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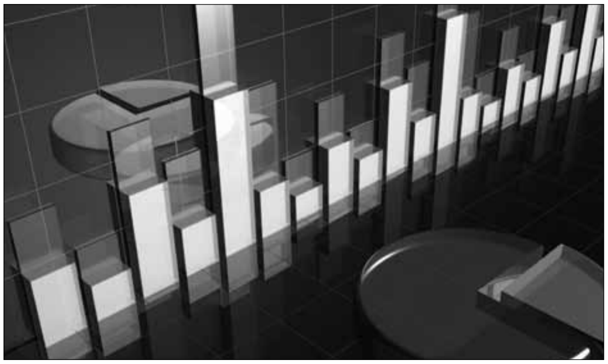

THOSE PESKY ARITHMETIC MEANS (PART 2)

By Richard R. Joss

I wrote an article for the February, 2010, edition of *Risks and Rewards* entitled "Those Pesky Arithmetic Means." The article showed how an arithmetic mean calculation extracted from any given actual investment history can vary significantly just depending upon the fiscal period used for the calculation. To deal with the problem of multiple arithmetic means, the article suggested increasing the number of data points to improve the accuracy of the calculation. But this process, when fully carried out, yields the geometric mean return, which is the rate of actual growth exhibited by the investment. Calculating the arithmetic mean in the more traditional ways by using only a relatively small number of observations of the changes in wealth produces a wide range of arithmetic mean results, all of which must exceed the actual rate of wealth growth for the investment.

In addition to writing the article, I participated in a session about arithmetic means at the most recent Society of Actuaries Annual Meeting. The other participant in the panel discussion was Alex Kane, Professor of Finance at UCSD and a co-author of the widely-used textbook *Essentials of Investments.* During the session I presented the issues outlined above, stressing the conditional nature of historical investment returns. Dr. Kane took a different approach, saying that the best estimate for a geometric return depended upon the investment horizon. A recap of both presentations is to be included in the next issue of *Pension Section News.*

HORIZON-BASED FORECASTS

In his portion of the panel discussion, Dr. Kane offered a very specific formula for the best estimate of a geometric mean. The formula depended upon the length of investment horizon being considered (H) and the length of the history that is used to supply the data for the estimate (T). The specific formula is as follows:

$$E(G_H) = (G_T)x(H/T) + (A_T)x[(T-H)/T]$$

where A_T is an arithmetic mean of historical data and G_T is the geometric mean from the same data set.

As an example of how the formula could be applied, consider the small company stock return data from the *2008 Stocks Bonds Bills and Inflation Yearbook.* This document provides details for an 82-year history of returns, where the calendar-year arithmetic mean is 17.1 percent and the geometric mean is 12.5 percent. For a 10-year investment horizon, the above formula yields 16.5 percent as the best estimate for the geometric return. This specific result is derived as follows:

$$16.5\% = (12.5\%)x(10/82) + (17.1\%)x(72/82)$$

But the same data source shows a sample standard deviation of small company stock returns of about 32.5 percent, and this result when combined with the arithmetic mean of 17.1 percent actually puts limits on the possible range of geometric returns. The chart below shows six different sets of data, each with an arithmetic mean of 17.1 percent and a standard deviation of 32.5 percent. These six different sets of data produce geometric returns that range from 8.1 percent to 14.3 percent.

Comparison of Geometric Average Returns for Sets of 10 Data Elements Each Set Having an Arithmetic Mean of 17.1 percent and a Standard Deviation of 32.5 percent

Year	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
1	27.4%	86.0%	56.9%	72%	47.9%	108.9%
2	27.4	17.1	56.9	55	47.9	6.9
3	27.4	17.1	56.9	40	47.9	6.9
4	27.4	17.1	17.1	30	47.9	6.9
5	27.4	17.1	17.1	20	47.9	6.9
6	27.4	17.1	17.1	10	-13.7	6.9
7	27.4	17.1	17.1	0	-13.7	6.9
8	27.4	17.1	-22.7	-10	-13.7	6.9
9	27.4	17.1	-22.7	-18	-13.7	6.9
10	-75.4	-51.8	-22.7	-28	-13.7	6.9
Geometric Average	8.1%	12.2%	12.9%	13.0%	13.0%	14.3%

Particular attention should be paid to the first and last columns in the above chart as these results present the absolute minimum and maximum values for a geometric mean for any set of 10 returns that have an arithmetic mean of 17.1 percent and a standard deviation of 32.5 percent. Even though the formula offered a "best estimate" of a geometric return of 16.5 percent, the only way that this result can actually be obtained is if in the future, the arithmetic average of small stock returns exceeds its historical average or the standard deviation of small stock returns falls short of its historical average.

SUMMARY

Actuaries working with pension plans face a difficult challenge when choosing an investment return assumption to use for valuation purposes. At the time this article is being written, there is an Exposure Draft of an Amendment to Actuarial Standard of Practice No. 27, Selection of Economic Assumptions for Measuring Pension Obligations, which specifically references arithmetic means as a possible basis for assumption selection. During the panel discussion both panelists encouraged that the Exposure Draft not be adopted. As shown above, historical arithmetic means may significantly overstate true expected returns, unless the future returns exhibit significantly different characteristics than that particular type of investment exhibited in the past.

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