SOCIETY OF ACTUARIES Investment Section

Risks & Rewards

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2013 SYMPOSIUM PRESENTER'S DIARY: RESPONSES TO THE GLOBAL FINANCIAL CRISIS

By J. Scott Colesanti

It is not debated that the consequences of the global financial crisis—and responses thereto by varied governments—have been unprecedented. Canada's bailout is said to have topped $100 billion. Round one of the U.S. bailout alone cost more than $700 billion. And the European Union has now been described as spending $2 trillion in various forms of member state relief.

But a question garnering much less of a consensus centers on the fundamental causes of the crisis. Could a particular breed of investment vehicle simply have proven to be at once diabolically alluring and unquestionably toxic? Did storied Wall Street greed reach new proportions? Were regulators both under-resourced and outpaced by nimbler market participants? Or have events since the Fall of 2013 merely evidenced the perfect storm that provides uncharted swells every 100 years or so?

During the 2013 SOA Life and Annuity Symposium held in Toronto last month, I broached these and other questions on ideal regulatory

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To join the section, SOA members and non-members can locate a membership form on the Investment Section Web page at *www.soa.org/investment*

CHAIRPERSON'S CORNER 2012-2013 COUNCIL YEAR IN REVIEW

By Thomas Anichini

This year we initiated a few new efforts to keep you engaged and contribute to your continuing investment education syllabus. Here are a few highlights.

EBSCO BUSINESS SOURCE CORPORATE PLUS (EBSCO)

At the 2012 Annual Meeting, the Investment Section Council agreed to purchase access to EBSCO for section members. (Read my piece describing EBSCO elsewhere in this issue, "Your newest member benefit…") If you do not otherwise have access to EBSCO outside the SOA Investment Section, I think you will find this one of the most valuable aspects of your section membership.

Response to this new benefit has been stellar. The week we rolled out EBSCO, I received an unsolicited email from a section member that began: "Tom: Wow – this is truly awesome. Thank you so much for getting it done. And BTW I've never written the SOA to complement them before ..." So if you have not yet explored EBSCO, take some time to explore it and discover what one of your fellow section members finds so "truly awesome."

INVESTMENT SYMPOSIUM AUDIO RECORDINGS

For the first time, the Investment Symposium sessions have been recorded, thanks to the generous sponsorship of the Investment Section, and these audio recordings are available for purchase ($20 for non-members, $0 for Investment Section members) via the SOA web store. Visit the SOA website's Professional Development / Presentation Archives / 2013 Presentations page, where you will find a link to the 2013 Investment Symposium presentations.

The SOA's media team synchronized the audio recordings with the presentations, which I find makes the viewing experience virtually like that of a webcast. If you like what you hear make sure you attend next year's Symposium in person.

INVESTMENT CONTEST

Back in the fall, the council was contemplating some ideas to prompt social networking. Some ideas we considered included prediction contests, e.g., guess the price of gold, the price of oil, the level of the Dow, the yield on the 10-year Treasury, etc. Then we eschewed the notion of merely guessing numbers in favor of actually building portfolios and decided to hold an asset allocation contest.

In April, we solicited entries from section members and received about 120 entries. Entrants had to build portfolios using combinations of 10 exchange-traded products (ETPs). We are offering prizes in three categories: cumulative return, lowest volatility, and highest reward/risk ratio, measured over nearly a six-month period. We will announce winners at our hot breakfast at the 2013 Annual Meeting.

The original intent of the contest was as a catalyst to social activity—so visit the section home page and look for the "2013 Investment Contest" link. You can see all the entrants' names, results to date, and if you unhide all the sheets, their allocations and predictions.

INTERACT WITH YOUR FELLOW SECTION MEMBERS AND YOUR COUNCIL

Chances are, on the contest workbook or our LinkedIn sub-group page, you will see some names of friends and former colleagues—ping them via email or say hello via LinkedIn. You may find most council members listed in the SOA member directory, and several of us are members of our LinkedIn sub-group. Share your feedback and suggestions with the council. As a team of volunteers, we are here to facilitate your ideas.

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reforms. My presentation offered as models the regulatory responses of Canada, the United States, and the European Union to three possible causes of the financial crisis: 1) concentration in over-the-counter (i.e., non-exchange traded) derivatives, 2) questionable imprimaturs by the major credit rating agencies, and 3) anonymous trading in "dark pools." This article both summarizes and supplements that presentation.

OTC DERIVATIVES

A considerable amount of regulatory response since 2009 has been focused on credit default swaps, the oft-blamed but seldom understood hedges to many CDO trading strategies of the last decade. The nearly uniform regulatory response of requiring "transparent" trading of these instruments on regulated exchanges or trading facilities is designed to, among other things, both increase competition and provide for better pricing.

Nearly five years after the onset of the crisis, final rules governing the new transparency remain debated. At first glance, the varied approaches to regulatory rulemaking provide some insight into the delay: The European Union employs an extra-territorial process that begins with the European Commission and often ends with legislative action at the Member State level. Canadian regulation, while driven by the Canadian Securities Administrators (CSA), might vary from province to province. Even the United States has seen relevant turf wars between Congress, the SEC, and the CFTC.

But the political promises of repeal of these reforms now appear quixotic, and mandatory measures aimed at greater disclosure are being rolled out in the United States and the European Union between 2013 and 2015. Meanwhile, in Canada, the provincial responses such as the Quebec Derivatives Act of 2009 have highlighted the need to provide exemptions for sophisticated entities serving as counterparties to the subject swap trades.

One audience member during my session opined that the press had recently reported that U.S. default swap trading today so nearly resembles pre-crisis levels that Wall Street employers are once again recruiting new graduates with an expertise in the field. Indeed, while the crisis succeeded in highlighting the almost unfathomable degree to which institutions trusted this business line, the practice continues on a weighty world scale.

CREDIT RATING AGENCIES

The Securities Exchange Commission was authorized by the Dodd-Frank Act of 2010 to require greater disclosures from nationally registered credit rating agencies. Subsequent Commission rulemaking mandated that the agencies, among other things, improve the quality of ratings and provide more transparency in attendant methodologies. But perhaps the more interesting reform contributed by Dodd-Frank paves the way for potential class actions against the agencies by private plaintiffs. Specifically, by eradicating its Rule 436(g) and concurrently clarifying the required proof of mental state for suits against agencies, the SEC invited the private class action Bar to the table set by Congress in 1933 for plaintiffs against issuers, underwriters, and broker-dealers.

Reforms in the European Union and Canada, while also guaranteeing agency registration and greater disclosure, do not similarly empower private plaintiffs. A pending EU reform asks commenters for feedback on whether fines against certain entities would deter misleading ratings.

A U.S. Department of Justice civil action from early 2013 did allege that a major rating agency overvalued CDOs in 2007, perhaps succeeding foremost in raising issues of timely government response than rating accuracy or earnestness. Overall, reforms to date have done little to alter the "issuer pays" model of compensation, while also stopping short of subjecting the agencies to the degree of oversight reserved for broker-dealers and issuing companies.

One audience commenter openly asked whether stricter regulation of the agencies is even possible, given the lack of the direct customer relationship that fuels broker-dealer supervision.

DARK POOLS

Earlier this year, *The New York Times* reported that such "off Board" trading may have peaked near 40 percent of the activity of some exchange listed issues in 2013. Nonetheless, there is no immediate plan for American regulators to fit that genie back into the bottle. Likewise, the operational requirements imposed by the EU on dark pools in recent years have actually been credited for their growth.

Canada alone has taken direct action to decrease the flow of trading away from established stock exchanges. CSA rules imposed in October 2012 obligate firms to demonstrate that customer trades filled internally were completed at a price commensurate with the market. Dark pool trading was said to have decreased in excess of 30 percent in the month immediately following, thus raising questions of whether greater regulation may succeed foremost in driving business to other markets.

CONCLUSION

To be sure, reasonable minds can differ on the wisdom of greater scrutiny of credit rating agencies, slowing the wave of off-board stock trading, and publishing details of credit derivative transactions akin to publically available information about stock trades. Questions of agency capture, the dearth of criminal penalties, and the lingering moral hazard occasioned by both add to the debate. What might best restore confidence in world markets would be closure on the present slate of reforms. In a nutshell, approaches—even where crystallized—remain somewhat undetailed and futuristic at present, perhaps precluding meaningful evaluation of results. After my presentation, I was of firmer conviction that measures such as Dodd-Frank need to be implemented with finality and publicized with force. As an old adage posits, Wall Street can handle anything except uncertainty itself.

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SOA 2013 LIFE & ANNUITY SYMPOSIUM UPDATE

By Frank Grossman & Ryan Stowe

ALBERT MOORE (L) PLAYING CHESS WITH CALEB BOUSU (R).

The late May weather that greeted actuaries attending the 2013 Life & Annuity Symposium in Toronto was particularly clement: sunny with temperatures in the mid-60s. Maple Leafs fans were gleefully walking on air, as their team returned to the Stanley Cup playoffs after a nine-year hiatus. Yet Mother Nature has a way of asserting herself when presented with anomalous phenomena. In the seventh game of their preliminary round series with the Leafs, the Beantown Bruins scored twice in the final 1:22 of regulation time to force overtime, and duly sent the Toronto side and their supporters packing in the extra frame—leaving the Ottawa Senators as the sole Canadian squad to advance to the quarter-finals. And the north wind delivered a frost warning to parts of southern Ontario during the week following the symposium, *after* most out-of-town attendees had safely returned home.

THOMAS C. BARHAM III SPEED CHESS TOURNAMENT (SESSION 01)

A dozen chess enthusiasts met on the Sunday afternoon prior to the meeting for the inaugural Thomas C. Barham III Speed Chess Tournament. This networking event is named for a prominent chess-playing member of the SOA, and was jointly sponsored by the Technology and Investment Sections as an opportunity for members to make new acquaintances, and have fun playing 10-minute speed chess too.

A local chess organizer, Ted Winick, and his colleagues at the Annex Chess Club were enlisted to run the event. Tournament directors Alex Ferreira, and Shabnam Abbarin, handled the pairings and myriad details with aplomb. Interestingly, Ted's day job as the president of the Chess Institute of Canada is devoted to establishing chess—as a teaching tool for fundamental concepts such as pattern recognition, abstract thinking, and problem solving—on the grade school mathematics curriculum.

Congratulations to Caleb Bousu who finished first after five rounds with a clean 5:0 record. And second place went to Eduard Nunes (all the way from Tokyo!) with a 4:1 score. Here is their game from round four:

Caleb Bousu v Eduard Nunes (Closed Sicilian) 1 e4 c5 2 Nc3 d6 3 f4 Nf6 4 Nf3 Nc6 5 Bb5 g6 6 O-O Bg7 7 d3 O-O 8 Bxc6 bxc6 9 Qe1 Rb8 10 Qh4 Ne8 11 e5 dxe5 12 fxe5 Rb4 13 Ne4 f5 14 exf6 exf6 15 c3 Rb5 16 a4 Rb7 17 Nxc5 Qb6 18 d4 Re7 19 Re1 Rxe1+ 20 Qxe1 Nd6 21 Bf4 Re8 22 Qd1 Nc4 23 b4 a5? 24 Qb3! Be6 25 Nxe6 Rxe6 26 Qxc4 Kf7 27 Re1 c5 28 Rxe6 Qxe6 29 Qxe6+ Kxe6 30 dxc5 Kd7 31 b5 f5 32 Be5 Bf8 33 Bd4 h6 … 1:0

Plans for the second speed chess networking tournament, to be held on the Tuesday evening of the upcoming 2013 Annual Meeting in San Diego, are already underway. [FG]

INVESTMENT SECTION HOT BREAKFAST (SESSION 04)

A good mix of both larks and owls turned out for this early Monday morning (7:00 am!) session. Section news and views were delivered by Ryan Stowe and Frank Grossman, including a look back to the March Investment Symposium in New York, and a look forward to the Investment Section-sponsored sessions at the 2013 Annual Meeting in San Diego. Nino Boezio arrived with perfect timing to share some observations about *Risk & Rewards* from his perspective as our long-standing co-editor, inviting those in attendance to consider becoming contributors. And the section's new staff partner, David Schraub, was briefly introduced.

Geoff Hancock then gave a short talk titled, "Economic Scenario Generators: All models are wrong—so now what?" in which he offered a pungent commentary for those actuaries able to smell the coffee at that early hour. During his engaging presentation, Geoff surveyed things that actuaries have done fairly well (e.g., increased facility with stochastic modeling), some things that haven't been done that well (e.g., over-reliance on point estimates when delivering results rather than ranges) and areas where we've really fouled-up (e.g., making the use of ESGs and

FROM LEFT TO RIGHT: TOURNAMENT CO-ORGANIZER ALBERT MOORE, AND TOURNAMENT DIRECTORS SHABNAM ABBARIN AND ALEX FERREIRA, AWARD FIRST PLACE CERTIFICATE AND PRIZE TO CALEB BOUSU.

HANS AVERY (L) PLAYING CHESS WITH RYAN STOWE (R).

stochastic models "too academic" and opaque for senior management to trust). The breakfast session concluded with a book-draw, and Leonard Mangini won a copy of Dan Areily's *Predictably Irrational*. [FG]

PERSPECTIVES ON LIFE INSURANCE AND ANNUITIES IN THE MIDDLE MARKET (SESSION PD-25)

This session was a panel discussion delivered by Douglas Bennett, Robert Buckingham, and Walter Zultowski, moderated by Ryan Stowe, and co-sponsored with the Marketing and Distribution Section. The life insurance portion of this session focused on recent research from the second half of 2012. The life insurance middle market (defined as "Young Families" age 25–40 with annual household income between $35,000–$125,000, and at least one dependent in the household) was segmented into three groups, each having different attitudes and beliefs about their need for life insurance, as well as different motivations for purchasing (or not purchasing) life insurance. "Protectors" buy life insurance to meet a need rather than based on a strong belief in the product. "Planners" most likely perceive and appreciate the long-term value of life insurance. "Opportunistic buyers" have less belief (e.g., perceived value) in the product, and typically purchase less coverage than other segments. They also tend to buy most products at their place of employment. The presentation provided insight into the different methods that companies can use to segment their target markets through data mining and predictive modeling, all with a view to making their marketing efforts more effective by targeting their customers' preferred buying style.

