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Using Monte Carlo Simulation to Understand Mortality

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This is the first year for appointed actuaries in the United States to review X factors and evaluate actual mortality experience that is emerging. Monte Carlo simulation has been suggested as a useful tool in this evaluation. Considerable attention has been given to the subject in seminars, in a draft practice note, and in an exposure draft Actuarial Standard of Practice. This article attempts to provide a simplified explanation of how Monte Carlo simulation can be used by a small company to evaluate its mortality experience.

Anticipated mortality, according to the proposed Actuarial Standard of Practice, is best evaluated in terms of face amount, not policies. There is little statistical theory for developing a distribution of benefits incurred by a block of policies. Monte Carlo methods make it possible to test actual experience of face amount paid out in death claims against the distribution of expected claims suggested by the mortality table, which are 20-year select factors and X factors for a specific block of business. For a small company or a small block of business, Monte Carlo methods overcome concerns about whether the data set is large enough, because the results converge to the underlying distribution given enough simulation trials. The method is useful whether the purpose of the analysis is (1) to derive anticipated mortality over several years in order to set X factors, or (2) to evaluate the past year's mortality in order to certify adequate X factors for the current year.

In the following description of a Monte Carlo simulation, the Monte Carlo testing simulates death claims of a particular business for many repetitions. The results provide a distribution of death claim payments and further provide

confidence intervals that can be used to establish a rejection region for particular X factors. The simulated distribution is unique and appropriate to the particular policies being analyzed.

In this analysis, I used Microsoft Access 97, with database tables of policyholder records, and also a Visual Basic for Applications module that performed the calculations in the study. I used several years of experience for understanding anticipated mortality, and a single year for evaluating emerging experience.

Input variables are listed below:

1. For each policy, the effective date, paid through date, and factors required to determine the X factor group to be analyzed.
2. For each insured, the birth date, gender, and benefit amount.
3. Overall, the beginning and ending date of the study, and actual benefits paid.
4. Life table assumptions, such as life table, select factors, gender, smoker status, and X factor assumed.

The program consisted of several steps:

1. Initialize each policy record, computing duration, and exposure period.
2. Initialize a random generator over the 0 to 1 uniform distribution.
3. For each X factor individually to be tested (or group of X factors in aggregate), simulate benefits for each selected policy in the block of business, and sum the total dollars of benefits for each simulation.



4. Repeat the simulations for 1000, or as many times (N) as needed, to obtain a stable distribution.
5. Sort the distribution from lowest to highest, and save the Y^{th} percentile result.
6. Repeat these results for each X factor to be tested.

To simulate the benefits, determine the number of years to simulate. For each policy and each year:

- 1) Compute the exposure between 0 and 1 years.
- 2) Compute the duration at the beginning of the year.
- 3) Look up the q_x , select factor, and X factor for that year.
- 4) Let qx be tabular qx times tabular 20-year select factor times X factor.
- 5) Let simulated q_x ($\text{Sim}Q_x$) be a random number.

Now determine whether a death was simulated as follows:

- 1) There is no death if exposure is zero;
- 2) For a whole year of exposure, there was a death if $\text{Sim}Q_x < q_x$.
- 3) For a partial year of exposure, there was a death if $\text{Sim}Q_x < q_x$ times exposure.

If a death is simulated, that policy is ended. Otherwise, an additional year is simulated if the exposure is longer than a year. At the end of the study period for a policy, the total benefit paid is summed into a simulation total, and the next policy is simulated until all selected policies are simulated.

After a simulation run is completed, the resulting N values are sorted from lowest to highest. Assuming that the company's regulator desires the Yth percentile as the rejection region, the Yth percentile can be calculated, and comparisons can be made to actual benefits incurred.

If the number of policies sampled is too small, the Yth percentile of benefits may be zero. For example, consider policies sold to an age 30 female nonsmoker on an age nearest birthday basis. The q_X is 0.00124 and the select factor is .31, for

expected death rate of 0.0003844. If the rejection region is the 95th percentile, and if the expected deaths follow a Poisson distribution, one year of experience for 133 policies still has a 95th percentile of zero. Note that (1 - q) ^ 133 = .95015, whereas (1 - q) ^ 134 = .949785. If the X factor cell being tested consists of only age 30 female non-smokers, sales of at least 134 policies are needed to give a one-year 95th percentile larger than zero dollars.

A simplified example illustrates the Monte Carlo method below (Table 1). The example uses only 25 runs, which is normally too few, but which is adequate to illustrate the method. This example used 4,000 lives over one year. Assume that actual benefits incurred were \$350,000. These benefits are at the 53.8th percentile if the X factor is 1.00, at the 61.5th percentile

if the X factor is 0.70, and at the 76.9th percentile if the X factor is 0.40. If the rejection percentile is set at the 75th percentile, an X factor of 0.40 would be rejected, whereas an X factor of 0.70 or 1.00 would not be rejected.

Monte Carlo simulations provide a simple but powerful way to analyze mortality distributions with limited experience. In the simulation with X = 1.0, the average cost of benefits was \$437,000, ranging from 0 to \$1,020,000. This method took only a couple of days to program, and could be applied to as many samples of business as needed.

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Table 1. Example of Monte Carlo simulations

Order of Runs from Lowest to Highest	Percentile	Benefits X = 0.40	Benefits X = 0.70	Benefits X = 1.00
1	3.8%	0	0	0
2	7.7%	0	0	55,000
3	11.5%	0	0	197,527
4	15.4%	0	0	200,000
5	19.2%	0	90,000	200,000
6	23.1%	0	200,000	225,000
7	26.9%	0	225,000	250,000
8	30.8%	0	250,000	250,000
9	34.6%	0	250,000	260,402
10	38.5%	0	250,000	300,000
11	42.3%	75,000	250,000	300,000
12	46.2%	100,000	260,402	320,000
13	50.0%	115,737	300,000	330,000
14	53.8%	125,000	300,000	350,000
15	57.7%	150,000	320,000	400,000
16	61.5%	200,000	350,000	536,000
17	65.4%	260,402	365,737	590,737
18	69.2%	295,000	375,000	600,000
19	73.1%	300,000	400,000	650,000
20	76.9%	350,000	500,000	700,000
21	80.8%	356,000	506,000	750,000
22	84.6%	400,000	600,000	800,000
23	88.5%	500,000	750,000	815,000
24	92.3%	565,000	815,000	850,000
25	96.2%	600,000	820,000	1,020,000