

Notation and Terminology Used on Exam MLC

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Introduction

This notation note completely replaces similar notes used on previous examinations.

In actuarial practice there is notation and terminology that varies by country, by application, and by source. The purpose of this study note is to present notation and terminology that will be used on the MLC examination for situations where notation or terms differ from that in *Actuarial Mathematics for Life Contingent Risks* (2nd edition) (*AMLCR*), the textbook for the examination and notation or terms that are unique to the examination. For notation and terms not discussed here, the meaning in *AMLCR* will apply.

The format of this note is to list common alternative notations for a given item. The specific notation(s) that will be used on the examination will then be provided.

Notation and Terminology

The **force of mortality** may be represented by μ_x or $\mu(x)$ or μ_{x+t} or $\mu(x+t)$ where x and $x+t$ are attained ages. The symbol $\mu_{[x]+t}$ indicates selection at age x and attained age $x+t$. The symbols μ_x , μ_{x+t} , and $\mu_{[x]+t}$ will be used on the examination.

Both the **survival function**, **salary rate function** and **salary scale** may be represented by S or s . On the examination, the symbol $S_x(t)$ will be used to indicate the survival function; the symbols \bar{s}_x and s_x will indicate the salary rate function and salary scale respectively.

The **number of lives** at age x can be represented by ℓ_x or l_x . The symbol l_x will be used on the examination.

The **complete future lifetime of (x) random variable** can be represented by T_x or $T(x)$. Similarly, the symbols used for joint life status can be T_{xy} or $T(xy)$, and for last survivor status can be $T_{\overline{xy}}$ or $T(\overline{xy})$. The symbols T_x , T_{xy} and $T_{\overline{xy}}$ will be used on the examination.

The **curtate future lifetime of (x) random variable** can be represented by K_x or $K(x)$. Similarly, the symbols used for joint life status are K_{xy} or $K(xy)$, and for last survivor status are $K_{\overline{xy}}$ or $K(\overline{xy})$. The symbols K_x , K_{xy} and $K_{\overline{xy}}$ will be used on the examination.

The **present value of future losses random variable** may be represented by L or ${}_0L$ or L_0 for loss at issue and ${}_tL$ or L_t for loss from t years after issue. Superscripts may be included. When the symbol L is used to represent present value of future losses random variable the symbol including any subscripts or superscripts will be defined in the text of the question.

Duration subscripts can be used differently. For example, something happening in the first duration (between ages x and $x+1$) may be identified with a 0 or 1. The text of the question will define any notation used.

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If benefits can vary continuously, the **benefit** at time t is represented by b_t . If benefits vary but as a step function, the **benefit** at the end of period k is represented by b_k . The text of the question will define the benefit either by formula or in words.

Actuarial present value and **expected present value** are terms used for the expectation of the random variable representing the present value of one or more contingent future payments. Either term may be used on the examination.

Fully discrete insurance is an insurance where both the premiums and the benefits are paid only at discrete time points. **Semi-continuous** insurance is an insurance where the premiums are paid at discrete time points and the death benefits are paid at the moment of death. **Fully continuous** insurance is an insurance where the premiums are paid continuously and the death benefits are paid at the moment of death. Unless stated otherwise in the text of the question discrete time points are the beginnings of years for premium payments and the ends of years for death benefit payments.

Special insurance is an insurance that has either non-level benefits or non-level premiums or both. The non-level aspects of the insurance will be described in the text of the question. If an insurance is not defined as “special” then premiums and benefits are assumed to be level, unless there is explicit information in the text of the question to the contrary.

Net premium is the premium determined by the equivalence principle and assuming no expenses. In previous exams this was called benefit premium. The term benefit premium will no longer be used on the examination.

The **net premium for fully discrete insurances** will be represented by P with the appropriate symbols attached. P_x , $P_{x:\overline{n}|}$, $P_{x:\overline{n}|}^1$, and $P_{x:\overline{n}|}^{\perp}$ may be used on the exam.

The symbols are defined in terms of an insurance, A , and an annuity, \ddot{a} , as follows:

$$P_x = \frac{A_x}{\ddot{a}_x}, \quad P_{x:\overline{n}|}^1 = \frac{A_{x:\overline{n}|}^1}{\ddot{a}_{x:\overline{n}|}}, \quad P_{x:\overline{n}|}^{\perp} = \frac{A_{x:\overline{n}|}^{\perp}}{\ddot{a}_{x:\overline{n}|}}, \quad P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

The symbol P will be defined within the text of the question if it is not one of the symbols shown above.

