SOCIETY OF ACTUARIES

Article from

Risks & Rewards

August 2017

Issue 70

Expected Returns on Indexed Credits

By Gary Hatfield

The opinions and views expressed in this article are my own and do not reflect those of my employer. I speak for myself as an actuary and a mathematician. Any errors are mine alone.

In 2014, and into early 2015, there was controversy regarding the illustration practices for Indexed Universal Life (IUL). The controversy had been brewing for a while, and in large part because there were significant differences in practices among insurers regarding the appropriate illustrated rate for an IUL policy. Most used a method called "look-back," where the crediting mechanism's returns over a prior period of history were used to establish the Assumed Indexed Credit (AIC) for the purposes of illustration. However, companies were free to "cherry-pick" details, such as the number of years, the days of the year (say January to January, or June to June), etc. These concerns were legitimate, and ultimately resulted in AG 49, which limited the maximum allowed illustrated rate for indexed credits.

However, there were other concerns that were expressed at the time that seemed to come from a poor understanding of derivatives-based strategies. In particular, claims were made that indexed crediting couldn't work as illustrated because it implied a long-term expected return of 20 percent to 50 percent (or more) on hedge budgets. The underlying premise of the argument appeared to be that a strategy that sustains returns of 20 percent to 50 percent is absurd. As actuaries, we can substitute demonstration for impressions, and I intend to do exactly that.

The organization of this paper is as follows: In the first section, I will review indexed crediting and suggest three possible criteria for the AIC. In the next session, I will discuss alternative approaches to setting assumptions for future returns, both for the underlying index and the AIC. Finally, I will assert that the corresponding returns of 20 percent to 50 percent (or more) for "hedge budgets" are not unreasonable. Along the way, I will point out limitations of my analysis.

Note that AG 49 specifically relates to the maximum allowed illustration rate for IUL products, which is not the same as the "best-estimate" return on an indexed credit. While related, I emphasize this distinction, as the former entails a host of issues that are out of scope. As such, the reader should assume that I am neither endorsing, nor criticizing, AG 49.

INDEXED CREDITS AND AIC

For the purposes of this article, I will assume an indexed credit takes its most common form: 100 percent participation in the price return of an equity index (most often the S&P500), subject to a 0 percent floor, and a cap C declared by the insurer beforehand. Written out mathematically, if S(t) is the equity price index, at time t, then the indexed credit (IC) for the period t to t+1 is given by

$$IC(t,t+1) = Min\left(Max\left(\frac{S(t+1)}{S(t)} - 1,0\right), C\right).$$

Equivalently, the indexed credit is the payoff of a call spread consisting of two call options. A long call option struck at 100 percent, and a short call option struck at 1+C. That is, if I denote the payoff of call option expiring at time t+1 with underlying S, and struck at K as

$$Call_{payoff}\,(S(t+1),K) = \;Max(S(t+1)-K,0)$$

then

$$IC(t,t+1) = \frac{Call_{payoff}\,(S(t+1),S(t)) - Call_{payoff}\,(S(t+1),(1+C)*S(t))}{S(t)}\;.$$

What happens mechanically to the policy Account Value (AV) during the period t to t+1 is something like:

$$AV(t+1) = (AV(t) + PremiumApplied - Charges) * (1 + IC(t,t+1)).$$

The actual indexed credits will vary from year to year, and are not known upfront. So the illustration uses an AIC *vis*:

$$AV(t+1) = (AV(t) + PremiumApplied - Charges) * (1 + AIC)$$

An important question here is, "If the investor has a subjective view on the returns of the index, how should this translate into a choice for the AIC?"

To assist in answering this question, I propose three potential criteria to evaluate the choice for AIC:

In the absence of premiums and charges,

C1: The AV compounded at this rate represents the expected future account value.

C2: The rate represents the expected compounded annual return.

C3: The AV compounded at this rate represents the median future account value.

Or, if you prefer mathematical notation, for a large number of annual periods N:

C1: $E[AV(N)] \cong AV(0) * (1 + AIC)^N$

C2: $E\left[\left(\frac{AV(N)}{AV(0)}\right)^{\frac{1}{N}} - 1\right] \cong AIC$

C3: $\Pr[AV(N) \geq AV(0) * (1 + AIC)^N \cong \frac{1}{2}$.

LIMITATION: In reality, the pattern of charges versus premiums matter. Typically, premiums are greater than charges early, and then opposite later. But there is not, to my knowledge, a good way to account for that in choosing the AIC because the impact depends on the pattern of higher returns versus lower returns, and other factors in a complex manner. All else being equal, this complication will not generally bias things.