The annuity portion of the session offered a different perspective on the middle market. The majority of the people who purchase annuities are age 50 or older. In fact, 26 percent of Americans are baby boomers, and each day 10,000 of them turn 65. What does this mean? That the pre-retiree and retired population need the guaranteed benefits that annuities can deliver. By definition, the middle market has fewer assets (on a per client basis; the middle market itself is quite large) than the affluent and mass affluent markets, and this is one of the challenges of serving the middle market. The panelists focused on the challenges advisors face in allocating assets of the middle market to maximize the retirement income outcome dilemma through a concept similar to the efficient frontier in the investment world. [RS]

DISCOUNT RATES FOR FINANCIAL REPORTING PURPOSES: ISSUES AND APPROACHES (SESSION PD-32)

This session, presented in conjunction with the Financial Reporting Section, dealt with the International Actuarial Association's (IAA) recent work developing an educational monograph on discount rates for financial reporting purposes. David Congram lead off with some background about the project and its sponsors, as well as the IAA's broader educational mandate. Andy Dalton then provided an overview of the monograph itself, followed by some comments contrasting risk free rates in theory and in practice—accompanied by the session's most memorable slide. Next, Derek Wright spoke about setting discount rates for pass-through products. David returned to the lectern to discuss the evolving influence of sovereign and political risk, and Frank Grossman concluded the session with some brief comments about replicating portfolios. [FG]

DO LONG-TERM GUARANTEES IN INSURANCE PRODUCTS MAKE SENSE? (SESSION D-69)

The format of this well attended session was unique; a "facilitated debate" between four panelists, moderated by Emile Elefteriadis. Jeff Adams and David Harris offered a Canadian perspective, while Michael Downing and Michael LeBoeuf delivered their comments from an American vantage point. The panelists addressed various questions posed, and attendees were also invited to share their views through the use of handheld interactive voting devices.

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THOMAS C. BARHARM III (R) WAS A MEMBER OF THE SOA AND PRESIDENT OF THE MANHATTAN CHESS CLUB, AS WELL AS THE SPEED CHESS TOURNAMENT'S NAMESAKE.

The following question was posed to the audience at both the beginning and the end of the session: "Do you believe long-term guarantees in insurance products make sense?" At the outset, 71 percent of the audience answered, "Yes, but only if the guarantees and risks are costed for appropriately." Even after substantial discussion regarding Canadian and American insurance and annuity products and the different long-term guarantees that they can provide, the audience response was virtually unchanged at the end of the session. A large majority of those in attendance still believed that long-term guarantees make sense if priced appropriately.

Key stakeholders' perception of the transparency of long-term guarantees was also broached, and the audience was polled on the following question: "Do you believe long term guarantees are transparent for management and other stakeholders?" The overwhelming response (again, 71 percent) was, "No, transparency is lacking for management and stakeholders." This presents an interesting conundrum for actuaries; the responsibility to conservatively price the promises and guarantees that are embedded in insurance products, while also effectively communicating the financial implications of those same guarantees to senior management, who may not be actuaries. As economic times change, and product development evolves, this will continue to challenge our profession. [RS]

RESPONSES TO THE GLOBAL FINANCIAL CRISIS: A TRANSNATIONAL PERSPECTIVE (SESSION L-80)

Christine Lagarde, head of the IMF, was recently asked whether enough has been done to reform the financial system so as to avoid another crisis. Her response, in part, was:

> … if you turn to over-the-counter derivative markets, for instance, it hasn't been done. It's still very obscure and not transparent at all. Plenty of work has been done, but international cooperation is going to be critically important, because otherwise you'll have people having done what they think is their job in their respective corner but it will not be consistent with what others will have done. Bankers, traders, financiers are very smart and astute people; they will find out what is the right channel to optimize the system—which is fine, as long as risks are taken care of and as long as, at the end of the day, it's not the taxpayer who picks up the bill.[1]

Scott Colesanti, who is an associate professor at Hofstra University Law School, addressed this and related issues during his presentation about the regulation of financial markets which was co-sponsored with the Joint Risk Management Section. Please refer to a separate article elsewhere in this issue of *Risk & Rewards* which provides additional details about Scott's session. [FG]

[1] "En Garde! As head of the IMF, Christine Lagarde must be ready for any financial crisis. What worries her now?" by David Wessel, ***WSJ.Money***, page 14, May 18, 2013.

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PRESENT VALUES, INVESTMENT RETURNS AND DISCOUNT RATES – PART 1

By Dimitry Mindlin

This is the first of a two-part article. The first provides the groundwork for exploring the different formulations of the discount rate, to support various sorts of objectives. It provides some useful rule of thumb for estimating quantiles in the distribution of discount rates and for relating geometric and arithmetic discount assumptions in a defined series of returns. The second part, to follow in early 2014, applies this approach to examples of a stochastic distribution of returns.

The concept of present value lies at the heart of finance in general and actuarial science in particular. The importance of the concept is universally recognized. Present values of various cash flows are extensively utilized in the pricing of financial instruments, funding of financial commitments, financial reporting, and other areas.

A typical funding problem involves a financial commitment (defined as a series of future payments) to be funded. A financial commitment is funded if all payments are made when they are due. A present value of a financial commitment is defined as the asset value required at the present to fund the commitment.

Traditionally, the calculation of a present value utilizes a discount rate—a deterministic return assumption that represents investment returns. If the investment return and the commitment are certain, then the discount rate is equal to the investment return and the present value is equal to the sum of all payments discounted by the compounded discount rates. The asset value that is equal to this present value and invested in the portfolio that generates the investment return will fund the commitment with certainty.

In practice, however, perfectly certain future financial commitments and investment returns rarely exist. While the calculation of the present value is straightforward when returns and commitments are certain, uncertainties in the commitments and returns make the calculation of the present value anything but straightforward. When investment returns are uncertain, a single discount rate cannot encompass the entire spectrum of investment returns, hence the selection of a discount rate is a challenge. In general, the asset value required to fund an uncertain financial commitment via investing in risky assets—the present value of the commitment—is uncertain (stochastic).[1]

While the analysis of present values is vital to the process of funding financial commitments, uncertain (stochastic) present values are outside of the scope of this paper. This paper assumes that a present value is certain (deterministic)—a present value is assumed to be a number, not a random variable in this paper. The desire to have a deterministic present value requires a set of assumptions that "assume away" all the uncertainties in the funding problem.

In particular, it is generally necessary to assume that all future payments are perfectly known at the present. The next step is to select a proper measurement of investment returns that serves as the discount rate for present value calculations. This step—the selection of the discount rate—is the main subject of this paper.

One of the main messages of this paper is the selection of the discount rate depends on the objective of the calculation. Different objectives may necessitate different discount rates. The paper defines investment returns and specifies their relationships with present and future values. The key measurements of investment returns are defined in the context of return series and, after a concise discussion of capital market assumptions, in the context of return distributions. The paper concludes with several examples of investment objectives and the discount rates associated with these objectives.

1. INVESTMENT RETURNS

This section discusses one of the most important concepts in finance—investment returns.

Let us define the investment return for a portfolio of assets with known asset values at the beginning and the end of a time period. If PV is the asset value invested in portfolio P at the beginning of a time period, and FV is the value of the portfolio at the end of the period, then the portfolio return R_p for the period is defined as

$$R_P = \frac{FV - PV}{PV} \tag{1.1}$$

Thus, given the beginning and ending values, portfolio return is defined (retrospectively) as the ratio of the investment gain over the beginning value. Definition (1.1) establishes a relationship between portfolio return R_p and asset values PV and FV.

Simple transformations of definition (1.1) produce the following formula:

$$FV = PV\left(1 + R_P\right) \tag{1.2}$$

Formula (1.2) allows a forward-looking (prospective) calculation of the end-of-period asset value FV. The formula is usually used when the asset value at the present PV and portfolio return R_p are known (this explains the notation: PV stands for "Present Value"; FV stands for "Future Value").

While definition (1.1) and formula (1.2) are mathematically equivalent, they utilize portfolio return R_p in fundamentally different ways. The return in definition (1.1) is certain, as it is used retrospectively as a measurement of portfolio performance. In contrast, the return in formula (1.2) is used prospectively to calculate the future value of the portfolio, and it may or may not be certain.

When a portfolio contains risky assets, the portfolio return is uncertain by definition. Most institutional and individual investors endeavor to fund their financial commitments by virtue of investing in risky assets. The distribution of uncertain portfolio return is usually analyzed using a set of forward-looking capital market assumptions that include expected returns, risks, and correlations between various asset classes. Later sections discuss capital market assumptions in more detail.

Given the present value and portfolio return, formula (1.2) calculates the future value. However, many investors with future financial commitments to fund (e.g., retirement plans) face a different challenge. Future values—the commitments—are usually given, and the challenge is to calculate present values. A simple transformation of formula (1.2) produces the following formula:

$$PV = \frac{FV}{1 + R_P} \tag{1.3}$$

Formula (1.3) represents the concept of discounting procedure. Given a portfolio, formula (1.3) produces the asset value PV required to be invested in this portfolio at the present in order to accumulate future value FV. It must be emphasized that return R_p in (1.3) is generated by the actual portfolio P, as there is no discounting without investing. Any discounting procedure assumes that the assets are actually invested in a portfolio that generates the returns used in the procedure.

Formulas (1.2) and (1.3) are mathematically equivalent, and they utilize portfolio return in similar ways. Depending on the purpose of a calculation in (1.2) or (1.3), one may utilize either a particular measurement of return (e.g., the expected return or median return) or the full range of returns.[2] The desirable properties of the future value in (1.2) or present value in (1.3) would determine the right choice of the return assumption.

Future and present values are, in a certain sense, inverse of each other. It is informative to look at the analogy between

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future and present values in the context of a funding problem, which would explicitly involve a future financial commitment to fund. Think of an investor that has $\$P$ at the present and has made a commitment to accumulate $\$F$ at the end of the period by means of investing in a portfolio that generates investment return R.

Similar to (1.2), the future value of $\$P$ is equal to

$$FV = P(1+R) \tag{1.4}$$

Similar to (1.3), the present value of $\$F$ is equal to

$$PV = \frac{F}{1+R} \tag{1.5}$$

The shortfall event is defined as failing to accumulate $\$F$ at the end of the period:

$$FV < F \tag{1.6}$$

The shortfall event can also be defined equivalently in terms of the present value as $\$P$ being insufficient to accumulate $\$F$ at the end of the period:

$$P < PV \tag{1.7}$$

In particular, the shortfall probability can be expressed in terms of future and present values:

$$\text{Shortfall Probability} = \Pr(FV < F) = \Pr(PV > P) \tag{1.8}$$

If the shortfall event happens, then the shortfall size can also be measured in terms of future and present values. The future shortfall $F - FV$ is the additional amount the investor will be required to contribute at the end of the period to fulfill the commitment. The present shortfall $PV - P$ is the additional amount the investor is required to contribute at the present to fulfill the commitment.

Clearly, there is a fundamental connection between future and present values. However, this connection goes only so far, as there are issues of great theoretical and practical importance that distinguish future and present values. As demonstrated in a later section, similar conditions imposed on future and present values lead to different discount rates.

Uncertain future values generated by the uncertainties of investment returns (and commitments) play no part in financial reporting. In contrast, various actuarial and accounting reports require calculations of present values, and these present values must be deterministic (under current accounting standards, at least). Therefore, there is a need for a deterministic discounting procedure.

Conventional calculations of deterministic present values usually utilize a single measurement of investment returns that serves as the discount rate. Since there are numerous measurements of investment returns, the challenge is to select the most appropriate measurement for a particular calculation. To clarify these issues, subsequent sections discuss various measurements of investment returns.

2. MEASUREMENTS OF INVESTMENT RETURNS: RETURN SERIES

This section discusses the key measurements of series of returns and relationships between these measurements. Given a series of returns $r_1,\ldots, r_n$, it is desirable to have a measurement of the series—a single rate of return—that, in a certain sense, would reflect the properties of the series. The right measurement always depends on the objective of the measurement. The most popular measurement of a series of returns $r_1,\ldots, r_n$ is its arithmetic average A defined as the average value of the series:

$$A = \frac{1}{n}\sum_{k=1}^{n} r_k \tag{2.1}$$

As any other measurement, the arithmetic average has its pros and cons. While the arithmetic average is an unbiased estimate of the return, the probability of achieving this value may be unsatisfactory. As a predictor of future returns, the arithmetic average may be too optimistic.

Another significant shortcoming of the arithmetic return is it does not "connect" the starting and ending asset values. The starting asset value multiplied by the compounded arithmetic return factor (1 + A) is normally greater than the ending asset value.[3] Therefore, the arithmetic average is inappropriate if the objective is to "connect" the starting and ending asset values. The objective that leads to the arithmetic average as the right choice of discount rate is presented in Section 5.

Clearly, it would be desirable to "connect" the starting and ending asset values—to find a single rate of return that, given a series of returns and a starting asset value, generates the same future value as the series. This observation suggests the following important objective.

Objective 1: To "connect" the starting and ending asset values.

The concept of geometric average is specifically designed to achieve this objective. If A_0 and A_n are the starting and ending asset values correspondingly, then, by definition,

$$A_0\left(1+r_1\right)\ldots\left(1+r_n\right)=A_n \tag{2.2}$$

The geometric average G is defined as the single rate of return that generates the same future value as the series of returns. Namely, the starting asset value multiplied by the compounded return factor $(1 + G)^n$ is equal to the ending asset value:

$$A_0\left(1+G\right)^n=A_n \tag{2.3}$$

Combining (2.2) and (2.3), we get the standard definition of the geometric average G:

$$G=-1+\prod_{k=1}^{n}\left(1+r_k\right)^{1/n} \tag{2.4}$$

Let us rewrite formulas (1.2) and (1.3) in terms of present and future values. If A_n is a future payment and $r_1,\ldots, r_n$ are the investment returns, then the present value of A_n is equal to the payment discounted by the geometric average:

$$A_0=\frac{A_n}{\left(1+r_1\right)\quad\left(1+r_n\right)}=\frac{A_n}{\left(1+G\right)^n} \tag{2.5}$$

Thus, the geometric average connects the starting and ending asset values (and the arithmetic average does not). Therefore, if the primary objective of discount rate selection is to connects the starting and ending asset values, then the geometric average should be used for the present value calculations.

