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ENGAGING THE FEAR GAUGE
EMPIRICAL OBSERVATIONS ON COUNTERINTUITIVE VIX BEHAVIOR

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During the recent years, the CBOE Volatility Index® VIX (commonly referred to as "the fear gauge"), a measure of short-dated equity markets' volatility implied from listed S&P 500® Index options, has received a significant amount of attention, and its daily value has been used almost synonymously as an indicator of whether markets are in state of fear or calm. Due to its growing acceptance and usage by investors and journalists as a "fear gauge," a mention of price action in the VIX index is often linked to major news reports (Booton and Egan, Kiernan). Further, price action in the VIX index that is in the same direction as price moves in the S&P 500® index, defying the "fear gauge" definition, often is highlighted as a curious occurrence (Gammeltoft and Kisling). The purpose of this article is to examine the VIX definition in closer detail and attempt to provide rational explanations for some of the seemingly aberrant daily behavior of the VIX index at various recent occasions

1. OVERVIEW OF THE VIX INDEX

The VIX index as of any point in time provides a forward-looking estimate for 30-day volatility of the daily returns[1] of the S&P 500® Index (SPX). This estimate is obtained based on the live bid and ask prices of SPX options listed on CBOE and it relies on the assumption that option prices at any point in time, via their implied volatilities, embed information about the expected realization of volatility of the SPX until the option's maturity. However, as options with the same maturity, but with different strikes, tend to imply different volatility numbers (i.e., volatility skew exists), an estimate for the expected volatility independent of a specific option strike needs to take all this information into account to produce a single volatility number. Such an estimate has been studied by many and is well-described by Demeterfi, Derman, Kamal and Zou, and is calculated in practice by taking all available sufficiently-liquid options (calls for strikes above the forward, puts for strikes below) of the two maturities nearest 30 days, computing a weighted average of their prices to obtain a single estimate for the expectation of the SPX variance for each of these two maturities, and interpolating/extrapolating these two numbers to obtain the expected 30-day variance. Finally, the square root of this number is reported as the VIX.

Calculation of the VIX index can be very sensitive to the data, and thus, CBOE provides a detailed step-by-step description of the process in a VIX White Paper. An abbreviated summary has been included in the Appendix to assist the reader in understanding some of the case studies provided later.

2. A DEEPER DIVE INTO VARIANCE

This section covers a brief theoretical discussion to provide intuition behind the process involved in the calculation of the VIX. It tends to be more technical than the rest of this article and is provided as a reference for the technically-inclined reader; a less curious reader may safely omit this section without lack of continuity in the exposition.

Much has been written by researchers about variance swaps and volatility swaps. As the calculation behind the VIX index is based on variance swap pricing theory, this discussion focuses on the former. Demeterfi, Derman, Kamal and Zou (DDKZ) provide a way to decompose a variance swap into a static portfolio of options, a forward, and a continuously rebalanced delta hedge. Heuristically, the argument is made to extract an expression for the average variance, $AverageVar(0,T) = \frac{1}{T}\int_0^T \sigma_t^2 \, dt$, using a simple application of Ito's Lemma on the Geometric Brown Motion process for a stock with price S_t, which does not pay dividends, has an instantaneous volatility σ_t, and exists in an economy with an instantaneous risk-free rate r_t. The below provides a brief heuristic theoretical outline based largely on the approach taken by DDKZ.

If the stock process under the risk-neutral measure Q is represented by the stochastic differential equation

$$\frac{dS_t}{S_t} = r_t dt + \sigma_t dW_t^Q \quad (1)$$

then by Ito's Lemma, $Ln[S_t]$ follows the following process:

$$d\,Ln[S_t] = \left[r_t - \frac{\sigma_t^2}{2}\right]dt + \sigma_t dW_t^Q \quad (2)$$

Thus, subtracting (2) from (1) yields the expression:

$$\frac{dS_t}{S_t} - d\,Ln[S_t] = \frac{\sigma_t^2}{2}dt \quad (3)$$

When integrated, (3) gives an expression for the total variance, where the last equality follows after plugging (1) under the integral:

$$AverageVar(0,T) \times T = \int_0^T \sigma_t^2\,dt = -2\,Ln\left[\frac{S_T}{S_0}\right] + 2\int_0^T \frac{dS_t}{S_t} = -2\,Ln\left[\frac{S_T}{S_0}\right] + 2\int_0^T r_t dt + \int_0^T \sigma_t dW_t^Q \quad (4)$$

