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**PRICING AND THE
STATEMENT OF ACTUARIAL OPINION**

Moderator: ARNOLD A. DICKE
Panelist: RICHARD F. FISHER
 RONALD S. LEVIN
 MICHAEL R. TUOHY
Recorder: HENRY B. RAMSEY, III

- o Coordinating pricing and valuation roles
- o Communicating product specifications
- o Setting mutually acceptable assumptions
- o Reflecting risk in pricing and valuation
- o Investment policy and interest-crediting policy
- o Reflecting a volatile economic environment

MR. ARNOLD A. DICKE: Our first speaker will be Rick Fisher, who is associate actuary at Northwestern Mutual Life and has prepared the opinion for that company relative to New York's Regulation 126. Rick is also chairing the Traditional Life Subgroup of the Special Advisory Council to the NAIC, which is considering changes in the standard valuation law.

Our second speaker will be Mike Tuohy, who heads the New York office of Tillinghast/TPF&C. Mike now concentrates on valuation issues but previously was deeply involved in the pricing of variable and universal life products.

Our third speaker will be Ronald Levin. Ron has specialized in asset-liability matching, first at the Equitable, then at the Metropolitan Life, and now at J. P. Morgan Securities. He now concentrates in option products and asset-liability strategy.

PANEL DISCUSSION

MR. RICHARD F. FISHER: I have been asked to relate my company's experience in complying with Regulation 126 and comment on the impact of these efforts on our pricing.

Our first decision had to be whether to do the work or to set up the penalty reserves and defer the work one year. Of course, Regulation 126 applies to annuity reserves. The company had \$2.1 billion of annuity reserves as of year-end 1986, which was 13% of reserves. Although annuities were definitely a minority product line for us, we decided not to put off doing the work, since it was a substantial amount.

The \$2.1 billion includes all years of issue. We decided to file the opinion for all years of issue to keep the work simple. We knew that we had conservative reserves above statutory minimums and high quality assets, so we expected no problems. These annuity reserves are all individual annuities; \$0.8 billion are variable annuities, for which no calculations are needed, and \$1.3 billion are fixed dollar annuities.

During 1986, we had been working on a plan of segmentation of assets for annuities. Segmentation is desirable because it permits a tailored investment strategy for each segment and facilitates calculations required for actuarial opinions. We divided the general account into two segments: one for annuities, and the other for life insurance and disability income. We wrote the segmentation plan for New York approval in approximately six months and filed it on October 31, 1986; it was approved February 26, 1987.

The next step was to select the assets to be in the annuity segment and to tag them in the database. Assets selected had a market value equal to the annuity reserves times the company's overall ratio of market value to book value of assets. This was our method for ensuring equitability to all policy-owners in the segmenting of assets. The actual selection and tagging was handled by the investment department, and, I am told, was a minor effort. The assets that we put into the annuity segment were all debt instruments -- mainly publicly traded. The Macaulay duration of these assets was about 4.7 years.

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The next step was to develop new computer programs to prepare the opinion required under Regulation 126. We developed three liability cash flow projection models that were PC programs: one for front-end loaded dividend paying deferred annuities, another for back-end loaded current interest rate deferred annuities, and a third for immediate annuities and supplementary contracts. This was about a three-month effort. The cash flow projection for existing assets was supplied by the investment department, which coincidentally had been developing a dynamic model of their own over the preceding nine months. We also developed a PC program that invested the net cash flows and produced income statements and balance sheets into the future. That was, perhaps, a two-month effort.

There were many assumptions to be made in order to produce the opinion. We had to make assumptions about future premiums, lapses, our competitor's rates, calls, prepayments, defaults, investment strategies, crediting strategies, expenses, federal income taxes, and interest scenarios. (I will make just a brief comment about each.)

We assumed no future premium payments on flexible premium contracts because that is the assumption used in computing reserves. Note that this valuation assumption differs from what might be the pricing assumption.

We had to make some assumptions about our policyowners of fixed dollar annuities exercising their put option by lapsing. This is a variable option for them if interest rates rise. Our dynamic lapse assumption was a formula from another actuarial presentation that looked reasonable to me. Mike Tuohy pointed out to me that I picked it up from one of his presentations. It is based on the difference between an assumed competitor's rate, our credited rate, and the surrender charge. We assumed that the competitor's rate was a function of treasury yields.

The following chart shows the formula we used, and the lapse rates that the formula produces for a 0% and 7% surrender charge.

Our crediting strategy was one that produced a fairly current rate so that the highest lapse rate under the seven New York prescribed paths was only 21%. Under one extra extreme scenario that we tested, we did hit 49%.

PANEL DISCUSSION

DYNAMIC LAPSES

$$10\% + 2 (X - Y)^2 - Z$$

Where X = Competitor's Rate (%)
 Y = Our Rate (%)
 Z = Surrender Charge (%)

X - Y	Lapse Rate	
	Z = 0%	Z = 7%
0%	10%	3%
1	12	5
2	18	11
3	28	21
4	42	35
5	60	53

Our call and prepayment assumptions were as follows: For bonds, private placements, and mortgage loans, we assumed 100% calls and prepayments whenever market rates fell below coupon rates by more than a certain amount. This was done on a security-by-security basis. The certain amount varied by year to reflect call protection features. For mortgage pass-through certificates, we assumed a schedule of accelerated annual prepayment rates based on the difference between the coupon rates and market yields.

Our default assumption was that cash flows would be reduced due to default, by 10 to 150 basis points, depending on the scenario. We relied on the investment department's call, prepayment, and default assumptions. The investment and crediting strategy assumptions were constant over the projection period. This was a simplifying assumption.

Our assumed expense factors were higher than our pricing expense factors because the opinion states that we should allow reasonable margins for adverse deviations. We assumed that the current federal income tax formula would remain unchanged over the projection period.

The scenarios we tested were the seven prescribed paths plus another 13 more extreme paths. The 13 more extreme paths had the same pattern, but with larger and faster changes in interest rates and some flattening and inversions of the yield curves in the rising scenarios. The ratios of yields on corporate debts to treasury yields was kept constant over the interest paths. New York Regulation 126 refers to a 4% floor. Different interpretations of that 4% floor produced

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very different scenarios. We decided to interpret that to mean a floor on treasury yields at all durations.

The results of our testing were that we had adequate reserves and assets under all 20 scenarios. There were some offsetting risks between deferred and immediate annuities. For example, a given interest path might result in a higher ending surplus for deferred annuities and a lower ending surplus for immediate annuities as compared to another interest path.

The ending surpluses under all of the varying scenarios were worse than under the level scenario. The ending surpluses varied from 14% to 95% less than the ending surplus under the level scenario. On average, the ending surplus was 65% of the ending surplus under the level scenario. It is obvious that a pricing assumption of level interest rates is overly optimistic.

Now, I will make a few comments about the impact of this Regulation 126 work on pricing. I think that the relationship between valuation actuarial opinions and pricing can be described as a loop. Pricing practices drive the actuarial opinion. That is, a valuation actuary has to observe the pricing practices in forming his opinion. Thus, this starts the movement through the loop. His opinion may result in extra reserves being required which puts pressure on management. By management, I mean the CEO and the Board of Directors. The surplus strain caused by the extra reserves should force management to apply pressure upon the pricing actuary for more pricing discipline. This will cause him to change the pricing practices, and that starts the next round through the loop. Thus, ideally the exercise of complying with Regulation 126 will result in companies revising their pricing practices and more pricing discipline for most companies.

So far, our company is traveling through the loop for the first time, and we are just learning the relationships. These relationships require good communication. The valuation actuary has made assumptions regarding investments and rate setting strategy. He must communicate them to the appropriate people and follow up by checking management reports. We have created new internal management reports dealing with investment income by segment and subsegment, cash flows, durations, quality of assets, annuity net income by month, and so on.

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In conclusion, there are definite benefits from this work. Management will understand risks better. We have found the investment department people to be very cooperative and supportive. They are concerned about C1 and C3 risks; the kinds of analyses that we are doing for Regulation 126 are of interest to them. We hope to do more risk analysis in the future so that the risks are better understood.

Rate setting and competition are always a problem. Educating the marketing people as to the product risks is important. They do not want to see the company lose money either, and they will let up on the marketing pressure a little if they understand there is good reason.

I believe that valuation actuary work such as first required by Regulation 126 will lead life insurance companies to sounder pricing practices and reduction of investment risks, thereby reducing future insolvencies.

MR. DICKE: Rick has covered the work he did to actually produce the opinion for Regulation 126. We will follow up with Mike, who is going to discuss a pricing model that takes into account the factors involved in doing the actuarial opinion and the ways that they affect the pricing of the product.