To present certain relationships between arithmetic and geometric averages, let us define variance V as follows:[4]

$$V=\frac{1}{n}\sum_{k=1}^{n}\left(r_k-A\right)^2 \tag{2.6}$$

If $V = 0$, then all returns in the series are the same, and the arithmetic average is equal to the geometric average. Otherwise (if $V > 0$), the arithmetic average is greater than the geometric average ($A > G$).[5]

There are several approximate relationships between arithmetic average A, geometric average G, and variance V. These relationships include the following relationships that are denoted (R1) – (R4) in this paper.

$$G\approx A-V/2 \tag{R1}$$

$$\left(1+G\right)^2\approx\left(1+A\right)^2-V \tag{R2}$$

$$1+G\approx\left(1+A\right)\exp[\,(-1/2)\mathrm{V}(1+\mathrm{A})^{-2}] \tag{R3}$$

$$1+G\approx\left(1+A\right)\left(1+V\left(1+A\right)^{-2}\right)^{-1/2} \tag{R4}$$

These relationships produce different results, and some of them work better than the others in different situations. Relationship (R1) is the simplest, popularized in many publications, but usually sub-optimal and tends to underestimate the geometric return.[6] Relationships (R2) – (R4)

CONTINUED ON PAGE 14

are slightly more complicated, but, in most cases, should be expected to produce better results than (R1).

The geometric average estimate generated by (R4) is always greater than the one generated by (R3), which in turn is always greater than the one generated by (R2).[7] Loosely speaking,

$$(R2) < (R3) < (R4)$$

In general, "inequality" (R1) < (R2) is not necessarily true, although it is true for most practical examples. If $A > V/4$, then the geometric average estimate generated by (R1) is less than the one generated by (R2).[8]

There is some evidence to suggest that, for historical data, relationship (R4) should be expected to produce better results than (R1) – (R3). See Mindlin [2010] for more details regarding the derivations of (R1) – (R4) and their properties.

Example 2.1.

$n = 2, r_1 = -1\%, r_2 = 15\%$. Then arithmetic mean *A*, geometric mean *G*, and variance *V* are calculated as follows.

$$A = \frac{1}{2}(-1\% + 15\%) = 7.00\%$$

$$G = \sqrt{(1-1\%)(1+15\%)} - 1 = 6.70\%$$

$$V = \frac{1}{2}\sum_{k=1}^{2}(r_k - A)^2 = 0.64\%$$

Note that $(1+G)^2 = (1+A)^2 - V$, so formula (R2) is exact in this example.

Given $1 at the present, future value *FV* is

$$FV = 1 \cdot (1-1\%)(1+15\%) = 1.1385$$

If we apply arithmetic return *A* to $1 at the present for two years, we get

$$(1+7\%)^2 = 1.1449$$

which is greater than future value $FV = 1.1385$.

If we apply geometric return *G* to $1 at the present for two years, we get

$$(1+6.70\%)^2 = 1.1385$$

which is equal to future value *FV*, as expected.

Given $1 in two years, present value *PV* is

$$PV = \frac{1}{(1-1\%)(1+15\%)} = 0.8783$$

If we discount $1 in two years using geometric return *G*, we get

$$\frac{1}{(1+6.70\%)^2} = 0.8783$$

which is equal to present value *PV*, as expected.

If we discount $1 in two years using arithmetic return *A*, we get

$$\frac{1}{(1+7.00\%)^2} = 0.8734$$

which is less than present value $PV = 0.8783$.

3. CAPITAL MARKET ASSUMPTIONS AND PORTFOLIO RETURNS

This section introduces capital market assumptions for major asset classes and outlines basic steps for the estimation of portfolio returns.

It is assumed that the capital markets consist of n asset classes. The following notation is used throughout this section:

m_i mean (arithmetic) return;
s_i standard deviation of return;
c_{ij} correlation coefficient between asset classes *i* and *j*.

A portfolio is defined as a series of weights $\{w_i\}$, such that $\sum_{i=1}^{n} w_i = 1$. Each weight w_i represents the fraction of the portfolio invested in the asset class *i*.

Portfolio mean return A and variance V are calculated as follows:

$$A = \sum_{i=1}^{n} w_i m_i \tag{3.1}$$

$$V = \sum_{i,j=1}^{n} w_i w_j s_i s_j c_{ij} \tag{3.2}$$

Let us also define return factor as $1 + R$. It is common to assume that the return factor has lognormal distribution (which means $\ln(1 + R)$ has normal distribution). Under this assumption, parameters μ and σ of the lognormal distribution are calculated as follows:

$$\sigma^2 = \ln\left(1 + V(1 + A)^{-2}\right) \tag{3.3}$$

Using σ calculated in (3.3), parameter μ of the lognormal distribution is calculated as follows:

$$\mu = \ln(1 + A) - \frac{1}{2}\sigma^2 \tag{3.4}$$

Given parameters μ and σ, the P^{th} percentile of the return distribution is equal to the following:

$$R_P = \exp\left(\mu + \sigma\Phi^{-1}(P)\right) - 1 \tag{3.5}$$

where Φ is the standard normal distribution. In particular, if $P = 50\%$, then $\Phi^{-1}(P) = 0$. Therefore, the median of the return distribution under the lognormal return factor assumption is calculated as follows.

$$R_{0.5} = \exp(\mu) - 1 \tag{3.6}$$

Example 3.1.

Let us consider two uncorrelated asset classes with mean returns 8.00 percent and 6.00 percent and standard deviations 20.00 percent and 10.00 percent correspondingly. If a portfolio has 35 percent of the first class and 65 percent of the second class, its mean and variance are calculated as follows.

$$A = 8.00\% \bullet 35\% + 6.00\% \bullet 65\% = 6.70\%$$

$$V = (20.00\% \bullet 35\%)^2 + (10.00\% \bullet 65\%)^2 = 0.9125\%$$

It is interesting to note that the standard deviation of the portfolio is 9.55 percent ($= \sqrt{0.9125\%}$), which is lower than the standard deviations of the underlying asset classes (20.00 percent and 10.00 percent). Assuming that the return factor of this portfolio has lognormal distribution, the parameters of this distribution are

$$\sigma = \sqrt{\ln\left(1 + 0.9125\% / (1 + 6.70\%)^2\right)} = 0.0893$$

$$\mu = \ln(1 + 6.70\%) - \frac{0.0893^2}{2} = 0.0609$$

From (3.5), the median return for this portfolio is

$$R_{0.5} = \exp\left(0.0609 + 0.0893 \cdot \Phi^{-1}(0.5)\right) - 1 = 6.27\%$$

From (3.5), the 45th percentile for this portfolio is

$$R_{0.45} = \exp\left(0.0609 + 0.0893 \cdot \Phi^{-1}(0.45)\right) - 1 = 5.09\%$$

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END NOTES

[1] There are exceptions, e.g. an inflation-adjusted cash flow with a matching TIPS portfolio.
[2] See Mindlin [2009] for more details.
[3] That is as long as the returns in the series are not the same.
[4] For the purposes of this paper, the concerns that the sample variance as defined in (2.6) is not an unbiased estimate are set aside.
[5] This fact is a corollary of the Jencen's inequality.
[6] For example, see Bodie [1999], p. 751, Jordan [2008], p. 25, Pinto [2010], p. 49., Siegel [2008], p. 22., DeFusco [2007], p 128, 155.
[7] That is, obviously, as long as the returns in the series are not the same and $V > 0$.
[8] Mindlin [2010] contains a simple example for which (R1) > (R2).

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THE U.S. INFLATION MARKET WHAT IT IS AND HOW TO PROFIT FROM IT

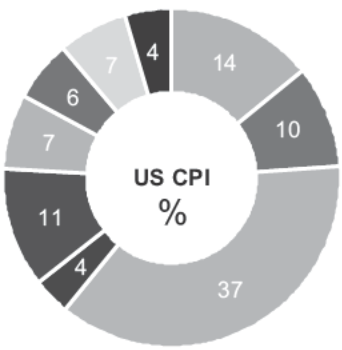
By Allan Levin

This article first appeared on the Clear Path Analysis website as part of their report, "Inflation Hedging & Real Return, North America 2013." It is reprinted with permission.

BENEFITING FROM STRUCTURAL BIASES IN THE U.S. INFLATION MARKET

Executive summary: Market participants can obtain direct exposure to CPI via a variety of instruments including TIPS, inflation swaps and inflation options. The need for inflation protection is universal—and arguably even greater for under-funded pension obligations. Liquidity in U.S. Headline CPI, as well as DB Core U.S. CPI, means that these can be used as proxies for correlated or more specific inflation measures such as Canadian Inflation or U.S. Property Inflation. Risk premiums can be earned by taking advantage of structural biases in the U.S. Inflation Market including: cheapness in one-year TIPS and inflation swaps, richness of inflation options, and the relative value between TIPS and inflation swaps.

Breakdown of CPI



	WEIGHTS
FOOD	14.243
ENERGY	9.759
HOUSING	36.793
APPAREL	3.665
TRANSPORTATION	11.331
MEDICAL CARE	7.144
RECREATION	5.99
EDUCATION & COMMUNICATION	6.763
OTHER GOODS & SERVICES	4.311
ALL ITEMS	100
ALL ITEMS LESS FOOD AND ENERGY	75.998

- **Food**
- **Energy**
- **Housing**
- **Apparel**
- **Transportation**
- **Medical Care**
- **Recreation**
- **Education and Communication**
- **Other Goods & Services**

Source: Bureau of Labor Statistics

1. MEASURING INFLATION

Each month, the U.S. Bureau of Labor and Statistics releases the consumer price index (CPI), a measure of the price level of consumer goods and services. Changes in CPI can be used to measure inflation on a month-over-month or year-over-year basis. Two widespread measures of inflation are headline inflation, components shown in Graph 1, and core inflation, which excludes food and energy. The statistics are available on the BLS website, at http://www.bls.gov/cpi/.

Housing has the largest weighting in Headline CPI, accounting for almost 40 percent. However, much of the monthly volatility in the index is driven by short-term changes in energy prices, despite energy only accounting for 10 percent of the index. Over longer periods, changes in energy prices tend to average out, reducing the impact on long-term inflation trends.

2. ROLE OF INFLATION IN A PORTFOLIO

U.S. pension plans may have direct or indirect inflation-linked liabilities. Many Defined Benefit plans determine benefits based on the final wages earned by the retiree, or an average of the wages during the last few years of service. Defined Benefit pension plans also often contain a cost-of-living allowance (COLA) that compensates beneficiaries for the erosion in value of benefits due to inflation. Pension asset portfolio returns often do not have a strong correlation to inflation, leaving the plan with a net short exposure to inflation. The under-funded status of many public and private U.S. pensions exacerbates the inflation exposure. Assuming a pension plan is 25 percent under-funded, 133 percent of the asset portfolio value would need inflation protection to offset the impact of inflation on the plan's liabilities.

Even if a plan has no explicit inflation-linked obligation, the beneficiaries generally need distributions to be sufficient in order to cover their cost of living. Accordingly, it is in the interest of the beneficiaries that returns earned should be positively correlated to inflation.

3. INFLATION IN A TRADITIONAL INVESTMENT PORTFOLIO

A traditional equities/bonds portfolio relies on stable correlation assumptions to produce diversification. Stocks and bonds should produce diversification in an economy dominated by growth. However, periods of increased inflation or inflation uncertainty can have a negative impact on both equities and bonds.

Equities: Dividend streams may be pressured if inflation leads to slower economic growth while the discount rate is increased due to higher nominal rates and uncertainty.

Bonds: A higher discount rate would similarly lead to lower valuations on fixed rate bonds.

The stagflation period of the late 1970s and early 1980s in the United States was characterized by low single-digit portfolio returns coupled with high correlation (above 50 percent) between equities and bonds.[1]

4. RISK FACTOR APPROACH TO ASSET ALLOCATION

In asset allocation circles, an increasingly favored approach is to focus on diversification among different sources of risk premiums rather than simply diversifying among asset classes.

Not only can CPI-linked instruments provide inflation protection, but it is also possible to use these tools to source risk premiums due to a number of features of the CPI-linked market. These features include:

- Deflation risk premium in short-dated TIPS and swaps,
- Asset swap premium in TIPS, and
- Tail risk premium in inflation options.

5. ACCESSING EXPOSURE TO THE INFLATION MARKET

Inflation can be traded through a variety of instruments. These include: TIPS, Total Return Swaps, ETFs, Inflation-linked Notes, Inflation Swaps and Inflation Options.

TIPS (TREASURY INFLATION PROTECTED SECURITIES)

TIPS are securities issued by the U.S. government that offer investors inflation protection. The principal is accredited daily based on the Headline CPI index and is repaid at maturity subject to a minimum of par, thus providing deflation protection. Semi-annual coupons paid on TIPS are based on the inflation-adjusted principal. The TIPS market is the largest inflation-linked market in the world. Regular auctions are conducted in 5y, 10y and 30y TIPS.

The U.S. government issued approximately $150 billion of TIPS in 2012 and is expected to issue the same or more in 2013. Total market value of outstanding TIPS exceeds $900 billion and average daily trading volume is $11 billion (Source: Federal Reserve Bank of New York).

In addition to investing in TIPS, funds can also obtain exposure to TIPS as an overlay, by entering into a Total Return Swap on a TIPS Index. In a total return swap, one party pays the return of the index in exchange for a funding rate, which could be quoted either as a fixed rate or as a spread to a floating rate. TIPS indices are typically market-weighted, and are available for both the aggregate TIPS market and for specific maturity buckets.

Some portfolio managers have employed total return swaps to synthetically replicate inflation-linked credit portfolios. The cash is utilized to invest in high-yield bonds to obtain a credit risk premium, and a total return swap on a TIPS index provides inflation exposure as well.

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// FOCUS ON DIFFERENT SOURCES OF RISK PREMIUMS. //

A third way to obtain exposure to TIPS is via ETFs that reference TIPS indices[2] or ETNs that reference Inflation Expectations as implied by the difference in yield between TIPS and Treasuries.[3]

5.2. INFLATION SWAPS AND OPTIONS

Inflation swaps offer a mechanism to trade inflation over a given time horizon, with mechanics similar to nominal interest rate swaps. At maturity, one party pays the cumulative percentage increase in the reference inflation index over the life of the swap in exchange for an annually compounded fixed rate, known as the breakeven inflation rate.

