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# NEW APPROACH TO CONTINGENCIES

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- o This session will be based on the syllabus synopsis of the same title released to the general membership.
  - -- Time until death random variables
  - -- Present value random variables
  - -- Prospective loss random variables
- o This session will be of particular interest to members who received their Associateship before 1984.

MR. WARREN R. LUCKNER: One of the old cliches about making presentations is that a good presentation contains three parts. In the first part, you tell the audience what you're going to say; in the second part you say it; and in the third part you tell them what you said. I'm going to try to make this a good presentation, so this is the part where I tell you what I am going to say.

First, I will identify two objectives for this session. Then I will try to meet those objectives by providing some discussion of the fundamentals of the "new" approach to contingencies. I will discuss the use of random variables and the resulting impact on the determination of net premiums and reserves. Then I will discuss the challenge and opportunity presented by the new approach and try to illustrate with examples. Finally, I'll briefly summarize what I consider the major points of what I told you, and thus hope to satisfy the third part of a good presentation.

Now comes the critical part -- this is where I try to say what it is I told you I was going to say!

The first objective of the session is to try to make some sense of the Syllabus Synopsis "'New' Approach to Contingencies -- A Summary." You should have received a copy in the mail, and copies have been distributed on the chairs at this session. I'll try to accomplish this objective by taking you through the fundamentals of the stochastic approach, in much the same way the synopsis does. The second objective is to try to provide further insight through additional examples.

Has anyone had the opportunity to read the Syllabus Synopsis? Did it make sense? Anyone find the mistakes in the synopsis? I know of at least one that could be classified as substantive (although it does not affect the conceptual developments), and a few typos. I'll let you know what they are at the end of the session -- and maybe we'll discuss others.

Okay, let's get going on the fundamentals.

A key element of the "new" approach to the study of contingencies is the use of random variables -- variables that take on each possible value (or each interval of possible values in the case of continuous random variables) with a certain probability. Use of random variables allows for use of standard statistical concepts, such as expected value (the "average" value) and variance (the "average" value of the squares of the deviations of the possible values of the random variable from the expected value of the random variable), and terminology, such as distribution function and probability (density) function. The distribution function gives probabilities of obtaining values of the random variable less than or equal to any given value; the probability (density) function gives probabilities of obtaining a given value (or a given interval of values in the case of a continuous random variable) of the random variable.

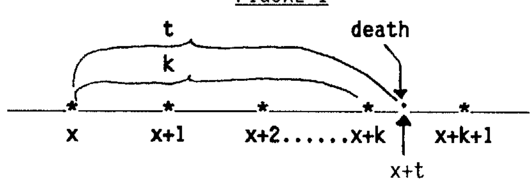
Several random variables are developed and discussed in *Actuarial Mathematics*. The random variable for "time-until-death" for a person age x is useful in developing such standard actuarial concepts as expectation of life and, more importantly, is the basis for the development of the present-value random variables, the prospective loss-at-issue random variables, and the prospective loss-at-times-after-issue random variables.

The present-value random variables are important to the development of net single premiums or actuarial present values. The prospective loss-at-issue random variables are important to the development of premiums. The prospective loss-at-times-after-issue random variables are important for the development of reserves.

**TIME-UNTIL-DEATH RANDOM VARIABLES**

Two "time-until-death" random variables are developed in *Actuarial Mathematics*: a continuous random variable, denoted T(x) or T, for total time until death for a life age x, and a discrete random variable, denoted K(x) or K, for complete years of age lived until death for a life age x. Figure 1 illustrates.

FIGURE 1




That is, if a life age x dies at age x+t, the value of the random variable T is t and the value of the random variable K is k, where k is the greatest integer in t.

