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THE MYTH OF "THE MYTH OF TIME DIVERSIFICATION"

By Rowland Davis

Editors Note: *The following two articles were submitted in response to the Investment Section's call for essays to explore and expose investment fallacies. The editors chose them as being generally suitable for the newsletter, but contest winners have not yet been selected as Risks and Rewards goes to print. The Section is in the process of reviewing all the essays received and determining the winners in accordance with the contest criteria.*

In 1963 Paul Samuelson published a paper titled "Risk and Uncertainty: A Fallacy of Large Numbers." Thus was born the phrase: "the myth of time diversification."

The purpose of this essay is not to challenge the accuracy of Samuelson's work, but to challenge the expansive misuse of his findings—an abuse that has substantial implications for actuaries. As an example, a Google search of the phrase yields this quote:

> It sounds nice in principle, but it's actually an example of the "time diversification" fallacy. Investments do not become safer the longer they are held. Time reduces the variance in the average *annual* return, but it actually increases the variance in the cumulative return. In other words, smoothing won't bring more certainty to retirement savings. For any given portfolio, collective DC plans face the same risk-return tradeoff as ordinary 401(k) plans.
> —Jason Richwine in the *National Review* blog

To understand the abuse occurring here, we must return to Samuelson's work. The specific application to investment risk was first developed in his 1969 paper "Lifetime Portfolio Selection by Dynamic Stochastic Programming." It was, in fact, a mathematical proof—of the general nature "***if this***, then ***that***," where ***that*** is essentially the statement that time horizon should not affect an investor's risk tolerance. (The corollary to this is more frequently used—that the risk of stock investing does not decrease with longer time frames.)

Unfortunately, the ***if this*** conditions are almost universally ignored, and the proof only holds with those conditions in place. There are two important conditions that Samuelson uses to frame the whole analysis: 1) that the investor's utility function is isoelastic (i.e., a single continuous utility function covers the entire spectrum of outcomes, without conditional sensitivity to any particular values of the outcome); and 2) that the only issue at stake is an individual investor's terminal wealth based on the investments alone. In this case, and only in this case, is it wrong to assume that stringing together a sequence of risky bets is superior to a single risky bet (i.e., time does not diversify risk)?

Actuarial work involves collective systems, so can the same logic be applied? Is it wrong for a group of investors saving for retirement to collectively take more risk over a longer time frame than they would over a shorter time frame? This essay shows that it is ***not*** wrong to do so in the real world (i.e., free of the narrow constraints on the Samuelson proof).

Since I am not an academically trained economist, I will construct an actual example to make the point. Although the words used are somewhat opaque to a non-economist, Samuelson acknowledges that real-world investors might indeed have more risk tolerance in the early stage of their career: "Note: if the elasticity of marginal utility … rises empirically with wealth, and if the capital market is imperfect as far as lending and borrowing against future earnings is concerned, then it seems to me likely that a doctor of age 35-50 might rationally have his highest consumption then, and certainly show his greatest risk tolerance then—in other words be open to a 'businessman's risk.' But not in the frictionless isoelastic model!" (The reference here to a "businessman's risk" is explained elsewhere in the paper as the ability to take more investment risk.) Because the "frictionless isoelastic model" is not very relevant in the real world, the door is immediately open to investment policies that do, in fact, depend on time frame. Target date funds are one simple example, based on the concept of including the value of human capital as part of the investor's wealth.

My example will assume two assets: a safe asset with an expected real return of 2 percent and a standard deviation of 5 percent; and a risky asset with an expected real return of 4.5 percent and a standard deviation of 20 percent. For the Samuelson base case, I use a standard risk-averse utility function that meets his ***if then*** conditions: U(w) = ((w^λ)-1)/λ, with λ = -2. With this function, utility is maximized with a risk asset allocation of around 25 percent. And as Samuelson proved with his equations, a stochastic simulation verifies that this same allocation is the utility-

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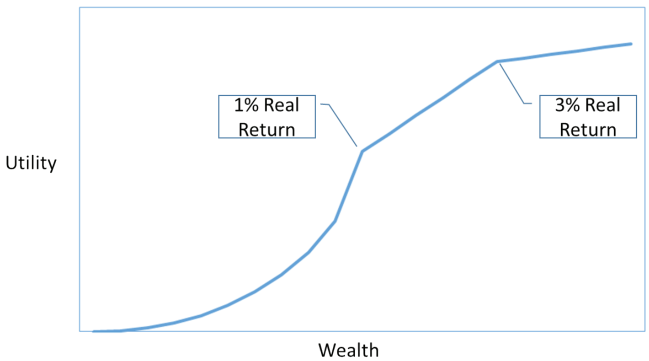

maximizing allocation with both a 10-year horizon and a 30-year horizon.

