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Stochastic Simulation for C3 Risk: A Statistical Review

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In 2003, the American Academy of Actuaries Life Capital Adequacy Subcommittee (LCAS) published a set of 10,000 stochastic scenarios that was intended to be used to support the publication, "Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Products with Guarantees (Excluding Index Guarantees)." Any practitioner may download the prepackaged scenarios from the AAA Web site.[1] The prepackaged scenarios include 360 months of simulated data for three interest rates and nine asset classes. The public availability of this data provides a unique opportunity for analysis.

The primary purpose of this article is to illustrate statistical measures that can be used to evaluate stochastic simulations. Given the public availability of the data, readers may download the data and replicate the calculations. The secondary purpose of this article is to use those measures to compare the pre-packaged scenarios to current forecasting practice.

The prepackaged scenarios were created to satisfy the recommendations of the LCAS for simulations with tails that satisfy specified distributional requirements. Those requirements were based on Mary Hardy's regime-switching lognormal stochastic model (RSLN2) for the S&P 500. The LCAS model was developed by extending the RSLN2 model from a single asset class model to a multiple asset class model. While the Hardy model was described in an extensive paper in the NAAJ and an accompanying Excel spreadsheet, the LCAS did not publish comparable documentation on the extension of the model to multiple asset classes. Due to the complexity of the Hardy model and the LCAS extension, this writer was not able to determine if the LCAS model extension has the same degree of validation as the original Hardy model. Nevertheless, it is possible to analyze the prepackaged scenarios as a set of data observations.

Description of the Data

The prepackaged scenarios consist of 10,000 scenarios of 360 monthly returns for nine asset classes and 361 beginning-of-month yields for three interest rate categories.

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1) *The url for the files and supporting information is on the Academy Web site at* http://www.actuary.org/life/phase2.html.

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The modeled asset classes are:

Asset Class	Market Proxy
Money Market	3-month Treasury Returns
Intermediate Term Government Bonds (U.S. IT GVT)	U.S. Intermediate Term Government Bonds
Long-Term Corporate Bonds (U.S. LT CORP)	U.S. Long-Term Corporate Bonds
Diversified Fixed Income	65% USITGVT + 35% USLTCORP
Diversified Balanced	60% U.S. Equity + 40% Fixed Income
Diversified U.S. Equity	S&P 500 Total Return Index
International Equity	MSCI-EAFE $USD Total Return Index
Intermediate Risk	U.S. Small Capitalization Index
Aggressive or Specialized Equity	Emerging Markets, Hang Seng

Interest Rate	Market Proxy
Short-Term Rates	3-month U.S. T-Bill Yields
Medium-Term Rates	7-year U.S. Treasury Yields
Long-Term Rates	10-year U.S. Treasury Yields

For the purposes of this analysis, the asset returns have been converted to annual returns for 30 years and the yields have been converted to beginning-of-year yields for the initial date and the beginning of each subsequent year. As a 30-year annual simulation, the converted simulation is consistent with the common practice of expressing simulation assumptions and historical statistics in annual terms. The statistics referenced by LCAS are in annual or multi-year terms.

Since the simulation was based on a monthly model, translation to annual statistics tests the connections among the months, as well as the monthly simulation.[2] The data is available in csv files; since current versions of Excel are able to read only 256 of the 361 columns, the author used APL2 for the calculations and Excel to create the graphs.

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2) Please refer to my article, "Time Track: Analyzing Historical Returns" in the September 2000 issue of Risks and Rewards *for examples of the differences between monthly and annual returns.*

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Statistics For Stochastic Forecasts

Readers are undoubtedly familiar with the commonly used statistics that are used for economic modeling: mean returns, standard deviation of returns and correlation of returns. Less well known is that there are at least two variations of "mean," and seven variations of both "standard deviation" and "correlation." These variations exist because the simulation consists of a matrix of 10,000 scenarios, unlike history, which is a single scenario. The added dimensionality of the simulation creates the opportunity for these alternative measures, which are described in the following paragraphs. In each case, the statistics are calculated for an N year time horizon, where N can be as large as the length of the simulation—30 years for the pre-packaged scenarios.

The year-by-year standard deviations can be used to determine whether the average volatility changes over time.

Mean return: Mean return can be expressed either as arithmetic mean or annualized compound mean. The annualized compound mean is also known as "geometric mean" or "compound mean." Arithmetic mean return for the N year horizon is the simple average of the N x 10,000 observations. For annualized compound mean, the annualized compound return is calculated for each scenario and then the arithmetic mean of the 10,000 annualized compound returns is calculated. (Note that the compound mean is actually the arithmetic mean of the geometric means for all scenarios.)