Traditionally, actuaries have used a life table to summarize the probability distribution for integral age at death for a life age 0, and then developed probabilities for the discrete random variable K using values from such a table. (Although many actuaries may not have been thinking in those terms.) *Actuarial Mathematics* directly discusses the distribution function and probability, or probability density, functions for the random variables K and T. The distribution function for the continuous random variable T turns out to be the familiar $_tq_x$, and the probability density function is $_tp_x\mu_{x+t}$. It can also be shown that $_{k|}q_x$, which can be evaluated using values from the life table, is the probability function for the discrete random variable K, and thus the distribution function for K is given by $_{k+1}q_x$. Thus, the distribution functions and probability, or probability density, functions for K and T are not new concepts in the study of contingencies -- just new, and perhaps better, ways of looking at some familiar probabilities and actuarial symbols.

The curtate expectation of life, $e_x$, is a simple illustration of how directly considering probability functions may help in understanding actuarial concepts. The traditional approach tends to emphasize the definition:

$$e_x = \sum_{k=1}^{w-1-x} {}_kp_x \quad \text{(where w = last age of the life table)}$$

While the "new" approach emphasizes that $e_x$ represents the expected value of the number of complete years lived until death for a life age x, because $e_x$ is set equal to the sum of the products of the possible number of complete years lived and the probability of living that number of years; i.e.:

$$e_x = \sum_{k=0}^{w-1-x} k \cdot {}_{k|}q_x$$

The mathematical equivalence of these two expressions for $e_x$ can be easily demonstrated.

**PRESENT-VALUE RANDOM VARIABLES**

In the Syllabus Synopsis there are two examples to illustrate how the "time-until-death" random variables are used to develop present-value random variables, and how the use of random variables changes the calculations for net single premiums (NSP) or actuarial present values (APV). (Note: *Actuarial Mathematics* emphasizes the use of the term "actuarial present value," especially with respect to annuities; "actuarial" is to emphasize that other factors are involved besides interest; "present value" is used to refer to values based on adjusting for interest only.)

In the interest of time, I'll use only one example. Pedagogically it would probably be better to use the endowment insurance example at this point because it is the only example used later; however, I think the annuity example is more fun, so I'm going to use it!

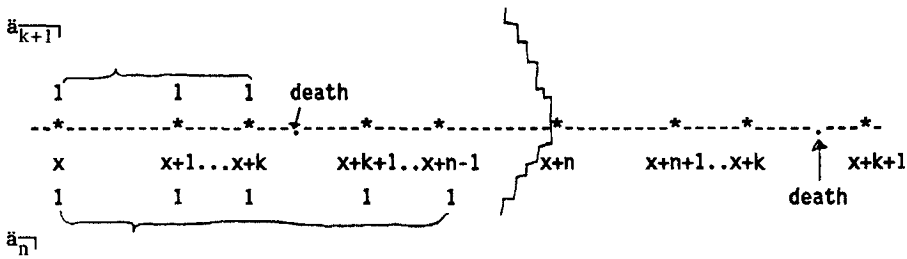
Consider the calculation of the APV for an n-year temporary life annuity due with annual payments of 1, issued to a life age x. The traditional approach (referred to as the "current payment" technique in *Actuarial Mathematics*) gives the value at age x by:

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \cdot {}_kp_x$$

In the context of the random variable K, we will consider a present-value random variable Y for this benefit. This situation is illustrated by Figure 2:

FIGURE 2

$0 \leq K < n$




$n \leq K \leq w\text{-}1\text{-}x$

If $0 \leq K < n$, then the present-value random variable is $\ddot{a}_{\overline{K+1}|}$ (see top of Figure 2); if $n \leq K \leq w\text{-}1\text{-}x$, then the present-value random variable is $\ddot{a}_{\overline{n}|}$ (see bottom of Figure 2).

That is:

$$Y = \begin{cases} 1 \cdot \ddot{a}_{\overline{K+1}|} & 0 \leq K < n \\ 1 \cdot \ddot{a}_{\overline{n}|} & n \leq K \leq w\text{-}1\text{-}x \end{cases}$$

Then, the expected value of Y is given by:

$$E(Y) = \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1}|}\, {}_{k|}q_x + \sum_{k=n}^{w-1-x} \ddot{a}_{\overline{n}|}\, {}_{k|}q_x$$

because ${}_{k|}q_x$ is the probability function for the random variable K. It can be shown that this expected value is equal to the traditional $\ddot{a}_{x:\overline{n}|}$, and that

$$\mathrm{Var}\,(Y) = (2/d) \cdot (\ddot{a}_{x:\overline{n}|} - {}^{2}\ddot{a}_{x:\overline{n}|}) + {}^{2}\ddot{a}_{x:\overline{n}|} - (\ddot{a}_{x:\overline{n}|})^2$$

Theoretically, use of the variance emphasizes the fundamental nature of the APV; practically, its use provides some additional information that may be helpful in making decisions.

