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VA GMxB AND DELTA HEDGING IN OCTOBER '08 AND BEYOND

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Variable Annuity products with significant guaranteed minimum benefits have been one of the biggest sales successes of the US insurance sector over recent years. These guarantees must be 'marked-to-market' under US GAAP (FAS 133), and this has encouraged firms to hedge the market risk exposures created by such guarantees.

There are a variety of hedging strategies that can be implemented by VA GMxB writers, and each of these strategies has its own relative merits with respect to costs and risks. These trade-offs can be brought into sharp relief by volatile trading conditions such as those in October 2008. This note produces some high-level analysis of the likely strains that such conditions would have placed on the performance of a delta hedging strategy over that period, and discusses the implications this may have for the sector.

Market risk management strategies for VA GMxB vary across the sector and include:

- *Do nothing but hold capital.* Not a common approach, but leaving some GMxB exposures 'naked' may be considered by firms that are not particularly concerned with US GAAP earnings volatility and believe they have sufficient capital to fund the economic risks.

- *Reinsure.* Pass these market risk exposures on via a reinsurance treaty. This has been significantly used by some firms in the past, but the reinsurance sector appears to have a limited appetite for these risks, especially at the prices at which the guarantees are being sold.

- *Structured OTC hedging solution.* Pass the risk on to the capital markets through the purchasing of a hedging solution that has been tailored to (permanently) match the specific characteristics of the GMxB liabilities. Note this will leave policyholder risks (lapse behaviour and its impact on the amount of GMxB that is in play). Like the reinsurance solution, firms may find it difficult to achieve a solution at a cost commensurate with the product pricing of the guarantees.

- *Dynamic internal hedging.* The firm dynamically manages a portfolio of exchange-traded vanilla derivatives in a way that matches the short-term market value sensitivities of the GMxB liabilities. This may be, *on average,* a cheaper solution than the structured approach, but it can leave material residual market risks behind.

The dynamic internal hedging approach is the most commonly used risk management strategy (particularly if weighted by $ exposure). Firms using the dynamic strategy may make different choices as to which market risk sensitivities are hedged. For example, firms may hedge the sensitivity of the liability value to changes in the underlying asset values *(delta)*; its sensitivity to changes in risk-free interest rates *(rho)*; its sensitivity to changes in option-implied volatility *(vega)*. Some firms may choose to hedge all three (and perhaps more) of these, but many will choose to focus solely on the biggest sensitivity and will hedge only the *delta* exposure.

A delta-hedging strategy recognises that the guarantee value will change as the underlying fund value changes, and so the hedge portfolio takes an off-setting position in the underlying fund to neutralise this impact. The *delta* of the guarantee cost is the ratio of how much the guarantee cost changes relative to a small change in the fund value. For example, if a $1 fall in the underlying fund value results in the market-consistent guarantee cost increasing by $0.1, the *delta* is -0.1 (i.e. -$0.1/$1). If the hedge portfolio's position in the underlying fund portfolio is *delta* of the underlying fund value of the policy, the hedge portfolio is *delta-neutral*, and we are hedged – a change in the fund value does not affect the net position of the hedge portfolio less the guarantee value. Okay, great. So what's the problem?

WHAT'S LEFT BEHIND BY DELTA HEDGING

Generally, any delta-hedging strategy will leave a couple of key risks 'on the table':

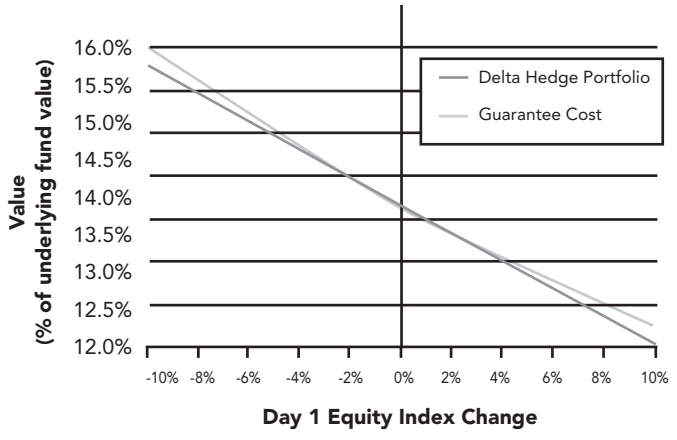
- From a mark-to-market valuation perspective, the balance sheet remains exposed to changes in option-implied volatility. This will impact on the liability mark-to-market value, but will not impact on the value of the (pure) delta-hedging portfolio

(the short position in the underlying fund portfolio will not be increased in value by an increase in option-implied volatility).

