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MEASURING THE COST OF DURATION MISMATCH USING LEAST SQUARES MONTE CARLO (LSMC)

By Casey Malone and David Wang

Duration matching is perhaps the best-known strategy for asset-liability management (ALM) in insurance companies today. Duration is a measure of the sensitivity of an asset or liability to a change in interest rates. Matching the duration of the assets in a portfolio to the duration of the liabilities backed by that portfolio immunizes the company's equity to changes in interest rates.

Duration matches are transitory—the durations of the assets and the liabilities change as time passes and interest rates change (due to convexity). Generally, companies rebalance their asset portfolios to recalculated liability durations on a monthly, quarterly, or perhaps even less frequent basis. The duration mismatch between rebalancing leads to ALM breakage, and there is a cost associated with this, especially when there is a large change in rates, and the company's equity is subjected to unwanted interest rate risk.

Knowing the daily mismatch position may help quantify how much ALM breakage the company is exposed to. Least-squares Monte Carlo (LSMC) proxy modeling provides a methodology for generating daily liability values including duration, convexity, and other higher-order sensitivities if needed. The company can use this information for setting and monitoring rebalancing thresholds and measuring the impact of the ALM breakage over a reporting period. Knowing the financial impact of ALM breakage thus enables the company to incorporate ALM risk into its ERM framework.

LSMC is a proxy modeling approach that replaces stochastic calculations with closed form solutions. With the closed form solution (or polynomial in this case), an instantaneous calculation replaces a full-blown stochastic run. This can be used to monitor a stochastic calculation in real time or to replace a nested stochastic calculation when runtimes are prohibitive.

A number of mathematical techniques are used to reduce the required runtime and increase the speed of convergence of the polynomial to the model results. The process begins with smart selection of calibration scenarios. You must understand your model and what factors move the results, so that the proxy model can survive a wide range of future environments. On the back end, the polynomial is fit to avoid econometric pitfalls such as collinearity and overfitting.

In this article, we will focus on using LSMC to measure and manage ALM breakage due to duration mismatch. For the following case study, we modeled a hypothetical $1 billion fixed deferred annuity block as of May 31, 2013. We calibrated a polynomial for present value of future benefits (PVFB) as a function of key swap rates. We tested the one-year, two-year, three-year, four-year, five-year, seven-year, 10-year and 30-year key swap rates. For intermediate points on the starting yield curve, we used a cubic spline technique for interpolation. The PVFB is assumed to be the average over 1,000 stochastic interest rate scenarios, generated with parameters consistent with the starting yield curve. Our polynomial replaces the 1,000-scenario stochastic calculation so that PVFB calculations can be performed in real time. Below, we track the block over the following month.

Our calibrated proxy is a 39-term polynomial. It should be noted that we use Legendre polynomials since they are orthogonal to each other on the range [-1, 1]. This is how we correct for collinearity between explanatory variables. The table below shows the coefficients in the left column and the degree of the Legendre polynomials for each key rate to the right.

OUR CALIBRATED PROXY IS A 39-TERM POLYNOMIAL. ... [W]E USE LEGENDRE POLYNOMIALS SINCE THEY ARE ORTHOGONAL TO EACH OTHER. ...

Table 1: Coefficients for Proxy Function Polynomials

$mil	Exponents							
Coefficient	kr1	kr2	kr3	kr4	kr5	kr7	kr10	kr30
900								
1	1							
-71						1		
-50					2			
32				1				
-29							1	
43				1		1		
15					1			
53				1	1			
48					1	1		
-27				1			1	
-14						2		
-14				2				
18						1	1	
23				2	2			
8		1						
-21		1				1		
24				1	1	1		
-18				2	1			
12		1			1			
8							2	
-11	1				1			
-2						2	2	
8			2					
-30			2		1			
17					2	2		
-12		1			2			
8	1						1	
-11				1		2		
-10				1	2			
6						3		
-15					1	3		
9			2	1				
9					2		1	
9					1		2	
11				2		2		
10			2		2	0		
8	1					2		
9			2				2	

For example, the last term of the polynomial is:
9 * L(kr3, 2) * L(kr10, 2), where
L(X, y) is the Legendre polynomial of degree y for variable X.