Finally, taking the expectation under Q, and assuming for simplicity that $r_t = r$ is constant, gives us the expression, in which the stochastic term disappears since W_t^Q is a martingale under Q.

$$E^Q[AverageVar(0,T)] = \frac{2}{T}E^Q\left[-Ln\left[\frac{S_T}{S_0}\right]\right] + 2\,r \quad (5)$$

In other words, the expected variance is linked directly to the expected value of the $-Ln\left[\frac{S_T}{S_0}\right]$ term, the short log-contract, which is nothing more than the negative of the continuously compounded return of the stock until time *T*. Since this contract itself does not trade in the market, however, DDKZ show that the short log-contract is identical in payoff (and thus in price and expectation) to a carefully-chosen portfolio composed of calls, puts, and a forward on the stock, all with maturity *T*. Thus, the term $E^Q\left[-Ln\left[\frac{S_T}{S_0}\right]\right]$ in (5) can be replaced simply by the undiscounted price of the replicating portfolio—a price easily observable as it is based on liquid instruments traded in the market. While theoretically, this replicating portfolio is based on a continuous set of strikes, in reality the strike space is often discretized to use available listed options, leading to the following expression, in which K_i is the strike of option, i, ΔK_i is the spacing between adjacent strikes, and $Q(K_i)$ is the price of the option with strike K_i, (assuming calls for strikes above some cut-off level and puts for the rest)

$$E^Q[AverageVar(0,T)] = \frac{2}{T}\left[\sum_i \frac{\Delta K_i}{K_i^2}e^{r\,T}Q(K_i) - E^Q\left[\frac{S_T}{S_0} - 1\right]\right] + 2\,r = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2}e^{r\,T}Q(K_i) \quad (6)$$

The expression in the expectation is simply the undiscounted value of the forward contract, which can be further simplified to $E^Q\left[\frac{S_T}{S_0} - 1\right] = e^{r\,T} - 1 \approx rT$. Making this substitution leads to the right-hand expression in (6).

Expression (6) looks very similar to the VIX formula, with a small exception that the latter includes an adjustment factor to account for the discreteness of the strike chosen to be the forward level.

This can be further simplified by setting $w_i = \frac{2}{T} \times \frac{\Delta K_i}{K_i^2}e^{r\,T}$ to conclude that the expected variance is simply a weighted average of the existing options prices.

$$E^Q[AverageVar(0,T)] = \sum_i w_i\,Q(K_i) \quad (7)$$

3. PRACTICAL OBSERVATIONS

This section provides examples of several instances, where the VIX calculation formula yields results that may be viewed as perplexing, and which, however, have a rational explanation in the context of the formula. An example is also shown of an occasion when pure market dynamics can drive the VIX to react contrary to common expectations.

As the theoretical underpinnings behind the VIX formula are based on the assumption that one can trade and observe prices for options of infinitely many strikes, several simplifications are made in the VIX formula to accommodate the limitations of options trading in practice

The first simplification is based on the need to use discrete option strikes since for short-dated maturities, listed SPX options exist only in strikes in multiples of five. For a given maturity, starting from the strike closest to the forward level, the VIX selects all calls with strikes higher than the

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> "AMONGST EQUITY INVESTORS, THE VIX, A MEASURE OF MARKET VARIANCE, IS WIDELY INTERPRETED AS THE MARKET FEAR GAUGE."