MR. MICHAEL R. TUOHY: We have gone through a real life situation with Rick. I am now going to move us through a hypothetical situation. We are going to look at a block of single premium deferred annuities (SPDA) business, put it through the Regulation 126 ringer, then look at a few more scenarios and feed back that information into the pricing.

First of all, let me describe the block of business we are dealing with. It is a group of males, age 50, buying SPDAs maturing at age 70. The average single premium is \$20,000. The SPDA has a typical surrender charge -- 7% grading down to zero in year eight, and a guaranteed interest rate of 4%. We are holding a statutory reserve equal to the full account value. We have been writing this business since 1984. Ten million dollars of reserves is left from the 1984 issues and so on through 1985 and 1986. The initial credited rate was 10.5% in 1984 and is now down to 8.5% on all blocks of business. As with Northwestern Mutual, we decided not just to do the 1986 business, but all years of issue.

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The basic experience assumptions that we put into the projection were rather standard: \$20 per thousand maintenance expense, withdrawals starting at 5% in the early years, and withdrawals up to 10% when the surrender charge goes away. Once the lives reach age 65, we move them up to 25%. We used the 1965-70 Ultimate Mortality Table, and assumed an income tax rate of 34%. One assumption that is quite important is when you are going to distribute the profits to your stockholders or your policyholders. If during the good years, you are going to take 50% of the profits and distribute them, they are not going to be sitting around for the later bad years. The stockholder dividend assumption is quite important. Jim Geyer and Mike Mateja at Aetna have done a lot of work on the impact of this particular assumption.

These are standard assumptions. Others are more interest-sensitive assumptions. First of all, we assumed a very simplistic investment strategy. Everything is invested in 10-year A rated bonds with 5-year call protection. I think that in the event of the bonds being called, there was a 3% premium, but they can be called after five years. The interest crediting strategy was set in a traditional way, planning to get a steady spread of 140 basis points. We found that this is not easy. After further thought, we felt we needed other constraints on interest crediting strategy. We looked at the competition and insisted that we be within a corridor around the competition. Although we aim for the 140 basis point spread, we never want to credit greater than 50 basis points above, nor less than 200 basis points below the competition. We set these limits when we found that if you get too far away from your competition on any reasonable assumptions, you are going to see business disappearing rather fast. We did not want that to happen to too great an extent. But, if you are going to stay with the competition when that gets ahead of you, you have to reap high rewards when you can credit more than the competition to make any money at all in this business.

We need to define the competition. The competition in this test was the greater of the five-year treasury yield less 25 basis points, or in the case of extreme inversion, 80% of the one-year treasury yield less 25 basis points. This may seem a bit low, but it has reasonably tracked experience over time. For the additional withdrawal formula, we used the same formula Rick showed you earlier: $2 \times (\text{Competitive Rate} - \text{Credited Rate})^2$. Some examples of this are: if you are 1% below the competition, then withdrawals go up 2%; if you are 2%

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below, the withdrawals go up 8%. Because of our crediting strategy, we never get worse than that because we never fall more than 2% below the competition.

There were two other interest-sensitive assumptions. If cash flow is negative, we do not cash in the assets. We borrow at 2% above the 90-day treasury rate, payable quarterly. We assumed the inflation rate to be the three-year treasury rate less 5%. Those are the base assumptions other than the future interest scenarios.

The first work was just what New York told us to do. There are seven interest scenarios that are defined. One is level, three increase, and three decrease. The first increasing one goes up steadily over ten years; then there is the "up/down" one that goes up over five years, and down over five; and finally the "pop up" one that goes up 3% immediately, then stays level. The minimum rate does not apply to those going up, but does apply to the last three, which are sort of the mirror image of scenarios two through four. We were a bit lax in that New York told us to put some inversions in a couple of these scenarios, but in fact, we did not. So, we assumed that the yield-curve went up and then down in parallel fashion.

Now, these scenarios relate to treasuries, and we happen to be investing in A rated bonds, so we developed a formula to take you from a treasury to an A rated bond.

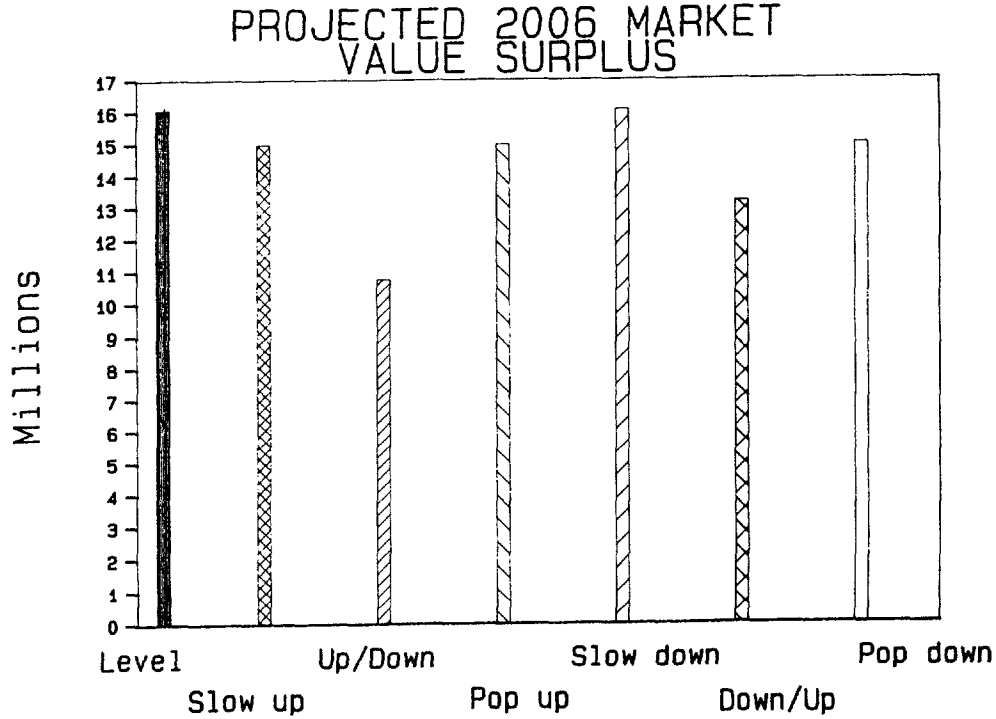
A-RATED BONDS

	<u>90-Day</u>	<u>3-Year</u>	<u>10-Year</u>
Spread	0.500%	0.600%	0.750%
Multiplier	1.024	1.033	1.049

$$\text{A-RATED YIELD} = \text{TREASURY YIELD TIMES MULTIPLIER} + \text{SPREAD}$$

These factors are in the ballpark of experience.

The results we got from those seven scenarios are shown in Graph 1, which shows the market value surplus at the end of 20 years. On this particular block of business, all the liabilities are gone by then because they were maturing at age 70. We assumed that everything was paid out in cash rather than



GRAPH 1

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annuitized, so that this is the market value of the assets at the end of 2006. We started with \$45 million of reserves, and we see a range of \$10.8 to \$16.3 million of surplus from this business. Now, this is 20 years out -- to approximate the present value, divide by four. This gives you a range of \$2.7 to \$4.1 million. Also, remember that we were holding the full account value as the statutory reserve. If we had been holding the surrender value, which may be about 6% less than the account value, the reserve would be in the order of \$2.5 million less, and that would have put the up/down scenario pretty to close to zero. But, we were holding the account value, and the results made us feel pretty comfortable. With the particular investment strategy we have, testing those scenarios put out by the New York Department made us feel that we had easily sufficient reserves.

However, we decided, as did Northwestern Mutual, that we ought to look at some more scenarios. We decided not to hand pick those scenarios. If you do, you can hand pick the scenario that will give you the result you expect. The whole process becomes very subjective. What we prefer to do is randomly generate the scenarios we are going to use by some sort of stochastic process. We randomly generated 40 different scenarios to test this block of business.

We accomplished this process by assuming the ratio of the interest rate in one period to the interest rate in the previous period to be a log normal distribution. A lot of work and research has gone into justifying that assumption. However, you do have to put some constraints on the interest rates. First of all, we randomly projected the 90-day treasury and the ten-year treasury independently, but then put some constraints on their relationship. For instance, we made a constraint that those two rates should never be more than 4% apart. We also set some upper and lower boundaries. The 90-day, we said, would never fall below 2% or be greater than 28%, and the ten-year rate never would fall below 4% or be greater than 28%. We also determined a coefficient of correlation with respect to the direction that each interest rate moves; something like two thirds of the time those interest rates would move in the same direction. So, although we started off with two randomly generated sets of interest rates, the constraints kept them within what we thought were bounds of reasonableness. The last thing we had to decide, having put in those boundary constraints, was the volatility, by which we mean the standard deviation of the log normal distribution.