Now we move into the real world. First we develop a new utility function that reflects an investor (or a group of stakeholders in a collective plan) with a 3 percent real return target. For this investor, real returns in excess of 3 percent have a decreased marginal value, and real returns less than 1 percent become painful very quickly. Here is a graph of the utility function I use for this case.

This kind of utility function has been shown by behavioral finance research to represent the way that humans make decisions in the real world (i.e., prospect theory, developed by Kahneman and Tversky).

With this utility function, a 10-year investor will maximize utility with a risk asset allocation of about 20 percent—very similar to the Samuelson base case. But a 30-year investor will maximize utility with a risk asset allocation of about 60 percent. For this investor, the time frame does matter, with more risk becoming appropriate over longer time frames. (For a similar example see, "The Fallacy of Large Numbers Revisited" by De Brouwer and Van den Spiegel, *Journal of Asset Management,* 2001.)

Now let us proceed to the issue of human capital. Assume that this investor, seeking a 3 percent real return, adopts a strategy of dynamic adjustment for his saving plan. After 10 years, if savings fall below 90 percent of his real return target, he will make additional contributions over the next five years with a total value equal to the shortfall relative to the 90 percent threshold. If savings after 10 years exceed 120 percent of the real return target, then part of the surplus will be withdrawn. The amount withdrawn is sensitive to the asset allocation, but will always be set so that the expected value of the adjustment process is zero (i.e., expected withdrawals will equal expected additional contributions). The investor is comfortable with this adjustment strategy because his human capital is sufficient to absorb any required additional contributions.

With this dynamic adjustment process, the 30-year investor will now find maximum utility with a 75 percent risk asset allocation, instead of 60 percent. Interestingly, even with the standard utility function this adjustment process will move the optimal risk asset allocation for the 30-year investor up to 35 percent, from the 25 percent level that applies to the 10-year investor with no adjustment process. Once again, real-world details matter when thinking about the relationship between risk and time frame.

Collective systems involve spreading risks among stakeholders and across age cohorts in ways that allow for efficient risk-taking. Human capital is not only recognized, it is pooled—within a single closed cohort, human capital diminishes in value over time, but the aggregate human capital across the full range of cohorts remains constant. Unlike the fund for an individual investor, which builds from a level of zero to ever-larger dollar totals, a mature collective fund is expected to remain relatively constant in real terms. A dynamic self-adjustment process (through variable contribution inflows and/or variable benefit outflows) can create a sustainable fund where the risky bet can be repeated time after time with controllable risk. There will always be risk over any specific time frame, but a properly designed system can manage these risks through time in a sustainable way. Risk is no longer measured simply by some value of terminal wealth (as in the Samuelson paper),

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but by more complicated metrics of ongoing financial risk exposure to various cohorts of stakeholders. Samuelson never said anything different.

The bottom line on this is that critics have the right to say that risks do exist, and need to be carefully measured and managed. And critics also have the right to express their honest opposition to collective systems (i.e., those involving intergenerational risk-sharing) on political grounds.

But they do not have the right to invoke Samuelson's proof within any blanket statement asserting that collective systems can't work because they are based on a fallacy. Implicit in any argument of this type is an assumption that a collective system can be simply decomposed into segments consisting of "classical" individual investors—but then they are no longer talking about a collective system, which is far more complicated in its risk dynamics.

Rowland Davis is a nationally known actuary and pension consultant who heads his own firm—RMD Pension Consulting, created in 1997—that specializes in asset/liability modeling, risk management and asset allocation studies for pension funds. In addition to his consulting activities, Rowland has recently engaged in a number of activities related to public policy for retirement systems, including the Retirement 20/20 project sponsored by the Society of Actuaries. He is a Senior Fellow at the Center for American Progress specializing in retirement policy issues. Formerly a Vice President and consulting actuary at Towers Perrin Forster & Crosby, he was also the national director of the asset and liability forecasting practice at TPF&C. Prior to this national role, Rowland was the Senior Actuary in TPF&C's Chicago office and lead consultant for several of their largest clients. He can be reached at RowlandDavis@mindspring.com.

TECHNICAL ENDNOTE:

Samuelson himself acknowledged in a 1989 paper ("The √N Law and Repeated Risktaking" included in *Probability, Statistics, and Mathematics, Papers in Honor of Samuel Karlin*) three separate cases, using different assumptions, where time frame would change a rational investor's risk tolerance. One of these is the simple one of including human capital in wealth. A second one recognizes that the original argument does not hold if markets are mean-reverting (and there is substantial evidence that they are). The third involves an assumption set using a utility function that incorporates some minimum required threshold for terminal wealth, similar in concept to the one used in this essay. Samuelson was well aware of his own ***if then*** criteria.