An example of the above is if the fixed rate quoted on a five-year inflation swap was 2 percent, and the CPI index rose from a level of 220 to 255 during the five-year period (3 percent per annum inflation), a net payment of 5.50 percent of notional would be paid to the buyer of inflation. This is slightly more than 1 percent per annum due to the effect of compounding.

It is also possible to obtain exposure to a measure of core inflation via inflation swaps that reference the DB Core U.S. CPI Index, thus mitigating volatility due to fluctuations in energy prices.

U.S. CPI can be used as a proxy hedge for other correlated markets such as Canadian Inflation. Similarly, DB Core U.S. CPI can be used as a proxy for more specific measures of inflation such as property inflation.

Inflation Options provide asymmetric returns relative to CPI. Caps provide payoffs when inflation exceeds a strike (e.g., 4 percent), and Floors pay out when inflation is below an agreed strike (e.g., 0 percent in the case of a "deflation" floor). There are two-types of inflation options that regularly trade. "Year-On-Year" Options pay annually based on each year's inflation rate, whereas "Zero Coupon" Options pay out on the final maturity date based on cumulative inflation over the period.

Banks and other parties may issue Inflation-linked Notes that include embedded inflation swaps and inflation options. This provides access to these markets for clients who do not typically participate in the underlying derivative markets.

Inflation swaps and options have grown significantly over the past several years and are actively traded across a wide range of maturities and strikes.

6. INFLATION RISK PREMIUM STRATEGIES

6.1. SHORT-DATED INFLATION-LINKED BONDS AND SWAPS

Short-dated inflation-linked bonds and swaps tend to have an embedded deflation risk premium. Hence, the implied inflation rate in TIPS and inflation-linked swaps has tended to systematically under-predict realized inflation over short-term horizons.

A structural reason for this effect is that most TIPS funds are benchmarked to indices that only include TIPS with greater than one-year to maturity. Accordingly, most TIPS funds are forced to sell TIPS as soon as their maturities fall below one year in order to reduce tracking error. Money-market funds cannot buy these short-dated TIPS as they are limited to fixed-rate debt, and TIPS interest is floating (with inflation). Accordingly, the lack of natural buyers of short dated TIPS results in implied inflation (TIPS breakeven rates) being underpriced at the front-end of the curve.

Graph 2 (pg. 21, top) shows historical1y inflation swaps and one-year realized inflation over the corresponding periods.

Inflation swaps, which typically imply higher inflation rates than TIPS, have under-predicted realized inflation by more than 0.50 percent. Note: past performance is no guarantee of future results.

Volatility of returns can be affected by energy prices. Accordingly, returns are less volatile if energy moves are hedged-out. For example, this can be achieved by buying one-year DB Core U.S. CPI Inflation Swaps.

IT IS POSSIBLE TO ARBITRAGE THE RELATIVE VALUE BETWEEN THE SWAPS AND TIPS MARKET.

6.2. TIPS ASSET SWAP PREMIUM

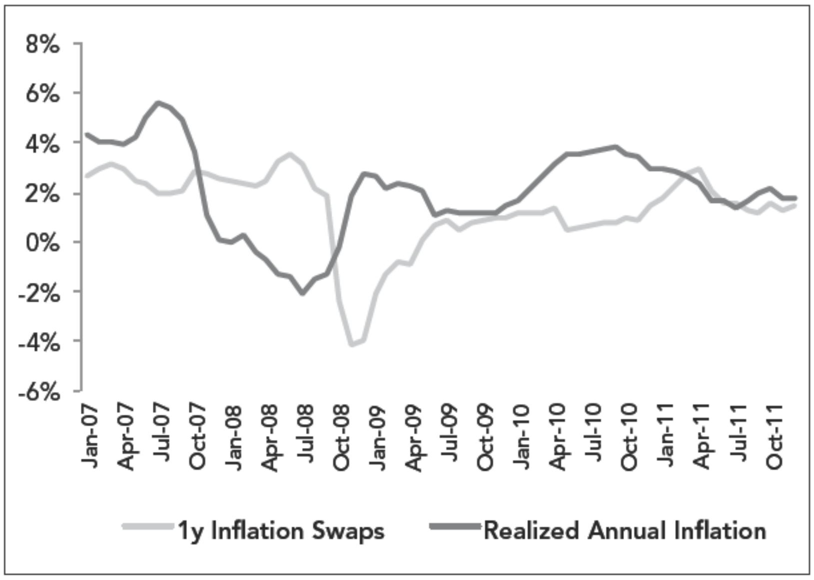
Future levels of CPI can be implied from TIPS and Treasury Yields as well as from inflation swap levels. Interestingly, inflation swaps typically imply higher levels of CPI by about 0.2 percent to 0.4 percent per annum. As both reference the same measure of CPI, it is possible to arbitrage the difference[4] if the position is held to maturity.

Inflation swaps are generally a rich relative to TIPS. This is because there are many natural buyers of inflation via inflation swaps, but far fewer natural sellers of inflation. In fact, the vast majority of inflation supply in the U.S. market is provided by the U.S. Government by virtue of TIPS issuance. Accordingly the strong demand for inflation swaps and large supply of TIPS causes a supply/demand imbalance that results in Inflation swaps being generally rich compared to TIPS.

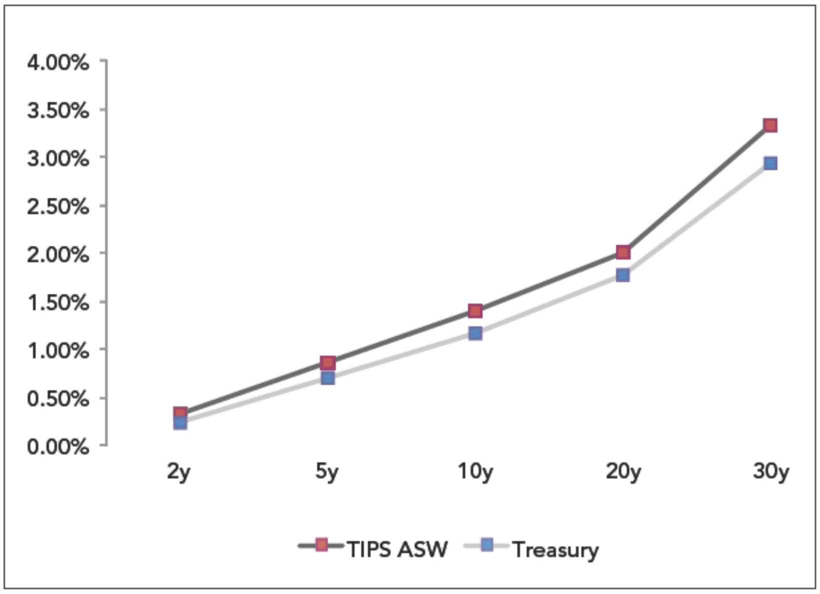
As suggested above, it is possible to arbitrage the relative value between the swap and TIPS market. The spread is not fully arbitraged away because profits are less certain if the position is unwound prior to maturity. Therefore, "mark-to-market" risk limits, combined with balance-sheet limits, restrict the extent to which market participants can implement the arbitrage.

The most common way in which this strategy is implemented is via a TIPS Asset Swap, i.e., buying a TIPS issue and entering a swap to pay out the inflation-linked coupons and redemption amount, in exchange for non-inflation linked payout profiles. Typically, these are either Fixed TIPS Asset Swaps, where the investor earns a higher fixed yield relative to comparable "nominal" treasuries (see graph 3); or Floating TIPS Asset Swaps which pay a spread relative to three-month Libor, and the investor earns the difference between the floating rate and the repo cost of funding the purchase of the TIPS issue.

GRAPH 2



GRAPH 3



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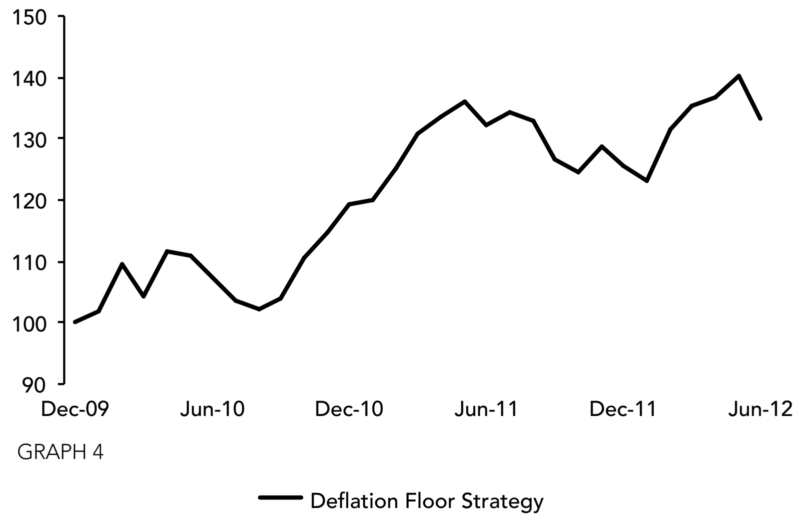
6.3 DEFLATION FLOOR RISK PREMIUM

Inflation options are often bought as tail-risk hedges. Equity macro hedge funds have purchased deflation floors as an alternative to equity put options due to their relatively low premiums and expected good performance in deflationary markets.

As a result of these purchases, inflation options appear rich under various metrics, such as comparing realized with implied volatility; or implied deflation probabilities relative to the distribution of inflation rates predicted by surveys of professional forecasters. Accordingly, inflation caps embed an inflation risk premium, and inflation floors embed a deflation risk premium. Generally, floors are considered richer than caps, especially since the FOMC is averse to deflation and would likely take extreme steps to prevent deflationary scenarios.

GRAPH 4



One way in which the deflation risk premium embedded in inflation floors can be earned is by systematically selling year-over-year deflation floors.

Graph 4 (below) reflects the performance of selling 5y year-over-year 0 percent-strike deflation floors and rolling the position monthly. Note: past performance is no guarantee of future results.

The above strategy can be implemented by either selling the deflation floors directly or by entering into a total return swap on an index which is designed to replicate this strategy.

7. CONCLUSION

Large and liquid markets for U.S. CPI-linked exposure can be accessed to obtain inflation protection. Regularly traded CPI-linked instruments include TIPS, Inflation Swaps and Inflation Options. Products based on these instruments include Total Return Swaps and Asset Swaps, as well as ETFs, ETNs and Inflation-linked Notes.

Not only can these instruments be used to obtain inflation protection, but they can also be used to earn risk premiums that arise due to structural imbalances in the CPI market. Examples include cheapness in the front-end of the inflation curve, relative value between inflation swaps and TIPS, and richness of deflation floors.

Much of the content for this paper was sourced from the Deutsche Bank presentation and webinar titled, "Inflation Risk Factor and Risk Premia Strategies." For access to the presentation or webinar, or for further information, please contact the author: email: allan.levin@db.com.

Risk Magazine ranked Deutsche Bank No.1 for US Inflation Swaps and No.1 for US Inflation Options for 2012.

Greenwich Associates ranked Deutsche Bank No.1 for Global Fixed Income for 2012, 2011 & 2010 and No.1 Overall US Fixed Income for 2012, 2011 & 2010.

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END NOTES

[1] Deutsche Bank, "Hedging Inflation," Fixed Income Special Report, July 29, 2009.

[2] Tickers include: TIP, WIP, STPZ, IPE, LTPZ, STIP, TIPZ, TDTF, TDTT, ITIP, GTIP, VTIP, ILB, UINF, SINF, RINF, and FINF.

[3] Tickers: INFL and DEFL.

[4] Fleckstein, Longstaf, Lustig, "Why does the Treasury issue TIPS? The TIPS-Treasury Bond Puzzle," NBER Working Paper, September 2010

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LAYERING YOUR OWN VIEWS INTO A STOCHASTIC SIMULATION—WITHOUT A RECALIBRATION

By Tony Dardis, Loic Grandchamp and David Antonio

Editor's Note: This article summarizes the authors' presentation at the May 2013 Life and Annuity Symposium.

There is a useful technique that is gaining some popularity amongst practitioners that enables "own views" to be superimposed on a stochastically generated scenario set without having to recalibrate the underlying model(s). The technique won't be appropriate for all applications, but it may be effective for some, such as superimposing alternative estimates or creating "what-if" scenarios and stress tests.

The approach involves the use of a statistic known as entropy. The value of entropy is maximized when equal weighting is given to each individual scenario in a given scenario set. Thus the objective of the "own views" exercise is to re-weight scenarios so that they hit your specific target, while maximizing the value of entropy.

Mathematically, for a set of N scenarios each with weight W_i the entropy S of a scenario set is defined as follows:

$$S = -\sum_{i=1}^{N} w_i \ln w_i$$

There is potentially an infinite combination of weightings that would hit any specified target that we may have. The objective of the entropy technique is to find the optimal weights W_i so that we maximize S while hitting our specific target.

A simple example will help reinforce the concept.

Let us assume that we have calibrated a one-year interest rate model so that on average it hits 3 percent. We then use the model to generate five scenarios as follows:

SCENARIO RATE	ONE-YEAR PROJECTED
1	2.0%
2	2.0%
3	3.0%
4	4.0%
5	4.0%
Average	3.0%

The maximum entropy for these scenarios will always be achieved by equally weighting these scenarios, and is calculated as follows:

FIVE SCENARIOS EQUALLY WEIGHTED

Scenario	One-year projected rate	Weight	Entropy
1	2.0%	0.20	0.3219
2	2.0%	0.20	0.3219
3	3.0%	0.20	0.3219
4	4.0%	0.20	0.3219
5	4.0%	0.20	0.3219
Avg/total	3.0%	1.00	1.6094

Let us now say that we have an alternative view as to what will transpire in the future and instead would like to re-weight the scenarios so that on average we hit a lower rate—say, 2.5 percent. Our first inclination might be to give weighting only to the lowest rates from our original set of five scenarios, with entropy calculated as follows:

TARGET = 2.5 PERCENT;
CHOOSE ONLY THE VERY LOW WEIGHTS

Scenario	One-year projected rate	Weight	Entropy
1	2.0%	0.25	0.3466
2	2.0%	0.25	0.3466
3	3.0%	0.50	0.3466
4	4.0%	0.00	0.0000
5	4.0%	0.00	0.0000
Avg/total	2.5%	1.00	1.0397

However, weighting only the very low rates is missing a lot of very important information about the overall distribution of the rates and this becomes apparent from the entropy value—a much lower number than what we started with for the original scenario set that as equally weighted. So let's now consider what happens if we give some weighting to all the scenarios, while still hitting the "own views" target of 2.5 percent:

TARGET = 2.5 PERCENT;
SOME WEIGHTING TO ALL SCENARIOS

Scenario	One-year projected rate	Weight	Entropy
1	2.0%	0.35	0.3674
2	2.0%	0.35	0.3674
3	3.0%	0.10	0.2303
4	4.0%	0.10	0.2303
5	4.0%	0.10	0.2303
Avg/total	2.5%	1.00	1.4256

This simple example demonstrates the key features of using the maximum entropy technique:

- Maximum entropy is where equal weighting is given to all scenarios.
- Minimum entropy is where one scenario is given a weight of one, and all other scenarios a weight of zero.
- The optimization algorithm favors solutions where the weight is as evenly distributed across scenarios as possible. This ensures we don't overweight any particular batch of scenarios and thus ensures we retain as much as possible the features of the original probability distribution.