This polynomial may seem daunting at first, but it is very easy to code into MS Excel or any modeling software, and a computer can calculate this value in a trivial amount of calculation time. Each of these terms is statistically significant, as we use the Akaike information criterion (AIC) for model selection. The AIC is a common measure to quantify the trade-off between model fit and model complexity. This is how we avoid over-fitting the model.

The following graph shows our daily proxy values for PVFB, as well as the seven-year swap rate for reference. The darker bars at the beginning and end of the month show the full stochastic values for validation of the proxy model. The difference at the end of the month will be due to sampling error in scenario selection and model drivers that are not adequately captured by the polynomial. This can be overcome by generating more scenarios for the full stochastic runs and more calibration scenarios for the LSMC fitting.

Chart 1: Dollar Sensitivity of PVFB to Swap Rates

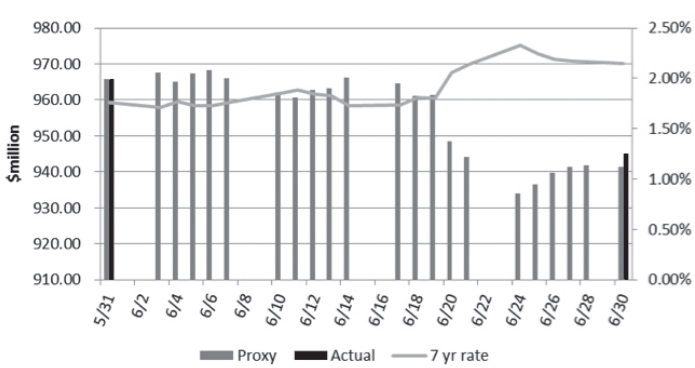

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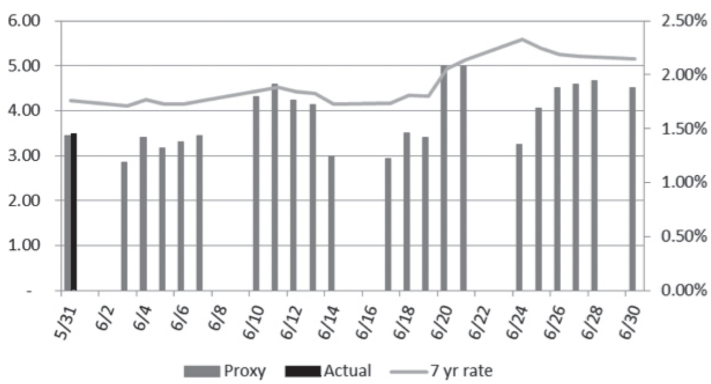
Chart 2: Duration Sensitivity of PVFB to Swap Rates



As expected, the liability values move inversely to interest rates.

The graph above shows the duration of PVFB, measured as 100 times the percentage change in PVFB per 1 basis point (bp) parallel shock to the yield curve.

Table 2: Calculated DV01 Series

	DV01								
$thousand	1 yr	2 yr	3 yr	4 yr	5 yr	7 yr	10 yr	30 yr	Duration
5/31/2013	(24)	137	(0)	368	332	(806)	(332)	-	3.45
6/3/2013	(22)	139	1	376	404	(833)	(335)	-	2.86
6/4/2013	(24)	134	0	372	340	(814)	(331)	-	3.41
6/5/2013	(24)	139	1	377	363	(819)	(335)	-	3.16
6/6/2013	(26)	140	0	376	345	(810)	(338)	-	3.31
6/7/2013	(25)	137	(0)	371	331	(806)	(333)	-	3.45
6/10/2013	(26)	127	(6)	355	236	(772)	(322)	-	4.32
6/11/2013	(27)	124	(10)	359	200	(763)	(317)	-	4.59
6/12/2013	(27)	128	(6)	361	245	(777)	(326)	-	4.24
6/13/2013	(27)	130	(5)	364	256	(781)	(327)	-	4.13
6/14/2013	(23)	138	1	381	376	(824)	(333)	-	3.00
6/17/2013	(21)	136	1	380	382	(827)	(329)	-	2.93
6/18/2013	(23)	130	1	382	303	(806)	(319)	-	3.51
6/19/2013	(23)	131	2	385	312	(809)	(320)	-	3.41
6/20/2013	(21)	94	(23)	364	75	(704)	(268)	-	5.00
6/21/2013	(17)	78	(33)	368	28	(672)	(241)	-	4.99
6/24/2013	2	34	(47)	411	(12)	(587)	(155)	-	3.24
6/25/2013	(4)	54	(36)	387	16	(641)	(191)	-	4.07
6/26/2013	(10)	68	(32)	383	27	(669)	(214)	-	4.52
6/27/2013	(11)	72	(29)	366	48	(671)	(226)	-	4.58
6/28/2013	(13)	74	(29)	372	34	(669)	(229)	-	4.67
6/30/2013	(11)	76	(24)	379	70	(700)	(231)	-	4.51