Numerical values are fairly easy to calculate from tabulated values, or by recursive methods.

**PROSPECTIVE LOSS RANDOM VARIABLES**

Combining present-value random variables for benefits and those for net premiums, random variables for the value of the prospective loss at issue, or at any time after issue, can be developed. Verbally, the value of the prospective loss is the value, as of the age of valuation, of the future benefits less the corresponding value of future net premiums. These prospective loss random variables are critical to the development of net premiums and reserves. The following development illustrates.

**Random Variable for Prospective Loss at Issue**
Consider a 1 unit n-year endowment insurance policy with immediate payment of death claims and annual payment of premiums, issued to a life age x. The random variable for the value of the prospective loss at issue is given by:

$$L = \begin{cases} 1 \cdot v^T - P \cdot \ddot{a}_{\overline{K+1}|} & 0 < T \le n;\ 0 \le K < n \\ 1 \cdot v^n - P \cdot \ddot{a}_{\overline{n}|} & n < T \le w\text{-}x;\ n \le K \le w\text{-}1\text{-}x \end{cases}$$

Figure 3 demonstrates.

FIGURE 3

$0 < T \le n;\ 0 \le K < n$ | $n < T \le w\text{-}x;\ n \le K \le w\text{-}1\text{-}x$

Value of Benefits: $1 \cdot v^t$ | Value of Benefits: $1 \cdot v^n$

```
            1 death                                          1        death
*-----*-------*---*----*-------*-----*-       *----*-----*-------*---*----*-
x    x+1 ....x+k x+t x+k+1...x+n-1  x+n       x..x+n-1  x+n.....x+k x+t x+k+1
P     P ....  P                               P .. P
```

Value of Premiums: $P \cdot \ddot{a}_{\overline{k+1}|}$ | Value of Premiums: $P \cdot \ddot{a}_{\overline{n}|}$

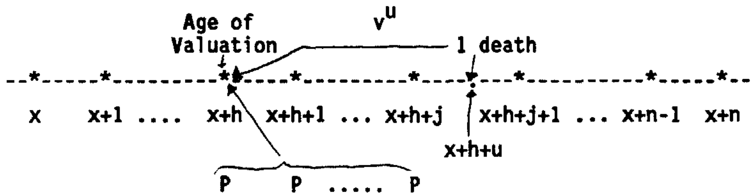
If $0 < T \le n$, $0 \le K < n$, then, as of age x, the value of the benefit is $1 \cdot v^t$ and the value of the premiums is $P \cdot \ddot{a}_{\overline{k+1}|}$, because the benefit is paid at time of death and premiums are paid until the beginning of the year of death. Similarly, if $n < T \le w\text{-}x$, $n \le K \le w\text{-}1\text{-}x$, then, as of age x, the value of the benefit is $1 \cdot v^n$ and the value of the premiums is $P \cdot \ddot{a}_{\overline{n}|}$, because the benefit is paid at time n and the premiums are only paid until age x+n-1.

**Random Variable for Prospective Loss at Any Duration after Issue**
The random variable for the prospective loss as of any duration h after issue is denoted ${}_hL$. To define ${}_hL$, we must first define random variables U and J to represent, respectively, the total time lived until death for a life age x+h, and the complete years lived until death for a life age x+h.

Again considering a 1 unit n-year endowment insurance policy with immediate payment of death claims and annual payment of premiums, issued to a life age x, two cases must be considered for ${}_hL$: case 1 for $h < n$, and case 2 for $h = n$. In case 1, there are two ranges for the random variables U and J which yield different expressions for ${}_hL$. In case 2, there is only one range. The two ranges for case 1 are shown in Figures 4A and 4B.