Let's now consider a practical, and very topical, example. The American Academy of Actuaries provides a basic interest rate and equity scenario generation capability on its website *actuary.org*. This is made available to practicing actuaries as a means of meeting the reserving and capital requirements of variable annuity business under Actuarial Guideline 43 and C3 Phase II which require a stochastic valuation. Related to this, the Academy has also posted a set of 10,000 interest rate and equity scenarios, which practitioners can download without needing to use the generator itself. Some actuaries have gone on to use the generator and/or scenarios for applications beyond meeting the statutory requirement, and while there are very significant limitations to this (the Academy generator was originally developed as a "starter pack" for purposes of enabling companies with relatively simple asset-liability profiles to meet

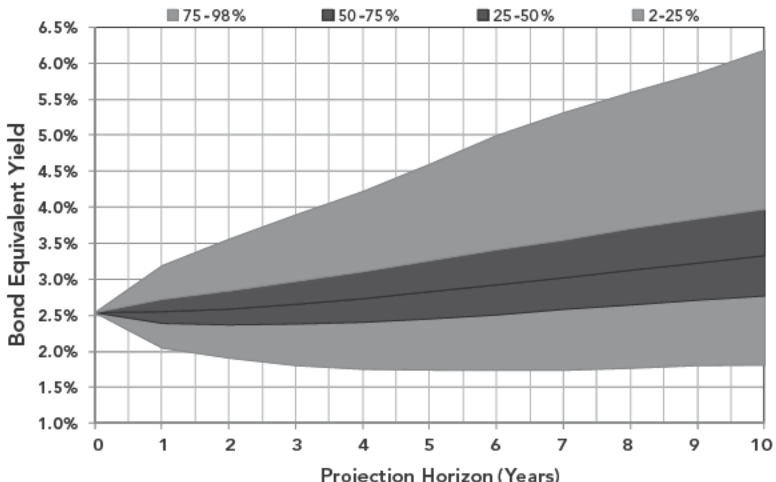

CHART 1:
10-YEAR PROJECTION OF THE 20-YEAR TREASURY BOND EQUIVALENT YIELD USING THE AMERICAN ACADEMY OF ACTUARIES' SCENARIO GENERATOR INITIALIZED TO THE TREASURY YIELD CURVE AT 12/31/2012, 10,000 TRIALS.

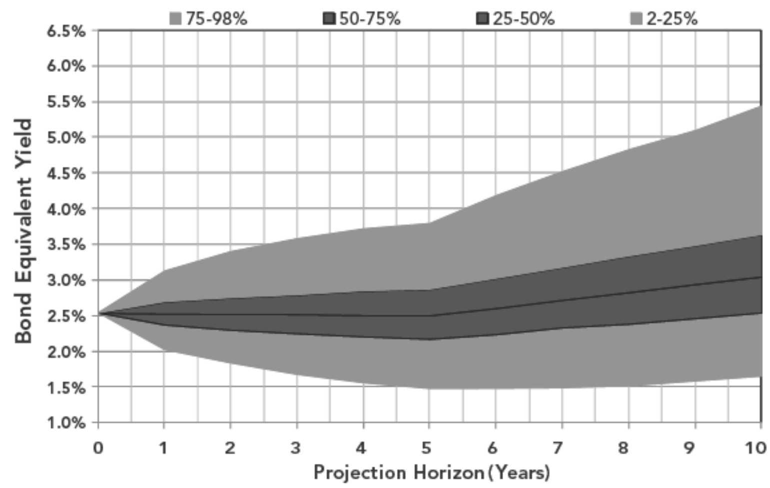

CHART 2:
10-YEAR PROJECTION OF THE 20-YEAR TREASURY BOND EQUIVALENT YIELD USING THE AMERICAN ACADEMY OF ACTUARIES' SCENARIO GENERATOR INITIALIZED TO THE TREASURY YIELD CURVE AT 12/31/2012, 10,000 TRIALS, BUT REWEIGHTED USING MAXIMUM ENTROPY TO KEEP RATES CONSTANT ON AVERAGE FOR THE NEXT FIVE YEARS.

CONTINUED ON PAGE 26

the newly emerging statutory stochastic requirement), there may be some applications, such as testing of a new product campaign where interest rate risk is the only market driver of the business, where this is appropriate. This in turn leads to the natural question: if I download the Academy's 10,000 scenarios, is there a way I can rebalance these so that they produce an average interest rate path that is different to what is assumed in the Academy calibration?

The entropy technique can be used extremely effectively in such an example, and has great flexibility. In Chart 1 (pg. 25, top) we show the distribution of the 20-year Treasury bond equivalent yield projected over a 10-year horizon under 10,000 Academy scenarios. These scenarios were generated from the Academy generator, initialized to the Treasury yield curve at 12/31/2012.

As will be immediately apparent, the average path of the 20-year rate under the Academy calibration immediately sets off on an upward trend which persists throughout the projection period. What if our "own view," however, was that given the current economic climate, and the very high expectation that the government will persist in a monetary policy that continues to keep interest rates at extremely low levels, a much more realistic expectation is that on average rates will remain at or very close to today's levels for at least the next five years?

Our first port of call might be to download the Academy's interest rate generator from actuary.org and recalibrate that so that it hits our target. The path of the 20-year rate in the generator can be controlled using two parameters: long-term mean reversion level and a speed of mean reversion. There isn't, therefore, sufficient flexibility to target a more general path for interest rates and the user is also constrained because he can only directly control the evolution of the 20-year rate. It may also not be obvious to the user what parameters should be input to achieve the desired path. Perhaps entropy can help?

In Chart 2 (pg. 25, bottom) we show a revised distribution of the 20-year Treasury bond equivalent yield projected over a 10-year horizon that starts with the 12/31/2012 10,000 Academy scenarios, but reweights using maximum entropy in order to maintain the current level of the 20-year rate over the next five years.

There are some very interesting features of the new distribution that should be highlighted:

- The entropy technique has worked beautifully in hitting our "own views" path on average.
- The overall characteristics of the probability distribution in terms of dispersion and tails are similar under the original and the reweighted scenario sets.
- It will be noted that the lower band of the reweighted set at the second percentile level is well below the original set. This doesn't mean to say that we are weighting scenarios that were outside the original scenario set, but rather that we are now giving a lot more weighting to scenarios that were originally outside the second percentile level. This highlights another important characteristic of the entropy method—it will not work where our target falls outside any of the scenarios that were originally generated. In this example, this would mean that we may not be able to use entropy to target ultra-low 10-year rates, e.g., at or close to 1 percent for a prolonged period.
- The entropy of our reweighted scenario set corresponds to a set of 8,353 equally weighted scenarios. This number, called the effective number of scenarios is a useful statistic of the entropy method which allows practitioners to gauge how far apart the original and reweighted sets are. However, the technique should not be viewed as a scenario reduction technique.

Note that while this article has focused on looking at interest rate scenarios and how we can reweight according to "own views" around a target path, other variables and target metrics could be used equally effectively. For example, we might be more interested in setting a target for returns rather than yield, and perhaps it is equity rather than interest rate scenarios that are of most interest. Indeed, theoretically it would even be possible to take a pre-generated real-world scenario set and optimize the weights to pass martingale tests and hence create a set of risk neutral scenarios. While we wouldn't necessarily recommend such an approach—this would be a considerably more complex exercise than having a relatively simple target such as a different long-term interest rate path, and also creating a set of risk neutral scenarios directly from a genuine calibration is a much easier task than creating a set of real-world scenarios directly from a full calibration process—it still highlights how flexible the entropy approach can be.

Another point to make about the attraction of the entropy technique is that it avoids the scope for negative scenario weights, which other methodologies might not handle so satisfactorily. That is to say, because of how the entropy value is calculated, looking at the log of weights, it won't accept negative weights. So trying a "trick" such as weighting a scenario you really like by, say, 1.2, and one you don't like by -0.2, just wouldn't work.

While the entropy technique holds much promise for certain uses, it comes with a number of words of warning:

- It is not a model recalibration, and is not a substitute for recalibration.
- Although the integrity of each individual scenario from the original set is maintained, validation work is needed when more than one risk variable is being modeled, e.g., equities as well as interest rates. What does a new target for one variable mean in terms of targets for other variables, and what is the impact on correlations?
- As we get further out into the tails, we need to be increasingly careful. Generally, the entropy technique will be very effective for metrics that are close to the central estimate, e.g., CTE(70), but less effective for metrics that are focused on the tails, e.g., CTE(95).
- A large original scenario set will be needed for effective re-sampling.
- Not all "own views" targets can be achieved, i.e., they may fall outside the range of the original set.

With that said, if these limitations are recognized and understood, there may be a number of applications for which the technique can be effective:

- Weighing up the relative merits of strategic decisions where risk and return and focused on the inner tails of the distributions, e.g., decision to launch product X versus product Y, or testing of a variety of different asset mixes.
- Testing the relative impact of different "own views.
- Stress testing and sensitivity analysis.
- Ensuring ownership and consistency of economic assumptions used throughout various business units.

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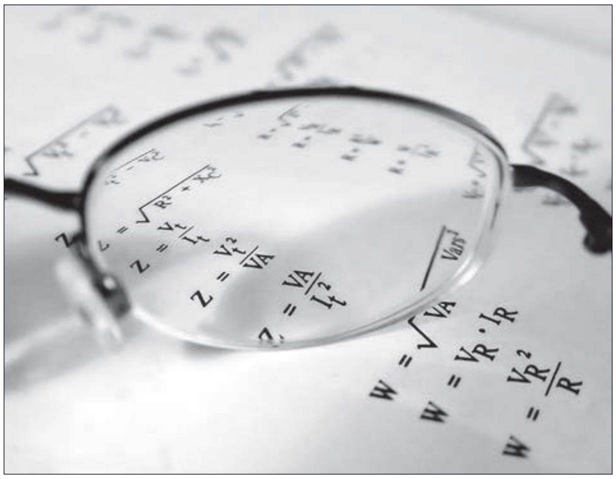

AUMANN-SHAPLEY VALUES: A TECHNIQUE FOR BETTER ATTRIBUTIONS

By Joshua Boehme

"HAPPY IS THE ONE WHO KNOWS THE CAUSES OF THINGS." –VIRGIL[1]

Actuaries encounter attribution problems on a regular basis. Indeed, any situation where results change, whether due to changes in assumptions, market conditions, or even just the passage of time, often leads to the natural follow-up question: why did the results change? To answer this question, actuaries use various techniques, each with its own strengths and weaknesses. However, a technique not widely known among actuaries—the Aumann-Shapley value from game theory—in many cases can produce attributions that better satisfy our intuitive expectations of a good attribution.

Some authors have already applied the Aumann-Shapley approach to various financial problems in non-actuarial settings. Denault (1999), for example, used it to allocate margin requirements among portfolios of options. However, the Aumann-Shapley approach has yet to receive widespread exposure within the actuarial community.

FORMALIZING THE PROBLEM

Suppose we have a multivariate function f and two vectors of parameters u and v representing the previous and latest parameters respectively. In the most general form of the problem we place almost no restrictions on the function f or its parameters. In some applications f may contain discontinuities or may lack a closed-form solution. The function f could take non-continuous parameters as well. For example, a binary variable could indicate whether to use one method or another, such as curtate versus continuous mortality.

In an attribution problem we seek to explain the difference f(v)-f(u) by assigning to the i^{th} variable an amount a_i representing its contribution to the difference, where i ranges from 1 to the number of inputs to the function f.

Ideally the total of the a_i values would equal f(v)-f(u), but in practice that does not always happen. Any remaining difference, which sometimes goes by the term untraced or unexplained, represents some portion of the change that the attribution method in question could not allocate to one of the input variables.

Although likely few of us have ever tried to formally list the properties we want a "good" attribution to satisfy, intuitively we have an idea of how a reasonable method should behave. For example, if the i^{th} variable did not change (so $u_i = v_i$), we would expect its contribution to the difference to equal zero. Similarly, if the i^{th} variable has no impact on the value of f (meaning that f(u) = f(v) whenever $u_i \neq v_i$ and $u_j = v_j$ for all $j \neq i$), then we again expect its contribution to equal zero.

ATTRIBUTION TECHNIQUES

Aumann-Shapley

The technique that this article focuses on, the Aumann-Shapley value, requires f and its parameters to satisfy a few conditions. Specifically, f must have partial derivatives[2] in all of its parameters along the vector between u and v. We do not need a closed-form version of f, but we must have a way to compute its partial derivatives at any given point on the path.

For attribution problems that satisfy these requirements, the Aumann-Shapley approach produces some valuable results. It always produces an attribution with no unexplained amount.[3] As we will see in later examples, its results also show a certain desirable stability with respect to how we set up the problem.

For each variable we calculate the attributed amount a_i as:

$$a_i = (v_i - u_i)\int_0^1 \frac{\partial f}{\partial x_i}\big((1-z)u + zv\big)dz$$

The resulting integral does not always have a closed-form solution, but we can evaluate it numerically.

Alternatively, we can view the Aumann-Shapley approach as a three-step process:

1. Find the partial derivative of f with respect to its i^{th} parameter, denoted $\frac{\partial f}{\partial x_i}$ here.