- There is a more fundamental risk exposure that is left behind by the *delta* hedging strategy: the delta of the guarantee continuously changes as the underlying fund value changes. In particular, as the fund value rises (falls) and it becomes less (more) likely that the guarantee will 'bite', the delta will decrease (increase) in magnitude. This second-order sensitivity is so important that it has its own greek name, *gamma*.

The trouble with *gamma* is that it creates a convexity in the guarantee cost such that a delta-hedging strategy loses money in *both* market directions. As markets fall, the guarantee cost delta rises in magnitude and become more negative, and our net position develops a positive *delta* exposure which loses money in falling markets. Conversely, when markets rise, the guarantee cost *delta* falls in magnitude and becomes less negative, which creates a net negative *delta* exposure which loses money as markets rise.

Exhibit 1: Guarantee Cost and Delta Hedge Portfolio Behaviour



The bigger the market rise or fall, the more money the delta hedger loses. In this sense, delta hedging has removed exposure *to directional* equity market changes, but leaves an exposure to unexpected changes in *realized fund volatility*. This is illustrated by the chart below, which is based on an example we discuss further later.

This mis-match can be mitigated by frequently re-balancing the size of the *delta* exposure in the hedge portfolio (i.e. increasing the size of the short equity position when the underlying fund value falls).This dynamic re-balancing can, in theory, perfectly capture this non-linearity in the *delta*, if the hedge portfolio is re-balanced continuously. But, in real life, continuous re-balancing isn't practically possible, and the discrete re-balancing frequency leaves the hedger exposed to the risk that the underlying fund value moves a lot before the hedger gets the chance to re-balance the hedge portfolio to the required exposure. In this case, the hedger will make a loss (irrespective of whether the big price change was up or down).

This risk is naturally most pronounced when markets are very volatile or 'jumpy'. Markets were especially volatile and jumpy in October 2008 – realized daily volatility of the S&P 500 over this month was 78%! We now use a simple GMWB case study to estimate what damage this volatility may have done to hedging performance during this period.

A SIMPLE VA GMWB CASE STUDY

The case study makes the following key assumptions:

- 5% GMWB for life, starting immediately for a 60 year-old male;
- 5% lapse rate that dynamically reduces to 0% as the underlying fund value approaches 0;

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• 0.75% AMC and 1.5% guarantee charge deduction;

• 100% of underlying funds invested in S&P 500 index.

Using the B+H VA Hedging ESG, the starting market-consistent guarantee cost for the GMWB is estimated at 13.7% of the starting underlying fund value on October 1st. The accompanying delta is estimated at -0.16.

We now consider how a daily-rebalanced delta-hedging program would have fared in the following weeks. In particular, our analysis focuses on estimating the losses that arise from the *gamma* slippage in this particularly volatile market environment. Exhibit 2 charts the daily S&P 500 price change between October 1st and November 20th and an estimate of the cumulative daily losses that would arise from the hedge strategy's *gamma* exposure that is highlighted in Exhibit 1 on page 19.