I will use this equivalence to answer the following question: If I know the cap C, and have some belief on the distribution of expected returns for the index, what is the expected indexed credit $E_P[IC(t, t+1)]$? Here, I use P to represent the subjective or real-world probability measure. There is no "known" or "correct" P for the index S; by necessity, it is a belief. However, most investors harbor a belief, such as, "The expected return for the S&P500 is 8 percent with annual volatility of 15 percent." So how should I interpret this in a way that allows me to make a statement about the expected return on an index credit?

INVESTOR VIEW AND AIC

The above question and criteria assumes that the investor holds a "view" on market returns and then asks how to translate that into an appropriate AIC. In this section, I will propose two methods for developing an AIC given such a view—one of which satisfies C1 (but not C2 and C3) and the other C2 and C3 (but not C1). For the investor, however, developing a view on the return distributions that can be so translated is a highly non-trivial task. So I will also propose three approaches an investor may take (and certainly this list is not exhaustive) and show how to estimate AIC in each case.

The first method for setting AIC is simply to set the AIC to the one-period expected value of the indexed credit.

$$AIC_1 = E[IC(t, t+1)]$$

Under the *assumption* that annual price returns are Independent and Identically Distributed (IID), Condition 1 is met (exactly, not approximately).

The IID assumption is not true of course. Consider auto-correlation. Historically, the SP500 shows strong positive auto-correlation over short horizons (daily, monthly and quarterly). However, this dissipates quite a bit over longer horizons. In fact, for a one-year horizon, auto-correlation appears to have mostly dissipated, and is close to zero.[1] Similarly, return distributions are not time invariant. But since we are concerned here with long-term returns, it seems unnecessarily complicated to impose a "term structure" of return distribution into an illustration. Hence, for our purposes, I am content with the conclusion $E[AV(N)] \cong AV(0) * (1 + AIC_1)^N$ and C1 is met.

I will now show why AIC_1 most likely will not satisfy C2 or C3, and then propose a choice that will satisfy C2 or C3.

First, some notation. For each interest credit $IC(t-1, t)$, which we are treating as a random variable, define the following new random variables:

$$R_t = 1 + IC(t-1, t)$$

and

$$y_t = \ln(R_t),$$

so that

$$\frac{AV(N)}{AV(0)} = R_1 \cdot R_2 \cdot R_3 \cdots R_N = Exp(y_1 + y_2 + y_3 + \cdots + y_N)$$

The RVs y_t are certainly not normally distributed (not even "kind of"), but we will assume[2] that they are IID with some mean μ_y and some standard deviation σ_y. If N is large enough[3], the Central Limit Theorem (CLT) tells us that the sum $Y_N = y_1 + y_2 + y_3 + \cdots + y_N$ is approximately normal with mean $N * \mu_y$ and standard deviation $\sqrt{N} * \sigma_y$. From this observation, we see that

$$(1 + AIC_1)^N \cong E\left[\frac{AV(N)}{AV(0)}\right] \approx Exp\left(N\left(\mu_y + \frac{1}{2}\sigma_y^2\right)\right).$$

The *median* value of $\frac{AV(N)}{AV(0)}$ is approximately *Exp* $(N\mu_y)$ and

$$Pr\left[\frac{AV(N)}{AV(0)} \geq (1 + AIC_1)^N\right] \cong Pr\left[Y_N \geq N\left(\mu_y + \frac{1}{2}\sigma_y^2\right)\right] = Pr\left[\frac{Y_N - N\mu_y}{\sqrt{N}\sigma_y} \geq \frac{\sqrt{N}}{2}\sigma\right] < \frac{1}{2}$$

via the CLT. For example, for $\sigma_y = .15$ and $N = 30$, the probability of achieving the mean return is about 34 percent. In short, C3 is not satisfied by AIC_1

Moving on to C2, and again using the CLT,

$$E\left[\left(\frac{AV(N)}{AV(0)}\right)^{\frac{1}{N}} - 1\right] = E\left[Exp\left(\frac{y_1 + y_2 + y_3 + \cdots + y_N}{N}\right) - 1\right] \approx Exp\left(\mu_y + \frac{1}{2}\frac{\sigma_y^2}{N}\right) - 1.$$

$$\neq Exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right) - 1 = AIC_1$$

The second term in the exponent will be small for reasonably large N (in fact, typically $\frac{1}{2}\sigma_y^2$ will already be somewhat small since it is related to the volatility of the indexed credit). Hence, I propose

$AIC_2 = Exp(\mu_y) - 1$, where

$$\mu_y = E[y_t] = E[ln(1 + IC(t-1, t))].$$

It is immediately evident that C2 is satisfied by this choice. But also from the observation about the median above, C3 is also satisfied.