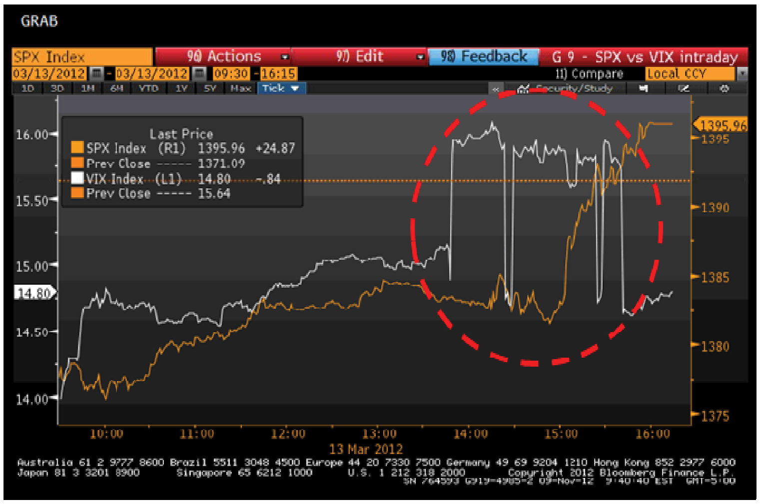

Figure 1. Source: Bloomberg

Ticker	Strike	Bid	Ask	Last	IVA	IVB
263) SPX 4/21/12 P985	985	.05	1.00	.40y	48.87	35.21
264) SPX 4/21/12 P990	990	.05	1.00	1.00y	48.24	34.73
265) SPX 4/21/12 P995	995	.05	1.00		47.61	34.26
266) SPX 4/21/12 P1000	1000	.20	.30	.30	40.33	38.57
267) SPX 4/21/12 P1005	1005	.05	1.00		46.36	33.31
268) SPX 4/21/12 P1010	1010	.05	1.00	2.15y	45.73	32.84
269) SPX 4/21/12 P1015	1015	.05	1.00		45.12	32.37
270) SPX 4/21/12 P1020	1020	.05	1.00	2.00y	44.50	31.91
271) SPX 4/21/12 P1025	1025	.20	.40	.40	38.90	35.95
272) SPX 4/21/12 P1030	1030	.05	1.00	1.36y	43.27	30.98
273) SPX 4/21/12 P1035	1035	.05	1.00	3.70y	42.66	30.52
274) SPX 4/21/12 P1040	1040		1.00	.75y	42.05	
275) SPX 4/21/12 P1045	1045		1.00	3.60y	41.45	
276) SPX 4/21/12 P1050	1050	.40	.50	.50	37.16	36.14
277) SPX 4/21/12 P1055	1055	.15	1.00	2.00y	40.24	31.87
278) SPX 4/21/12 P1060	1060	.10	1.10	1.75y	40.20	30.12
279) SPX 4/21/12 P1065	1065	.10	1.10		39.60	29.64
280) SPX 4/21/12 P1070	1070	.25	1.15	1.80y	39.26	32.13
281) SPX 4/21/12 P1075	1075	.30	.75	.85	36.32	32.30
282) SPX 4/21/12 P1080	1080	.30	1.20	.75	38.30	31.78

Figure 2. Source: Bloomberg

forward, for which a non-zero bid exists, until two consecutive zero-bids are discovered, which sets the upper bound for the highest call strike; similarly, puts with non-zero bids are selected with decreasing strikes starting from the forward until two consecutive zero-bids are found. Thus the sudden appearance and disappearance of zero-bids across various strikes at different times during the day may have noticeable effects on the value of the VIX (the VIX calculation is refreshed continuously throughout the day). Case Study 1 in this section provides an example of this effect.

The second necessary approximation is driven by the fact that the VIX represents 30-day volatility at all times without regard to whether options with 30-day maturity are actually traded in the market at that moment. To allow for this, a linear interpolation in the total variance space[2] is used to approximate the 30-day maturity based on two traded maturities that are close to 30-days. Most frequently, one such maturity will be shorter than 30 days, while the other will be longer than 30 days, and thus interpolation is used. Occasionally, however, due to the VIX requirement that the near maturity has at least one week to expiration, it may occur that both maturities used have more than 30 days remaining, thus requiring *extrapolation*. This may, at times, lead to counterintuitive results, as illustrated in Case Study 2.