Arithmetic Mean = average (10,000 x N observations, taken individually)

Compound Mean = average (Annualized Compound Return of each scenario, for 10,000 scenarios)

Mean Cumulative Return: Mean cumulative return is calculated by taking the arithmetic average return for each scenario and then taking the average of the 10,000 averages. The mean cumulative return equals the arithmetic mean return for the same time horizon. The standard deviation of the mean cumulative return will approximate the standard deviation of compound returns.

Mean Cumulative Return = average (arithmetic average of each scenario, for 10,000 scenarios)

Mean Wealth:[3] Return can also be expressed as the total of initial principal and growth, or accumulated value, with investment growth reinvested. This is equal to 1.0 plus the non-annualized compound return. Note that annualizing is non-linear and the Nth root of mean wealth is generally not equal to the mean compound return.

Mean Wealth = average (accumulated value of each scenario, for 10,000 scenarios)

Standard Deviation: There are three variations of standard deviation that relate to annual returns and one variation that relates to compound returns. The simplest measure is the global standard deviation, which is the standard deviation[4] of the N x 10,000 observations, taken individually. Two other measures access the simulation data as a two dimensional matrix. The longitudinal standard deviation represents the average standard deviation over N years (however, it is calculated by averaging variances); the cross sectional standard deviation represents the average standard deviation over the 10,000 simulations. Where the simulation data is a result of an i.i.d. process, all three annual measures will be approximately equal. When there are trends or other connections among the years, the statistics may vary. Since the standard deviation of the N years of a scenario is conceptually identical to the standard deviation of an N year historical period, the longitudinal standard deviation is typically considered to be the best comparator to history. Note that the longitudinal standard deviation uses the standard deviation

3) The distribution of wealth, particularly at the tails, is the focus of the RBC requirements.

4) Opinions differ as to usage of the sample or population variations of the standard deviation. The sample standard deviation has the benefit that the global, cross sectional and longitudinal standard deviations would all be equal for data consisting of white noise.

or variance of each scenario, and it is also possible to calculate the standard deviation of the standard deviations. Ideally, the distribution of standard deviations in the simulation scenarios would be comparable to the range of standard deviations seen in history. The year-by-year standard deviations (i.e., the basis of the cross sectional standard deviation) can be used to determine whether the average volatility changes over time. (Although means are linear, readers should note that standard deviations are not.)

The standard deviation of compound returns takes the standard deviation of the annualized compound return in each scenario. If the observations are i.i.d, then the standard deviation of compound returns would be approximately equal to the global standard deviation of annual returns, divided by the square root of N, the time horizon. In the presence of positive (negative) serial correlation, the standard deviation of compound returns will be higher (lower) than that approximation.

Global Standard Deviation = Standard Deviation (N x 10,000 observations, taken individually)

Longitudinal Standard Deviation =
(average (variance of each scenario for 10,000 scenarios))$^{.5}$

Cross sectional Standard Deviation =
(average (variance of each year for N years))$^{.5}$

Standard Deviation of Compound Returns =
standard deviation (compound return of each scenario, for 10,000 scenarios)

Standard Deviation of Wealth: The standard deviation of wealth is the standard deviation of the accumulated value, over 10,000 scenarios.

Correlations: The variations of the correlation measure are parallel to the standard deviation measure. There are three measures of annual correlation and one measure of compound correlation. Of course, the correlation matrix is computed for each pair of asset classes, with parallel observations for each asset class —global, longitudinal, cross sectional or compound.

In the case of returns with i.i.d., correlation measures would be approximately equal. As with standard deviations, the longitudinal correlation is most comparable to historical statistics.

Global Correlation = correlation (N x 10,000 observations, taken individually)

Longitudinal Correlation = average (correlation for each scenario, for 10,000 scenarios)

Cross Sectional Correlation = average (correlation for each year, for N years)

Compound Correlation = correlation (compound return of each scenario, for 10,000 scenarios)

Serial Correlation: The serial correlation of annual returns may also be calculated. Typically, the global form of the statistic is calculated.

Serial Correlation = correlation (each return to the return of the prior year, for all 290,000 feasible observations)

Percentile Ranking of Results: It is very common for stochastic results to be presented in the form of percentile. For example, the 10th, 25th, 50th, 75th and 90th percentile of the return in each year or the compound return can be calculated. Percentiles are typically calculated on a global, cross sectional basis or compound basis; the longitudinal variant is rarely seen.

