Multivariate Dependence Modeling Using Pair-Copulas

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Abstract

In the copula literature there are many bivariate distribution families but very few higher dimensional ones. Moreover, most of these are difficult to work with. Some of the bivariate families can be extended to more dimensions but in general the construction of distribution functions with more than two variables is a difficult problem. We introduce a construction method that is straightforward to implement and can produce multivariate distribution functions of any dimension. In essence the method takes an arbitrary multivariate density function and decomposes it into a product of bivariate copulas and marginal density functions. Each of these bivariate copulas can be from any of the available families.

We also highlight the power of a graphical display known as a *chi-plot* to help us understand the dependence between pairs of variables. One illustration, based on changes in the exchange rate of three currencies, shows how we can specify the pair-copulas and estimate their parameters. In another illustration we simulate data that exhibits complex dependencies as would be found, for example, in enterprise risk management or dynamic financial analysis.