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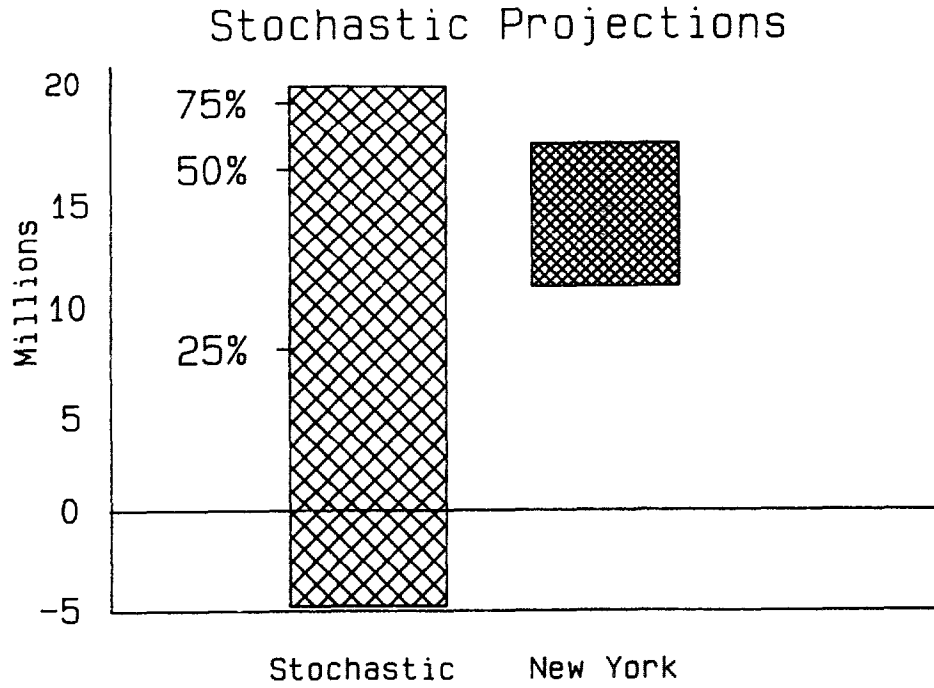
MEASURE OF VOLATILITY

	<u>1965-71</u>	<u>1972-78</u>	<u>1979-85</u>	<u>Assumed</u>
90-Day	.139	.143	.191	.160
10-Year	.072	.045	.098	.800

Here we have some examples of how volatile interest rates have been over the last 20 years. You can see that the volatility in the period 1979 through 1985 is significantly greater than prior years, particularly on the 90-day rates. You can see the 90-day is more volatile than the 10-year. What we assumed in generating our 40 scenarios was an average of what we had experienced over the last 20 years.

What results did we get from putting the projections through these 40 scenarios? Well, it was a much broader spread than we were getting from New York (see Graph 2). The maximum there is 18.3, and the minimum goes well below that negative five. Now, let's just compare that with the narrow range of possibilities suggested by the New York scenarios. Although New York gets us into the mode of doing scenario testing, the narrowness of that band of results suggests that one ought to look at other scenarios in addition to those that are required by New York. The New York scenarios come in around the midpoint and go down as low as the 30% range and get as high as about the 65% range. So, they only cover a middle band. The valuation actuary was so worried that there were so many results significantly below the New York scenarios and that five of them were actually negative, that he for the first time in his life, actually did communicate with the pricing actuary.

The pricing actuary was also worried. He decided to reprice this business using the scenario approach. He needed additional assumptions, so he put in initial commissions and initial expenses. His pricing had traditionally been assuming a 140 basis point spread which gave him a profit margin (discounting present value of profits at issue) of about 1 3/4% of the premium. But, when he put it through the stochastically-generated scenarios, he got some very different pictures. He found that his 1 3/4% was close to the median of the results. However, although there were nearly as many instances of greater profitability as there were less profitability, the good was not very good, and the bad was awful. The mean was negative, and there were some really disastrous situations.



GRAPH 2

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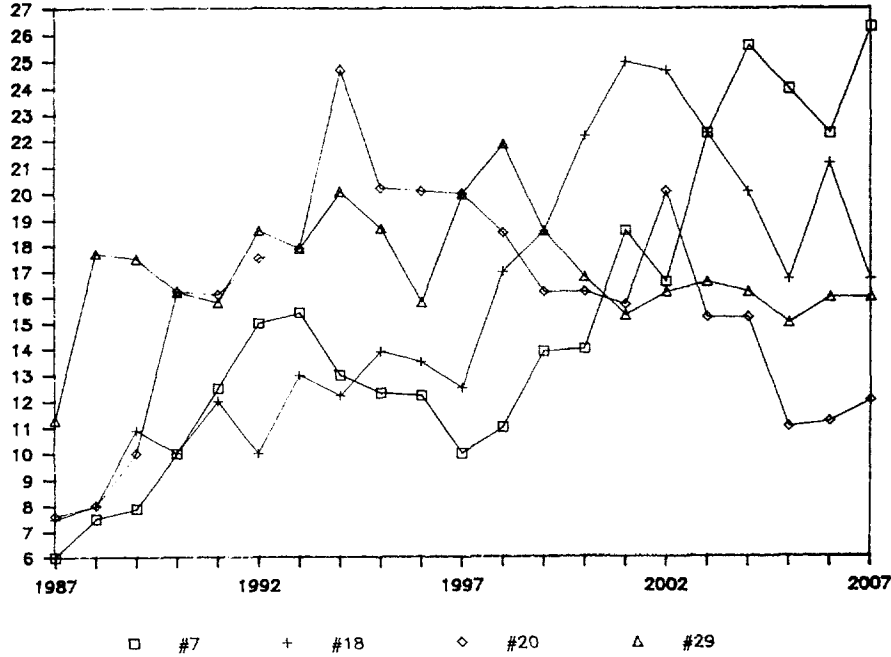
Now, having gotten these results, it became necessary to dig in and find the cause of the really bad ones. What I have pulled out are the four worst scenarios, and as you can see in Graph 3, each of those scenarios may be outside your realm of possibility. All of them have rates that go in excess of 20%. So these are the five-year treasury rates that produce the worst results. The first one produced a -14.4%, the second one down produced a -11.6%, the third, -19.3%, and the worst one of the lot was the number 20 that produced a -24.3%. In addition to these four, I looked at one or two other negative ones, and there was one that came in at a -.4 which did not have interest rates that went into the twenties. So, obviously, this is only as good as the assumptions we used, but I think we were getting some fairly horrific results with scenarios most people would think were within the realm of possibility.

So let us backtrack to look at those profit test results from a pricing actuary's point of view. I think most people would conclude, assuming that they agreed with these assumptions, that this was an unsatisfactory product to put on the market. Your expectancy was to lose money, and you had a chance of severe loss and not too much chance of making much. What you would not do there is throw the product out. I think the first things to look at are the strategies that are being used, either the investment strategy and/or the interest crediting strategy.

Of course, we used a very simplistic investment strategy of ten-year A rated bonds because we knew that it could produce more exciting results than if we had used five-year A rated bonds. Clearly, that is the first thing to revisit with regard to this product. Is that an appropriate strategy, or should we come a bit shorter and see what a five-year investment strategy would give us? This would just be a first step in the pricing process, and clearly you must consider the investment strategy as part of the pricing process for this sort of product.

But, what does this reveal? This all started with a Regulation 126 exercise that showed us some things, and at least got us into doing some arithmetic that was useful. But it did not tell us the full story. As a first step to doing valuation work, I am very much in favor of 126. But to base one's pricing on those scenarios is treading dangerous water. One needs to get into more volatile scenarios to test products.

PROBLEM SCENARIOS (5 Year Treasuries)



GRAPH 3

PANEL DISCUSSION

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I prefer the approach of using stochastically generated scenarios because it can always be argued that you select the scenarios that you are selecting to make the point that you want to make. I think you get around that by stochastically generating those scenarios and picking a volatility that relates back to history. Do you want the last seven years' volatility or the last 20 years' volatility? That has more meaning to me than just hand picking a scenario and then not being sure how likely it is. So, I think a lot of good use can be made of the sort of work that the valuation actuary is doing in New York, and particularly in the pricing area.

MR. DICKE: Ron Levin will talk about some of the investment aspects of this pricing process.

MR. RONALD S. LEVIN: I am going to be speaking about pricing and investment considerations for interest-sensitive products, and at the start, I would like to make a couple of main points that I am going to elaborate on during my talk. The first is that although the topic that I am talking about in title is broken up into two parts -- *pricing considerations and investment considerations* -- they are really tied together. In other words, it would be a mistake to separate the pricing of a product from the investment strategy that should go behind that product. The second point is that the conventional tools that we have for asset liability management are inadequate to handle the subtleties in that the cash flow dynamics of interest-sensitive products.