Net premium reserves are reserves based on the net premium assuming no expenses, and using the same mortality and interest assumptions as the net premium calculation. In previous exams these were called benefit reserves. The term benefit reserve will no longer be used on the examination.

Gross premium reserves are reserves based on the gross premium. The mortality, interest and expense assumptions for the reserve would not necessarily be the same as those used in the gross premium calculation.

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Unless stated otherwise in the text of a question all expenses are equal to zero. If expenses are specified in the text of a question then the expenses need to be used in the solution to the question.

In a **multiple decrement model** $q_x^{(j)}$ is the probability a life age x fails in the next year due to decrement (j) and $q_x^{(\tau)}$ is the probability of failure due to all decrements. The associated single decrement probability of failure at age x due to decrement (j) is $q_x^{\prime(j)}$. The probability $q_x^{(j)}$ is also called the dependent probability. The probability $q_x^{\prime(j)}$ is also called the independent probability. A multiple decrement model is a special case of a multi-state model. Any of the multi-state model notation of the next paragraph may be used with a multiple decrement model on the examination.

In a **multi-state model** ${}_t p_x^{ij}$ is the probability that a life currently age x and in state i is in state j at age $x+t$. The symbol μ_x^{ij} is the force of transition between states i and j at age x . The symbol ${}_t \bar{p}_x^{ii}$ is the probability that a life currently age x and in state i remains in state i through the period x to $x+t$.

The **asset share** at time t may be represented by ${}_t AS$ or AS_t . The symbol ${}_t AS$ will be used on the examination. Asset shares for universal life insurance are calculated like those for traditional life insurance.

The **reserve** at time t may be represented by ${}_t V$ or V_t . The symbol ${}_t V$ will be used on the examination.

In practice, the financial statements of an insurance company will include a liability amount in respect of future outgo on a policy in force, and this amount is called the reserve. *AMLCR* calls this “the actual capital held in respect of a policy” and uses the term reserve only in this context. The exam will use the term reserve both for this context, and also for the expected value of a future loss random variable, even where this is not related to the provision in the financial statements. *AMLCR* calls the expected value of the future loss random variable a policy value. The term policy value will not be used on the exam. (This paragraph keeps the meaning of reserve on the examination the same as its meaning in examinations before 2014. *AMLCR* discusses its distinction between reserve and policy value on page 185 and in chapter 12.).

Reserves may be calculated based not only on mortality, but also with possible inclusion of other decrements (e.g. surrender/lapse, morbidity). Unless otherwise noted in the problem, reserves should be calculated based on the decrements given in the problem.

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A **modified reserve** is a reserve computed without expenses but adjusting the valuation premiums to allow implicitly for initial expenses. A full preliminary term reserve is an example of a modified reserve. All modified reserves have the expected present value at issue of the benefits equal to the expected present value at issue of the valuation premiums; valuation premiums are typically lower in the first year or first few years than in later years. Any modified reserve questions on the examination other than full preliminary term reserves will specify the modification basis in the question.

If a **table of select and ultimate values** is presented in a question the format of the table will either follow the convention of (i) reading across the row of select rates and then down the column of ultimate rates for the values corresponding to each age at selection or it will follow the convention that (ii) all row entries indicate a current age but differ as to the age at selection. On the examination, the table method can be inferred from the table headers.

On the examinations the **transition probabilities** for a multi-state model may be presented in a matrix. For example, for a model with two states, 0 and 1, the transition probabilities would be presented in a matrix as follows:

$$\begin{bmatrix} p_x^{00} & p_x^{01} \\ p_x^{10} & p_x^{11} \end{bmatrix}$$

Euler's method can be used to numerically obtain an approximate solution to a differential equation. The method produces a piece-wise linear approximate solution. There are two versions, called the forward method and the backward method. In both methods the next line segment joins the current point in time and next point in time. In the forward version, the slope is determined at the beginning of the time interval, whereas in the backward method, the slope of the curve is evaluated at the other endpoint.

That is, suppose we have a differential equation $\frac{d}{dt}f(t) = d(t)$ for some functions f and d .