It should be clear that satisfying all three conditions is not possible. But for purposes of illustrations, my opinion is that C2 and C3 are more appropriate for what the investor is trying to understand.

I now suggest three possible approaches for establishing a view on the distribution of returns:

A1: Parametric-subjective

This method would assume a parametric form for return distributions, (e.g., lognormal), and then allow the investor to choose the parameters to best represent their view on expected future returns. Since the price return is what matters for index credits (not total return), care must be taken that the investor understands what it is that they are providing. So for example, if the investor says, "I think the stock market will grow 10 percent per year on average" and the dividend yield is 2 percent, I would assume that they are talking about total return and the corresponding parametric distribution should have a mean return of about 8 percent (not 10 percent). But it would be better to understand exactly what they believe.

Once we have specified the parametric distribution and a parameter choice, we can calculate AIC_1 or AIC_2 via integration. In particular, if we denote the probability density function (pdf) for $X = \frac{S(t+1)}{s(t)}$ as $f(x)$, then

$$AIC_1 = E[IC(t, t+1)] = \int_0^\infty Min(Max(x-1, 0), C)\, f(x)\, dx.$$

And $$AIC_2 = Exp(\mu_y) - 1$$

where $$\mu_y = E[\ln(1 + IC(t, t+1)] = \int_0^\infty ln\big(Min(Max(x, 1), 1 + C)\big)\, f(x)\, dx.$$

In some cases, for example when the distribution is lognormal, AIC_1 can be calculated explicitly in closed form via a Black-Scholes-like formula. However, in general this will not be true. I have personally implemented the integration for AIC_2 for the lognormal case in VBA using Simpson's rule, and it works quite well. So the need to numerically integrate shouldn't be an obstacle.

A2: Parametric-historical

This is very similar to the above, except that parameters are estimated from historical price data as opposed to subjective inputs. For example, let $S_0, S_1, S_2, \ldots S_M$ be the index prices for a period of M years. If we were to assume a lognormal return distribution, we would then assume that $u_t = S(t)/S(t-1)$ are distributed normally and estimate the parameters to be the sample mean and sample standard deviation of $u_1, u_2, \ldots, u_M$

The rest is the same as above.

A3: Non-parametric historical

One might believe that annual stock returns are not well-represented by any parametric distribution (or, at least, not any common one). This is particularly true because returns show skew and kurtosis. One approach for dealing with this is to take the empirical distribution of returns and simply fit a curve to it (via some kind of spline). For the full distribution, this is problematic in terms of fitting the tails because we just don't have that many data points.

However, the middle of the distribution is another matter. And when it comes to index credits, that is where we are playing. Because instead of empirically estimating the full distribution, we only need to estimate the probabilities that the returns fall outside the floor/cap range (both probabilities) and the distribution when returns are in between. The data point requirements for doing this are much less daunting.

The suggestion here is the following:

For a given set of historical prices $S_0, S_1, S_2, \ldots S_M$, calculate the corresponding index credits $IC_1, IC_2, \ldots IC_M$. We would then estimate

$$AIC_1 = \frac{1}{M}\sum_{k=1}^{M} IC_k$$

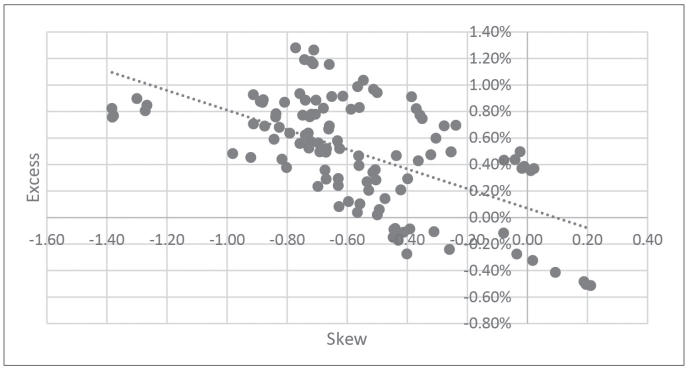
and

$$AIC_2 = Exp\left(\frac{1}{M}\sum_{k=1}^{M} \ln(1 + IC_k)\right) - 1$$

The astute reader will no doubt realize that under this method, AIC_2 is the "look-back" method.