As expected the duration of the liability moves over time, demonstrating the convexity of the block. The darker bar at the beginning shows a full stochastic calculation of the duration. Assuming monthly rebalancing of assets, the duration of assets would have been set to the duration of liabilities at the beginning of the month. The change in the duration of the liabilities over the month will lead to ALM breakage as the liabilities become more or less sensitive to interest rates versus the assets.

LSMC also allows us to break up the duration into key rate durations on a daily basis. Table 2 (left, bottom) shows the dollar value of one basis point (DV01) for the key rates in the polynomial. The overall duration is shown as well for comparison.

The DV01s change over the month since the key rates appear in the polynomial in terms of higher order than 1. The 30-year rate has no statistically significant bearing on the PVFB; or at least, it has no bearing that is not better explained by changes in the other rates. As the overall duration changes over the month, the key rate durations shift slightly between each key rate.

Assuming the assets are calibrated to the beginning-of-month key rate durations, we can track the ALM breakage as the daily difference between the change in assets based on constant key rate durations and the change in liabilities based on the proxy function. Table 3 (page 33) shows the daily tracking: the change in asset value, the change in liability value, and the difference between the two (i.e., the ALM breakage).

WITH LEAST SQUARES MONTE CARLO, DAILY LIABILITY MONITORING CAN BE A REALITY.

Table 3: Daily P&L Tracking

$million	Change in:		
Date	Assets	Liabilities	Difference
6/3/2013	1.98	1.70	(0.28)
6/4/2013	(2.77)	(2.50)	0.28
6/5/2013	2.36	2.26	(0.09)
6/6/2013	1.03	1.08	0.05
6/7/2013	(2.39)	(2.34)	0.04
6/10/2013	(3.86)	(4.01)	(0.16)
6/11/2013	(1.24)	(1.44)	(0.20)
6/12/2013	1.91	2.15	0.25
6/13/2013	0.33	0.38	0.05
6/14/2013	3.11	3.00	(0.11)
6/17/2013	(1.61)	(1.63)	(0.02)
6/18/2013	(3.62)	(3.37)	0.25
6/19/2013	0.31	0.29	(0.02)
6/20/2013	(11.87)	(13.19)	(1.32)
6/21/2013	(3.84)	(4.55)	(0.71)
6/24/2013	(10.37)	(10.35)	0.02
6/25/2013	2.16	2.76	0.60
6/26/2013	3.13	3.07	(0.06)
6/27/2013	1.50	1.67	0.17
6/28/2013	0.67	0.50	(0.17)
6/30/2013	(1.12)	(0.45)	0.67
Total	**(24.19)**	**(24.96)**	**(0.77)**

In total, this shows a $0.8 million (8 bps of account value) mismatch over the month. This mismatch can be reduced by convexity matching. In that case, this analysis can be extended into higher-order sensitivities and alert the asset managers when the convexity match breaks down and duration thresholds are breached. The thresholds can be set in terms of overall duration, key rate duration mix, convexity, or higher-order sensitivities.

This simple, hypothetical demonstration illustrates how LSMC proxy models might be used to improve and benchmark ALM and even enable companies to quantify ALM risk as a component of an economic capital framework. With LSMC, daily liability monitoring can be a reality, and with that knowledge, companies can manage risk exposures in real time.

LSMC can be used a proxy for any stochastic calculation. However, extreme care must be taken to ensure that all risk drivers in the model are captured and a thorough validation exercise is performed.

Casey Malone is a consulting actuary with Milliman Inc., Seattle, Wash. He can be contacted at casey.malone@milliman.com.

David Wang is a consulting actuary with Milliman Inc., Seattle, Wash. He can be contacted at david.wang@milliman.com.