## FIGURE 4A

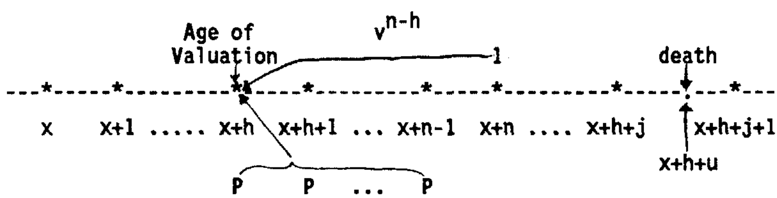
$h < n$; $\quad 0 \le J < n-h$
$\qquad\quad\; 0 < U \le n-h$




If $h < n$ (i.e., the duration since issue is less than n) and $0 \le J < n-h$, $0 < U \le n-h$, then, as of age $x+h$, the value of the benefit is $1 \cdot v^u$ and the value of the premiums paid since age $x+h$ is $P \cdot \ddot{a}_{\overline{j+1}|}$ because the benefit is paid at the time of death, age $x+h+u$, and the premiums are paid from age $x+h$ until age $x+h+j$.

## FIGURE 4B

$h < n$; $\quad n-h \le J \le w-1-(x+h)$
$\qquad\quad\; n-h < U \le w-(x+h)$




If $h < n$ and $n-h \le J \le w-1-(x+h)$, $n-h < U \le w-(x+h)$, then, as of age $x+h$, the value of the benefit is $1 \cdot v^{n-h}$ and the value of the premiums paid since $x+h$ is $P \cdot \ddot{a}_{\overline{n-h}|}$ because, in this case, the benefit is paid at age $x+n$ and the premiums are paid from age $x+h$ until age $x+n-1$. Thus, for $h < n$:

$$
{}_hL = \begin{cases} 1 \cdot v^U - P \cdot \ddot{a}_{\overline{J+1}|} & 0 \le J < n-h; \quad 0 < U \le n-h \\ 1 \cdot v^{n-h} - P \cdot \ddot{a}_{\overline{n-h}|} & n-h \le J \le w-1-(x+h); \quad n-h < U \le w-(x+h) \end{cases}
$$

Figure 5 illustrates the situation for $h = n$.

FIGURE 5

$h = n; \quad n\text{-}h \leq J \leq w\text{-}1\text{-}(x+h)$
$\qquad\qquad n\text{-}h < U \leq w\text{-}(x+h)$

```
                                    Age of
                                   Valuation
                                      1          death
-----*-------*--------*------*--------*-----*-*-----
     x      x+1.....x+n-1    x+n ... x+h+j   x+h+j+1
                             =x+h            x+h+u
```

If $h = n$ and $0(= n\text{-}h) \leq J \leq w\text{-}1\text{-}(x+h)$, $0(= n\text{-}h) < U \leq w\text{-}(x+h)$, then, as of age x+h, the value of the benefit is 1 and the value of the premiums paid since age x+h is 0, because the benefit is paid at age x+n = x+h and there are no premiums paid since x+h. Thus, for h = n:

$$_{h}L = 1\text{-}0 = 1$$

Of course, if $h > n$, $_{h}L = 0$.

**Premiums**

The traditional principle for determination of net premiums is to determine net premiums such that the APV of the benefits equals the APV of the net premiums. For premium determination, *Actuarial Mathematics* emphasizes the use of the prospective loss-at-issue random variable. In that context, the traditional principle is equivalent to what is referred to as the equivalence principle, which determines net premiums such that the expected value of L is 0. It can be shown that using the equivalence principle on the n-year endowment insurance example yields a net premium:

$$P = \overline{A}_{x:\overline{n}|} \;/\; \ddot{a}_{x:\overline{n}|}$$

confirming the equivalence of this principle to the traditional principle. The equivalence principle is one important principle that can be used to set net premiums, but one of the important points made in *Actuarial Mathematics* is that it is not the only legitimate principle for premium determination. Example 6.6 in *Actuarial Mathematics* (pages 170-73) illustrates this and the potential value of using a variance calculation:

> Example 6.6: Consider a 10,000 fully discrete whole life insurance. Let $\pi$ denote an annual premium for this policy and $L(\pi)$ denote the loss-at-issue random variable for one such policy on the basis of the Illustrative Life Table, 6% interest and issue age 35.
>
> (a) Determine the premium, $\pi_a$, such that the distribution of $L(\pi_a)$ has mean 0. Calculate the variance of $L(\pi_a)$.
>
> (b) Approximate the lowest premium, $\pi_b$, such that the probability is less than 0.5 that the loss $L(\pi_b)$ is positive. Find the variance of $L(\pi_b)$.

(c) Determine the premium, $\pi_c$, such that the probability of a positive total loss on 100 such independent policies is .05 by the normal approximation.

The results are:

| | $\pi$ | Prob $L(\pi)>0$ | $Var(L(\pi))$ | Std Dev $(L(\pi))$ | $E(L(\pi))$ |
|---|---|---|---|---|---|
| (a) | 83.62 | .276 | 2,412,713 | 1,553.29 | 0.06 |
| (b) | 50.31 | .487 | 2,171,630 | 1,473.65 | 512.79 |
| (c) | 100.66 | .217 | 2,540,907 | 1,594.02 | -262.22 |

Management of financial security systems may be improved by having an understanding of the implications of such values.

Finally, Chapter 4, Variance of the Present Value of Future Profits, from Study Note 210-21-87 (or 7BA-114-87), Gross Premiums and Asset Shares, by David B. Atkinson, FSA, demonstrates how the random variable approach can be used to address questions such as:

1. Given that a company issues 100 policies, what level of Present Value of Profits (PVP) can be achieved with a probability of 95%?

2. Given that a company issues 100 policies, what is the probability that PVP will be greater than 0?

3. How many policies must be issued to achieve PVP greater than 0 with a probability of 95%?

**Reserves**

The traditional expression for the reserve is, prospectively, the APV of future benefits less the APV of future net premiums. *Actuarial Mathematics* defines the reserve at duration h after issue to be the expected value of the random variable ${}_hL$. This makes sense because the reserve should be that amount which together with the future net premiums is sufficient to fund future benefits, and the expected value of ${}_hL$ is the expected value of the amount that will not be funded by future premiums. It can be shown that for the endowment insurance example this expected value gives the traditional prospective formulation of the reserve.

It is, of course, possible to calculate the variance for ${}_hL$ to further analyze the variability of reserve values. Example 7.12 from *Actuarial Mathematics* (pages 218-19) illustrates:

Example 7.12: Consider a portfolio of 1,500 annual premium 5-year term life insurance policies issued to lives age 50 with claims paid at the end of the year of death. Assume all policies have premiums due immediately. Further assume 750 policies are at duration 2, 500 at duration 3 and 250 at duration 4, and that the policies in each group are evenly divided between those with 1,000 face amount and those with 3,000 face amount.

(a) Calculate the aggregate reserve.

(b) Calculate the variance of the prospective losses over the remaining periods of coverage of the policies, assuming such losses are independent. Also, calculate the amount which, on the basis of the normal distribution, will give the insurer a probability of .95 of meeting the obligations to this block of business.

(c) Calculate the variance of the losses associated with the 1-year term insurance for the net amounts at risk under the policies and the amount of supplement to the aggregate reserve which, on the basis of the normal distribution, will give the insurer a probability of 0.95 of meeting the obligations to this block of business for the 1-year period.

(d) Redo (b) and (c) with each set of policies increased 100-fold in number.

