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On The Importance Of Hedging Dynamic Lapses In Variable Annuities

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Variable annuities (U.S.) and segregated funds (Canada) are life insurance contracts offering benefits that are tied to the returns of a reference portfolio. These policies include various forms of capital and income protection in the event of market downturns such as a guaranteed minimum death benefit (GMDB) or a guaranteed minimum withdrawal benefit (GMWB).

An important feature of variable annuities is the possibility for the policyholder to lapse or surrender the contract. In the latter case, the policyholder gives up the underlying insurance protection, ceases to pay fees to the insurer and receives a surrender value. Lapse assumptions are critical inputs in pricing and hedging models of variable annuity guarantees and can be divided into two types: deterministic (or static) and dynamic lapses (see Eling and Kochanski, 2013, for more details). Deterministic lapses are due to unforeseen events in the policyholder's life (for example, loss of employment creating liquidity needs) and are generally seen as diversifiable. On the other hand, dynamic lapses result from an investment decision on the part of the policyholder. For instance, when the guarantee is deep out-of-the-money, the policyholder has a strong incentive to lapse the contract and choose an alternative investment product. This is simply because the insured is paying high fees (fees are generally deducted in proportion to the sub-account's value) for a guarantee that is very unlikely to be triggered in the future. Therefore, dynamic lapses are generally driven by the moneyness of the guarantee and since the evolution of markets affects most VA contracts in a similar fashion, these lapses are clearly very difficult to diversify.

There is growing evidence that dynamic lapsation is important to take into account in variable annuities. For example, Milliman (2011) and Knoller et al. (2015), found a strong statistical relationship between lapse rates and the moneyness of the guarantee in empirical data. Moreover, the Canadian Institute of Actuaries (2002) and the American Academy of Actuaries (AAA) (2005) both recommended to take dynamic lapsation into account by varying the lapse rate with the moneyness of the guarantee. According to a survey from the Society of Actuaries performed in 2011, approximately 60 percent and 80 percent of participating insurers followed this practice when modeling death and living benefits, respectively.

The objective of this article is to investigate the importance of hedging dynamic lapses in variable annuities. More precisely, we aim to answer one very practical question, that is, what is the impact on hedging effectiveness when an insurance company chooses not to hedge dynamic lapses, or alternatively, to hedge them but with the wrong assumptions.

GMMB CONTRACT

Suppose that an insured invests in a guaranteed minimum maturity benefit (GMMB) product with a set maturity T. The sub-account value is credited with the returns of an underlying reference portfolio and fees are continuously deducted from the sub-account as a percentage of the account balance. Denoting the value of the reference portfolio at time t by S_t, the sub-account value at time t is given by

$$A_t = S_t e^{-at}$$

where a is the aforementioned annual fee rate, and $A_0 = S_0$ is the initial investment.

If the policyholder does not surrender his contract before maturity, he is entitled to $\max(A_T, G)$ at time T where G denotes the amount of the guarantee (for a return-of-premium guarantee, we have $G = A_0$). If $A_T < G$, the guarantee matures in-the-money and the insurer is responsible for the shortfall, i.e., its liability is the payoff of a put option: $\max(G - A_T, 0)$.

If the policyholder surrenders his contract at any time prior to the maturity of the policy, he receives the balance of the sub-account value minus a surrender charge which we suppose is expressed as a fraction κ κ of A_t. Therefore, the surrender value at time t corresponds to $A_t(1-\kappa)$.

DECOMPOSITION OF THE PAYOFF TO THE POLICYHOLDER

We integrate dynamic lapsation into the GMMB contract by assuming that the policyholder will surrender his contract at the first moment (before maturity) the sub-account value net of surrender charges hits a predetermined barrier known as the moneyness threshold or level. We will use the term moneyness ratio when this moneyness threshold is expressed relative to the guarantee G. Table 1 shows that the

Table 1

Decomposition of the payoff of a GMMB contract with dynamic lapsation risk

Components of the portfolio	Barrier is *hit* before maturity	Barrier is *not hit* before maturity
(I) Up-and-out put	0	$\max(G - A_T, 0)$
(II) Rebate option	Moneyness level paid upon surrender	0
(III) Up-and-out call with zero strike	0	A_T
Total payoff	Moneyness level paid upon surrender	$\max(A_T, G)$

payoff of a GMMB contract with dynamic lapsation can be viewed as a basket of barrier options.