The third example in this section is slightly different in nature as it addresses the use of the VIX as a "fear gauge." The belief that the VIX should move in the opposite direction of the SPX is rooted in the existence of implied volatility skew in SPX options, i.e., options with lower strikes trade with higher implied volatilities than options with higher strikes. However, whether the SPX at-the-money volatility moves along this skew (implied volatility levels are linked to actual SPX levels) so that an SPX decline is accompanied by an at-the-money implied volatility increase, depends on whether SPX volatilities follow a "sticky-strike" dynamics. In contrast, when the behavior of the volatility surface resembles a "sticky-delta" dynamics, i.e., as the SPX declines, at-the-money volatility does not change; rather the volatility skew simply moves along with the SPX, and a rise in volatilities may not be observed Case Study 3 shows a practical example of a time when SPX volatilities not only did not behave in a "sticky-strike" manner, thus not exhibiting the "fear-gauge" effect, but even declined together with the SPX.

Case Study 1: *Impact of the Vanishing Option*
March 13, 2012 illustrates a scenario in which VIX suddenly begins to whipsaw intraday for no apparent reason. As shown in the Bloomberg snapshot in Figure 1 (pg. 36, top), for most of the day, the VIX seems to slide along the skew, rising when the market falls and falling when the market rises. However, in the last two hours of trading, as the SPX begins to set up for a strong 1 percent rally into the close, the VIX suddenly becomes a series of four discontinuous 1 VIX point jumps – a tremendous relative move given that average volatility levels at the time were 15. (See fig. 1, pg. 36, top)

As mentioned above, this stochastic effect is due not to sudden changes in sentiment, but it is rather a technical of the VIX calculation itself. Specifically, while in theory the hedge for a variance swap calls for the purchase of the entire range of option strikes from zero to infinity, in practice, reasonable provisions must be made to account for the lack of liquidity of deep, out of the money options. Investment houses have varied proprietary approaches to define and model these "wings"; the VIX methodology employs a consecutive zero bid test. In short, the premiums for all options with strikes starting from at the money are included in the calculation until two consecutive strikes with zero bids are encountered. At that point, deeper out of the money option strikes are excluded from the calculation.

In the Bloomberg screenshot in Figure 2 (pg. 36, bottom), circled is the breakpoint at the 1040 and 1045 strikes.

Note in Figure 3 (pg. 37, top), which depicts the front

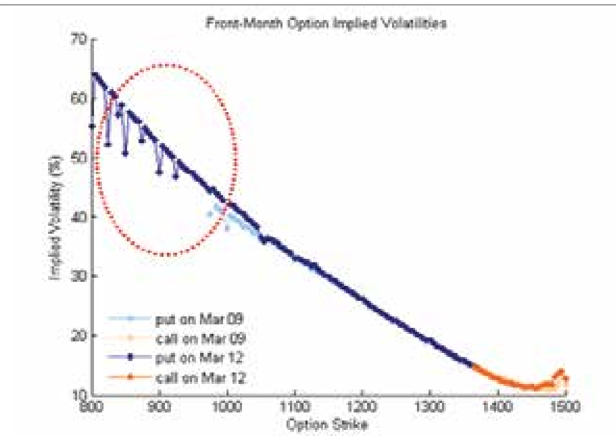

Figure 3. Source: Credit Suisse

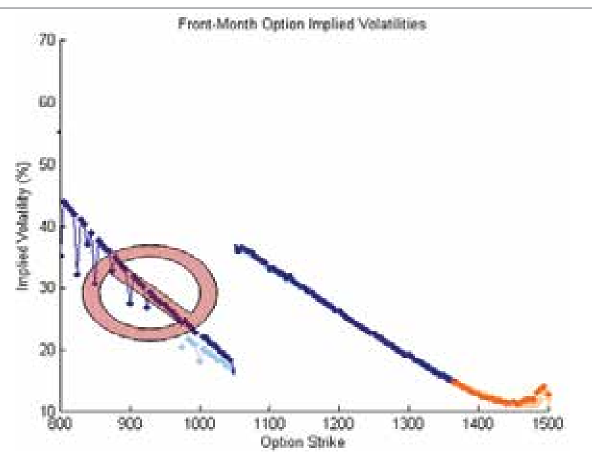

Figure 4. Source: Credit Suisse

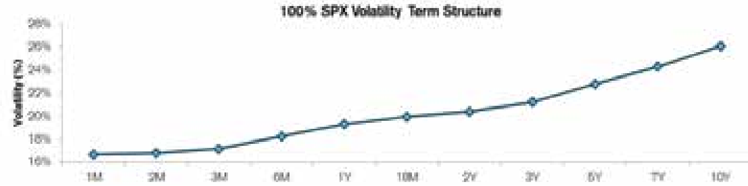

Figure 5. Source: Credit Suisse

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// OPTION LIQUIDITY IN A PARTICULAR SINGLE STRIKE CAN HAVE A MEANINGFUL IMPACT ON THE VIX VALUE //