I would like to suggest an approach that has more promise. Let me start out by taking a look at Regulation 126. I think Regulation 126 makes a couple of points that really get to the crux of the problem. The first point is that the cash flows of these interest-sensitive products are not fixed, they are uncertain. They depend primarily on interest rate paths, but they are not known in advance, and the products that we are talking about here, single premium deferred annuities, single premium life, universal life, defined contribution, guaranteed investment contracts (GICs), certainly have that characteristic. The second point is that a valuation regulation is concerned with what amount of assets is necessary to back the particular liabilities. The important thing here is that it is impossible to divorce from that process the investment strategy involved and the crediting rate strategy. Those two are fundamental to both the pricing and the valuation.

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The regulation raises two questions which it leaves unanswered. The first question is about reserving and pricing methodology. Are there really any objective standards to the pricing and reserving of these liabilities? Let me take a minute to explain what I mean by an objective standard here. If we were looking at a liability with fixed cash flows, for example a five-year defined benefit GIC, what we would probably do is look at the assets that we would use to back that cash flow. In other words, we would look at five-year noncall investments. The price of those assets would then lead us directly to an objective standard, and in a sense you can think of it as the cost of manufacturing. The way we manufacture that liability is to put together an asset portfolio which mimics those cash flows. That would be an objective standard. The second question that the regulation leaves unanswered is one of asset liability management. The classical immunization tools that we usually use for analyzing asset liability problems are inadequate. They miss the uncertain cash flow nature of the problem. What this points to is the need for a different approach. An approach which addresses the cash flow dynamics of the particular liabilities.

What I would like to suggest is an approach which is based on an option pricing theory. Let me start out by saying that the reason that option pricing is so appropriate for this is that the basic characteristic of an option is that the cash flows are unknown. The cash flows depend on some future event. If it's an option on a stock, it will depend on the stock price at the point at which the option expires. If it's an option of bond to an interest rate, the performance of that option depends on future interest rates. That's essentially the situation that we find ourselves in with interest-sensitive products. The primary difference is that they are much more complicated than ordinary options.

The fundamental principle of option pricing theory is that the value of an option or the value of an unknown cash flow is its manufacturing cost. There is one major difference between what that means with regard to option pricing theory and what it means to the five-year defined benefit GIC, something with fixed cash flows. The difference is that we cannot put together a single asset, just one portfolio, that is going to mimic the variable cash flows of the liability or the options. What we can do is put together an investment strategy which will replicate the performance of the option -- or the liability in this case. By investment strategy, what I mean is a systematic decision rule for buying and selling securities in reaction to market moves. So, for example, if the market

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moves up, getting greater exposure to either interest rates or some underlying security could be one possible strategy. The key is that we are not trying to predict market moves, but we are reacting to them in order to replicate an option.

The rest of my talk is going to be focusing primarily on SPDAs. I think it is useful to gain a little bit of intuitive understanding of the options in an SPDA policy. There are options on the policyholder's side and options on the issuer's side. The policyholder has the right to cash value withdrawal, and in terms of options, this is a put option. More specifically, the policyholder has a put option to sell a fixed income security when rates rise. The company, on the other hand, has the right to lower its crediting rate when interest rates fall. This option is very similar to the call feature of corporate bonds where the issuer of the bond, the borrower, has the right to call back the bond, and in effect lower its borrowing rate when interest rates fall.

Having made this option analogy, I want to caution you not to take it too literally. There are two reasons for that. First, the policyholders are not exercising their option with perfect efficiency. When interest rates rise, there is a certain amount of policyholder inertia which will mean that the majority of policyholders will not lapse the policy. The second complicating feature here is the interaction between the policyholder rights and the issuer rights. The policyholder is really reacting to how the issuer exercises its options -- changing its crediting rates. So, it is a chain reaction. The first part of that reaction is that interest rates change. Once interest rates change, then the issuer moves by resetting its crediting rate. Once the issuer resets the crediting rate, the policyholders then react by either lapsing or maintaining their policies.

What this analysis points to is the critical role that the reset strategy plays in asset liability management. It is akin to restructuring the liabilities, in the same way that you might restructure assets to match new liabilities. By changing the crediting rate, we are in effect restructuring the liability side of the balance sheet. That decision is certainly as critical in the asset liability scheme as the asset management itself. In fact, each reset transforms the old liability into a new liability.

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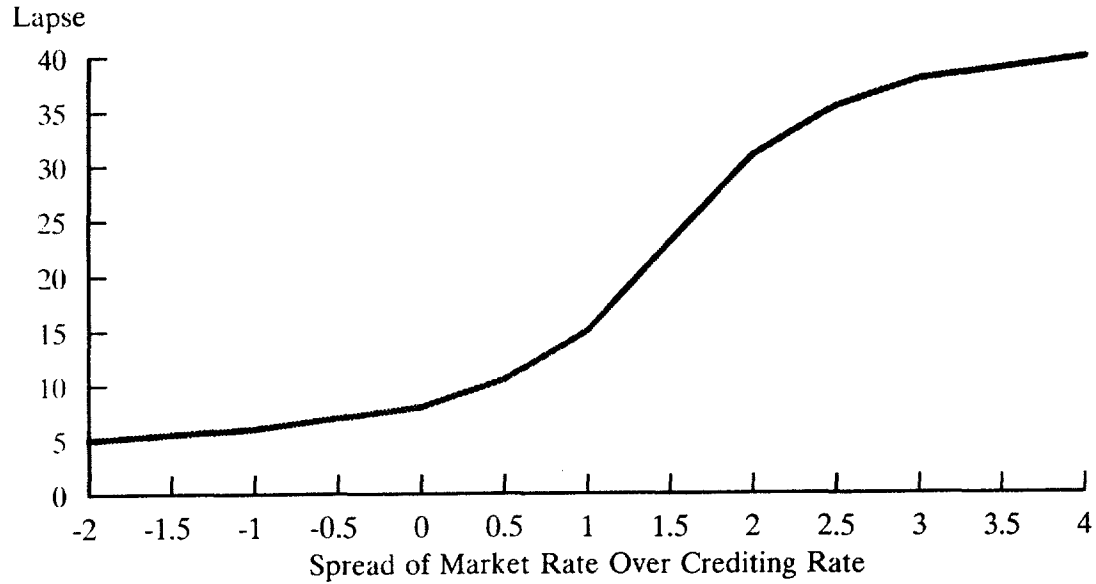
Looking at the problem this way can lead us to our objective in the rate reset strategy. The objective should be a strategy which will minimize the value of the liability. One thing that I am going to be talking about over the remainder of this presentation is the idea that the liabilities themselves have a market value in the sense that you would value any set of cash flows, whether it be an asset or a liability. This reset strategy problem involves a combination of dynamic programming and option pricing theory. Most of my work at J. P. Morgan Securities involves option pricing, hedging, and arbitrage strategies for the option traders, in addition to the asset liability work that I do. I find the option problems presented by interest-sensitive products to be among the most fascinating and complex that I have come across. However, I am not going to be speaking in detail about the rate reset strategy because of its complexity.

I would like to look at an SPDA example with very simple assumptions; the first assumption being an 8% rate which is guaranteed for seven years. Since the rate is guaranteed for seven years, there is no question of a reset strategy. So that takes one moving part out of the picture. The withdrawal penalty is a typical withdrawal penalty -- 7% in the first year of grading down to 0% after seven years. We have seven-year investments available at a 9% yield. Acquisition cost is 5% of premium. For \$100 of premium, the company is receiving \$95 that it has to invest. The liability that goes with that \$100 premium has to be worth less than \$95 if we expect to make profit on this product. So, in terms of the way we look at the market value or the value of that liability, its value should be less than \$95. Lapses are critical and are assumed to be a function of prevailing interest rates.

The particular assumption that I am using is described by Graph 4. Lapses are a function of the spread of the market rate over the crediting rate. So, at the outset, (we are starting at 0) lapses are about 8%. If interest rates drop by 1 to 2%, lapses do not change very much -- they go down to about 5% with a 200 basis point drop in interest rates. On the other hand, if interest rates rise by 1%, lapses will go up to 15%. If they rise by 2%, lapses will go up into the 25 to 30% range. If interest rates go up by 4% or more, lapses start to cap out in the 40% range.