2. Integrate that partial derivative along the line segment between u and v. Here, the dummy variable z represents the linear interpolation between u (at z = 0) and v (at z = 1).

3. Multiply the result by the change in that parameter $(v_i - u_i)$

Step-Through

Many actuaries faced with an attribution problem will solve it by stepping through the parameters one at a time, a technique with several important advantages. As long as we can evaluate the function f at each combination along the step-through, f can have any number of discontinuities and it can lack a closed-form solution. We can use a step-through even when f has non-continuous inputs. Furthermore, a step-through also produces an attribution with no unexplained amount.

Step-throughs have one well-known disadvantage, though: the results depend on the arbitrary order we use to step through the parameters. In the examples in this article we will use a modified technique to overcome this issue: we will perform the step-through for every possible order, then average the attributions together.[4] This removes the dependency on an arbitrarily chosen order. As we will see with a later example, though, even this modified step-through method still has a significant weakness. Despite that, in situations where we cannot satisfy the requirements of the Aumann-Shapley approach, a step-through remains a viable alternative.

Partial Derivatives

Actuaries already frequently use derivatives or approximations to the derivative to perform attributions. Some partial derivatives come up so often that they have specific names, such as the "Greeks" (delta, gamma, vega, rho, theta, etc.) or duration. In some cases, we use formulas to directly calculate the partial derivatives; other times, we shock one of the parameters a small amount to numerically estimate the derivative.

Partial derivatives have as one major advantage their frequent ease of computation and interpretation. For example, from the duration of a bond, a relatively intuitive concept, we can quickly estimate the change in its value due to a change in interest rates.

The main difference between a partial derivative attribution and the Aumann-Shapley approach comes from *where* we evaluate the partial derivative. In the Aumann-Shapley approach, we evaluate it along the entire path between u and v. For the partial derivative, we evaluate it at a single point, usually the beginning point.[5] This difference, though, leads to the major drawback of a partial derivative approach: the attribution generally has a nonzero unexplained amount. This may suffice for a quick estimate. Other times, though, we may want a complete attribution of the difference.

EXAMPLES

Example 1: *Zero-Coupon Bond*

Consider a zero-coupon 10-year bond with a maturity value of $1 million. For a given yield to maturity y, the following formula gives its value at time t:

$$f(y,t) = e^{-y(10-t)} \cdot 10^6$$

Suppose we have the following parameter sets:

	y	t	f(y, t)
u	5.00%	1	637,628
v	8.00%	2	527,292
		Difference	-110,336

We then get the following attributions from the three methods discussed above:

	y	t	Total Attributed	Unexplained
Aumann-Shapley	-147,619	37,284	-110,336	0
Step-through	-146,952	36,616	-110,336	0
Partial derivative	-172,160	31,881	-140,278	29,942

CONTINUED ON **PAGE 30**

As expected, both the step-through and the Aumann-Shapley approach fully attribute the change. They also produce comparable results. The partial derivative results, though easily calculable,[6] do not accurately capture the total change in value.

Example 2: *Zero-Coupon Bond, Revisited*
Many times, we can formulate a problem in multiple ways. Suppose that instead of expressing the yield for the zero-coupon bond in terms of a single variable, as in the previous example, we express it in terms of two components: a prevailing interest rate r, and a credit spread c. As a formula:

$$f(r,c,t) = e^{-(r+c)(10-t)} \cdot 10^6$$

Given the modified parameter sets:

	r	c	t	f(r, c, t)
u	4.00%	1.00%	1	637,628
v	5.00%	3.00%	2	527,292
		Difference		-110,336

The initial and final values have not changed from the previous example—we have merely separated the yields to maturity into two components. Now that we have shifted perspective, what happens to the attributions?

	r	c	t	Total Attributed	Unexplained
Aumann-Shapley	-49,206	-98,413	37,284	-110,336	0
Step-through	-49,162	-98,027	36,853	-110,336	0
Partial derivative	-57,387	-114,773	31,881	-140,278	29,942

Note in particular the step-through values. By formulating the problem in a slightly different way, the value attributed to the time variable t has changed! In contrast, the amounts attributed to t by both the Aumann-Shapley approach and the partial derivative have not changed from before.

Now, some readers may object that this change in the attributed value for t comes from the fact that we stepped through every possible order of variables. In practice, most actuaries would use only a single order, and most likely we would step through the two interest rate components consecutively. Under those circumstances, the value attributed to t would come out equal under both formulations. For example, if we use the order y, t in the first example and c, r, t in the second, then in both cases we attribute $40,540 to t. However, any particular order comes from an arbitrary choice on our part. Nothing intrinsic in the order itself would lead us to conclude that we should choose one order over another. The equally natural order t, y (or t, c, r) would lead us to attribute $32,692 to t.

In the end, when using a step-through, we must either accept that we have chosen an arbitrary order, or we must accept that the results could vary if we re-formulate the problem in an equivalent way. Either way, step-throughs produce non-unique results.

Example 3: *Binary Call*
Suppose we own a binary call on a particular security, with a strike K at 100. For illustrative purposes we will hold the interest rate r constant at 2 percent and the volatility σ constant at 25 percent, and we will assume the underlying security pays no dividends. For the current asset spot price S and time to option maturity t, the value of our call equals:

$$f(S,t) = e^{-rt}\Phi(d_2)$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}u^2}\, du$$

and

$$d_2 = \frac{\ln\frac{S}{K} + (r - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

At the boundary where t = 0, its value equals

$$f(S,0) = \begin{cases} 1 \text{ if } S > K \\ 0 \text{ if } S \le K \end{cases}$$

Given the parameters:

	S	t	f(S, t)
u	90	1	0.314
v	110	0	1.000
Difference			0.686

The three methods disagree significantly about the nature and magnitude of the time component's contribution:

	S	t	Total Attributed	Unexplained
Aumann-Shapley	0.435	0.251	0.686	0
Step-through	0.653	0.033	0.686	0
Partial derivative	0.312	-0.060	0.252	0.434

The price crosses the strike during the attribution period, but the call does not reach full value immediately at that time. Its value still includes a discount for the probability of a subsequent decrease. The passage of time eventually drives that probability to zero, bringing the option to its full value. At the beginning of the attribution period, though, the opposite pattern holds: the possibility that volatility will cause the asset price to exceed the strike recedes as we approach maturity, meaning that the passage of time reduces the option's value. The partial derivative results reflect the latter effect.

Thus, f's sensitivity to time varies considerably over the attribution region. By only considering the edges of the region, the step-through does not accurately capture the full sensitivity and ends up attributing little of the change to the time component.

AN INTUITIVE ARGUMENT FOR WHY AUMANN-SHAPLEY PRODUCES A COMPLETE ATTRIBUTION

Although the examples have shown that the Aumann-Shapley approach produces a complete attribution in those cases, they do not explain why it works in general. A quick (though non-rigorous) argument will help illustrate the logic underpinning the technique.

We have defined f as a multivariate function of the x_i variables. However, over the attribution region we can also view f as a function of z alone, denoted $f^{(z)}$ for clarity. The linear interpolation between u and v connects the two: $f^{(z)}(z) = f((1\text{-}z)u+zv)$. Thus $f^{(z)}(0)$, for example, would mean to calculate the values of each x_i for z=0, then evaluate f at those values—in other words, $f^{(z)}(0)$ equals our initial value f(u). We can now write out an equation for the difference we seek to attribute:

$$f(v) - f(u) = f^{(z)}(1) - f^{(z)}(0)$$

Assuming that we have a sufficiently smooth function, we can express the difference as an integral:

$$= \int_0^1 \frac{df^{(z)}}{dz}(z)dz$$

We still need to relate this back to the original x_i variables, and we do this by applying the chain rule. Note that we have switched back to the original multivariate function f:

$$= \int_0^1 \left[\sum_i \frac{\partial f}{\partial x_i} \cdot \frac{dx_i}{dz}((1-z)u+zv) \right] dz$$

Since z represents our linear interpolation variable, the derivative of x_i equals the difference between the final and initial values. With that substitution, and separating out the individual terms inside the integral, we finally obtain:

$$f(v) - f(u) = \sum_i (v_i - u_i) \int_0^1 \frac{\partial f}{\partial x_i}((1-z)u+zv)dz$$

where the term on the right hand side corresponding to each x_i gives that variable's attribution.

From this argument, we can also see why jump discontinuities cause this approach to fail, since they cause changes in the value of f that do not get captured by the derivative. However, as long as we have a sufficiently smooth function (and a sufficiently accurate calculator for the integral) we will always get a complete attribution from the Aumann-Shapley approach.

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ACCURACY OF NUMERIC INTEGRATION

Since the Aumann-Shapley results come from a numeric integration, a natural question arises: how much confidence should we have in their accuracy? In practice we can obtain very rapid convergence using Simpson's rule.[7] Example 1's results earlier used 1,000 points to ensure a highly accurate result, but even if we evaluate the integrals at just *three* points and use Simpson's rule we get almost identical results:

						Contributions to integrals	
z	y	t	f	$\frac{\partial f}{\partial y} = -(10-t)f$	$\frac{\partial f}{\partial t} = yf$	$\int \frac{\partial f}{\partial y}$	$\int \frac{\partial f}{\partial t}$
0.00	5.00%	1.00	637,628	-5,738,653	31,881	-956,442	5,314
0.50	6.50%	1.50	575,509	-4,891,829	37,408	-3,261,219	24,939
1.00	8.00%	2.00	527,292	-4,218,339	42,183	-703,057	7,031
					Total	-4,920,718	37,283
					(v_i-u_i)	3.00%	1.00
					(v_i-u_i) Total	-147,622	37,283
					Previous Results for Comparison	-147,619	37,284

Thus, even a quick calculation can produce reasonable results. Furthermore, by using a spreadsheet or programming language we can easily evaluate more points to improve the accuracy.

ADDITIONAL CONSIDERATIONS

In this article we have looked at some artificially simple examples. In real life, though, we may face more complex attribution problems. For example, we may wish to reflect the fact that interest rates vary based on the time to maturity. Even if the yield curve does not shift, the yield on an asset could change simply due to the passage of time. In that case, it may make more sense to assign the change in value solely to time, not due to a nonexistent movement in the yield curve.

To further complicate matters, when valuing options we could view volatility as depending on both the time to expiration and the moneyness of the option, leading to a two-dimensional volatility surface.

The Aumann-Shapley approach applies in those more complex cases as well. We do need a way to interpolate smoothly between the observed points. However, to perform any attribution we generally need some form of interpolation anyway since our valuation points will rarely correspond exactly to the market-observable points. Some common methods include linear interpolation and cubic splines, both of which provide differentiable interpolations.

Although this article has focused on asset valuation examples, we can use the Aumann-Shapley approach for other applications. However, we do need to verify that our function f meets the requirements. Liabilities in particular, though, often contain features that can potentially create discontinuities, including:

1. Charges, guaranteed rates of return, or other features based on market values rounded to the nearest percent, nearest 25 basis points, or some other multiple.
2. Any feature involving rebalancing something back to a target, but only if outside some tolerance band. For example, due to an investment strategy we have adopted or due to a contractual agreement we have entered into, we might rebalance a particular asset allocation back to 80 percent equity at end of each quarter if the current asset allocation deviates from that by more than 5 percent.
3. Franchise deductibles.
4. Digital payoffs.
5. Features activated or deactivated at certain thresholds, including knock-in and knock-out features.

Keep in mind, though features such as these do not *automatically* imply a problem. In example 3 our function contained a discontinuity at the point (S, t) = (100, 0), but the path between u and v did not pass through that point. In general, the specific circumstances of the problem will dictate whether we can use the Aumann-Shapley approach or not.

Despite its requirements, the Aumann-Shapley approach offers a powerful way to solve attribution problems. By adding it to their toolkit, actuaries can produce more reliable and more complete attributions, and thus move that much closer to truly understanding the causes of things.

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END NOTES

1 Virgil. Wikiquote. Retrieved Jan. 27, 2013, from http://en.wikiquote.org/wiki/Virgil

2 The existence of partial derivatives also implies the continuity of f. The partial derivatives can contain non-removable discontinuities, but f itself cannot contain any along the path.

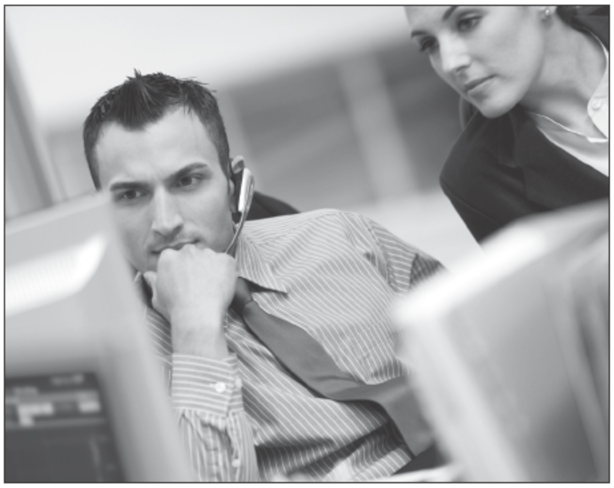
3 Except to the extent that whatever tool we use to calculate the results has finite precision, though this issue applies to any attribution technique.

4 This corresponds to the Shapley value from game theory.

5 Evaluating it at the end point instead or at both points does not eliminate its drawbacks. For the examples in this article we will use the partial derivative at the beginning point.