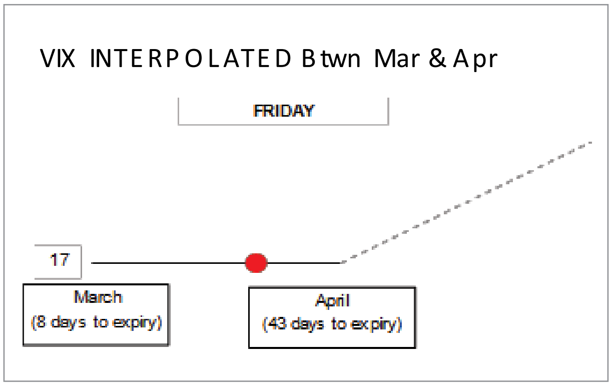
month SPX option skew, that not only do bids exist above the 1045, but there is also a solid chain of bids from the 1040 strike down to the 800 strike as well. The marginal contribution to the VIX of these sub-1040 strike options is one full VIX point.

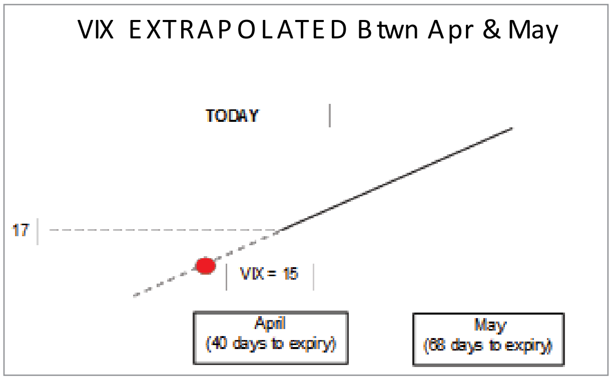
Thus, as illustrated in Figure 4 (pg. 37, middle), when bids for both the 1040 or 1045 strikes are missing, the marginal contributions by all these options and those to their left are sliced off, causing the VIX to plummet.

Likewise, a bid for any one or both of the strikes will cause the tail to "reattach" causing the VIX to shock upward.

Case Study 2: *The Contract Roll Effect*

Recall that the VIX is meant to represent the expected volatility for a 30-day option. However, since an SPX option with exactly 30 days to expiry is only available once per month, 30-day volatility is usually calculated as the (weighted) average of two contracts, a front month contract with less than 30-days to expiry, and a back month contract with more than 30-days to expiry. One day per month, the VIX initiates a contract roll in which the front month option is removed from the calculation and the VIX is then calculated using the second and third back month contracts. What is rather surprising to many, however, is that if the SPX volatility term structure is upward sloping, the contract roll whereupon the VIX calculation shifts from using the first and second month contract to the higher vol second and third month contracts, usually causes the VIX to decline! This is illustrated by using the March 2012 contract roll.





On that day, the SPX at-the-money term structure was upward-sloping as shown below with May expiry implied vols trading at a premium to April expiry implied vols and April expiry implied vols in turn trading at a premium to March expiry implied vols. (Figure 5, pg. 37, bottom)

During the trading session before the contract roll, March 9, 2012, the VIX was calculated using the March contract with eight days left to expiry and the April contract with 43 days to expiry. In this case, the VIX level was interpolated using the two contracts and as a result, the closing VIX level was between the March and April variance levels as shown in Figure 6 (left, top).

On the morning of the VIX contract roll date, March 12, 2012, however, the VIX was calculated using the April contract, now with 40 days left to expiry, and the new May contract with 68 days to expiry. Obviously, since the 30-day volatility number needed for the VIX is earlier than even the front month contract, interpolation is not possible. The VIX methodology prescribes that extrapolation be used along the gradient formed by the April and May contracts. As shown in Figure 7 (left, bottom), the newly extrapolated VIX level is significantly lower than both the April and May contracts.

