe in more detail what they are and show how they work. Then in the rest of this article, I give a practical example of their application.

Figure 1: Simple Model of a Guarantee



What are deflators?

To define deflators and to contrast them with a risk-neutral valuation, let us consider a very simple model—see Figure 1. This simple model provides a minimum guaranteed return to the policyholder at the end of one year. The premium is invested in assets assumed to have two equally likely outcomes. One pays out more than the minimum required, but the other leaves a shortfall. The question we wish to address is, given these assets, what is the value of the policyholder guarantee?

We start with the risk-neutral approach. Figure 2 outlines the construction of the risk-neutral probabilities that need to be assumed in this model to avoid the possibility of arbitrage. These probabilities are then applied in Figure 3 to derive the risk-neutral value of guarantee.

With this under our belt, I set out a definition of deflators in Figure 4. Technically speaking, deflators are path-dependent stochastic risk discount factors. Separate factors are associated with each real-world scenario. Their effect is to put a greater emphasis on those scenarios in which risky assets perform badly. The riskiness and downside aversion that is experienced in the market valuation of assets is absorbed within the deflator values. This contrasts with risk-neutral valuations, where it is absorbed within the economic scenarios themselves.

We can apply the definition of deflators in Figure 4 to construct the deflator values for our simple model. Applying them (see Figure 5) leads to a value of guarantee that, as we would expect, is the same as that calculated using a risk-neutral valuation. The value of the guarantee is the expected value of the deflated cash flows. You can think of this value as being equal to the value of the hedging portfolio that you would need, assuming that such a hedging portfolio is available to close out the risk completely.

Unfortunately, the construction of deflators is not normally this simple. They cannot just be derived on top of existing sets of scenarios as additional streams of values. You need a stochastic economic generator that has been purpose-built to generate the deflator values alongside its other simulated economic outcomes (interest rates, equity returns, inflation indexes, etc.). Given this, you're all set!

One hugely important property of deflators is that the values are dependent only on scenario and time. The values are independent of the assets and liabilities to which they are applied. This means that they can be used to put a value on any stream of cash flows that varies according to the economic assumptions used. The market-consistent valuation of these cash flows is always the mean value of the deflated cash flows.

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Figure 2: Risk-Neutral Probabilities

In order to obtain no arbitrage opportunities, two conditions for risk-neutral probabilities must be preserved:

- Normalization:

$$P_{RN}(\uparrow) + P_{RN}(\downarrow) = 1$$

- Conservation of market efficiency:

$$\text{initial portfolio value} * P_{RN}(\uparrow) * (1 + \text{return}(\uparrow)) + \text{initial portfolio value} * P_{RN}(\downarrow) * (1 + \text{return}(\downarrow)) = \text{initial portfolio value} * (1 + \text{risk-free rate})$$

Yielding the risk-neutral probabilities :

$$P_{RN}(\uparrow) = \frac{\text{risk-free rate} - \text{return}(\downarrow)}{\text{return}(\uparrow) - \text{return}(\downarrow)} \qquad P_{RN}(\downarrow) = \frac{\text{return}(\uparrow) - \text{risk-free rate}}{\text{return}(\uparrow) - \text{return}(\downarrow)}$$

Figure 3: Risk-Neutral Valuation

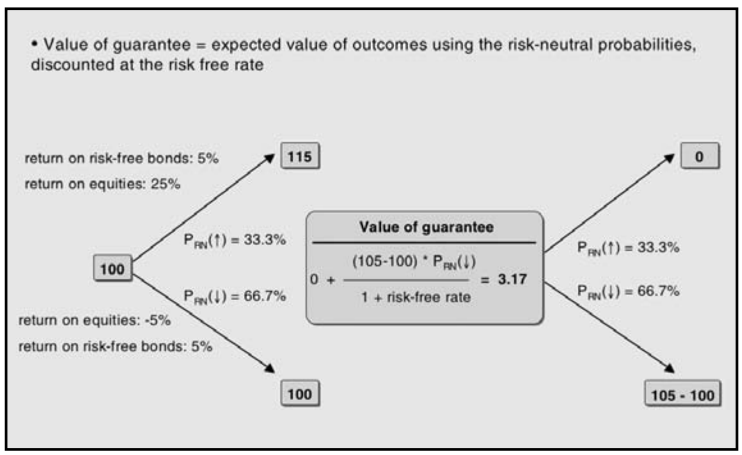

Figure 4: Definition of Deflators

Using risk-neutral probabilities P_{RN} and discounting with risk-free rates corresponds to using real-world probabilities P and discounting with deflators D:

$$D(\uparrow) * P(\uparrow) = \frac{1}{1 + \text{risk-free rate}} * P_{RN}(\uparrow)$$

$$D(\downarrow) * P(\downarrow) = \frac{1}{1 + \text{risk-free rate}} * P_{RN}(\downarrow)$$

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More background on deflators and the theory behind them may be found in "Modern Valuation Techniques," by Stuart Jarvis, Frances Southall and Elliot Varnell. This paper was presented to the Staple Inn Actuarial Society in the UK and copies of it may be downloaded from The Smith Model Web site at *www.thesmithmodel.com.* This award-winning paper is highly recommended.

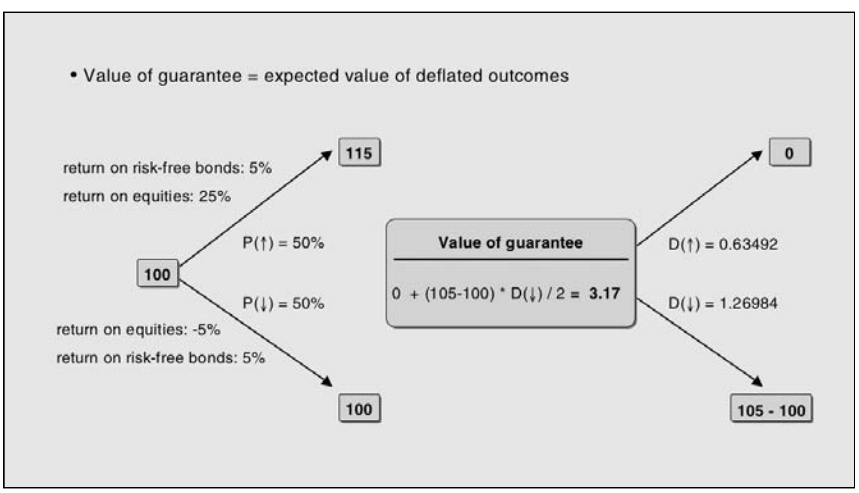
With the market downturn in 2000, the benefits which had been offered for little or no additional cost have moved significantly into the money and threaten to cause measurable financial pain to an industry coming out of several years of record sales.

I now move on to describe a practical application of deflators—the valuation of variable annuity guaranteed income and death benefits. This is based on a real project (the values have been changed) performed very rapidly by my colleagues, and I am indebted to Jason Grosse for his help in building the model.

Background to practical example

During the market boom of the late 1990s, the issuance of variable annuity contracts with rich guaranteed benefits thrived. With the market downturn in 2000, the benefits which had been offered for little or no additional cost have moved significantly into the money and threaten to cause measurable financial pain to an industry coming out of several years of record sales.

Figure 5: Deflator Valuation



As a consequence, many companies moved the valuation of these guaranteed benefits to the top of their to-do list and focused on the accurate valuation of these benefits. While a Monte Carlo approach is necessary in this exercise, questions remain as to the choice of scenarios and discount rates.

Some companies use real-world scenarios, discounting either at a risk-free rate plus a spread (e.g. the 90-day Treasury plus X bps), or at the spot rate, or at a level rate, representing the company's cost of capital for all years. The results from these scenarios are then analyzed to come up with a distribution of potential guaranteed benefit costs. This is reasonable for strategic planning and capital allocation. However, as discussed earlier in this article, it does not provide an accurate market consistent value.

Other companies use risk-neutral scenarios and discount at the risk-free rate. This approach gives a market-consistent valuation but has the disadvantage that it gives no strategic insight into the future. This is because the individual scenarios do not represent possible paths through the future, unlike the real-world scenarios.