The decomposition presented in Table 1 renders the analysis of the GMMB product tractable because closed-form expressions for each of the underlying options are available under the Black-Scholes model (see McDonald, 2006, Section 22). Therefore, the valuation of a GMMB contract (from a financial engineering perspective) under dynamic lapsation risk and the computation of Greeks required for establishing a dynamic hedging strategy are both straightforward to perform.

FAIR FEE

Having decomposed the payoff to the policyholder into a basket of barrier options, we now focus on how to compute the fee rate for the GMMB contract. Defining the insurer's net liability as the payoff of the contract net of fees and surrender charges, we say that the fee is fair if it is determined such that the net liability of the policy is zero at inception of the contract. This is similar to the equivalence principle in actuarial mathematics.

To analyze the effect of dynamic lapsation and surrender charges on the fair fee, we begin with a baseline contract in which surrendering is not allowed. For an initial investment of $100, a fixed guarantee of $100, a (continuously compounded) risk-free rate of 3 percent, an asset volatility of 16.5 percent (see below) and a contract maturity of 10 years, the fair fee rate is 1.07 percent per annum. This contract is equivalent to a plain vanilla put option financed by fees deducted periodically from the sub-account.

We now incorporate dynamic lapsation into the pricing framework and assume that there are no surrender charges. Figure 1 illustrates the behavior of the fair fee as a function of the moneyness ratio. As expected, if the policyholder only lapses when the moneyness ratio is extremely large, the fair fee converges to the one computed for the baseline case where surrendering was not allowed. However, if the insured lapses at smaller moneyness ratios, the fee needs to be increased to compensate the insurer for its lost future income. Indeed, when the guarantee is deep out-of-the-money, it is very unlikely that the guarantee will cost something to the insurer and surrender therefore leads to a loss for the insurer.

Figure 1

Fair fee as a function of the moneyness ratio assuming no surrender charges

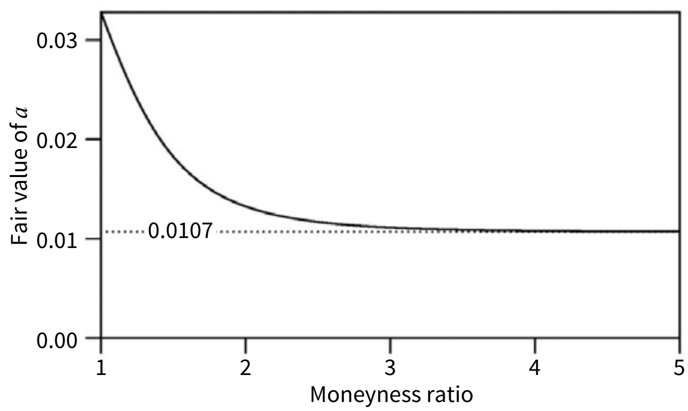

Figure 1 shows that dynamic lapsation can be priced into the contract by raising the fee rate. However, we observe that the required fee increase is rather steep: at a moneyness ratio of about 150 percent, the fair fee almost doubles. One way to reduce this fair fee is to include surrender charges. In fact, when a surrender charge of 4 percent is applied at the moment of surrender, the fair fee lies in between 1 percent and 1.2 percent for any given moneyness ratio. Therefore, the addition of a surrender charge has almost totally mitigated the effects of dynamic lapsation risk on the fair fee. In the following section, we examine how dynamic lapsation risk impacts hedging effectiveness.

HEDGING EFFECTIVENESS

When fees are collected as a percentage of the sub-account value, the fee income is affected by fluctuations in the value of the reference portfolio. For example, in a bear market, the sub-account value drops, the guarantee is in-the-money and the fee income decreases (at the worst possible time for the insurer). In contrast, the fee income is much greater in a bull market, but policyholders also tend to lapse more.

These observations show that both the payoff of the contract (at maturity or on surrender) and the fee income should be hedged if the objective of the hedge is to protect the insurer against changes in its net liability. In what follows, we lay down the main market and hedging hypotheses needed to analyze the impact of dynamic lapsation on hedging effectiveness.

MARKET ASSUMPTIONS

We will assess hedging effectiveness under two different types of market environments.