The drop to VIX based on this method is dependent upon the slope of the term structure. In this particular case, the extrapolation process resulted in a one point fall in the VIX in a 15 volatility environment.

Case Study 3: *VIX Action*

Generally, daily VIX moves can be attributed to five principal components: 1) the expected volatility move along the volatility skew, assuming the skew remains fixed to specific SPX strikes (sticky-strike dynamics); 2) parallel shifts up or down of the skew; 3) daily contract reweighting the back month contract (interpolation effect); 4) incremental demand for puts (steepening of downside skew); and 5) incremental demand for calls (steepening of upside skew). Historically, roughly 80 percent of the moves have been dictated by the first two components and, often, these two effects reinforce each other. Thus, when the SPX declines, at-the-money volatility slides up the skew (thus driving the VIX higher), and furthermore, the surface parallel-shifts upward. The end result is that a decline in the SPX drives the VIX upward. Thus, it is often considered bizarre behavior when these two effects move in opposite directions. The following example illustrates this effect on the day after the U.S. elections.

As shown in Figure 8 (right, top), on Nov. 8, 2012, following the U.S. presidential elections, the SPX fell 1.25 percent but yet the VIX also fell one point off a base of 19. On this day, the bulk of the VIX down-move derived from the SPX front month contracts—the November expiry options.

Figure 9 (right, bottom) shows a comparison between the SPX implied volatility skew for Nov. 7 (light orange, light blue) and Nov. 8 (dark orange, dark blue). Despite the sizeable SPX decline, the November-expiry SPX implied volatility skew from the 1,300 to 1,450 strikes parallel-shifted downward rather than upward as one would expect.

Figure 8. Source: Bloomberg

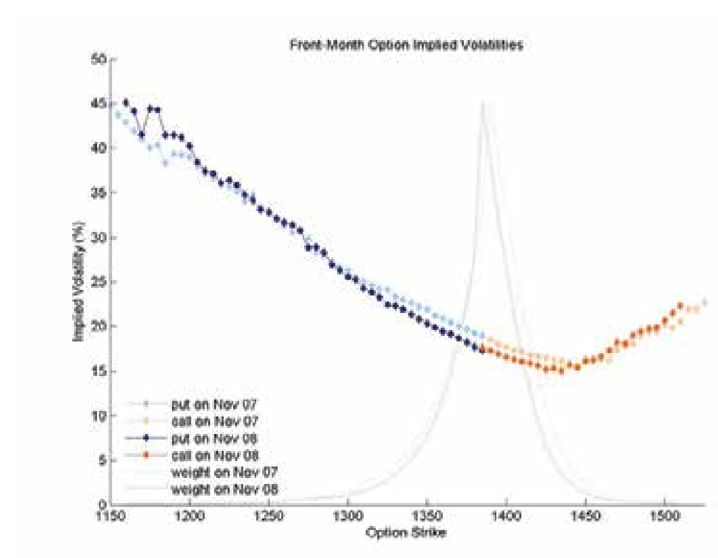


Figure 9. Source: Credit Suisse

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The key lies in the fact that the front month option for the VIX calculation was the November expiry contract. Heading into the first week of November, the U.S. presidential election was the major risk embedded into November expiry implied volatilities. With its solidification, traders with November expiry options, which now only had only six trading days remaining until expiration, now faced the following situation: with no foreseeable catalysts left, realized volatility was likely to decline. Thus, option traders remarked their SPX November expiry volatilities lower despite the market decline thereby causing the observed drop in the VIX.