The **Forward Euler Approximation**, with step size h , gives

$$\frac{f(t+h) - f(t)}{h} = d(t) \Rightarrow f(t+h) = f(t) + hd(t) .$$

The **Backward Euler Approximation**, with step size h , gives

$$\frac{f(t+h) - f(t)}{h} = d(t+h) \Rightarrow f(t) = f(t+h) - hd(t+h).$$

Generally, the forward method is used when the boundary condition is an initial value (for example, the Kolmogorov Forward equations for transition probabilities), and the backward method is used when the boundary condition is a terminal value (for example, the Thiele equation for policy reserves). In the case of Thiele's equation for policy reserves in a life/death model, the forward method might also be used even when the boundary condition is a terminal value. For example, see *AMLCR*, page 267 and Exam MLC sample question 306.

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On the examination, **retirement benefits** based on years of service include any fractional years, unless explicitly stated otherwise. For example, a pension plan that stipulates that annual retirement benefits are calculated by the participant's final salary $\times 0.02 \times$ the number of years of service would provide an annual benefit of 8200 for a participant with a final salary of 20,000 and 20.5 years of service ($20,000 \times 0.02 \times 20.5$).

As noted in Section 10.8 of *AMLCR*, the term **Normal Cost** is synonymous with the term **Normal Contribution**. Either term may be used on the examination.

Accrued Benefit

The term **accrued benefit** will be used on this examination to mean a benefit calculated using past service and average salary as of the determination date. This is consistent with U.S. practice.

In *AMLCR*, the term accrued benefit is used to mean a benefit based on past service and salary projected to exit:

- For the purpose of TUC, salary is projected to be level; and
- For the purpose of PUC, salary is projected using the assumed salary scale.

On the examination, we will refer to the benefits valued under the two funding methods as follows:

- For TUC, the benefit valued is the **accrued benefit**, having the meaning consistent with U.S. practice; and
- For PUC, the benefit being valued is the **projected benefit**, having the same meaning as accrued benefit in *AMLCR* for the PUC case.

Actuarial liability and **actuarial accrued liability** are synonymous terms and either term may appear on the exam.

Traditional Unit Credit

The **Traditional Unit Credit** (TUC) funding method for final salary pensions is described in *AMLCR* (Section 10.6.1). The text goes on to describe two possible variations, deriving from the different meanings of accrued benefit noted above. The two variations are described in a little more detail here, in the context of a pension benefit based on the average salary in the final k years of a member's service.

- 1) Accrued benefit consistent with the definition to be used on the exam: Value the accrued liability based on the k year average salary at the valuation date (this is loosely described in *AMLCR* (Section 10.6.1) as using z_x / s_x in place of z_{x+t} / s_x in the valuation formulas).
- 2) Accrued benefit based on level projection of salary: Value the accrued liability based on the salary at the valuation date, without averaging -- unless the member is within k -years of her exit date. In simple terms, this may be viewed as a PUC valuation, where the salary scale is assumed flat from the valuation date to the exit date.

Each of these variations is valid, depending on the meaning of accrued benefit. On the examination, we will use the first variation: For TUC, with a valuation date of, for example, 1/1/X (i.e., January 1, X):

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- To determine the **actuarial accrued liability** at the valuation date ($1/1/X$), use the salary at the valuation date, with averaging.
- To determine the **Normal Cost**, we also calculate the value at $1/1/X$ of the estimated actuarial accrued liability at $1/1/X+1$; for this calculation use the valuation salary assumption to project salaries one year, to $1/1/X+1$, and then continue as for the $1/1/X$ valuation. If there are exits with benefits between $1/1/X$ and $1/1/X+1$, the valuation salary assumption should be used to estimate the final average salary at exit.

On the examination, **pension reduction factors** may be presented in the context of benefit calculations for individuals who retire earlier than the normal retirement age. These factors are simple rates per unit time, rather than compound rates per unit time, unless explicitly stated otherwise.

For example, a pension plan may stipulate a 5% reduction for each year prior to normal retirement age, 65. For a participant retiring at age 60, this factor would result in a 25% reduction to the normal retirement benefit, rather than a reduction of $1 - (1 - .05)^5$.

Spot rates are the yield rates on zero-coupon bonds. Normally, spot rates vary by the maturity of the bond. Forward rates, if not given explicitly, are the forward rates implied by the spot rates. On the examination, both spot rates and forward rates are annual effective rates unless explicitly stated otherwise.