The results are:

| | (1) Aggregate Reserve | (2) Variance of Prospective Losses | (3) Amount for 95% Probability of Meeting Obligations | (4) (3)/(1) |
|---|---|---|---|---|
| 5-year term | $ 4,795 | $\$1.08 \times 10^{8}$ | $ 21,911 | 4.6 |
| 1-year term | 4,795 | $4.88 \times 10^{7}$ | 16,287 (total) | 3.4 |
| 100-fold 5-year | 479,500 | $1.08 \times 10^{10}$ | 650,659 | 1.36 |
| 100-fold 1-year | 479,500 | $4.88 \times 10^{9}$ | 594,418 (total) | 1.24 |

Again, the management of financial security systems may be aided by an understanding of the implications of such results.

## CHALLENGES AND OPPORTUNITIES

*Actuarial Mathematics* provides some new opportunities for greater insight into the fundamental nature of actuarial mathematics and presents some new challenges in applying this insight to improve the practice of actuarial science. Some actuaries would say that the emphasis in the new text should have been placed on considering the interest rate as the result of a random process because the interest factor has been the most volatile and troublesome for actuaries over the past 20 years. Jim Hickman addresses this issue in some detail in the February 1985 issue of *The Actuary*. This is perhaps the next major theoretical challenge that should be addressed in order to improve the theory and practice of actuarial science.

The main challenge presented by the new theoretical approach of the text *Actuarial Mathematics* is to bridge the theoretical and practical. The formulas and considerations can become very complicated when attempting to apply this approach to practical problems such as gross premium determination. The challenge is to take advantage of the insights and opportunities for further analysis provided by the random variable approach. To accomplish this requires first meeting the challenge of allocating the time and staff to study this approach and think through possible applications.

The following is a short list of questions that may help identify some of the specific challenges and opportunities:

- o Can we analyze mortality experience studies in more depth from the perspective of the random variable for time-until-death?
- o How do we analyze the results of variance calculations? What are acceptable levels? for present-value random variables? for loss random variables?
- o What premium principles are practical and acceptable?
- o How much more complex is it to extend the use of the loss random variable to include all factors involved in gross premium determination?
- o What is the probability of a positive loss for premiums determined by the equivalence principle? What is an acceptable level of that probability?
- o How does a change in interest affect the expected loss?

Other questions and ideas will come in the process of thinking about the implications of this approach.

To partly analyze the last two questions,I have used a Whole Life plan. Net Premiums are calculated using the equivalence principle,and issue ages 0, 20, 50, 65 and 80, both male and female, are analyzed.

For this Whole Life plan, Table 1 summarizes the probabilities that L is greater than 0 on the basis of 1980 CSO and 1980 U.S. Life mortality, 5% interest, male and female separately.

TABLE 1

Probabilities (L > 0)

| | 1980 CSO, 5% | | 1980 U.S. Life, 5% | |
|---|---|---|---|---|
| Issue Age | Male | Female | Male | Female |
| 0 | .1964 | .1799 | .1752 | .1540 |
| 20 | .2574 | .2495 | .2773 | .2759 |
| 35 | .3258 | .2925 | .3240 | .3114 |
| 50 | .3702 | .3368 | .3948 | .3393 |
| 65 | .4339 | .3945 | .4361 | .3901 |
| 80 | .5652 | .5229 | .5106 | .4978 |

Understanding the significance of the magnitude of these probabilities may require more experience analyzing loss random variables. However, the initial reaction may be that the probabilities are lower than one would have expected.

These results reflect the fact that, although the premiums were calculated such that E(L) = 0, the distribution of L is such that large positive values of L are associated with small probabilities for a number of years immediately after issue, and small negative values of L are associated with much larger probabilities later. With the relatively flat pattern of mortality, the effect of this pattern of L values is especially significant at the younger issue ages.

The magnitude of these probabilities should provide some assurance that, at least with regard to claims, there is a strong probability that the company will make money on a given individual policy. Of course, this conclusion would be more meaningful if the analysis was based on gross premiums, which is possible but more complex.

The pattern by issue age is as one might expect -- increasing probability of loss as issue age increases, perhaps implying that business at the older ages is less desirable from the company's standpoint. There does not seem to be a significant difference between male and female or between the two mortality bases, although the probability of L greater than 0 is consistently higher for males.

