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Canadian Dollar Time Series

by Joe Koltisko

What is going on with the exchange rate, anyway? In 2002, $1 U.S. bought $C 1.60, and now in May 2005 you only get $C 1.25—that's 20 percent less! Where's it going next?

One way to answer that question is with a time series model. Let's review some terms first, and then investigate the data. An **autoregressive** (AR) time series model has the form:

$$\text{where } \Delta X = a^* (MR - X_{t-1}) + \varepsilon_t$$

where MR gives the mean reversion target, and "a" gives the speed at which the series approaches its long-run target.

A moving average (MA) time series has the form

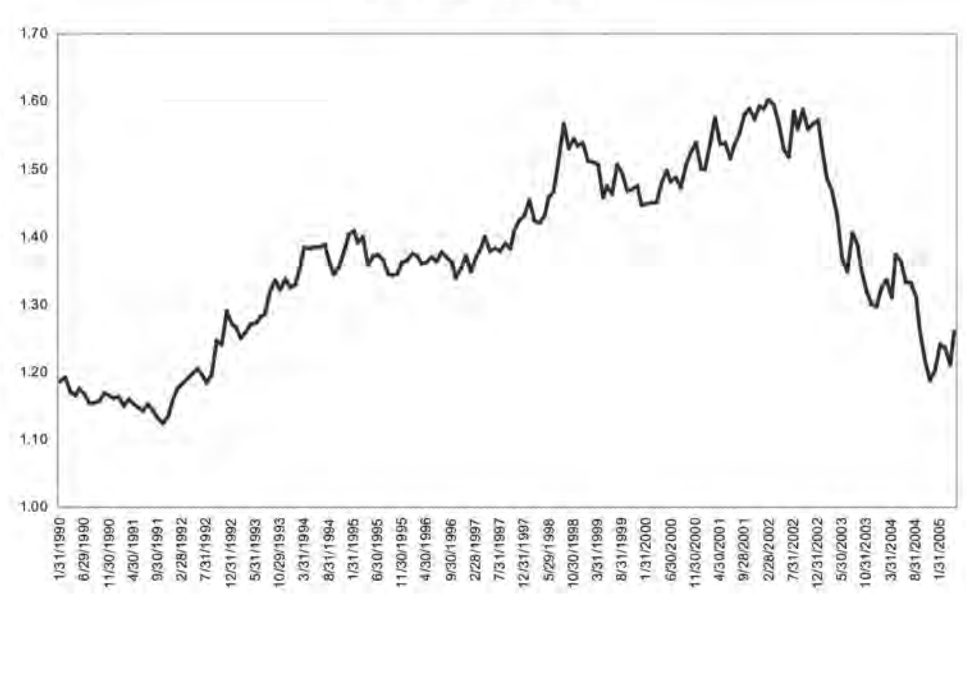
$$\Delta X = f * \varepsilon_{t-1} + \varepsilon_t$$

where "f" controls the autocorrelation of the resulting series. If f is large and negative, successive shocks are dampened, while if f is positive, shocks are magnified.

Technically, a model of the change in an economic variable is called an "integrated" (I) time series. The models may use data from one or more time periods. The series used here are ARIMA(1,1) time series, which means they are autoregressive moving average models of the change in the variable, both using one period of history.

Back to the loonie. The monthly change in the exchange rate (in absolute dollars) gives a familiar picture, similar to a seismogram. There are periods of high and low volatility. It's not obvious from the monthly change series, though, whether we're in an up trend or a down trend. To get at this, I also graphed the 12-month moving average of the change in the exchange rate (after filtering out some