The standard deviation of compound returns takes the standard deviation of the annualized compound return in each scenario.

Examples of Detailed Statistics for the U.S. Equity Class

Table 1 shows the year-by-year distribution of U.S. equity returns in the prepackaged scenarios. The reported statistics are the mean return in each year, the standard deviation of the returns in each year, the minimum and maximum return in each year and the percentile of the return in each year. The statistics in the bottom row of the table are the arithmetic averages of the annual statistics. The average statistics for the minimum, maximum and percentile are informational, but possibly ambiguous, e.g., the average of

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each year's minimum return is not a particularly useful statistic. It does, however, provide one measure of the overall distribution. The bottom portion of the table reports three standard deviation measures for the 30-year time horizon. All three statistics are very similar, which implies that there are minimal trends or serial correlation in the underlying data.

Table 2 on page 11 shows the distribution of annualized compound returns over various time horizons. For example, the average 30-year compound return is 11.13 percent and the standard deviation of the 30-year returns (10,000 observations) is 3.49 percent. Some practitioners believe that the standard deviation of compound returns provides a misleading portrayal of risk. Others, including the author, believe that it is one measure of risk that should be evaluated along with the other measures of risk. The fact that the standard deviation of compound return is 3.49 percent should not be interpreted to mean that the asset

Table 1: Distribution of Simulated U.S. Equity Returns – Annual Returns

				Percentiles							
Year	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
1	12.50	17.50	-57.16	-31.41	-9.96	1.65	12.74	23.61	34.64	53.49	91.03
2	12.91	17.35	-51.41	-30.04	-8.72	1.98	12.94	23.94	34.30	54.03	91.82
3	12.43	17.58	-50.03	-30.62	-9.86	1.27	12.59	23.85	34.29	53.78	94.71
4	12.96	17.49	-53.82	-29.09	-9.12	1.90	13.02	23.97	34.73	55.90	97.21
5	12.82	17.18	-51.48	-30.97	-8.54	2.05	12.85	24.03	34.32	54.38	87.85
6	12.36	17.65	-60.15	-32.81	-9.54	1.55	12.58	23.49	34.21	53.49	91.68
7	12.28	17.52	-50.28	-30.37	-9.70	1.27	12.45	23.61	34.20	54.46	94.85
8	12.54	17.52	-50.29	-31.25	-9.72	1.68	12.62	23.68	34.12	54.37	85.12
9	12.42	17.42	-54.83	-30.50	-9.52	1.37	12.51	23.70	34.09	54.33	87.77
10	12.30	17.40	-55.73	-30.84	-9.69	1.28	12.40	23.40	33.94	54.81	84.05
15	12.68	17.76	-54.87	-31.50	-9.94	1.43	12.90	24.17	34.89	54.85	83.18
20	12.01	17.56	-64.27	-30.84	-9.96	1.39	12.01	23.43	34.89	53.80	96.03
25	12.42	17.49	-55.23	-30.26	-9.92	1.36	12.67	23.84	33.48	53.47	87.52
30	12.24	17.72	-51.36	-31.78	-10.21	1.10	12.39	23.32	34.40	55.53	90.33
Avg	12.49	17.52	-53.99	-30.80	-9.60	1.52	12.59	23.78	34.33	54.22	95.33

Summary	30-Year Standard Deviation
Cross Sectional	17.52
Longitudinal	17.49
Global	17.52

class has low risk. In fact, Table 1 shows that the annual standard deviation is approximately 17.5 percent. The statistical measures of volatility are completely consistent, as long as they are appropriately compared.

Inspection of Table 2 indicates that the mean compound return decreases with time horizon, even though the year-by-year data shown in Table 1 is reasonably constant. This has often confused new practitioners, as the decreasing trend seems contrary to the stated assumptions. In fact, it is a simple artifact of a non-tended simulation. The mean compound return can be approximated from the arithmetic return by subtracting half of the variance. In this example,

Estimated mean compound return = $.1249 - .5{*}.1752^2$
= .1096,

which compares favorably to the actual 11.13 percent.