What I would like to look at is a very simple investment strategy of using the bond which matures in seven years to back this liability. From the \$100 of

Lapse Rates



GRAPH 4

PANEL DISCUSSION

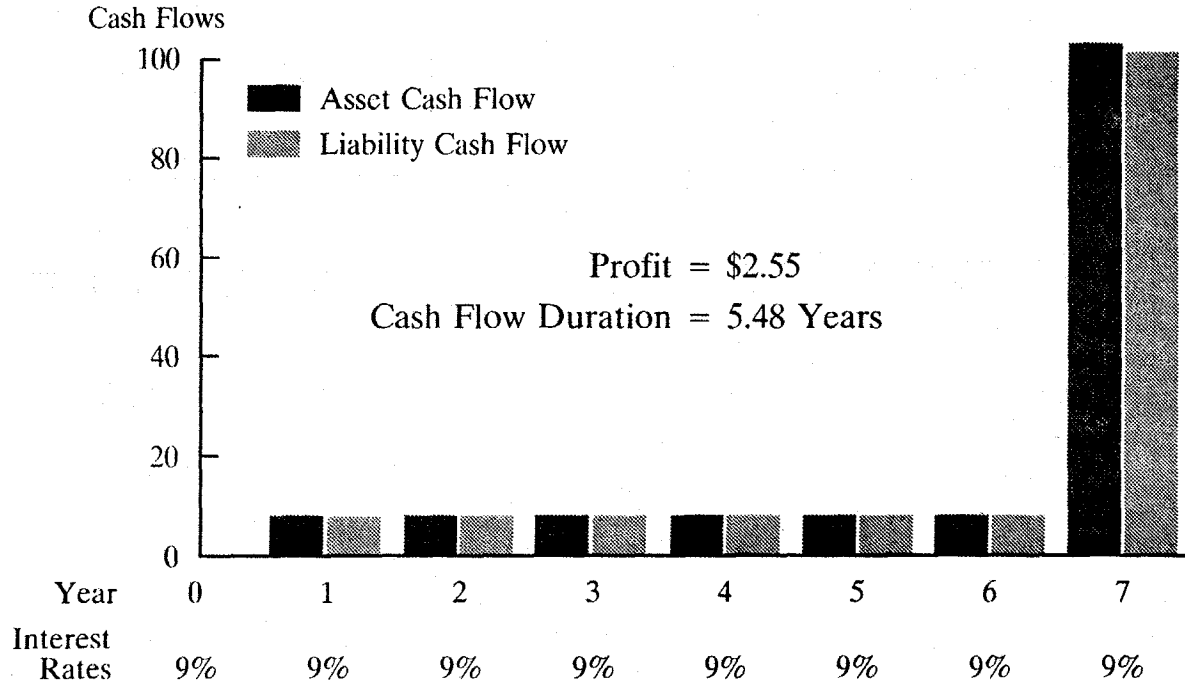
premium, the \$95 which we receive after acquisition costs is invested in the seven-year bond. In my first interest rate scenario, interest rates remain at 9% for seven years (Graph 5). In this scenario, it's almost an exact cash flow match, and in the end, we make a profit of \$2.55. The cash flow duration here is an after-the-fact duration, as we only know it after the block of business is off the books. The cash flow duration here for the liabilities is about 5 1/2 years. The duration for the assets is also about 5 1/2 years, since it is an exact cash flow match.

The problem is that when interest rates change, asset cash flows do not react, but the liability cash flows do. In fact, the liability cash flows are always going in the wrong direction. If interest rates go up as they do in Graph 6, what happens is the liability gets shorter. We are in a position of paying out cash flows which are in excess of the assets, but at a higher interest rate than we had initially invested. This is going to require selling the bond, which we bought at a 9% yield, at a 10% yield. As a result, the profits are reduced by about \$1.30. This profit is the profit at the end of seven years, just looking at the residual cash. Profits have dropped, but what happened to the cash flow duration? Well, for the assets it remains unchanged. It's still 5 1/2 years, but for the liabilities the duration after the fact is 4 1/2 years. In other words, we were backing a liability with assets which were actually longer than the liability when interest rates rose.

If interest rates go up from 9% to 10% after one year, and then to 11% after two years, and then remain at 11%, the lapses are a bit more extreme (Graph 7). As a result, the profit is now negative. Relative to the level interest rate scenario, we have lost \$5.50. (I have run interest rates backward starting at 1987, going back to 1980, which is much more extreme than this scenario. In that case, interest rates would go up to about the 14% level and then come down a little. When I ran interest rates backward, I believe the profit was -\$15.)

All of these analyses use a conventional asset liability approach to the problem. In fact, it is not only a duration match, it is a cash flow match assuming that interest rate scenario, and assuming the cash flows are exactly as we had projected. The problem is that cash flows will never be as we projected, and when

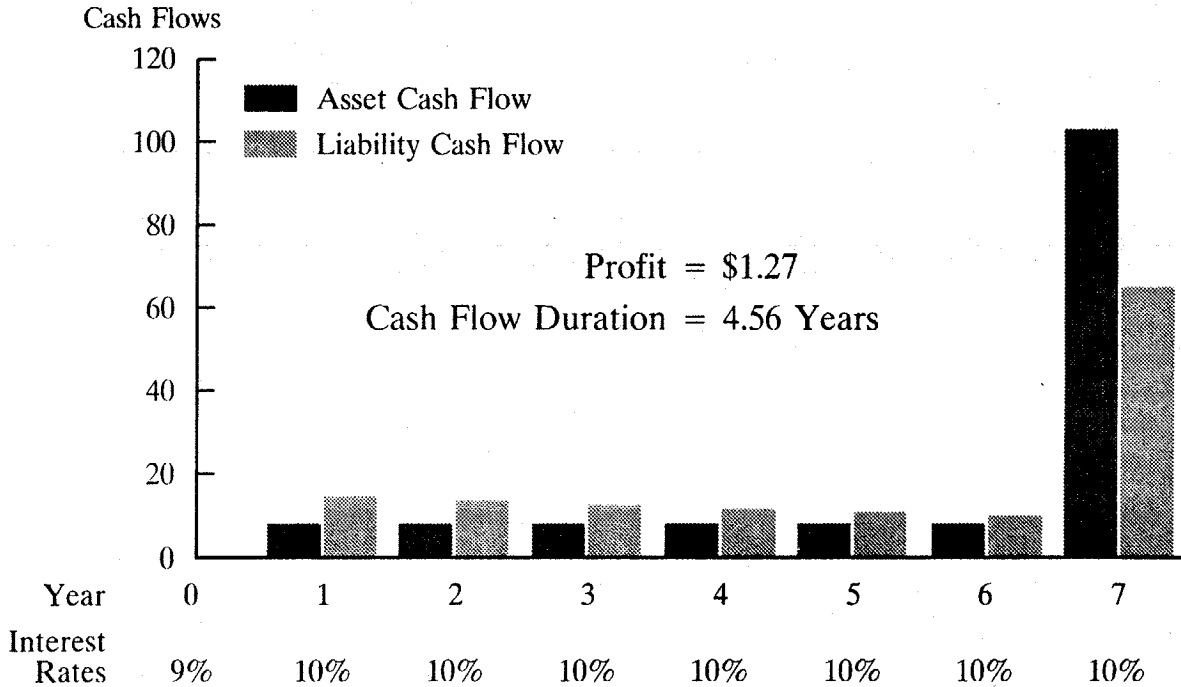
Stable Interest Rate Scenario



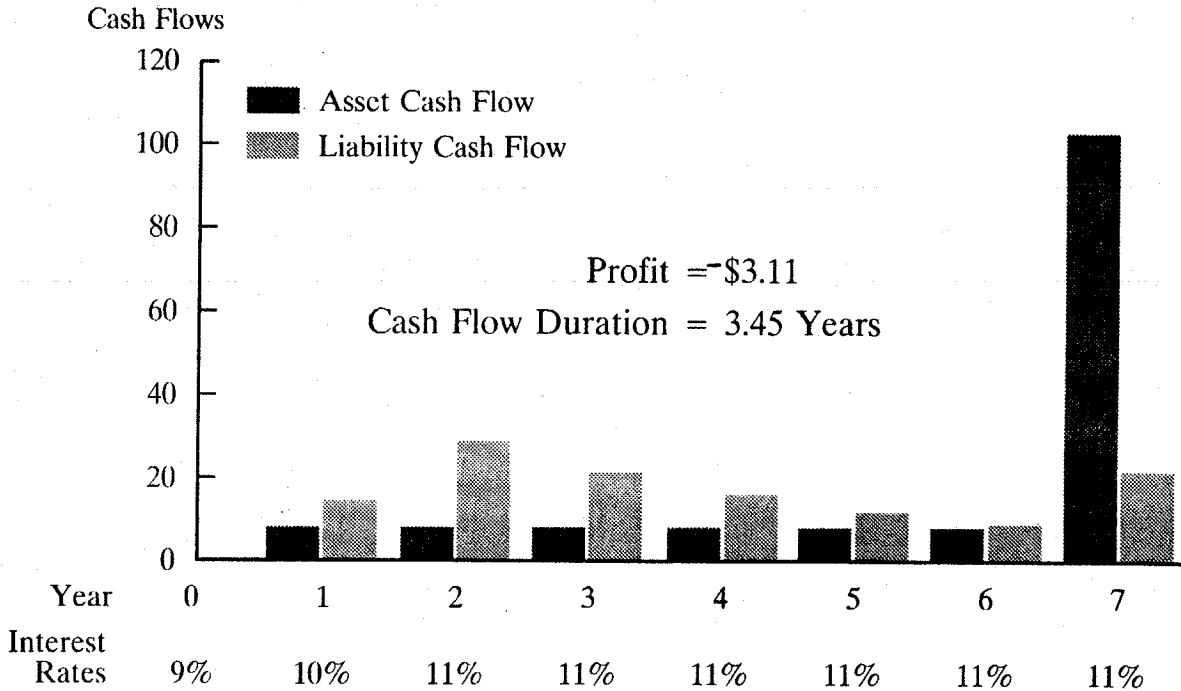
GRAPH 5

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Mildly Rising Interest Rate Scenario