6 The attribution for t, for example, equals 5 percent of the initial value.

7 $\int_{x-\Delta}^{x+\Delta} f(z)dz \approx 2\Delta \frac{f(x+\Delta)+4f(x)+f(x-\Delta)}{6}$



ENGAGING THE FEAR GAUGE

EMPIRICAL OBSERVATIONS ON COUNTERINTUITIVE VIX BEHAVIOR

By Bogdan Ianev and Edward K. Tom

During the recent years, the CBOE Volatility Index® VIX (commonly referred to as "the fear gauge"), a measure of short-dated equity markets' volatility implied from listed S&P 500® Index options, has received a significant amount of attention, and its daily value has been used almost synonymously as an indicator of whether markets are in state of fear or calm. Due to its growing acceptance and usage by investors and journalists as a "fear gauge," a mention of price action in the VIX index is often linked to major news reports (Booton and Egan, Kiernan). Further, price action in the VIX index that is in the same direction as price moves in the S&P 500® index, defying the "fear gauge" definition, often is highlighted as a curious occurrence (Gammeltoft and Kisling). The purpose of this article is to examine the VIX definition in closer detail and attempt to provide rational explanations for some of the seemingly aberrant daily behavior of the VIX index at various recent occasions

1. OVERVIEW OF THE VIX INDEX

The VIX index as of any point in time provides a forward-looking estimate for 30-day volatility of the daily returns[1] of the S&P 500® Index (SPX). This estimate is obtained based on the live bid and ask prices of SPX options listed on CBOE and it relies on the assumption that option prices at any point in time, via their implied volatilities, embed information about the expected realization of volatility of the SPX until the option's maturity. However, as options with the same maturity, but with different strikes, tend to imply different volatility numbers (i.e., volatility skew exists), an estimate for the expected volatility independent of a specific option strike needs to take all this information into account to produce a single volatility number. Such an estimate has been studied by many and is well-described by Demeterfi, Derman, Kamal and Zou, and is calculated in practice by taking all available sufficiently-liquid options (calls for strikes above the forward, puts for strikes below) of the two maturities nearest 30 days, computing a weighted average of their prices to obtain a single estimate for the expectation of the SPX variance for each of these two maturities, and interpolating/extrapolating these two numbers to obtain the expected 30-day variance. Finally, the square root of this number is reported as the VIX.

Calculation of the VIX index can be very sensitive to the data, and thus, CBOE provides a detailed step-by-step description of the process in a VIX White Paper. An abbreviated summary has been included in the Appendix to assist the reader in understanding some of the case studies provided later.

2. A DEEPER DIVE INTO VARIANCE

This section covers a brief theoretical discussion to provide intuition behind the process involved in the calculation of the VIX. It tends to be more technical than the rest of this article and is provided as a reference for the technically-inclined reader; a less curious reader may safely omit this section without lack of continuity in the exposition.

Much has been written by researchers about variance swaps and volatility swaps. As the calculation behind the VIX index is based on variance swap pricing theory, this discussion focuses on the former. Demeterfi, Derman, Kamal and Zou (DDKZ) provide a way to decompose a variance swap into a static portfolio of options, a forward, and a continuously rebalanced delta hedge. Heuristically, the argument is made to extract an expression for the average variance, $AverageVar(0,T) = \frac{1}{T}\int_0^T \sigma_t^2 \, dt,$ using a simple application of Ito's Lemma on the Geometric Brown Motion process for a stock with price S_t, which does not pay dividends, has an instantaneous volatility σ_t, and exists in an economy with an instantaneous risk-free rate r_t. The below provides a brief heuristic theoretical outline based largely on the approach taken by DDKZ.

If the stock process under the risk-neutral measure Q is represented by the stochastic differential equation

$$\frac{dS_t}{S_t} = r_t dt + \sigma_t dW_t^Q \quad (1)$$

then by Ito's Lemma, $Ln[S_t]$ follows the following process:

$$d\, Ln[S_t] = \left[r_t - \frac{\sigma_t^2}{2}\right] dt + \sigma_t dW_t^Q \ (2)$$

Thus, subtracting (2) from (1) yields the expression:

$$\frac{dS_t}{S_t} - d\, Ln[S_t] = \frac{\sigma_t^2}{2} dt \ (3)$$

When integrated, (3) gives an expression for the total variance, where the last equality follows after plugging (1) under the integral:

$$AverageVar(0,T) \times T = \int_0^T \sigma_t^2\, dt = -2\, Ln\left[\frac{S_T}{S_0}\right] + 2\int_0^T \frac{dS_t}{S_t} = -2\, Ln\left[\frac{S_T}{S_0}\right] + 2\int_0^T r_t dt + \int_0^T \sigma_t dW_t^Q \ (4)$$

Finally, taking the expectation under Q, and assuming for simplicity that $r_t = r$ is constant, gives us the expression, in which the stochastic term disappears since W_t^Q is a martingale under Q.

$$E^Q[AverageVar(0,T)] = \frac{2}{T} E^Q\left[-Ln\left[\frac{S_T}{S_0}\right]\right] + 2\,r \ (5)$$

In other words, the expected variance is linked directly to the expected value of the $-Ln\left[\frac{S_T}{S_0}\right]$ term, the short log-contract, which is nothing more than the negative of the continuously compounded return of the stock until time T. Since this contract itself does not trade in the market, however, DDKZ show that the short log-contract is identical in payoff (and thus in price and expectation) to a carefully-chosen portfolio composed of calls, puts, and a forward on the stock, all with maturity T. Thus, the term $E^Q\left[-Ln\left[\frac{S_T}{S_0}\right]\right]$ in (5) can be replaced simply by the undiscounted price of the replicating portfolio—a price easily observable as it is based on liquid instruments traded in the market. While theoretically, this replicating portfolio is based on a continuous set of strikes, in reality the strike space is often discretized to use available listed options, leading to the following expression, in which K_i is the strike of option, i, ΔK_i is the spacing between adjacent strikes, and $Q(K_i)$ is the price of the option with strike K_i, (assuming calls for strikes above some cut-off level and puts for the rest)

$$E^Q[AverageVar(0,T)] = \frac{2}{T}\left[\sum_i \frac{\Delta K_i}{K_i^2} e^{r\,T} Q(K_i) - E^Q\left[\frac{S_T}{S_0} - 1\right]\right] + 2\,r = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{r\,T} Q(K_i) \quad (6)$$

The expression in the expectation is simply the undiscounted value of the forward contract, which can be further simplified to $E^Q\left[\frac{S_T}{S_0} - 1\right] = e^{r\,T} - 1 \approx rT$. Making this substitution leads to the right-hand expression in (6).

Expression (6) looks very similar to the VIX formula, with a small exception that the latter includes an adjustment factor to account for the discreteness of the strike chosen to be the forward level.

This can be further simplified by setting $w_i = \frac{2}{T} \times \frac{\Delta K_i}{K_i^2} e^{r\,T}$ to conclude that the expected variance is simply a weighted average of the existing options prices.

$$E^Q[AverageVar(0,T)] = \sum_i w_i\, Q(K_i) \qquad (7)$$

3. PRACTICAL OBSERVATIONS

This section provides examples of several instances, where the VIX calculation formula yields results that may be viewed as perplexing, and which, however, have a rational explanation in the context of the formula. An example is also shown of an occasion when pure market dynamics can drive the VIX to react contrary to common expectations.

As the theoretical underpinnings behind the VIX formula are based on the assumption that one can trade and observe prices for options of infinitely many strikes, several simplifications are made in the VIX formula to accommodate the limitations of options trading in practice

The first simplification is based on the need to use discrete option strikes since for short-dated maturities, listed SPX options exist only in strikes in multiples of five. For a given maturity, starting from the strike closest to the forward level, the VIX selects all calls with strikes higher than the

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> "AMONGST EQUITY INVESTORS, THE VIX, A MEASURE OF MARKET VARIANCE, IS WIDELY INTERPRETED AS THE MARKET FEAR GAUGE."

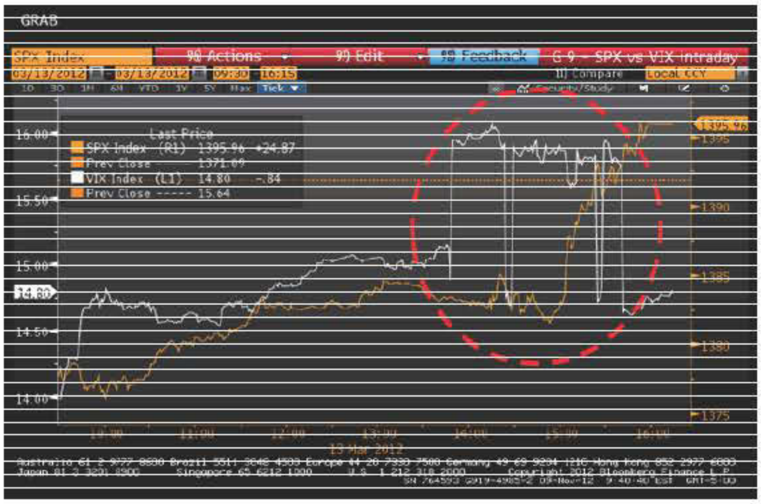

Figure 1. Source: Bloomberg

	Ticker	Strike	Bid	Ask	Last	IVA	IVB
263)	SPX 4/21/12 P985	985	.05	1.00	.40y	48.87	35.21
264)	SPX 4/21/12 P990	990	.05	1.00	1.00y	48.24	34.73
265)	SPX 4/21/12 P995	995	.05	1.00		47.61	34.26
266)	SPX 4/21/12 P1000	1000	.20	.30	.30	40.33	38.57
267)	SPX 4/21/12 P1005	1005	.05	1.00		46.36	33.31
268)	SPX 4/21/12 P1010	1010	.05	1.00	2.15y	45.73	32.84
269)	SPX 4/21/12 P1015	1015	.05	1.00		45.12	32.37
270)	SPX 4/21/12 P1020	1020	.05	1.00	2.00y	44.50	31.91
271)	SPX 4/21/12 P1025	1025	.20	.40	.40	38.90	35.95
272)	SPX 4/21/12 P1030	1030	.05	1.00	1.36y	43.27	30.98
273)	SPX 4/21/12 P1035	1035	.05	1.00	3.70y	42.66	30.52
274)	SPX 4/21/12 P1040	1040		1.00	.75y	42.05	
275)	SPX 4/21/12 P1045	1045		1.00	3.60y	41.45	
276)	SPX 4/21/12 P1050	1050	.40	.50	.50	37.16	36.14
277)	SPX 4/21/12 P1055	1055	.15	1.00	2.00y	40.24	31.87
278)	SPX 4/21/12 P1060	1060	.10	1.10	1.75y	40.20	30.12
279)	SPX 4/21/12 P1065	1065	.10	1.10		39.60	29.64
280)	SPX 4/21/12 P1070	1070	.25	1.15	1.80y	39.26	32.13
281)	SPX 4/21/12 P1075	1075	.30	.75	.85	36.32	32.30
282)	SPX 4/21/12 P1080	1080	.30	1.20	.75	38.30	31.78

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Figure 2. Source: Bloomberg

forward, for which a non-zero bid exists, until two consecutive zero-bids are discovered, which sets the upper bound for the highest call strike; similarly, puts with non-zero bids are selected with decreasing strikes starting from the forward until two consecutive zero-bids are found. Thus the sudden appearance and disappearance of zero-bids across various strikes at different times during the day may have noticeable effects on the value of the VIX (the VIX calculation is refreshed continuously throughout the day). Case Study 1 in this section provides an example of this effect.

The second necessary approximation is driven by the fact that the VIX represents 30-day volatility at all times without regard to whether options with 30-day maturity are actually traded in the market at that moment. To allow for this, a linear interpolation in the total variance space[2] is used to approximate the 30-day maturity based on two traded maturities that are close to 30-days. Most frequently, one such maturity will be shorter than 30 days, while the other will be longer than 30 days, and thus interpolation is used. Occasionally, however, due to the VIX requirement that the near maturity has at least one week to expiration, it may occur that both maturities used have more than 30 days remaining, thus requiring *extrapolation*. This may, at times, lead to counterintuitive results, as illustrated in Case Study 2.

The third example in this section is slightly different in nature as it addresses the use of the VIX as a "fear gauge." The belief that the VIX should move in the opposite direction of the SPX is rooted in the existence of implied volatility skew in SPX options, i.e., options with lower strikes trade with higher implied volatilities than options with higher strikes. However, whether the SPX at-the-money volatility moves along this skew (implied volatility levels are linked to actual SPX levels) so that an SPX decline is accompanied by an at-the-money implied volatility increase, depends on whether SPX volatilities follow a "sticky-strike" dynamics. In contrast, when the behavior of the volatility surface resembles a "sticky-delta" dynamics, i.e., as the SPX declines, at-the-money volatility does not change; rather the volatility skew simply moves along with the SPX, and a rise in volatilities may not be observed Case Study 3 shows a practical example of a time when SPX volatilities not only did not behave in a "sticky-strike" manner, thus not exhibiting the "fear-gauge" effect, but even declined together with the SPX.

***Case Study 1:** Impact of the Vanishing Option*

March 13, 2012 illustrates a scenario in which VIX suddenly begins to whipsaw intraday for no apparent reason. As shown in the Bloomberg snapshot in Figure 1 (pg. 36, top), for most of the day, the VIX seems to slide along the skew, rising when the market falls and falling when the market rises. However, in the last two hours of trading, as the SPX begins to set up for a strong 1 percent rally into the close, the VIX suddenly becomes a series of four discontinuous 1 VIX point jumps – a tremendous relative move given that average volatility levels at the time were 15. (See fig. 1, pg. 36, top)

As mentioned above, this stochastic effect is due not to sudden changes in sentiment, but it is rather a technical of the VIX calculation itself. Specifically, while in theory the hedge for a variance swap calls for the purchase of the entire range of option strikes from zero to infinity, in practice, reasonable provisions must be made to account for the lack of liquidity of deep, out of the money options. Investment houses have varied proprietary approaches to define and model these "wings"; the VIX methodology employs a consecutive zero bid test. In short, the premiums for all options with strikes starting from at the money are included in the calculation until two consecutive strikes with zero bids are encountered. At that point, deeper out of the money option strikes are excluded from the calculation.

In the Bloomberg screenshot in Figure 2 (pg. 36, bottom), circled is the breakpoint at the 1040 and 1045 strikes.