4. CONCLUSION

Amongst equity investors, the VIX, a measure of market variance, is widely interpreted as the market fear gauge. Although VIX typically has a coincidentally inverse relationship to SPX spot, which reinforces the fear gauge moniker, there are times when technical features of the variance calculation causes the VIX to move counter to intuition. As the VIX continues to receive press coverage and as an increasing number of investors begin to express market sentiment via exchange traded VIX products, an understanding of the factors used in the formula can be helpful for being able to distinguish the real price action in the index from the technical artifact resulting from the calculation methodology. Thankfully, CBOE has provided excellent transparency in their calculation, which allows anyone to replicate the numbers and perform the necessary analysis. This article illustrated examples of situations where VIX moves on three separate occasions defied intuition, but after a closer examination a rational explanation was found. In one case, it was made apparent that option liquidity in a particular single strike can have meaningful impact on the VIX value, especially if it cuts off a meaningful tail. Another example showed that, at times, the extrapolation across the two SPX option contract maturities used in the calculation may lead to unexpected results. Lastly, it was shown that occasionally the VIX may move in tandem with the SPX, thus defying its commonly-used "fear gauge" description.

APPENDIX

The following is an abbreviated and simplified description of the step-by-step process used by CBOE for the calculation of the VIX®. For the full process with examples, the reader should refer to the original VIX White Paper.

1. Select the options to be used in the VIX calculation (only options with non-zero bid prices are used)
   T_1 is the time until the first S&P Option contract month expiry, at least a week away (near expiry).
   T_2 is the time until the first S&P Option contract month expiry after T_1 (next expiry).
   F_1 is the forward SPX level applicable to the near expiry; F_2 is the forward SPX level applicable to the next expiry. Both are calculated by finding the strike of the options with the respective maturity, where the call price is closest to the put price.
   $K_{0,1}$ is the first listed strike price below the forward index level, F_1 and $K_{0,2}$ is the first listed strike price below the forward index level, F_2.

a. Select near-expiry out-of-the-money put options with strike prices $<K_{0,1}$.
   i. Start with the put strike immediately lower than $K_{0,1}$ and move to successively lower strike prices.
   ii. Stop once two puts with consecutive strike prices are found to have zero bid prices.

b. Select near-expiry out-of-the-money call options with strike prices $>K_{0,1}$.
   i. Start with the call strike immediately higher than $K_{0,1}$ and move to successively higher strike prices.
   ii. Stop once two puts with consecutive strike prices are found to have zero bid prices.

c. Select both the near-expiry put and call with strike price $K_{0,1}$.

d. Repeat (a)-(c) for the next expiry.

2. Calculate the variances $\boldsymbol{\sigma_1^2}$ **and** $\boldsymbol{\sigma_2^2}$ for both near-expiry and next-expiry options as the weighted average of all existing options of the same maturity

$$\sigma_t^2 = \frac{2}{T_t}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_t T_t}\, Q_t(K_i) - \frac{1}{T_t}\left[\frac{F_t}{K_{0,t}} - 1\right]^2, t = 1,2$$

Where:
t represents the near-term and the next-term expiry.
i spans the calls and puts of the respective expiry, t, as selected in Step 1.
$Q_t(K_i)$ is the mid-point of the bid-ask spread of the option with strike K_i of the respective expiry, t (as selected in Step 1).
R_t is the bond-equivalent yield of the U.S. T-bill maturing closest to the expiry date, t.

3. Calculate the annualized 30-day weighted average of $\boldsymbol{\sigma_1^2}$ **and** $\boldsymbol{\sigma_2^2}$. Then, take the square root of that value and multiply by 100 to get VIX.

$$\boldsymbol{VIX = 100 \times \sqrt{\left[T_1\sigma_1^2 \times \frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} + T_2\sigma_2^2 \times \frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right] \times \frac{N_{365}}{N_{30}}}}$$

Where:
N_{T_1} and N_{T_2} are the number of minutes until the near-expiry and the next-expiry, respectively.
N_{30} and N_{365} are the number of minutes in 30 and 365 calendar days, respectively.

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END NOTES

[1] More precisely, it is the square root of the estimate for expected annualized variance of daily log-returns of the S&P 500 under the risk-neutral measure. The VIX is an expected variance estimator, reported as its square root, rather than an expected volatility estimator. This is worth noting, as by its definition, $VIX = \sqrt{E[\sigma^2]}$, which is not necessarily the same as $E[\sigma]$, where σ stands for volatility and is an unknown quantity.

[2] The interpolation is done on the quantities $\sigma_{T_1}^2 \times T_1$ and $\sigma_{T_2}^2 \times T_2$.

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