Rising Interest Rate Scenario



PANEL DISCUSSION

interest rates change in this strategy, they will always change for the worse, whether interest rates go up or down.

Dropping interest rates are illustrated in Graph 8 where interest rates have dropped from 9% to 8% after one year, to 7% after two, and then remained level at 7% for the next five years. Now what happens is that instead of having cash flows with a duration of 5 1/2 years, it's closer to six years. In other words, our assets were too short for the liabilities in an environment of falling interest rates. As a result, the profit is reduced by about \$1.30.

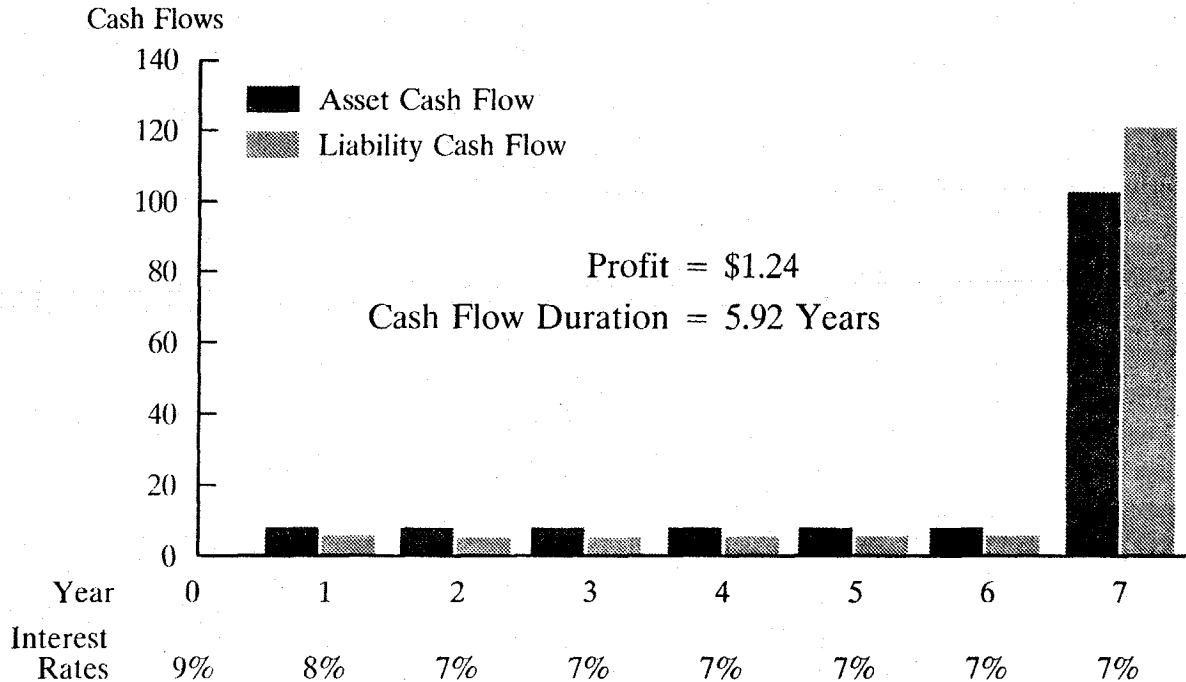
What we are seeing is that when writing options as we are when we issue these interest-sensitive products, volatility is the enemy. Any movement in interest rates will always be to our detriment. For the case of liabilities, puts expose the company to rising interest rates, or the risk of disintermediation. On the asset side, the calls and mortgage prepayments give us risk exposure when interest rates fall.

Next, I would like to take a look at the results of an option-based approach. There are two critical assumptions that go into the options-based analysis. The first is the lapse assumption. That is a very difficult assumption to make because it is hard to get reliable data for lapses as a function of the interest rate environment. The second critical assumption is the interest rate volatility. We measure interest rate volatility as the standard deviation in interest rates. For this analysis, we assumed an interest rate volatility of 12%. What this means is that for a 9% interest rate environment, a move from 9% to 10.1%, which is a 12% increase in interest rates, would be a one standard deviation move.

The SPDA value on this options-based analysis is exactly \$95. In other words, it breaks even in seven years, and all policyholders cash out at no penalty at the end of seven years. The SPDA duration is 4.6 years.

Let me just speak for a second about what duration means here. It has nothing to do with cash flows. What it is related to is interest rate sensitivity, which is really the main usefulness of the duration measure. In this case what it is saying is that here we have an asset which has an interest rate sensitivity which is the same as a zero coupon bond maturing in 4.6 years. This was versus a 5.5 year duration for our static analysis.

Falling Interest Rate Scenario



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GRAPH 8

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What is the cost of the option aspect of the product? It is the cost of the variable nature of the cash flows. What we have to do is look at what the price would be if there were no variations to the cash flows. If we knew the cash flows to be exactly the level interest rate scenario, we would just assume that interest rates have 0% volatility. In that case, the SPDA value is \$93.60. In other words, the option feature of the product is costing us \$1.40 in profit.

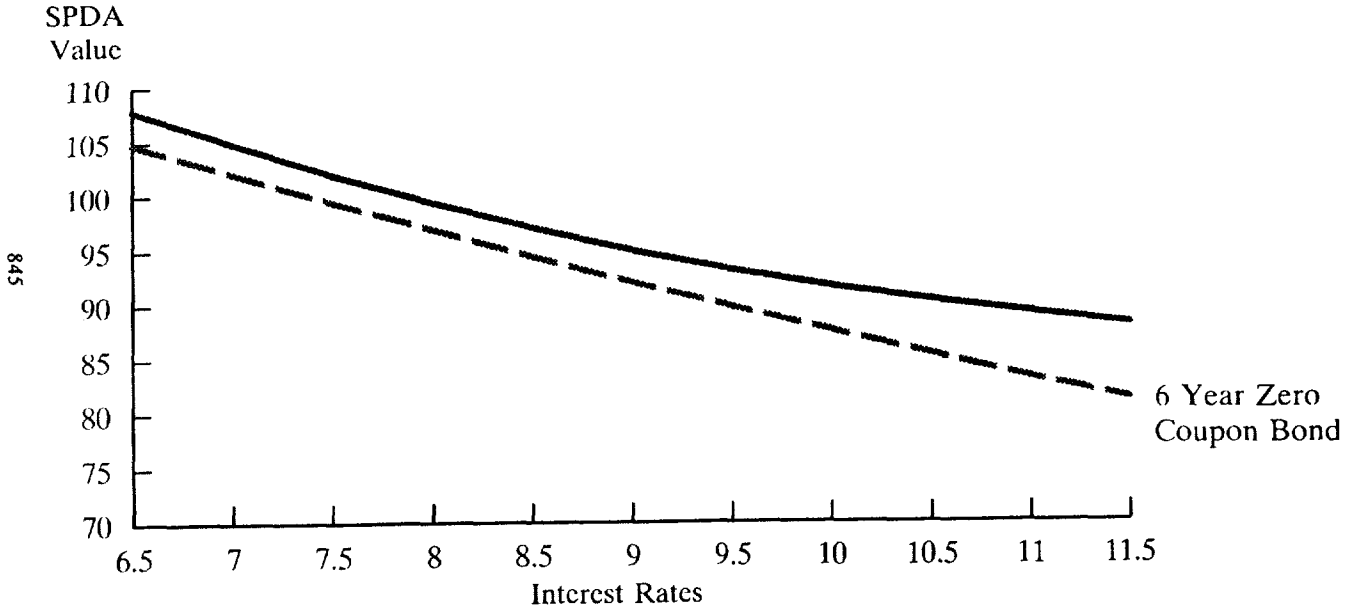
Let's take a look at its impact on yields on the investments. Now, a break even SPDA rate is not 8%, but it's 8.28%. So, in other words, 28 basis points in the crediting rate, or alternatively, 28 basis points in the yield on our assets was used just to cover the option features of this product.