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For a time period where all cash flows occur only at the beginning and end of the time period:

Profit for the time period occurs at the end of the time period and is (a) minus (b) where:

- (a) is the accumulated value of the sum of the reserve at the end of the previous period and the cash flows that occur at the beginning of the period; and
- (b) is the sum of the value of the reserve at the end of the period and the cash flows that occur at the end of the period.

Expenses at the inception of a contract may be classified as negative profit at time 0 or may be part of period 1 cash flow and included in the profit calculation for period 1. Any initial expenses that are not part of period 1 cash flow will be identified in the question as **Pre-contract expenses**. If a reserve is to be established at time 0, before the first premium is received, it would be part of the time 0 profit. Any such reserve will be identified in the question.

Expected profit is the profit calculated using the gross premium, expected cash flows at the beginning and end of the period, and accumulating beginning of year values at the expected interest rate. The expected interest rate, and the assumptions used to calculate the expected cash flows may or may not be the same as those used to calculate the reserve.

Actual profit is calculated using the gross premium, actual cash flows at the beginning and end of the period, and accumulating beginning of year values using the actual rate of interest earned during the period.

Gain is the actual profit minus the expected profit for the period. **Gain by source** is the gain calculated where the effect of the difference between the observed values and the expected values in the profit calculations from one source is reflected, while the differences for the other sources are not. Examples of sources are: expenses, interest, mortality and withdrawal. Often, gains from multiple sources are calculated sequentially. For example, the gain from mortality might be calculated first, reflecting the difference between the observed mortality and the assumed mortality, and the gain from interest calculated second, reflecting the difference between the observed and assumed interest, while using only the observed mortality.

The examination will only include questions asking for Gain by source where the reserves are gross premium reserves and expected profits are based on the reserve assumptions. Under those conditions, the expected profit is 0 and the sum of the gains by source is equal to the actual profit.

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Profit Measure Terms. These are defined and explained more completely in Chapter 12 of *AMLCR* (2nd edition).

Profit margin is the ratio of the actuarial present value at issue of expected profits divided by the actuarial present value at issue of gross premiums.

The **profit vector**, which may be written as a row vector or a column vector, is $\text{Pr} = (\text{Pr}_0, \text{Pr}_1, \dots, \text{Pr}_n)$, where Pr_0 is the time 0 profit, Pr_t is the expected profit for year t per policy in force at the start of year t ; n is the last year in the profit test period. The first term is Pr_0 even if there are no Pre-contract expenses and Pr_0 is 0. The examination may use the term profit vector and may use the Pr or Pr_t notation.

The **profit signature**, which may be written as a row vector or as a column vector, is $\Pi = (\Pi_0, \Pi_1, \dots, \Pi_n)$ where Π_0 is the Pre-contract expenses; Π_t is the expected profit for year t per policy issued; n is the end of the profit test period. The first term is Π_0 even if there are no pre-contract expenses and Π_0 is 0. The examination may use the term profit signature and may use the Π or Π_t notation.

Net present value (NPV) and **Partial net present value (NPV(k))** are the actuarial present values of expected profits at issue, including pre-contract expenses, per policy issued.

$$NPV = \sum_{i=0}^n \Pi_i (1+r)^{-i}$$

$$NPV(k) = \sum_{i=0}^k \Pi_i (1+r)^{-i}$$

In both forms, r is the **risk discount rate** or **hurdle rate**. The risk discount rate is a rate of interest, and not a rate of discount. In the first form, n is the number of years in the profit test period.

Internal rate of return is the risk discount rate such that the NPV is equal to 0.

The **Discounted payback period** is the first year k at which $NPV(k)$ becomes nonnegative.