Canadian Dollar / $US



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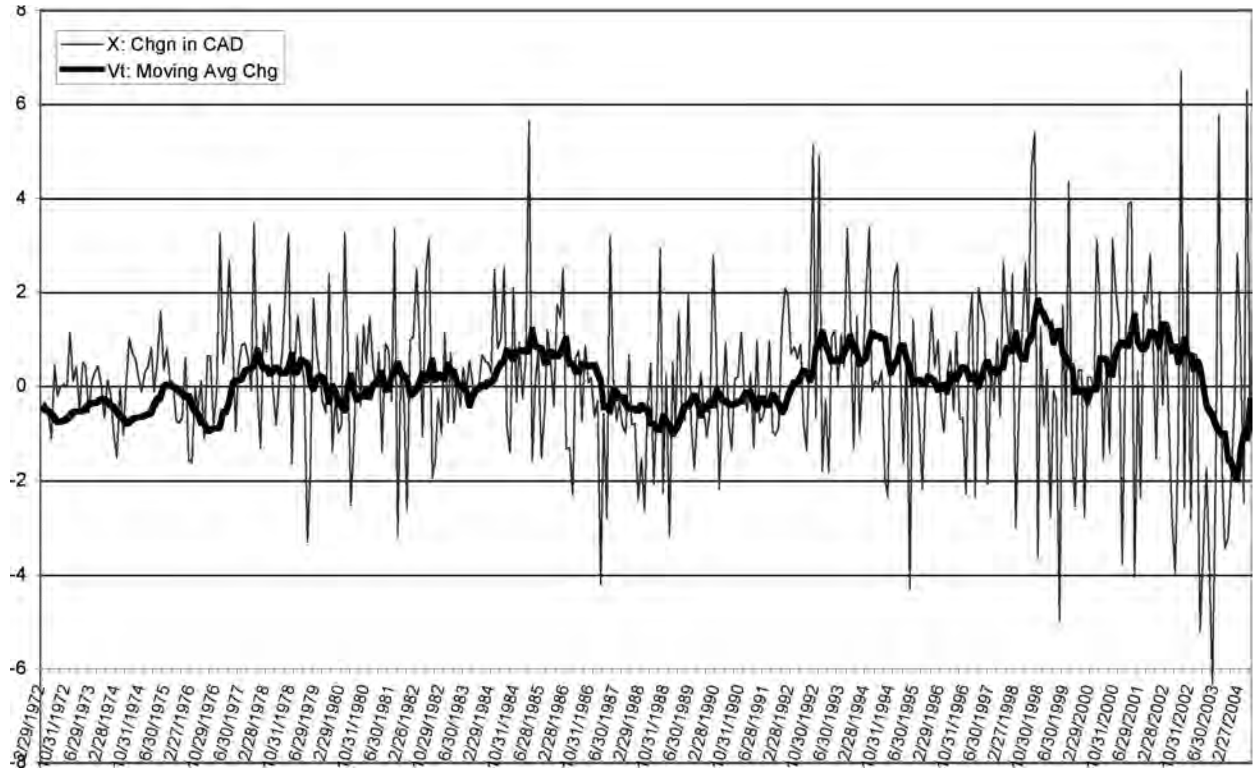
turn to page 14

From Page 13

Monthly Change in CAD and Moving Average



reversion to the long-run average since 1971 of about 125).

This suggests that we could reflect the frequency and severity of trends in the CAD series by modeling the mean change as a separate process. In contrast to popular "stochastic volatility" models, this would be a stochastic mean model. The form of the model is:

$$\Delta X = V_t + a_1 * (MR - X_{t-1}) + f_1\varepsilon_{t-1} + \varepsilon_t$$

Monthly change in CAD value

$$\Delta V = a_2*(0 - V_{t-1}) + f_2\varepsilon_{t-1} + \varepsilon_t$$

Change in mean of ΔX

That is, two autoregressive moving average models with one month of "memory." The second series, V, reverts to zero. The base series, X, experiences rising or falling trends when V is positive or negative, respectively.

I fit such a model, with least-squares minimization on the monthly data since 1971 from Bloomberg. The parameters are shown in the table on page 15. The mean reversion target, 125.16, is the average value from the data set. The series for V has a positive autocorrelation parameter, F2, which magnifies trends away from the long-term average. V also is modeled with a normal random deviate.

Once we filter out mean reversion and the process for the mean, the historical data for ΔX shows non-normal, fat-tailed residuals. To capture this behavior, the random term for X is modeled as a mix of two normal random deviates. The random term has a "low" standard deviation close to one for half of the time; the rest of the time it is over two. This fits the tail of the residuals reasonably well.

To check consistency over time, I recalibrated the model to the period after 1989. The parameters for V are comparable, which indicates that trends in the mean are not significantly different. The residual term for X shows higher volatility parameters in the more recent period. The average exchange rate was higher in the recent period, so the mean reversion target is higher as well.