Note in Figure 3 (pg. 37, top), which depicts the front

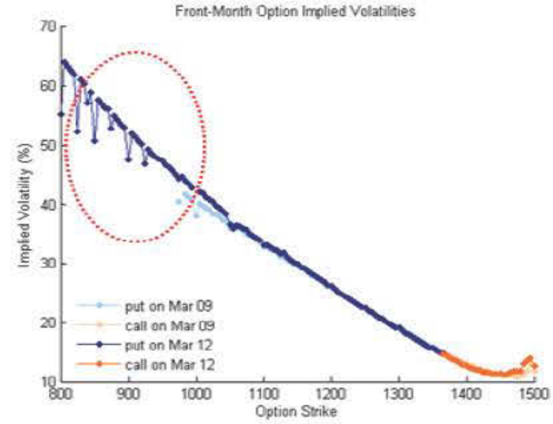

Figure 3. Source: Credit Suisse

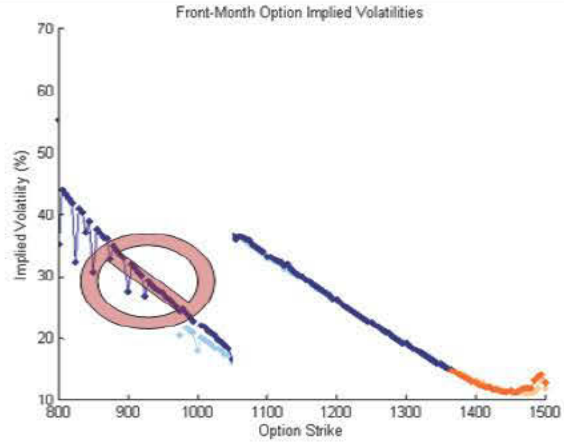

Figure 4. Source: Credit Suisse

Figure 5. Source: Credit Suisse

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> **OPTION LIQUIDITY** IN A PARTICULAR SINGLE STRIKE CAN HAVE **A MEANINGFUL IMPACT ON THE VIX VALUE**

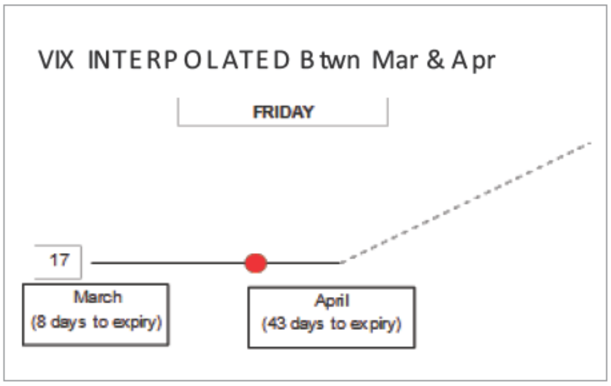
month SPX option skew, that not only do bids exist above the 1045, but there is also a solid chain of bids from the 1040 strike down to the 800 strike as well. The marginal contribution to the VIX of these sub-1040 strike options is one full VIX point.

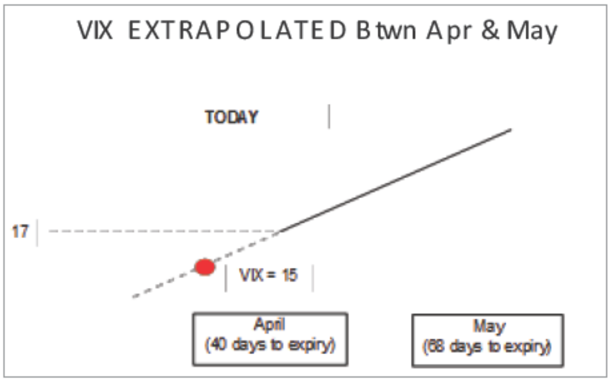
Thus, as illustrated in Figure 4 (pg. 37, middle), when bids for both the 1040 or 1045 strikes are missing, the marginal contributions by all these options and those to their left are sliced off, causing the VIX to plummet.

Likewise, a bid for any one or both of the strikes will cause the tail to "reattach" causing the VIX to shock upward.

Case Study 2: *The Contract Roll Effect*

Recall that the VIX is meant to represent the expected volatility for a 30-day option. However, since an SPX option with exactly 30 days to expiry is only available once per month, 30-day volatility is usually calculated as the (weighted) average of two contracts, a front month contract with less than 30-days to expiry, and a back month contract with more than 30-days to expiry. One day per month, the VIX initiates a contract roll in which the front month option is removed from the calculation and the VIX is then calculated using the second and third back month contracts. What is rather surprising to many, however, is that if the SPX volatility term structure is upward sloping, the contract roll whereupon the VIX calculation shifts from using the first and second month contract to the higher vol second and third month contracts, usually causes the VIX to decline! This is illustrated by using the March 2012 contract roll.





On that day, the SPX at-the-money term structure was upward-sloping as shown below with May expiry implied vols trading at a premium to April expiry implied vols and April expiry implied vols in turn trading at a premium to March expiry implied vols. (Figure 5, pg. 37, bottom)

During the trading session before the contract roll, March 9, 2012, the VIX was calculated using the March contract with eight days left to expiry and the April contract with 43 days to expiry. In this case, the VIX level was interpolated using the two contracts and as a result, the closing VIX level was between the March and April variance levels as shown in Figure 6 (left, top).

On the morning of the VIX contract roll date, March 12, 2012, however, the VIX was calculated using the April contract, now with 40 days left to expiry, and the new May contract with 68 days to expiry. Obviously, since the 30-day volatility number needed for the VIX is earlier than even the front month contract, interpolation is not possible. The VIX methodology prescribes that extrapolation be used along the gradient formed by the April and May contracts. As shown in Figure 7 (left, bottom), the newly extrapolated VIX level is significantly lower than both the April and May contracts.

The drop to VIX based on this method is dependent upon the slope of the term structure. In this particular case, the extrapolation process resulted in a one point fall in the VIX in a 15 volatility environment.

Case Study 3: *VIX Action*

Generally, daily VIX moves can be attributed to five principal components: 1) the expected volatility move along the volatility skew, assuming the skew remains fixed to specific SPX strikes (sticky-strike dynamics); 2) parallel shifts up or down of the skew; 3) daily contract reweighting the back month contract (interpolation effect); 4) incremental demand for puts (steepening of downside skew); and 5) incremental demand for calls (steepening of upside skew). Historically, roughly 80 percent of the moves have been dictated by the first two components and, often, these two effects reinforce each other. Thus, when the SPX declines, at-the-money volatility slides up the skew (thus driving the VIX higher), and furthermore, the surface parallel-shifts upward. The end result is that a decline in the SPX drives the VIX upward. Thus, it is often considered bizarre behavior when these two effects move in opposite directions. The following example illustrates this effect on the day after the U.S. elections.

As shown in Figure 8 (right, top), on Nov. 8, 2012, following the U.S. presidential elections, the SPX fell 1.25 percent but yet the VIX also fell one point off a base of 19. On this day, the bulk of the VIX down-move derived from the SPX front month contracts—the November expiry options.

Figure 9 (right, bottom) shows a comparison between the SPX implied volatility skew for Nov. 7 (light orange, light blue) and Nov. 8 (dark orange, dark blue). Despite the sizeable SPX decline, the November-expiry SPX implied volatility skew from the 1,300 to 1,450 strikes parallel-shifted downward rather than upward as one would expect.

Figure 8. Source: Bloomberg

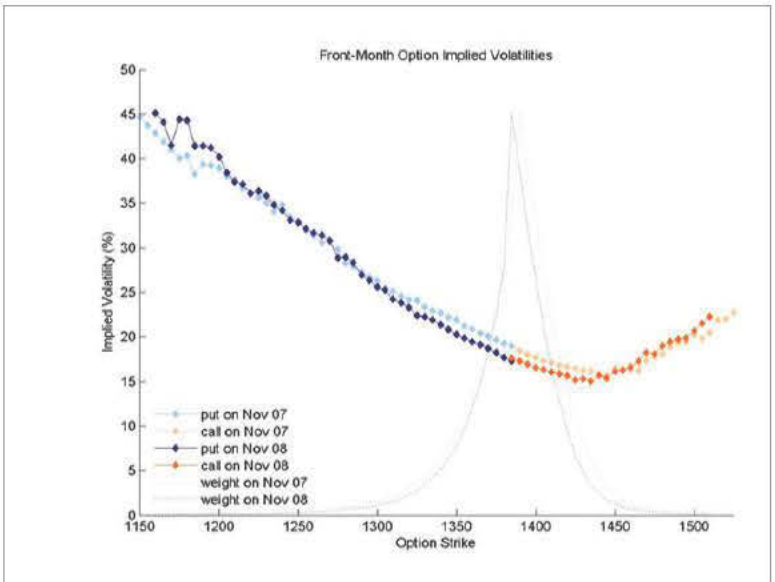


Figure 9. Source: Credit Suisse

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The key lies in the fact that the front month option for the VIX calculation was the November expiry contract. Heading into the first week of November, the U.S. presidential election was the major risk embedded into November expiry implied volatilities. With its solidification, traders with November expiry options, which now only had only six trading days remaining until expiration, now faced the following situation: with no foreseeable catalysts left, realized volatility was likely to decline. Thus, option traders remarked their SPX November expiry volatilities lower despite the market decline thereby causing the observed drop in the VIX.

4. CONCLUSION

Amongst equity investors, the VIX, a measure of market variance, is widely interpreted as the market fear gauge. Although VIX typically has a coincidentally inverse relationship to SPX spot, which reinforces the fear gauge moniker, there are times when technical features of the variance calculation causes the VIX to move counter to intuition. As the VIX continues to receive press coverage and as an increasing number of investors begin to express market sentiment via exchange traded VIX products, an understanding of the factors used in the formula can be helpful for being able to distinguish the real price action in the index from the technical artifact resulting from the calculation methodology. Thankfully, CBOE has provided excellent transparency in their calculation, which allows anyone to replicate the numbers and perform the necessary analysis. This article illustrated examples of situations where VIX moves on three separate occasions defied intuition, but after a closer examination a rational explanation was found. In one case, it was made apparent that option liquidity in a particular single strike can have meaningful impact on the VIX value, especially if it cuts off a meaningful tail. Another example showed that, at times, the extrapolation across the two SPX option contract maturities used in the calculation may lead to unexpected results. Lastly, it was shown that occasionally the VIX may move in tandem with the SPX, thus defying its commonly-used "fear gauge" description.

APPENDIX

The following is an abbreviated and simplified description of the step-by-step process used by CBOE for the calculation of the VIX®. For the full process with examples, the reader should refer to the original VIX White Paper.

1. Select the options to be used in the VIX calculation (only options with non-zero bid prices are used)
   T_1 is the time until the first S&P Option contract month expiry, at least a week away (near expiry).
   T_2 is the time until the first S&P Option contract month expiry after T_1 (next expiry).
   F_1 is the forward SPX level applicable to the near expiry; F_2 is the forward SPX level applicable to the next expiry. Both are calculated by finding the strike of the options with the respective maturity, where the call price is closest to the put price.
   $K_{0,1}$ is the first listed strike price below the forward index level, F_1 and $K_{0,2}$ is the first listed strike price below the forward index level, F_2.
   a. Select near-expiry out-of-the-money put options with strike prices $<K_{0,1}$.
      i. Start with the put strike immediately lower than $K_{0,1}$ and move to successively lower strike prices.
      ii. Stop once two puts with consecutive strike prices are found to have zero bid prices.
   b. Select near-expiry out-of-the-money call options with strike prices $>K_{0,1}$.
      i. Start with the call strike immediately higher than $K_{0,1}$ and move to successively higher strike prices.
      ii. Stop once two puts with consecutive strike prices are found to have zero bid prices.
   c. Select both the near-expiry put and call with strike price $K_{0,1}$.
   d. Repeat (a)-(c) for the next expiry.

2. Calculate the variances σ_1^2 and σ_2^2 for both near-expiry and next-expiry options as the weighted average of all existing options of the same maturity

$$\sigma_t^2 = \frac{2}{T_t}\sum_i \frac{\Delta K_i}{K_i^2} e^{R_t T_t}\, Q_t(K_i) - \frac{1}{T_t}\left[\frac{F_t}{K_{0,t}} - 1\right]^2, t = 1,2$$

Where:

t represents the near-term and the next-term expiry.

i spans the calls and puts of the respective expiry, t, as selected in Step 1.

$Q_t(K_i)$ is the mid-point of the bid-ask spread of the option with strike K_i of the respective expiry, t (as selected in Step 1).

R_t is the bond-equivalent yield of the U.S. T-bill maturing closest to the expiry date, t.

3. Calculate the annualized 30-day weighted average of $\boldsymbol{\sigma_1^2}$ **and** $\boldsymbol{\sigma_2^2}$. Then, take the square root of that value and multiply by 100 to get VIX.

$$\boldsymbol{VIX = 100 \times \sqrt{\left[T_1\sigma_1^2 \times \frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} + T_2\sigma_2^2 \times \frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right] \times \frac{N_{365}}{N_{30}}}}$$

Where:

N_{T_1} and N_{T_2} are the number of minutes until the near-expiry and the next-expiry, respectively.

N_{30} and N_{365} are the number of minutes in 30 and 365 calendar days, respectively.

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END NOTES

[1] More precisely, it is the square root of the estimate for expected annualized variance of daily log-returns of the S&P 500 under the risk-neutral measure. The VIX is an expected variance estimator, reported as its square root, rather than an expected volatility estimator. This is worth noting, as by its definition, $VIX = \sqrt{E[\sigma^2]}$, which is not necessarily the same as $E[\sigma]$, where σ stands for volatility and is an unknown quantity.

[2] The interpolation is done on the quantities $\sigma_{T_1}^2 \times T_1$ and $\sigma_{T_2}^2 \times T_2$.

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The landing page EBSCO designed for the SOA Investment Section includes several topical shortcuts. These allow members to browse by topic (e.g., Derivatives, Quantitative Methods, Risk Management) instead of searching. With your use and feedback, we can update this taxonomy to be even more relevant to your research preferences.

BENEFIT TO ALL OF US: SPECIFIC TOPIC SUGGESTIONS FOR FUTURE WEBCASTS AND CONFERENCE SESSIONS

In addition to serving as a clear benefit to your investment-related lifetime learning needs, your using EBSCO BSC+ will help the Section Council serve you better. Analytics on the topics and articles our members consume will clarify which topics are most relevant to them. We anticipate this enhanced clarity will assist us in planning articles, conference sessions, and webcasts that are more relevant to your needs than we do presently.

TELL US ABOUT TOPICS YOU WOULD LIKE TO SEE IN THE FORM OF WEBCASTS AND CONFERENCE TOPICS

As you peruse BSC+ and discover new articles, authors, and topics, you will run across some real gems. Tell us what they are. Mention them on LinkedIn. Help shape the agenda for our upcoming conferences, webcasts, and research by letting us know what you think your peers will find enlightening or provocative.

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