In Graphs 9, 10 and 11 I would like to give you a feel for the relationship between value of the SPDA, its duration (its interest rate sensitivity), and the concept of the term called convexity. What we are comparing here is the value of the SPDA and the value of the zero coupon bond (see Graph 9). This is a six-year zero coupon bond. Do not focus on the fact that the zero is a little bit below the SPDA. Look at the relative values and how they change. For that purpose, I am going to divide this graph into two halves. One is to the left and the other to the right of 9%. To the left of 9%, a six-year zero coupon bond is an extremely good match to the value of the SPDA if interest rates drop. If interest rates rise, what we see is that the zero coupon bond underperforms the SPDA by increasing rates. So by the time interest rates have risen by 250 basis points, it has dropped in value relative to the SPDA by about 5 to 6%.

What we are seeing is that although the zero coupon bond has constant interest rate exposure, the SPDA does not. The SPDA has increasing interest rate exposure. It is more sensitive to interest rates when interest rates fall, and it is less sensitive to interest rates as they rise. So it is a function of the cash flow dynamics. This liability is going to get longer when interest rates drop and shorter when interest rates rise. If you own the SPDA, it is a desirable feature. If you sold the SPDA (if you are the issuer), it is an undesirable feature. That is positive convexity; the fact that we get greater interest rate exposure when interest rates rise and less when they fall.

GRAPH 9

SPDA Value



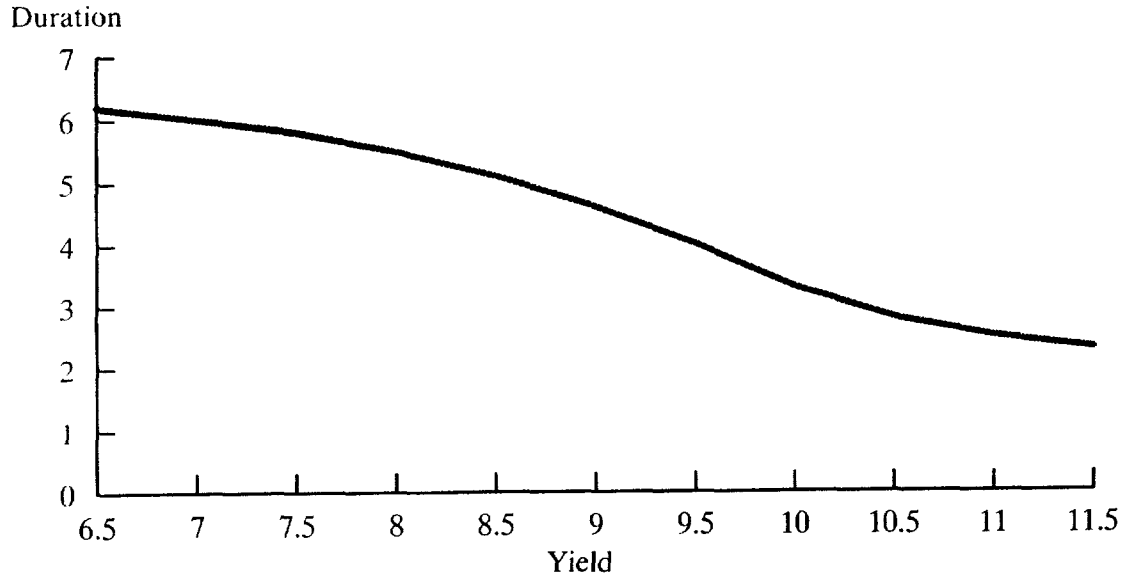
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In Graph 10 you have the SPDA duration that starts out at 4.6 years, but goes up to a little bit above 6 after interest rates have dropped by 250 basis points, and goes down to a little above 2 when interest rates go up by 250 basis points. If you were backing that with a portfolio of noncallable assets, we are matched if interest rates move in just a narrow range, otherwise we are mismatched in the wrong direction. That is going to be aggravated if we use callable assets such as a 9% Government National Mortgage Association (GNMA) security, which is going to be longer than we want. This security has a rising duration when interest rates rise and a falling duration when interest rates fall. That phenomenon is known as negative convexity. If you are the owner of that asset, that is something that you do not want. That's an undesirable feature. The reason that people buy mortgage backed securities is because they are compensated for this feature in the form of a higher yield. If you think about it, GNMA securities yield typically in excess of 100 basis points over treasury securities, but there is no default risk in these securities. They are guaranteed by the U.S. government. The reason the yields are higher is to compensate the investor for the undesirable cash flow characteristics, or in terms of price, its negative convexity.

What I have in Graph 11 is a profit profile of the SPDA over a one-year horizon. What we are doing is backing this liability with duration matched noncallable assets and, therefore, we realize our profit if interest rates do not move. We have a phantom profit margin which we can realize only in the face of unchanging interest rates. Rick Fisher describes this as "heads I win, tails you lose." You get to win only if the coin stands on its edge. That is what happens when interest rates stay exactly the same. That is where we are making our profit; but if interest rates vary significantly, we are facing a loss. I am using non-callable assets here. If we use callable assets, which is probably more typical of life companies, this situation is going to be aggravated.

The duration diagram points to what an investment management strategy might be for the SPDA product. What we want is to put together a strategy which matches the interest rate sensitivity of the SPDA. That means as interest rates fall, we are lengthening maturities (increasing interest rate exposure). As interest rates rise, we are decreasing the interest rate exposure. That is a dynamic strategy that will match the interest rate sensitivity of the liability. What we would be doing is trading in the middle of our profit profile for raised

Comparative Duration



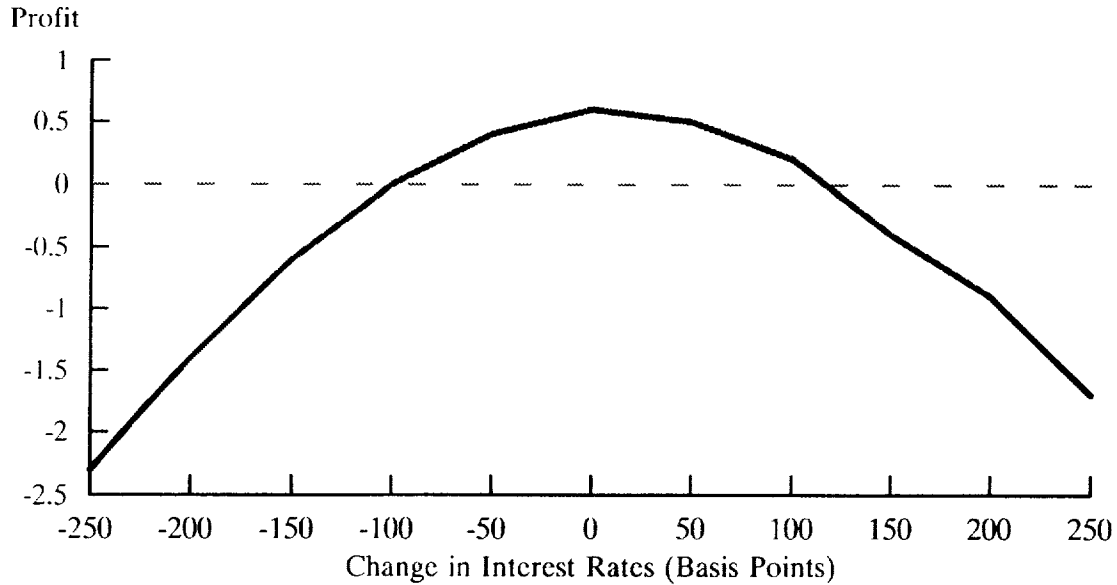
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GRAPH 10

SPDA Profit Profile

1 Year Time Horizon

(Assumes Assets Are Initially Duration-Matched)



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ends, and what we would get would be a straight zero. Hopefully, if we priced it properly and designed the product properly, we are going to be making some profit margin above zero.

In conclusion, what I want to leave you with is the importance of explicitly recognizing the options features of these products and pricing for them, and also, the investment strategy of adjusting interest rate exposure to match the interest rate sensitivity of the liability. This can be done several ways. I call one manual and the other automatic. The manual way would be where we actually buy and sell assets to reflect the change in duration. The other would be to put together assets which actually change in interest rate exposure as interest rates change, and that asset would have to be in the form of put and call options. The last thing I want to mention is the importance of the rate reset strategy. I feel that in the future we will see increasing work done in this area.