Exhibit 2: Equity returns and delta-hedging losses

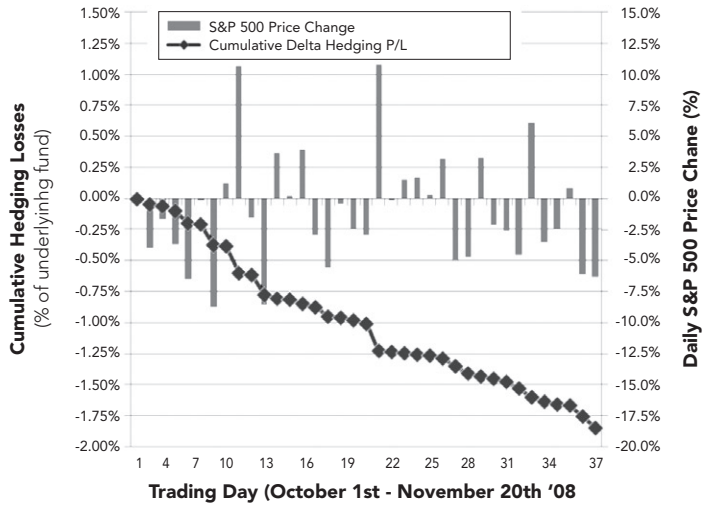


Exhibit 2 highlights how daily hedging losses were most significant when the magnitude of the equity index change is greater than 5%. Again, this can also be observed in Exhibit 1. Our analysis suggests that, for this particular variable annuity policy, the cumulative delta-hedging losses over this 7-week period were almost 2.0% of the underlying fund value, which is equivalent to several years' of the anticipated profit from the policy.

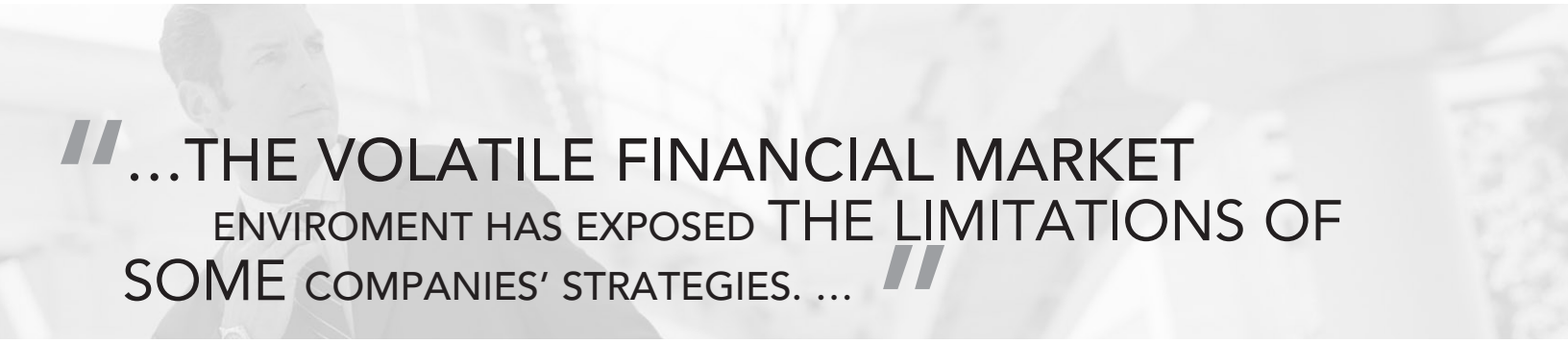
This analysis makes the key assumption that the hedge portfolio was re-balanced at the start of every trading day. Some firms may have been able to implement intra-day re-balancing that would have reduced these hedging losses. On the other hand, this analysis has not incorporated any allowance for trading costs or the difficulties implementing significant increases in equity short positions in these challenging market conditions.

It should also be noted that this is just *one* of the sources of hedging loss that has been quantified in this analysis. The increases in option-implied volatilities over the period will also have generated a significant mark-to-market hedging loss. The size of these calculated *vega* losses will depend on how firms are extrapolating option-implied volatilities beyond the liquid parts of the market. Also, unlike the above *gamma* losses, there is more opportunity to make *vega* gains as option-implied volatilities revert back to more normal levels in the future. However, we expect *vega* losses over the above measurement period to be at least comparable to the above *gamma* losses. Finally, falls in interest rates will have further exacerbated hedging losses.

Given that there is over $200bn of VA funds in-force in the US with attaching GMxBs, *we anticipate that total hedging losses over this period will total many billions of dollars*. These losses will be visible in Q4 2008 US GAAP earnings statements.