It is interesting to note that an increase in interest rates seems to decrease the probability of L being greater than 0. Table 2 illustrates this point.

TABLE 2

Probabilities (L > 0)

| | 1980 CSO, 10% | | 1980 CSO, 5% | |
|---|---|---|---|---|
| Issue Age | Male | Female | Male | Female |
| 0 | .0751 | .0664 | .1964 | .1799 |
| 20 | .1409 | .1361 | .2574 | .2495 |
| 35 | .2178 | .1880 | .3258 | .2925 |
| 50 | .3118 | .2592 | .3702 | .3368 |
| 65 | .3888 | .3541 | .4339 | .3945 |
| 80 | .4830 | .4479 | .5652 | .5229 |

Note that the decrease in Probabilities (L > 0) is most significant, relatively, at the youngest ages, and less significant, relatively, as age increases, reflecting, in part, the lesser impact a change in interest has over shorter periods of time.

Now comes the time when I'm supposed to tell you what I said. I'm going to cheat a bit on this point -- I'm going to highlight three words as a conclusion or summary of the presentation -- similarity, opportunity and challenge.

It is important to emphasize that in many respects the results of the traditional, deterministic approach and those of the "new" approach are the same, with perhaps some different terminology. That's why I've put the word "new" in quotations. However, the emphasis on the probabilistic and stochastic nature of the fundamental concepts in actuarial mathematics provides opportunity for further analysis and insight. The challenge is to translate this insight into better actuarial practice.

Since we have some time left, I'd like you to share your reactions, comments, suggestions for the benefit of all of us.

MR. MICHAEL J. COWELL: We measure our required capital by line of business, in terms of risk. One of the measures that we try to come up with is a standard deviation. We do a lot of modeling of mortality and morbidity and try to model some of the interest and expense factors,but as you say, the theory is less well developed. Is it correct to view the ratio in Column 4 of the table summarizing the results for Example 7.12, as the first stab at the additional amounts above the reserve that we should hold as capital or surplus, in order to protect that probability of loss?

MR. LUCKNER: I have to qualify my response by saying I have not been that involved in company financial analysis during my actuarial career. However, I think that the view you express in your question is a fair analysis because the probability does relate to the total amount you would need. If you have access to those funds, it would seem they should be able to be considered in determining the amount necessary for the 95% probability.

MR. COWELL: One of the opportunities that we have, or one of the challenges, is to go beyond mortality and morbidity, and to come up with comparable analysis for the other components in reserving techniques, particularly in GAAP reserves, where there is an expense factor, as well as interest and mortality and morbidity. I think that there is a lot of potential here for further development of the stochastic approach beyond the life and health contingencies.

MR. LUCKNER: Yes, I think that is important because our next step in terms of our basic education, hopefully, will be to get involved relatively soon with stochastic information about the interest risk. We do a little bit of that in our Fellowship exams. We talk about asset and liability matching, but our basic development of the interest mathematics is very much deterministic.

MR. RICHARD S. FOSTER: I appreciate your discussion, Warren. It is very helpful to know some of the potential uses of this. I also want to make sure that I understand some of the limitations. Am I correct in thinking that when you evaluate some of these probabilistic results that it is proper to interpret it by saying that you have a 95% probability of the specific result, assuming that the true, underlying mortality follows whatever mortality table you are using, and that in practice, if mortality improves, say over a period of years, that you no longer have really a 95% probability, but something else that is maybe not too far off 95%?

MR. LUCKNER: Yes, that is a good point, because not only are you talking about some inherent assumption about the basic mortality, but you are also talking about an approximate normal assumption in the procedure used to calculate the value.

MR. FOSTER: You have indicated that you have gotten started with incorporating interest variations. How complicated does the mathematics get when you want to incorporate both the mortality variation and interest variation? Does it get impossible?

