SOCIETY OF ACTUARIES

Article from:

Risks & Rewards

August 2010 – Issue 56

SHORT CUTS: EASY YIELD CURVE FIT

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With this edition we would like to kick off a special occasional section of numerical short cuts, rules of thumb, estimators, modeling tips—let's face it, fudges high and low which, like all great modeling tools are patently wrong but at the same time supremely useful at the right time and place. Please send your contributions to the editors or directly to Sam Phillips (sphillips@soa.org) at the SOA.

The problem is fitting the intermediate points for a real-world stochastic model of interest rates. For example suppose with much care and attention you have generated a range of paths for the 10-year interest rate, and a correlated set of paths for the two-year interest rate. You have worked out mean reversion strength, upper and lower limits, serial correlation, extent of inversion and time spent inverted. With these decided, the 80-20 rule would tell you to put down the pencil. But how do you splice together these pairs of 10s and twos into a set of yield curves?

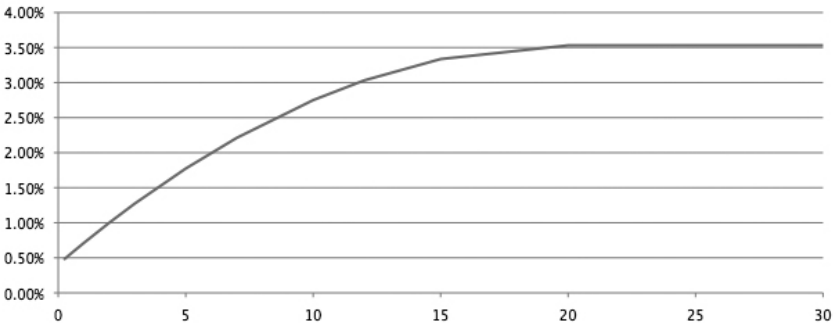
The short cut is to use a parabola $Y = aX2 + bX + c$, but some further assumptions are needed since the equation has three unknown coefficients, while for each yield curve you have only two points. The extra assumption is that the yield curve is flat, with slope zero, beyond some high point M such as 20 years. This implies interest rates are constant beyond year M. In this case, $dy/dx = 0 = 2aM + b$ so that $b = -2aM$. From there, if we know (X1, Y1) and (X2,Y2) we can substitute into the formula as follows.

The resulting curve connects the dots in a generally reasonable way without creating many discontinuities in the par or forward rate series.

When the curve is inverted, the output also looks like a yield curve. Since the 2nd derivative of a parabola is a constant, the slope always "decelerates" at a constant rate until it hits zero at year M by assumption. A new set of coefficients can be simply computed for each pair of inputs to interpolate and extrapolate.

Estimate yield curve as		$y = aX^2 + bX + c$
M	20 yr	Maximum x-axis point
X1	2 yr	lower point
Y1	1.00%	data for lower point
X2	10 yr	upper point
Y2	2.75%	data for upper point
a	-0.000078125	= [Y2 - Y1] / [(X2 - X1)(X2+X1-2M)]
b	0.003125	= -2aM
c	0.00406	= Y2 - a(X2)^2 - b(X2)

Yield Curve Fit



X	Y
0.25	0.48%
0.5	0.56%
1	0.71%
2	1.00%
3	1.27%
5	1.77%
7	2.21%
10	2.75%
12	3.03%
15	3.34%
20	3.53%
25	3.53%
30	3.53%

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