(1) The ideal case in which the value of the reference portfolio follows a geometric Brownian motion, exactly as in the Black-Scholes model. In this case, log-returns are independent and identically distributed as normal random variables. Because Greeks will be computed under the Black-Scholes model as well (see below), there will be no discrepancy between the hedging and market models in this scenario, i.e., there will be no model risk.

(2) A (two) regime-switching GARCH (RS-GARCH) market model that captures most of the stylized facts of asset returns (see Campbell et al., 1996; Tsay, 2012). In a RS-GARCH model, the state of the economy is driven by a latent Markov chain and in each state, the market follows a GARCH(1,1) model. This model encompasses the regime-switching log-normal (RSLN) model of

CONTINUED ON PAGE 14

Hardy (2001). Furthermore, Hardy et al. (2006) showed that the RS-GARCH model has a better overall fit than the stochastic volatility model of the American Academy of Actuaries. We believe that this better fit is achieved because the RS-GARCH model allows for jumps in the mean return and volatility dynamics.

The data set used to estimate the parameters of these two market models consists of weekly log-returns on the S&P500 index from Dec. 30, 1987 to Aug. 1, 2012. Data was extracted on Wednesdays to avoid most holidays. The time series includes 1283 observations and descriptive statistics are provided in Table 2 (the mean and standard deviation (abbreviated StDev) are given on an annualized basis).

Table 2:

Descriptive statistics of weekly log-returns on the S&P500 index from 12/30/1987 to 08/01/2012

Mean	StDev	Skewness	Kurtosis	Minimum	Maximum
7.0%	16.5%	-0.61	7.3	-16.5%	10.2%

Both market models were estimated by maximum likelihood (ML). Estimation of the Black-Scholes model by ML is straightforward as one only needs to compute the sample mean and variance of log-returns. The RS-GARCH model is more complicated to estimate because of a path-dependence problem. The most common ML estimation algorithm used for the RS-GARCH model is given by Gray (1996), but Augustyniak et al. (2015) generalized Gray's approach to reduce bias in the estimated parameters. R code for this technique is available on Maciej Augustyniak's website.

HEDGING ASSUMPTIONS

In what follows, we assume that the insurer uses delta-hedging under the Black-Scholes model to manage the risk of the GMMB contract in a frictionless market (no transaction costs, no constraints on short selling, lending, etc.). For the insurer to be delta-hedged at time t, it must ensure to hold a position of Δ_t in the underlying index. This can be accomplished using futures or, equivalently, by taking a long position in Δ_t shares of the underlying index and borrowing the cost or lending the proceeds. The Greek Δ_t corresponds to the first derivative of the insurer's net liability with respect to the asset price and can be computed in closed-form based on the decomposition presented in Table 1.

Four hedging scenarios are analyzed.

I. Baseline: The insurer hedges a GMMB product assuming that the policyholder will not surrender his contract and the policyholder conforms to this behavior. The fair fee in that case has already been calculated and corresponds to 1.07 percent.

II. Correct moneyness assumption: The insurer hedges a GMMB product assuming that the policyholder will lapse his contract if the moneyness ratio hits 150 percent and the policyholder conforms to this behavior. A surrender charge of 4 percent is applied in the event of surrender. This scenario allows us to better analyze the magnitude of the discrepancies in an inappropriate hedge scenario (see scenarios III and IV). The fair fee in this scenario is 1.17 percent per annum which is only slightly higher than in scenario I since surrender charges approximately cover the loss in fee income due to lapsation.

III. Dynamic lapsation is not hedged: The insurer hedges a GMMB product assuming that the policyholder will not surrender his contract but the policyholder does not conform to this behavior and lapses when the moneyness ratio hits 150 percent. A surrender charge of 4 percent is also applied. This situation allows us to assess the impact of dynamic lapsation on a hedging program when this risk is ignored. We assume that the product is correctly priced (1.17 percent per annum) even if the hedge is not properly constructed. This prevents hedging errors from being inflated because of a mispricing.

IV. Incorrect moneyness assumption: The insurer hedges a GMMB product assuming that the policyholder will lapse his contract if the moneyness ratio hits 175 percent, but the policyholder actually lapses his contract once the moneyness ratio hits 150 percent. A surrender charge of 4 percent is also applied. This situation allows us to assess the impact of incorrectly setting lapse assumptions on hedging effectiveness. As in scenarios II and III, the fee is set to 1.17 percent per annum which implies that the product is correctly priced but the hedge is not properly constructed.