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Table 2: Distribution of Simulated U.S. Equity Returns – Compound Returns

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
1	12.50	17.50	-57.16	-31.41	-9.96	1.65	12.74	23.61	34.64	53.49	91.03
2	12.05	12.74	-38.54	-21.15	-4.36	4.17	12.50	20.52	27.72	40.58	61.95
3	11.72	10.62	-33.40	-15.14	-2.32	4.96	12.12	19.03	24.85	35.03	50.79
4	11.68	9.26	-24.98	-12.01	-0.50	5.84	12.03	18.03	22.91	32.42	49.05
5	11.63	8.28	-20.63	-9.26	1.02	6.30	12.06	17.28	21.85	30.05	40.94
6	11.52	7.62	-19.65	-7.95	1.63	6.61	11.81	16.80	21.00	28.49	40.30
7	11.42	7.08	-16.24	-6.16	2.14	6.81	11.71	16.30	20.23	27.06	41.09
8	11.39	6.64	-18.14	-4.89	2.63	7.12	11.59	16.02	19.61	25.70	37.90
9	11.35	6.30	-16.97	-4.44	2.99	7.20	11.57	15.73	19.19	25.23	36.08
10	11.30	6.00	-14.48	-3.64	3.54	7.40	11.51	15.41	18.72	24.67	35.03
15	11.21	4.93	-8.35	-0.91	4.74	7.96	11.38	14.58	17.36	21.98	31.22
20	11.13	4.27	-7.62	0.78	5.57	8.36	11.23	14.01	16.53	20.69	28.18
25	11.14	3.82	-4.96	1.77	6.20	8.64	11.27	13.75	15.93	19.67	26.62
30	11.13	3.49	-2.40	2.84	6.60	8.81	11.16	13.50	15.56	18.96	22.75

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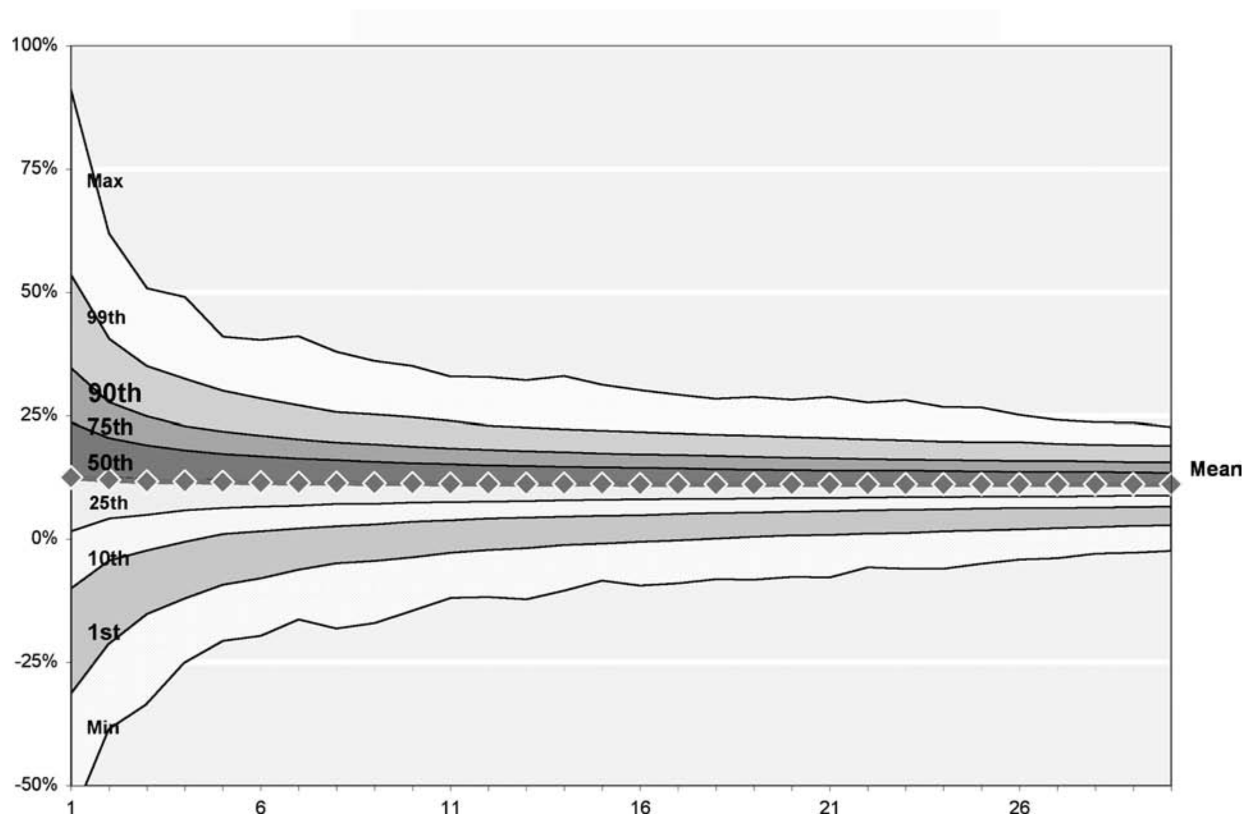
Chart 1 provides a graphic presentation of the distribution of compound returns for all years out to the 30th year. The chart shows that there is a fairly wide gap between the 99th percentile and the maximum observation and also between the first percentile and the minimum observation at each time horizon.