Our approach was to use deflators. The guaranteed benefits can be thought of as policyholder options, which are valued much like equity put options. A market-consistent valuation is the expected value of the deflated excess of the guaranteed benefit cash flows over the funded account values and represents the current cost of hedging all of the market risk associated with these guaranteed benefits. To achieve this valuation, we used The Smith Model (TSM) stochastic economic generator. This generator produces market-consistent, arbitrage-free scenario sets that include deflators. More information is available at The Smith Model Web site.

Comparative scenarios

We also used an alternative set of scenarios in this project. The second set was a subset of the scenarios recently published by the American Academy of Actuaries (AAA) and made available at *www.actuary.org/life/phase2.htm*. They were generated using a regime-switching lognormal generator and intended primarily to meet the recently published C-3 Phase 2 RBC requirements. The AAA scenarios are not arbitrage-free, and as stated in the documentation supplied by the AAA, should strictly not be used to price securities or derivatives, or in this case, liability cash flows. They were included in the project to demonstrate how the valuation result could differ based on the source of the scenarios. The cash flows produced using these scenarios were discounted at a flat rate of 8 percent, representing an assumed cost of capital.

The observed mean return and volatility assumptions for the equity fund modeled in each set of scenarios are shown in Table 1.

Table 1

Measure	Academy	TSM
Mean	10.62%	10.41%
Volatility	16.18%	23.60%

The mean values in this table reflect the geometric average annual rate over 30 years for all scenarios in each set. You can see that volatility of the TSM scenarios is much higher than that of the Academy scenarios. This is because the Academy scenarios were calibrated using historical volatilities whilst the TSM calibration used an implied volatility consistent with current market conditions at the valuation date. The observed value is slightly higher than the input assumptions due to the effect of convexity. We could have calibrated TSM using historical volatilities, though this would have been inappropriate for this project.

Table 2

Scenario Set: Discount Rate:	Academy 8%	TSM 8%	TSM Deflators
Measure	**Results**		
Mean	66.0	100.6	234.6
Min	24.3	23.3	1.4
Max	295.7	430.8	7374.4
Standard Deviation	46.3	66.0	642.1
CTE	**CTE of Combined GMDB & GMIB Benefit Costs**		
95 CTE	295.7	362.4	
90 CTE	215.1	287.3	
75 CTE	181.7	251.7	
50 CTE	129.8	196.8	
25 CTE	92.7	148.6	
5 CTE	76.4	120.4	
1 CTE	68.0	104.2	
	66.4	101.3	

Amount in $ millions

Results

The results of the calculations, using in-force policy data similar to that used in the real project, are displayed in Table 2. To determine a valuation from the Academy scenarios, we used CTEs. We also included a "middle" set of calculations to quantify the extent to which the results are driven by the scenarios themselves or by the discount rates. The higher CTEs calculated for the TSM scenarios discounted at 8 percent are primarily a result of the higher volatility.

One advantage of the deflator approach is immediately apparent. It gives a suitable value directly, without any need first to decide on an appropriate CTE level. It condenses the results to a single number, the mean value in this table.

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You can also see from the values in this table that the result obtained from using deflators places a far higher value on the guaranteed benefits than taking the mean value after discounting using a constant discount rate. It is equivalent to a CTE in excess of 95 percent using the Academy scenarios.

Conclusions

The use of deflators made the job of accurately estimating the cost of the guaranteed benefits very easy. The information we obtained by attempting a comparative valuation using an alternative methodology suggests that valuations based on a CTE approach may differ greatly from market-consistent valuations. This exercise also demonstrated that the cost of hedging the benefit guarantees may well be significantly higher than was previously thought. For reference, see Richard Q. Wendt's article, "An Actuary Looks at Financial Insurance" in the May, 1999 issue of *Risks and Rewards*.

Summary

In this article I have attempted to explain deflators in a simple, nontechnical way. Along the way I have:

- Shown how a stochastic valuation made using deflators differs from a risk-neutral valuation;
- Explained how deflators enable stochastic valuations to be made using real-world scenarios;
- Indicated why this is useful and may indeed be necessary in some circumstances;
- Highlighted a number of potential pitfalls that may arise if a CTE approach to valuation is used without due care;
- Demonstrated that the application of deflators is straightforward;
- Suggested that, if or when you start to adopt market-consistent valuation techniques for life insurance liabilities, you may find the results disturbing.