For these four hedging scenarios, we will analyze the distribution of the net hedging error at maturity. If the GMMB product is held until maturity, the net hedging error at maturity for a given scenario is

max(G - A_T, 0) + accumulated mark-to-market hedging gains/losses - accumulated value of fees.

If the GMMB is surrendered prior to maturity, the net hedging error becomes

accumulated mark-to-market hedging gains/losses - accumulated value of surrender charges and fees.

ANALYSIS OF HEDGING ERRORS

Table 3 shows the mean, standard deviation (StDev), 95 percent Conditional Tail Expectation (CTE) and 99 percent Value-at-Risk (VaR) of the net hedging error at maturity assuming weekly rebalancing of the hedge portfolio for each of the four scenarios that were presented and under the two market models considered (200,000 paths of the log-return process were generated for each model). As before, we assume an initial investment of $100, a fixed guarantee of $100, a risk-free rate of 3 percent, an asset volatility of 16.5 percent and a contract maturity of 10 years.

Table 3

Net hedging error at maturity for the four scenarios and two market models considered

	Mean		StDev		95% CTE		99% VaR	
Scenario	Black-Scholes	RS-GARCH	Black-Scholes	RS-GARCH	Black-Scholes	RS-GARCH	Black-Scholes	RS-GARCH
I	0.0	-0.7	0.7	1.8	1.6	3.6	1.9	4.4
II	-0.1	-1.1	0.8	2.0	1.9	3.6	2.2	4.4
III	1.2	0.0	3.8	4.0	8.2	7.7	8.6	8.3
IV	0.5	-0.6	1.7	2.4	2.7	3.7	2.9	4.3

We can first focus our analysis on the results obtained under the Black-Scholes model. By analyzing scenarios I and II, it is quite obvious that hedging under ideal conditions (no model or policyholder behavior risks) yields an important risk reduction (for example, the 95 percent CTE of the net unhedged loss at maturity is 28 if the policyholder does not lapse). However, the relevant practical issue is to determine whether it is advantageous for the insurer to hedge dynamic lapsation risk if he is unsure about the exact moneyness level at which the policyholder exercises his option to surrender. To address this issue, we must compare scenarios II, III and IV. For the Black-Scholes model, when there is no discrepancy between the hedging and market models, we observe that even if the moneyness ratio assumption is set wrong in the hedge, the risk measures in scenario IV are much lower than those obtained in scenario III where dynamic lapsation risk is not hedged at all. In fact, the standard deviation and risk measures in scenario IV (wrong moneyness ratio) are approximately twice as large as in scenario II (perfect hedge), but under scenario III (dynamic lapses are not hedged at all), they are five times larger.

Therefore, even if the assumption on the moneyness ratio is set wrong in the hedge, it is still possible to achieve a very significant risk reduction by hedging dynamic lapses.

The last question that remains is to determine whether the results that we obtain are robust to a more realistic market model. Comparing results for the Black-Scholes and RS-GARCH market models, it is not surprising to observe an increase in the standard deviation when hedging under the RS-GARCH model. However, even if the market model significantly deviates from the Black-Scholes model, we observe that the insurer is still much better off hedging dynamic lapses with the wrong moneyness ratio assumption, than not hedging them at all (for instance, the standard deviation and risk measures are halved).

Finally, it is comforting to note that even when assumptions used to construct the hedge strongly deviate from reality, dynamic hedging can still result in an important risk reduction relative to the actuarial approach. For example, under an RS-GARCH model, the standard deviation of the net unhedged loss at maturity is 13-15 percent of the initial investment (depending on whether the policyholder lapses or not) whereas it is between 2-4 percent when hedging is used. Tail risk measures also decrease by a very important margin in this context.

FURTHER READING

We note that Panneton and Boudreault (2011) have investigated the pricing of lapses in a simpler framework where lapses can only occur at specific dates during the contract. Moreover, we recommend reading Eling and Kochanski (2013) for a recent overview of the research on lapse in life insurance and Kling, et al. (2014), for a thorough analysis of the impact of policyholder behavior on hedging effectiveness in the context of guaranteed lifetime withdrawal benefits. ■

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