Table 3 illustrates the Cumulative Arithmetic Average Return. This statistic is similar to the compound return, except that the summation and division operations replace the product and root operations. The mean cumulative average return for each time horizon is exactly equal to the arithmetic average return for that time horizon. (For example, the 30-year average of 12.49 percent matches the 30-year average in Table 1.) The standard deviation of cumulative average return, on the other hand, is very close to the standard deviation of compound returns.

The next table on page 13 shows the distribution of accumulated wealth, which is the non-annualized

Chart 1: Distribution of U.S. Equity Returns Compound Returns



compound return. Note that the 10th root of the 10-year Mean Wealth is 12.70 percent, while the Mean 10-year Compound Return is 11.30 percent. This illustrates the principle that the average of the root is not equal to the root of the average, since the 10th root is a non-linear operation. On the other hand, the percentile of wealth and compound return line up one-one in rank order. Therefore, the 10th root of the median 10-year Wealth is 11.51 percent, which equals the median 10-year compound return. That relationship holds for all percentile.

The standard deviation of the 10-year Compound Return is 6.00 percent, while the global standard deviation in the 10th year (17.46 percent) divided by the square root of 10 is 5.52 percent. The fact that the standard deviation of compound return is greater than the estimate indicates the presence of a small amount of positive serial correlation.

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Table 3: Distribution of Simulated U.S. Equity Returns – Cumulative Arithmetic Average Return

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
10	12.55	5.82	-12.68	-1.61	5.09	8.75	12.70	16.51	19.79	25.78	36.16
15	12.52	4.78	-6.17	0.97	6.31	9.36	12.68	15.76	18.49	23.16	32.30
20	12.47	4.13	-5.80	2.75	7.13	9.72	12.57	15.24	17.72	21.83	29.15
25	12.50	3.70	-1.63	3.57	7.72	10.06	12.58	15.00	17.17	20.84	27.91
30	12.49	3.38	-0.47	4.50	8.12	10.26	12.49	14.80	16.81	20.15	24.30

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Table 4: Distribution of Simulated U.S. Equity Returns – Wealth (Initial Value = 100)

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
1	112.5	17.5	42.8	68.6	90.0	101.7	112.7	123.6	134.6	153.5	191.0
2	127.2	28.3	37.8	62.2	91.5	108.5	126.6	145.2	163.1	197.6	262.3
3	143.2	39.6	29.5	61.1	93.2	115.6	141.0	168.6	194.6	246.2	342.9
4	161.9	52.2	31.7	60.0	98.0	125.5	157.5	194.0	228.2	307.5	493.5
5	182.8	65.8	31.5	61.5	105.2	135.7	176.7	221.9	268.6	372.0	556.1
6	205.7	82.2	26.9	60.9	110.2	146.8	195.4	253.9	313.8	450.0	762.7
7	231.3	100.6	28.9	64.1	116.0	158.6	217.0	287.8	363.2	534.6	1112.8
8	260.6	121.5	20.2	67.0	123.1	173.4	240.4	328.2	418.9	623.4	1307.4
9	293.7	147.1	18.7	66.5	130.4	187.0	267.9	372.3	485.4	757.7	1599.9
10	330.4	176.6	20.9	69.0	141.6	204.3	297.2	419.1	556.3	907.2	2014.9
15	598.3	406.5	27.0	87.2	200.3	315.6	503.6	770.1	1103.3	1968.6	5890.9
20	1076.7	876.7	20.5	116.9	295.4	498.4	840.1	1376.0	2133.4	4301.7	14334.9
25	1961.1	1861.2	28.0	155.1	450.0	793.6	1444.5	2505.6	4028.6	8911.2	36519.8
30	3556.5	3754.6	48.2	231.4	681.2	1260.2	2392.5	4463.0	7658.1	18269.0	46817.8