LOOKING AHEAD

You may have read the above case study and asked yourself 'where's the news?'. After all, recognition that a delta-hedging strategy leaves *gamma* risk behind hardly represents a new breakthrough in option pricing theory. And equity markets



"...THE VOLATILE FINANCIAL MARKET ENVIROMENT HAS EXPOSED THE LIMITATIONS OF SOME COMPANIES' STRATEGIES. ..."

have produced a number of similar periods of high short-term volatility over the last twenty years. *Delta hedgers have lost money in a volatile market – so what?*

Well, insurance companies have not been in the dynamic hedging business for very long. This is the greatest period of financial market volatility that has been experienced since VA hedging programs were put in place. Senior managers, regulators and investment analysts may not have fully appreciated the possible scale of the 'residual' risks that sophisticated hedging strategies can leave behind. This experience may re-shape the sector's approach to product pricing and product design, and dial back the levels of tolerance for having these risks on insurance company balance sheets. It is very likely to lead to more scrutiny in the implementation of internal hedging strategies. In particular, we expect that there will be a number of key themes that will consistently arise in the development of more robust hedging analytics and strategies for 2009. Three of these are discussed below.

Hedge projection and evaluation: Recent experience will naturally lead to greater scrutiny of projected hedge performance and the quantification of the risks left behind. There is likely to be more stress test analysis of hedging strategies (both for internal and external purposes). There is also likely to be more interest in sophisticated stochastic equity models that can generate daily scenarios similar to those experienced in these market environments. Such models can help to estimate how frequently such environments can occur, and the probability of experiencing similar or greater losses in future periods. Using a stochastic model as well as historical stress tests has the additional benefit that it becomes harder to design a strategy that merely 'data-mines', i.e. that is designed to work very well in a particular historical scenario that will not specifically occur again.

Model risk in hedge analytics: The *greeks* of VA guarantees are assessed using (market-consistent) stochastic asset models – they are too complex to be found in a textbook or on a Bloomberg screen. At B+H we are clearly big fans of stochastic asset models. But we also know that a model is always an approximation. The art of developing and applying stochastic models lies in judging which approximations are unlikely to have a material impact on the results of interest, and which could have a big impact. This isn't always obvious, and sometimes the best way to quantify this model risk is to re-calculate results using more than one (reasonable) model. If the results are largely invariant to the model choice, the hedger can sleep more easily at night. We anticipate that greater scrutiny around hedge implementation risk will result in a more work being done in the area of *model* risk, and firms will be less reliant on a single model to provide them with the 'right answer'.

Allowing for credit risk in underlying VA funds: Historically high levels of credit spreads are resulting in greater investment of VA underlying funds in credit-risky bond funds that are offering attractive yields. The high level of credit spread volatility also means that exposure to credit risk in the underlying funds will have a greater impact on market-consistent guarantee valuations than previously. To date, few firms have fully incorporated allowance for credit risk in underlying funds into the assessment of the market-consistent costs of guarantees. The B+H VA Hedging ESG provides this capability, and we estimate that *making allowance for credit risk will increase market-consistent GMWB costs by 10%-25% for a product with a 50%/50% equity/bond allocation.*

CONCLUDING THOUGHTS

Recent months have clearly represented a challenging period for the global economy in general and the financial services sector in particular. The insurance sector has not been immune from such malaise. In particular, the volatile financial market environment has exposed the limitations of some companies' strategies for managing the market risk exposures created by the provision of significant long-term guarantees in VA products. We expect this will result in the sector incurring hedging losses of the order of many billions of dollars in Q4 2008, representing several years of the expected profit stream from these VA product lines.

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We also anticipate that this experience may trigger a significant evolution in the sector's approach to the pricing, design, risk management and (perhaps) regulation of the guarantees embedded in VA products. This experience may highlight - as similar experiences have highlighted in other markets around the globe - that, over the long life of such products, seemingly insignificant and residual market risks have a habit of emerging to impact significantly on insurance company balance sheets, and that the impact tends to be in one direction only. Whether in the US, Taiwan, UK or Continental Europe; and whether it be equity returns, equity volatility or long-term interest rate levels, over the last seven years global life insurance sectors have had to learn the hard way that apparently innocuous long-term market risk exposures demand very rigorous scrutiny and management.

We anticipate that the impact on the US sector will be similar to that experienced elsewhere around the globe during similar periods: a move to a more disciplined market-consistent approach to product pricing; consequently, less significant investment guarantees in products; more (truly) principle-based approaches to regulatory capital assessment; and more market risk hedging and risk management scrutiny. US VA hedging activity already represents the most sophisticated market risk management program in the global insurance sector, but there are still significant risks left behind that, over a 20-year product horizon, are going to cost a lot of money. 2009 will see better recognition, measurement and management of those risks, and B+H looks forward to supporting the sector in implementing these improvements.

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