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Other terms and common equivalents

Terms used on the examination	Equivalent or similar terms (not used on the examination)
annuity due, annuity-due	due annuity
annuity immediate, annuity-immediate	immediate annuity
temporary life annuity	term annuity
temporary expectation of life	term expectation of life
certain period	guarantee period
premium paying period	premium paying term
face amount	sum assured, sum insured
net amount at risk	death strain at risk, sum at risk, amount at risk
net premium	benefit premium
gross premium	contract premium, expense-loaded premium, expense-augmented premium
net premium reserve	net premium policy value, benefit reserve
gross premium reserve	gross premium policy value
cost of insurance, COI	mortality charge
μ_x^{ij}	$\mu_{ij}(x), \lambda_{ij}(x)$
${}_tP_x^{ij}$	${}_tP_{ij}^{(x)}$
Variance, Var	V
${}_tq_x^{(\tau)}$	${}_tP_x^{0\bullet}$
${}_tP_x'^{(j)}$	${}_tP_x^{*(j)}$
$\mu_x^{(\tau)}(t)$	$\mu_{x+t}^{0\bullet}$
Terms used to describe payments by policyholders on universal life policies	
Premium	contribution, deposit, consideration

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Universal Life Terminology and Account Value Mechanics

While all universal life insurance policies have similar structures, there are variations, especially with respect to cost of insurance calculations. The following structure applies to universal life policies on the exam unless otherwise specified. If variations to this structure occur on the exam the variation will be stated in the applicable question.

Account values are calculated at regular intervals. The question will indicate the calculation period.

For each calculation period the account value is calculated as follows:

$$AV_{end} = [AV_{start} + P(1 - f) - e - COI](1 + i^c), \text{ where } COI = \frac{DB_{end} - AV_{end}}{1 + i^q} (coi \text{ rate})$$

The symbols in the equations above will not be used on the exam unless the symbols are also defined in the question.

AV_{start} is the account value at the start of the period.

AV_{end} is the account value at the end of the period.

P is the premium paid (unless otherwise stated in the question, all premiums are paid)

f is the percent of premium charge.

e is the fixed expense charge.

DB_{end} is the projected death benefit at the end of the period, consistent with AV_{end} , reflecting the policy's death benefit description and the corridor factor if specified.

$ADB_{end} = DB_{end} - AV_{end}$ is the Additional Death Benefit, i.e. the amount paid on death in addition to the Account Value.

COI is the cost of insurance charge.

$coi \text{ rate}$ is the mortality rate used to determine the cost of insurance.

i^c is the credited interest rate per period.

i^q is the interest rate per period for discounting the net amount at risk in the COI calculation (unless otherwise stated in the question $i^q = i^c$)

Death benefits and surrender benefits are paid at the end of the period, after the account value at the end of the period has been calculated.

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There are two main death benefit structures for Universal Life.

Type A: the death benefit is a specified amount. The term Type A may be used on the examination. Any of these forms of description would be equivalent and any could appear on the exam:

- a death benefit of 10,000
- a Type A universal life of 10,000
- a Type A universal life with face amount 10,000

Type B: the death benefit is a specified amount plus the account values. The term Type B may be used on the examination. Any of these forms of description would be equivalent and any could appear on the exam:

- a death benefit of 10,000 plus the account value
- a Type B universal life of 10,000
- a Type B universal life with face amount 10,000
- a Type B universal life with additional death benefit of 10,000
-

The **cash surrender value** is the account value at the end of the period minus the applicable surrender charge, if any. The cash surrender value cannot be less than 0.

A universal life policy will lapse if the account value is 0. Except, a universal life policy may remain in force, even if the account value is below 0, if the policy has a **no-lapse guarantee** provision and satisfies the conditions of the no-lapse guarantee provision. You should assume that a universal life policy does not lapse unless you are told in the question to check if it lapses.

Corridor factors:

- Universal life policies may contain a provision that the death benefit will be the greater of: (1) the normal death benefit; and (2) the account value at the end of the period times a corridor factor. For example, if the specified amount on a Type A is 35,000, the account value is 30,000, and the corridor factor is 250%, then the death benefit would be 75,000 which is the greater of 35,000 and 250% of 30,000.
- Corridor factors can also apply to Type B contracts, and again would define the minimum total death benefit. For example, if the specified amount on a Type B is 35,000, the account value is 30,000, and the corridor factor is 250%, then the death benefit would be 75,000 which is the greater of 35,000+30,000 and 250% of 30,000.
- If a corridor factor provision would increase the death benefit, the cost of insurance charge calculation should reflect the increased death benefit in the calculation of the cost of insurance charge.
- If no corridor factors are stated in the question, assume the policy has none.
- If corridor factors are stated in the question, you should consider the corridor factors in any calculations.

Reserves for universal life policies are account values unless otherwise specified.