Does it matter whether we use A2 or A3? The answer is yes. I compared these approaches across many 30-year periods of the S&P500 in an effort to estimate AIC_2. I find that AIC_2 is typically higher via A3 than A2. Reviewing U.S. price index values from Robert Shiller *(http://www.econ.yale.edu/~shiller/data.htm)*, and calculating AIC_2 via 30-year lookback and 30-year historical parametric method beginning in 1901 (considering only each January), I found A3 indicates a higher estimate for AIC_2 every year since 1919. Why is this? I believe it is because the indexed credit mechanism cuts off negative returns. Without skew, this would not be very impactful; but with skew, it is significant[4], as illustrated in the following graph.

Excess of A3 over A2 versus skew



LIMITATIONS: All of the approaches above have weaknesses. All assume going forward that the return distributions are time invariant. The historical approaches assume time-invariance looking backward. The more mathematically tractable parametric approaches are typically too simple (lacking skew, for example). The truth is, there is no flawless way to choose the assumption. What you can do, however, is to make reasonable assumptions and then logically follow through on what those assumptions imply for indexed crediting while acknowledging their limitations.

RETURNS ON THE HEDGE BUDGET

Today, an indexed credit with a 12.5 percent cap can reasonably be purchased on a hedge budge of 4.5 percent. Let us apply our three approaches to calculate AIC_2 and see what kind of return on the hedge budget this corresponds with.[5]

A1: An equity investor is likely bullish on stocks (you would think, anyway). So they might say, "I expect the long-term total return on equities to be 8 percent to 12 percent." Prompted for a volatility, they might say something like "15 percent to 20 percent." These do not seem unreasonable. I will interpret their belief to mean that *median* total return is 8 percent to 12 percent (let's call it *tr*). That is, the median future value of the Total Return Index (*TRI*)[6] is $I(N)_{median} = (S(t)) * (1 + tr)^N$. Let's assume a lognormal model for the price index with drift *g*, volatility σ, and continuous dividend yield δ. Then the median long-term total return should satisfy $\frac{TRI(N)_{median}}{TRI(0)} = Exp(N * (g + \delta))$. This implies that $g = \ln(1 + tr) - \delta$

The range for tr and a choice of δ=.02 gives *g*=.057 or *g*=.093. Assuming further that σ=.15 or σ=.20, I calculate the following:

g	σ	AIC_2
5.7%	15%	5.5%
5.7%	20%	5.4%
9.3%	15%	6.6%
9.3%	20%	6.2%

This implies a "return" on the hedge budget between $\frac{5.4}{4.5} - 1 =$ 20 percent and $\frac{6.6}{4.5} - 1 =$ 47 percent.

A2: If we lookback at 30-years returns from various starting points going back to 1950 (similar to AG 49), but apply the parametric approach using a lognormal assumption, AIC_2 will range between 5.6 percent and 7.0 percent. This again implies "returns" on hedge budgets of 27 percent to 56 percent.

The crediting mechanism underlying IUL is a strategy that splits investments between a "safe" bond investment and a "risky" derivative strategy.

A3: Using the same returns as for A2 but using the empirical approach, AIC_2 will range between 6.4 percent and 7.8 percent. This implies "returns" on hedge budgets of 42 percent to 73 percent.

In short, there is nothing unreasonable about option strategies having high average returns. I want to add a couple of comments on this, however. These "returns" are not compounded returns. Indeed, a "fund" that fully invested in such strategy would go bankrupt with probability 1. The crediting mechanism underlying IUL is a strategy that splits investments between a "safe" bond investment and a "risky" derivative strategy. The derivative strategy may have what seems like a very high "average" return, but it loses 100 percent with great frequency. The key to resolve this paradox is to apply a long time horizon—keep playing! IUL accomplishes this with the large "safe" investment and a series of relatively small option trades over a long time horizon.[7] ■

Gary Hatfield, FSA, CERA, MAAA, is actuary—Enterprise Risk and Investments at Securian Financial Group, St. Paul, Minn. He can be contacted at *Gary.Hatfield@securian.com*

ENDNOTE

1 Looking at historical S&P500 data, the auto-correlation varies between -8 percent and 8 percent depending on which point in the calendar is used to compare point-to-point returns.

2 See the autocorrelation discussion above.

3 In the case of indexed credits, N = 15 seems to be large enough.

4 The estimated lognormal process and the emprical process have the same mean and volatility, but the empirical process has a lower expected return conditional on the return being negative. Therefore, cutting off negative returns has more value relative to the lognormal process with the same mean and volatility.

5 AIC_1 will be higher

6 The Total Return is simply the return on the price index assuming dividends are reinvested into the index

7 See for example the not-well-enough-known Kelly Criterion for allocating between "safe" and "risky" investments *https://en.wikipedia.org/wiki/Kelly_criterion*