MR. LUCKNER: I guess I am optimistic in saying that with the available computer technology it should not be too complicated to do the calculations. To work through the analysis and theory of the combination will probably be the bulk of the complication. At this point, I'm not expert enough to say just how complicated it gets. I have done a little bit of reading on the stochastic approach to interest and that's not too bad. When you do combine the stochastic approach to interest with the stochastic approach to mortality, it does get more involved. There are some references I have seen that do deal with that. I think that the calculations should not be that difficult, given our computer technology. Making sure that we're understanding and doing them correctly, and understanding the implications, will be the complicated part.

MR. COWELL: To some extent, Warren, we're already doing this in the kind of work that you do when you match assets and liabilities and you do scenarios. You use a Monte Carlo technique and you in effect determine your variability.

You probably don't even have, or think you need, the underlying math. We need some theoretician to come along and develop it. When you are doing financial projections and you are simulating variations in mortality and morbidity, and simulating variability in interest, for example, in asset and liability matching, or changes in the underlying asset value or expense, and you convolute all of those, the mathematics becomes certainly way beyond my ability, and beyond the ability of the average practicing actuary. The practical aspect of using Monte Carlo simulation in effect gives you a sense of the shape of the variation, and whether it is normal or otherwise. Usually it is not normal. Many of us are doing this and don't even realize it. The fundamental mathematics haven't really been developed. I'm sure somebody will develop them.

MR. LUCKNER: That's a very good point. One of the things I've noticed is that actuarial practice is, in a sense, ahead of what we do in basic education. That's unfortunate, but it's also encouraging that actuaries are doing these things. It also relates to one of the things that has been a pet peeve of mine -- a lot of times when we look at our basic education we focus on what we need to learn to do things the way we do them now, instead of focusing on what we need to learn to be able to do things better. I appreciate that comment because there's a lot more going on out there than we give ourselves credit for.

MR. FRANK E. KNORR: I understood that you were verifying all the things that we've learned in terms of expected values -- all the mortality tables we have are expected values and the formulas we've been using with those are correct. The new thing that we've incorporated now within actuarial mathematics is the variance of these expected values. Can we expect to see, in addition to standard mortality tables, variance tables?

MR. LUCKNER: That's an interesting thought. You're accurate in your understanding of what I was trying to convey, particularly for a group of actuaries who may have studied a different approach. There's not much difference in the sense that most of what you learned is still applicable -- it's just that we now have some new things to look at. In terms of the specific question about mortality, one thing that occurred to me is that maybe there is some way we can incorporate confidence intervals for the particular values that we use. That obviously involves standard deviations. I don't know how valuable they're going to be. I think this all relates to the issue of how important is the mortality component versus the interest component versus the expense component. How sophisticated should we get in each of those areas if the impact is not very great? Those are some things that we have to wrestle with, too.

MR. KNORR: It seems like it would be similar in each of those components, though. You would have to show the distribution function.

MR. LUCKNER: Yes. The life tables that we traditionally work with are really just probability distributions for the age-at-death of a person aged 0. We haven't really been talking in those terms, but that's really what it is.

MR. KNORR: I saw in your calculations that once you had the mortality table, you had your entire distribution. It just seems that just having those $q_x$ values does not define your variance, your variability, intuitively.

MR. LUCKNER: They do, indirectly, for the mortality component, but they do not do it for the interest component.

I think we're out of time, but I am going to tell you one joke because you've been a good audience! We've heard a lot of talk lately about the future of the actuary and the actuary of the future -- trying to do something to change our image, whatever that means. Different people have different perspectives on what that should be. However, one of the images of actuaries that I think is inappropriate is that actuaries are boring. I know a lot of exciting, creative actuaries. In fact, I heard recently of an actuary who created a new policy called the "Live it up Policy." It had the death benefit payable at the beginning of the year of death.

Note: The typos in the Syllabus Synopsis "'New' Approach to Contingencies -- A Summary" are as follows:

1. Blue heading of cover page: large "0" should be large lower case "n"

2. Page 2, expression for E(Z), second equality: "t" from "x+t" is too low

3. Page 4, expression for L: "$0 \leq T \leq n$" should be "$0 < T \leq n$"

4. Page 4, Figure 4, lower age line: "x+h-1" should be "x+n-1"