Chart 2: Distribution of Simulated U.S. Equity Returns
Wealth

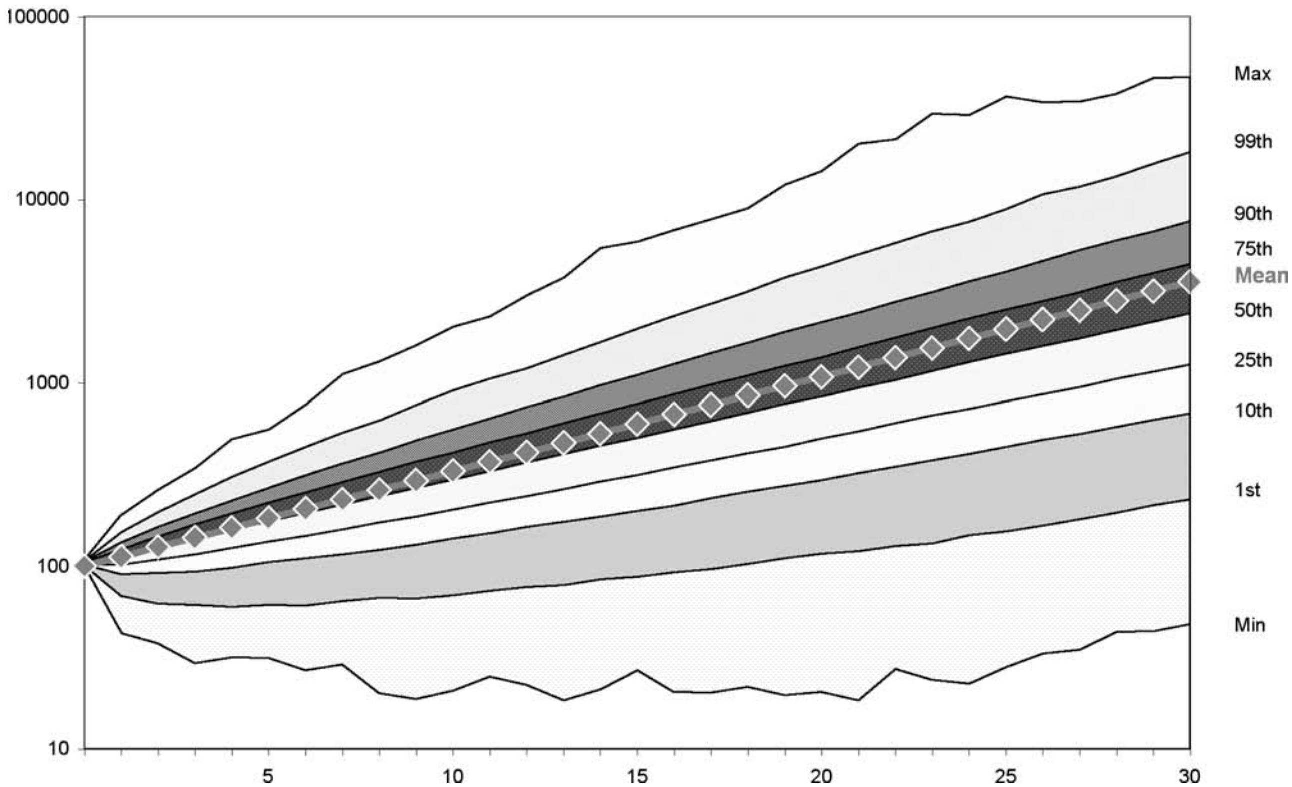

Chart 2 above illustrates the distributions of wealth for each time horizon.

As discussed in the definition of standard deviation, the standard deviation of the returns in each scenario can be calculated. For example, Table 5 shows that, over a 30-year horizon, the simulated standard deviations range from a minimum of 9.0 percent to a maximum of 27.7 percent. It is not surprising that a sample of 10,000 scenarios could have such a wide distribution; the 10th to the 90th percentile provide a more reasonable estimate of the range of the scenarios—from 14.3 percent to 20.9 percent. That is a relatively narrow range around the average of 17.5 percent. This table can be seen on page 16.