MR. DICKE: Let us look at Graph 11. If you are investing in options, is this writing some kind of a straddle? Is that a notably risky form of investing if we want to bet our money?

MR. LEVIN: Actually, if you go into the strategy with forethought, you are taking a bet on interest rates. Your view on interest rates is that they are not going to change very much. And if they do, you will be hurt by it.

MR. DICKE: Either up or down.

MR. LEVIN: Yes, what you are doing in the situation is you are writing a put option, so you get hurt when prices drop and interest rates rise, and you are also writing a call option so you get hurt when prices rise and interest rates fall. You get hurt both ways. The only way that you win is if nothing happens, in which case both options are worthless and what you are left with is the premium that you either implicitly or explicitly got for writing those options.

MR. JAMES A. GEYER: Are you assuming in Graph 11 that you do not lower the credited interest rate if rates fall?

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MR. LEVIN: I am assuming an 8% rate guaranteed for seven years in this situation. So, I really dodge the question of rate reset strategy in this example.

MR. GEYER: Realistically, most products today do not guarantee for that long. So, the curve is more flat on the left-hand side and the real risk is that rates rise and you are invested too long. Also, it seems to me that the lapse rates you had were much more extreme than I have seen anyone report.

MR. LEVIN: Yes. I would not necessarily attest to the realism of those rates, but what I would say is that in the last few years we have not had the experience of extremely high interest rates being much higher than the crediting rates. I think we need to go back to the very early 1980s to get that kind of experience.

MR. GEYER: I think the worst that our company experienced was in the 20% to 25% range.

MR. LEVIN: What kind of environment was that in?

MR. GEYER: Those were 14-15% interest rates with 9% credited rates.

MR. LEVIN: Was that broker-sold business or agent-sold business?

MR. GEYER: That was agent-sold business.

MR. LEVIN: I think you might get different experience from broker-sold business.

MR. GEYER: I have a final comment. You mentioned earlier the idea that you wanted to set the crediting strategy to somehow minimize the value of the liability. It does not seem to me that that can really be done in a vacuum. You would have to take into account the effect of the crediting strategy on new sales. So, maybe that just adds to the complexity.

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MR. LEVIN: Definitely. If you would just focus on one particular block of business, what you could do is maximize the profits on that block and assure that you never get any new business.

MR. KEITH J. DUBAS: Mr. Tuohy, in your stochastic scenario, you showed a range and a mean of profitability. You did not put a measure on the likelihood of the different scenarios. How could you do that to see what the expected return might be? I think the mean itself is misleading.

MR. TUOHY: Using the mean as the expected result assumes that each of those scenarios is equally likely. I would defer to Ron on how good an assumption that is. Basically, all we did was to randomly generate 40 scenarios assuming a lognormal distribution, and assume each of those was equally likely. Ron, in your opinion, how good or bad an assumption is that?

MR. LEVIN: I think you would have to use some overkill at the outset. Perhaps a few thousand scenarios are needed to really pinpoint what the mean is and then try blocks of much smaller samples to see what kind of deviation you are going to get.

MR. TUOHY: I think Mr. Dubas was concerned that even if you do 10,000, you are going to assume that each of those is equally likely, which is implicit using that mean assumption. I know that some of the work that Morgan Stanley has been doing says that they are not all equally likely. But, I cannot enumerate or even give you a feel from their work of how much that would shift their expected value away from the mean.

MR. LEVIN: There are two ways of running these scenarios. One is to assume that they are equally likely and the other is a very refined approach which relies on some new breakthroughs in financial theory, particularly option pricing, which allows you to give an adjusted probability weighting to each of these scenarios. And in that sense, what you are getting by running these large scenarios is an unbiased estimator of the true option value of the liability stream.

MR. DUBAS: Don't the more likely scenarios occur more often when you are generating them randomly?

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MR. JONATHAN E. MILLER: We know that the sample mean is an unbiased estimator of the population mean so that the only real question is, how large a sample do you have to have before you have a reasonable relationship? I think that is a real hard question to answer since you have an interest crediting strategy that changes with your interest rates, and you are not going to have an analytical relationship between the profits and the interest rate changes. I think you are probably going to have to run a thousand scenarios to get some idea of what your distribution of profits is after you have some idea of your distribution of interest rate scenarios.

MR. TUOHY: I think whether we run enough or not is a good point. We have, on occasion, run a lot more than 40 and found that we get pretty similar results to the 40. That's not a very statistically valid reason to using 40. Just for practical reasons, we generally do run 40 because we feel that is enough. The original questioner was really asking how valid the lognormal distribution is. If the lognormal distribution is valid, then those are all equally likely. I think the mean is reasonable.

MR. SHELDON EPSTEIN: I would just like to make a comment on that. The fact is that you can price noncallable paper using your interest rate paths, and you should come up with prices that are equal to what the market is pricing those noncallable bonds, etc. If that is the case, then all your paths are equally likely. If they are not, they do a good job of replicating noncallable paper. This is the test that we use when we select paths. We place all kinds of constraints on them, but basically, that is the procedure we use.

MR. LEVIN: If some of you are interested in the technology behind those interest rate generating paths, I recommend a paper by Thomas Hoe in the *Journal of Finance*, December, 1986.

MR. DICKE: Before we go further with the financial type things, let me throw out one question of a slightly different nature. Mike, do your clients find that this kind of an approach with scenarios is actually practical for doing pricing, as opposed to valuation work where everything is set all at once? In pricing, obviously you are trying to hunt for a way that you can be competitive in the market and so on, but you have to run 40 scenarios every time you come up with an idea. Is that really practical?

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MR. TUOHY: It is just as easy to run 40 as 1, once you have it in the machine. I think we are going through a phase. The original universal life development was done on the simplistic assumption that you were going to make a spread. I think now we know that to be a simplistic assumption that is rather optimistic. One has to go through this sort of process to get a more realistic estimate of how profitable these products are. It's a matter of what programs you have, how practical it is to do 40 or 10, 3 or 100. If you have the machine set up, it's just as easy to do 40 as it is 10.

MR. MORRIS SNOW: As a practical matter, most valuation or pricing actuaries are not going to become experts in this field, especially in option trading. The question really is where do they go if they want to get the resources they need to have to do all of this kind of research, either to price initially or find out about options trading? Is there literature? Are there consulting houses? Are there banks? What would the practical approach be to get the resources needed to set up products that you have to have or to value them?

MR. TUOHY: There are programs out there, obviously, packages that do the sorts of things that I was doing and have the ability to stochastically generate future interest rates. As far as the duration, Ron can talk to you. I think, for the pricing actuary, it's something that he has to get used to on these interest-sensitive products because their profitability is so much dependent on future interest scenarios. I think he has to understand the sensitivity of the results to those scenarios.

The approach I showed may be the simplistic approach: where you just make assumptions and you look to see what happens. And the initial problems that you get into is that you just get deluged with data that you do not know what to do with. We have been playing around for a long time, coming up with graphic ways of presenting that data, that we feel a lot more comfortable with; but when you start, there are a lot of numbers so that you do not feel you are any more informed than when you started out.

MR. LEVIN: I think there is a real need for actuaries who are involved in option-like products to get a much better understanding of options and the technology behind pricing those products. There has been very little written about pricing of those cash flows. Hopefully, I would like to get a chance to

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write a paper. In the meantime, I think the education of actuaries is very important in this area.

MR. MILLER: Maybe I am throwing a little bit of water on your party, but I have not noticed that these problems are coming down to the annual statements. I am just wondering if we are exaggerating the problem. Perhaps, as Mr. Fisher discovered, we have offsetting risks in several of our products. I do not want to say that we are not putting enough effort into it, because I think we need to put a lot more effort into it, but maybe we are putting a little more worry than we need to into this.

MR. TUOHY: Rick pointed out that if you have a block of immediate annuities and a block of SPDAs, they do help each other. I did an appraisal recently on a company that had equal stocks, and the volatility in the values of those numbers in future interest scenarios was much more stable than just looking at each block individually. So, that is definitely a plus if you have those two blocks.

I would disagree with your first point that maybe we are scaremongering. I think we have been lulled into a false sense of security by what has happened to interest rates over the last three years. Nothing better could have happened to interest rates as far as helping the bottom line of the companies writing these products. If we had a reverse of the last three years, I think that you would see red ink flying all over the place. If you look to the numbers from that simplistic model that I had, everything was good news if interest rates went down because I was a bit long. All the bad news came when interest rates came up. So, the test is going to be the next spike in interest rates.