Table 1: Canadian Dollar Time Series Parameters

	Since 1990	Since 1971	
A1	0.018226	0.008258	speed of mean reversion for X
F1	0.005241	(0.018521)	autocorrelation for X
MR1	136.857826	125.160000	mean reversion target for X
A2	0.068838	0.068179	speed of mean reversion for V
S2	0.233072	0.195848	std dev of V
F2	0.114252	0.056541	autocorrelation of V
P	0.667780	0.529337	chance of being in high state
S1a	1.160138	0.950646	low state st dev
S1b	2.735238	2.289211	high state st dev
X0	125.830000	125.830000	initial X 4/29/05 Value
VO	(0.800000)	(0.800000)	initial V 4/29/05 Value
max used	200.000000	200.000000	max X
min used	50.000000	50.000000	min X

As may be seen from the moving average graph, the mean term V has recovered from its historic lows of around -2, to around -0.8 as of the end of April. It's still negative, which means the model starts with the presumption that we are in a down trend. The Canadian dollar would continue to strengthen under this model. However, today's value of 125.83 is close to the long-run average.

Let's see what this model (1971 parameters) implies for the long term, and for the end of this year.

turn to page 16

From Page 15

Table 2: Distribution of CAD/$US, simulation of 1000 30-yr scenarios

	31-Dec-05	1	2	3	5	7	10	20	30
99.5%	134.93	137.29	143.55	150.68	165.00	172.00	181.33	186.34	197.10
99.0%	133.89	135.74	141.25	148.72	158.84	169.15	178.64	181.67	185.19
98.0%	132.26	133.76	138.61	146.34	154.91	164.30	174.57	172.97	175.73
95.0%	130.04	131.23	135.09	140.60	148.03	155.29	160.44	164.85	164.90
90.0%	128.12	128.88	132.12	135.16	142.84	145.99	151.02	156.04	157.43
80.0%	125.77	125.67	126.84	129.68	133.02	136.63	138.49	145.18	145.41
70.0%	123.98	123.43	123.49	124.86	127.61	129.70	132.10	138.08	137.77
60.0%	122.65	121.56	120.79	120.51	122.00	123.66	125.37	130.69	130.93
50.0%	121.35	119.76	117.77	117.29	117.90	118.26	120.06	123.05	125.56
40.0%	119.97	117.71	114.65	113.64	113.94	113.86	115.04	116.40	118.64
30.0%	118.49	115.76	111.62	109.58	107.80	107.94	108.33	109.42	112.06
20.0%	116.54	113.71	108.08	104.73	102.16	101.29	101.72	102.77	103.80
10.0%	114.31	110.69	103.57	98.58	93.51	93.64	92.27	92.07	92.69
5.0%	112.29	107.99	98.68	94.52	87.07	87.17	82.96	83.95	83.94
2.0%	109.86	105.69	93.84	89.47	80.18	80.02	74.00	74.23	72.51
1.0%	108.36	101.89	91.27	86.47	77.63	74.86	69.02	68.10	62.03
0.5%	107.65	101.20	90.13	83.30	73.85	67.96	66.10	64.11	57.14
Avg	121.25	119.69	117.47	117.22	117.93	119.33	120.65	123.74	124.86
Std	5.43	7.11	11.03	14.06	18.52	20.48	23.28	24.89	25.31
CV	0.045	0.059	0.094	0.120	0.157	0.172	0.193	0.201	0.203

This table shows the moderate strengthening trend continuing. Average CAD levels strengthen to 117, before turning back to the assumed long run level in 20 years. It takes that long to work off the trend!

The model could also be run under other initial conditions. Suppose the mean trend variable were at 1.6, its all-time high. In that case the model output is:

Table 3: Sensitivity Test: Historical Maximum for V

	31-Dec-05	1	2	3	5	7	10	20	30
99.5%	149.71	155.14	171.83	179.78	189.12	195.02	190.45	188.90	185.41
99.0%	148.90	153.64	167.54	176.78	184.94	190.01	186.21	187.64	180.58
98.0%	145.35	151.92	164.85	173.50	178.90	183.44	180.98	181.03	176.40
95.0%	143.66	149.03	159.67	166.51	172.36	171.97	171.63	169.71	167.98
90.0%	141.39	146.66	156.51	161.07	166.21	164.73	163.32	159.89	158.32
80.0%	139.15	143.47	150.60	153.72	157.37	155.57	153.20	149.36	148.51
70.0%	137.36	141.32	147.02	149.45	150.56	149.03	146.14	140.75	139.97
60.0%	135.85	139.05	144.00	145.60	144.51	143.52	139.88	133.57	132.51
50.0%	134.89	137.36	141.05	141.70	139.85	138.51	135.02	127.04	124.69
40.0%	133.43	135.50	138.32	138.46	135.22	134.07	129.83	120.02	117.30
30.0%	131.95	133.79	135.36	134.65	130.33	127.26	123.31	112.89	109.99
20.0%	130.27	131.80	132.41	129.88	123.68	120.95	114.91	104.67	102.68
10.0%	127.73	128.96	126.94	123.49	116.39	110.89	104.23	91.41	92.04
5.0%	125.99	126.06	123.23	118.63	109.85	101.31	95.26	82.27	82.47
2.0%	123.58	124.12	119.47	113.33	98.90	90.54	85.61	73.49	72.64
1.0%	121.37	122.30	115.75	108.90	92.82	84.52	80.96	70.65	67.40
0.5%	119.48	119.86	112.43	106.04	89.30	81.77	76.74	66.11	61.55
Avg	134.68	137.53	141.34	142.03	140.28	138.09	134.34	126.75	124.97
Std	5.42	6.92	11.24	14.49	19.58	21.42	22.75	26.26	26.04
CV	0.040	0.050	0.079	0.102	0.140	0.155	0.169	0.207	0.208

Again the trend works its way off in 30 years, but not before breaking 140 in the next three years. So, the model is sensitive to the initial condition for V. It may be more appropriate to start with a point half way between the observed 12-month moving average and zero. By the time we observe the trend, it may be over.

Finally, by running the model with parameters from 1990, we get

turn to page 18

From Page 17

Table 4: Parameters Calibrated Since 1990

	31-Dec-05	1	2	3	5	7	10	20	30
99.5%	140.71	144.89	157.07	164.54	176.51	179.70	187.52	187.40	188.20
99.0%	138.93	141.54	154.40	163.35	172.63	177.56	182.95	183.04	184.64
98.0%	136.52	139.59	150.21	158.33	167.60	173.66	177.80	178.76	181.17
95.0%	134.58	136.59	145.41	150.71	160.64	164.07	170.41	170.70	173.03
90.0%	132.17	133.73	139.73	144.47	153.72	158.04	161.14	163.88	165.59
80.0%	129.01	129.50	134.26	137.25	145.53	148.81	150.08	155.02	154.91
70.0%	126.45	127.17	130.04	132.78	139.60	142.41	143.49	147.77	148.29
60.0%	124.72	124.76	126.65	129.14	134.12	136.93	137.72	141.52	142.84
50.0%	123.35	122.71	123.61	125.39	128.03	130.82	133.22	136.28	137.14
40.0%	121.71	120.84	120.08	120.78	122.58	125.81	128.03	130.68	131.02
30.0%	119.80	118.15	116.36	116.43	117.49	121.05	122.87	124.89	125.72
20.0%	117.52	115.75	112.53	111.71	111.22	113.63	116.19	119.04	119.16
10.0%	114.57	111.12	107.19	104.05	103.88	103.55	108.24	107.14	110.20
5.0%	111.80	108.08	102.15	97.90	94.80	96.80	100.22	99.14	101.23
2.0%	109.74	104.67	96.73	91.95	85.54	88.75	91.45	92.11	91.96
1.0%	107.46	101.61	93.55	86.46	81.32	85.58	84.94	86.10	86.21
0.5%	105.93	99.66	91.49	82.43	77.82	78.63	81.90	83.12	82.59
Avg	123.22	122.63	123.39	124.67	128.36	131.09	133.58	136.11	137.03
Std	6.77	8.62	13.10	15.96	19.96	20.62	20.59	21.44	21.44
CV	0.055	0.070	0.106	0.128	0.155	0.157	0.154	0.158	0.156

It's interesting that the 1990 parameters give a distribution that is "fatter" in the early years, since the coefficient of variation (std. dev./average) is greater. After five years or so, the 1990 parameters give a tighter distribution of results (as well as reversion to a higher mean reversion level).

I'll hazard a forecast: 1 \$US = \$C 1.23 on December 31, 2005.

This sort of model may be useful for other sorts of financial data such as interest rates. An equity price series probably should be modeled with a non-zero mean reversion target for V, to reflect long-term growth.