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Table 5: Distribution of Simulated U.S. Equity Returns – Longitudinal Standard Deviations

				Percentiles							
Horizon	Mean	Std Dev	Min	1st	10th	25th	50th	75th	90th	99th	Max
10	16.74	4.45	3.99	7.95	11.61	13.92	16.77	19.93	22.96	28.89	37.25
15	17.15	3.62	5.78	9.81	12.85	14.86	17.22	19.72	22.12	26.54	32.56
20	17.33	3.13	7.57	10.81	13.58	15.28	17.39	19.53	21.59	25.27	29.31
25	17.42	2.80	8.20	11.53	14.01	15.59	17.47	19.38	21.16	24.52	28.99
30	17.49	2.57	9.00	11.99	14.34	15.81	17.52	19.30	20.93	23.84	27.66

Table 6: Distribution of Simulated U.S. Equity Returns – Correlations to Long Corporate Bonds

Horizon	Annual	Cross Sectional	Longitudinal	Global	Cumulative Average	Compound	Wealth
10	-0.02	-0.03	-0.02	-0.03	-0.14	-0.13	-0.13
15	-0.02	-0.03	-0.02	-0.03	-0.18	-0.17	-0.16
20	-0.04	-0.03	-0.03	-0.03	-0.20	-0.19	-0.16
25	-0.04	-0.03	-0.03	-0.03	-0.21	-0.20	-0.16
30	-0.02	-0.03	-0.03	-0.03	-0.21	-0.21	-0.15

Table 6 shows seven measures of correlation between U.S. Equity and Long Corporate Bonds. While the four correlations of annual returns are close to 0.0, the three correlations of compound returns are negative.

Comparison of Prepackaged Scenarios to Current Forecasting Practice

This section compares the summary statistics for the prepackaged scenarios to assumptions commonly used in ALM studies for pension plans and other institutional investors. The purpose of the prepackaged scenarios is to evaluate the risk for annuity guarantees at the extreme tails of the distribution, while pension plans and institutional investors are more concerned about the middle of the distribution. Therefore, it is not surprising that there are some incompatibilities between the prepackaged scenarios and assumptions typically used for pension forecasting. This analysis does not comment on the utility of the scenarios for their intended purpose.

Table 7 below shows the 10-year means and standard deviations for the seven basic asset classes in the prepackaged scenarios:

The average U.S. Equity returns are much higher than current mainstream forecasting assumptions and the fixed income returns are somewhat lower than current practice. The spread between the equity and fixed-income returns, which represents the equity risk premium, is much higher than current practice.

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Table 7: 10-Year Means and Standard Deviations

Asset Class	Arithmetic Average Return	Longitudinal Standard Deviation	Compound Return
U.S. Equity	12.6%	16.5%	11.3%
Intermediate Risk Equity	15.3%	22.8%	12.9%
Aggressive Equity	17.6%	28.3%	13.9%
International Equity	12.7%	18.7%	11.1%
Intermediate Term Government Bonds	4.0%	3.6%	3.9%
Long-Term Corp Bonds	4.2%	5.4%	4.0%
Money Market	2.6%	1.0%	2.6%

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On December 31, 2003, 30-year Treasury yields were about 5 percent. One reasonable approach for setting asset class assumptions would be to estimate 10-year long Treasury returns at the initial yield level. One would then expect long corporate bonds to be higher and intermediate term government bonds to be lower than the long Treasury return. Reasonable mainstream assumptions might be 4.5 percent for intermediate government bond returns and 5.5 percent for long corporate bond returns. Those estimates would indicate that simulated intermediate government returns are about 0.6 percent too low and that simulated long corporate bond returns are about 1.5 percent too low.

The risk premium for large cap stock over long T-Bond returns has typically been in the 2 percent to 4 percent range for the last 45 years. A common estimate for today's forecasting practice is about at the 3 percent level. The simulated U.S. Equity class has a spread of 7.4 percent and 7.3 percent over the intermediate government bond class and the long corporate bond class, respectively. That degree of spread is significantly above current forecasting practice. In nominal terms, expectations for large cap stocks are commonly in the 8 percent to 9 percent range; again significantly below the simulated nominal returns.

The historical standard deviation of long corporate bonds has ranged from about 1 percent in the 1940s to 19 percent in the 1980s. Volatility assumptions for long corporate bonds in current practice range from 7 to 13 percent, as compared to 5.4 percent for the prepackaged scenarios.

Table 8 shows the 10-year longitudinal correlations for the seven basic asset classes.

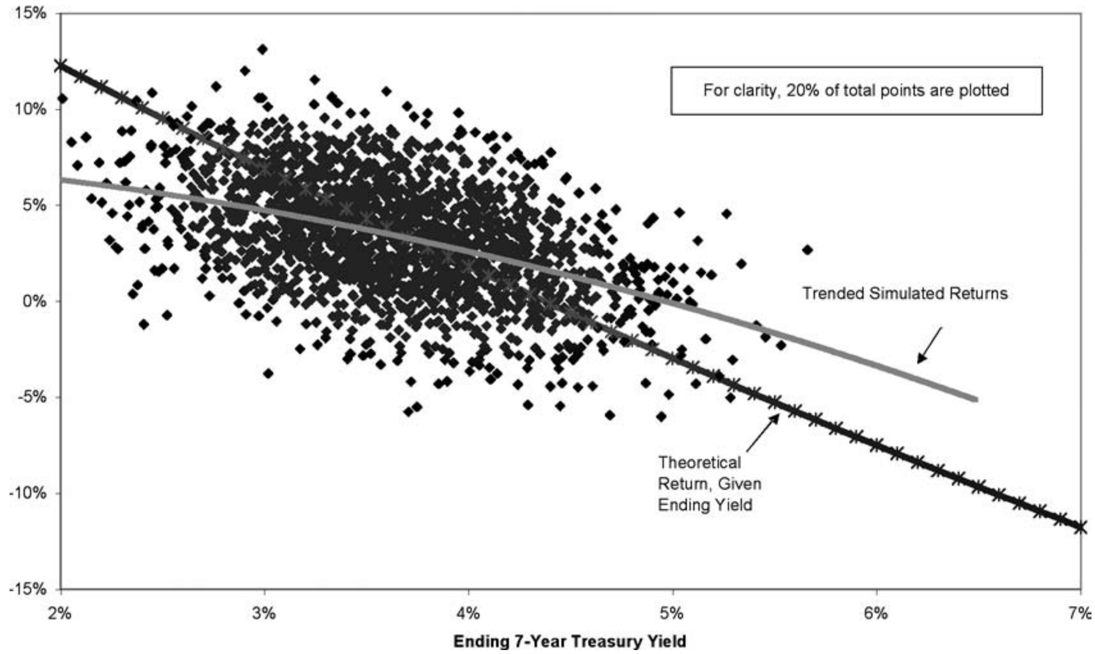
The correlation between U.S. Equity and the bond categories is significantly lower than current practice. For example, the prepackaged scenarios have a correlation of 0.00 between U.S. Equity and long corporate bonds, indicating a lack of correlation. However, most practitioners are using correlations in the 0.30 to 0.50 range, which indicates a lower degree of diversification than in the simulation.

The final analysis is to test the relationship of bond yields to bond returns. In most pension ALM

Table 8: 10-Year Longitudinal Correlations

Longitudinal 10-Year Correlations	U.S. Equity	Inter Risk Equity	Aggr Equity	Intl Equity	Inter Term Govt	Long Term Corp	Money Market
U.S. Equity	1.00						
Inter Risk Equity	0.75	1.00					
Aggressive Equity	0.65	0.65	1.00				
International Equity	0.60	0.50	0.55	1.00			
Intermediate Term Government Bond	0.10	0.10	0.05	0.15	1.00		
Long Term Corporate Bond	0.00	0.00	0.00	0.05	0.80	1.00	
Money Market	0.00	(0.05)	0.00	(0.05)	0.15	0.10	1.00

Chart 3: Bond Returns vs. Ending Yield
Year 1



forecasts, the liability discount rate is modeled by referencing a specified bond yield. If bond yields and bond returns are not synchronized with each other, then it is likely that the liabilities will not be synchronized with asset returns. This relationship is of utmost concern in a pension ALM forecast, as well as for other types of investors.

Chart 3 is an XY scatter chart of the first year simulated Intermediate Government Bond Return versus the simulated 7-Year Treasury yield at the end of the first year. Given the starting yield of 3.63 percent, it is possible to make reasonable estimates of the theoretical return for the various levels of ending yield.[7] In the prepackaged scenarios, the simulated bond returns have significant differences from the theoretical returns and the trend line has a concave shape instead of the desired slightly convex shape. Consequently, it does not appear that the yields should be used to model pension liabilities that act like bonds. This is not surprising, since the yields were designed for models of account balances and not for taking present values.

Summary

This article has illustrated and defined the multiple statistics that provide a summary of a stochastic forecast and has provided the actual results for the set of 10,000 prepackaged scenarios prepared by the LCAS. In addition, the article compared the simulation results to current practice for ALM studies for pension plans and other institutional investors and found that there were a number of differences. The expected return for U.S. Equity, the spread of U.S. Equity returns over bond returns and the correlation between U.S. Equity and bonds were the most notable differences. We also found that the simulated yields should not be used to model liability discount rates.

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*7) The theoretical yield is estimated by the equation (1+.25 * (0.363+ending yield)^2 * (Price of a six-year bond with coupon of 3.63 percent and yield equal to the ending yeld/100) - 1.*