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**FUNDING FOR INVESTMENT RISKS**

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Panelists: FAYE ALBERT  
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Recorder: STEPHEN J. O'BRIEN

- o This session will discuss funding for both default risks and yield curve risk. The specific topics are:
- Historic experience on bond defaults
  - Update on the NAIC proposals on the MSVR
  - Durational leverage: the hidden risk in duration management
  - Asset/liability management for total return

MR. ROBERT R. REITANO: I'm a 1980 Fellow of the Society and hold a Ph.D. in mathematics from MIT. I'm currently senior financial officer and director of research at John Hancock in Boston, in the investment policy and research department. My current research interests include option pricing, universal life investment policy development, asset/liability management, and in particular, analyzing the risk of nonparallel yield curve shifts on GIC and other duration-matched portfolios. I've published articles in *ARCH*, the *Transactions*, and most recently in the *Journal of Portfolio Management*. I also teach risk theory for the Boston Actuaries' Club.

Our panel discussion encompasses both credit risk and yield curve risk. Faye Albert will lead off with a discussion of her recent work in analyzing historic default experience. Following Faye, Dan O'Sullivan will provide an update on the NAIC activity on the mandatory securities valuation reserve (MSVR). I'll then switch gears from credit risk to yield curve risk and present some new work on duration analysis; specifically, on what I call durational leverage. Finally, Dave Hall will wrap up our program with a discussion of managing for total return.

Our first panelist, Faye Albert, is probably known to you already through her Society activities. She got her Fellowship in 1972, and her first work was primarily in the Education and Examination area. More recently she has been a member of the Research Management Committee and has been chairman of the group overseeing the Society's continuing care retirement communities (CCRC) research project. Faye's research in the investment field stems from her participation in the C-1 Risk Task Force, a subcommittee of the Committee on Valuation and Related Areas. She recently co-authored a paper for the *Transactions* with fellow subcommittee members Irwin T. Vanderhoof, Ralph Verni and Aaron Tenenbein. Here to review this research on historical default experience is Faye Albert.

MS. FAYE ALBERT: My presentation will be a brief review of the results on bond default rates that the C-1 Task Force prepared and which was published in the 1989 *Transactions* -- plus an update on that data that has recently been prepared by Moody's

## PANEL DISCUSSION

and just released. There's been tremendous interest in bond default results by rating class, although, as Ken Stewart indicated at the Investment Section luncheon, junk bonds have been studied to death. We still don't seem to know everything there is to know about bond defaults, and so I'll just review what we do know and hope that maybe I'll inspire some people to do additional research. I think we could still use some.

First, I'd like to point out that default risk is different from the risk of loss due to default experience. Just to emphasize that, I'm going to review how it's different. The amount of loss depends on the seniority of the debt and whether it is secured. For the same company that goes into default, a secured debt may pay off 60-65% of par value, while junior debt may pay only 35-40% of par value, while on even more subordinated issues, you can get nothing back. On the average, what we've noticed is that on default in the past, recoveries have been about 40%. So, default rates are more amenable to direct review than loss experience.

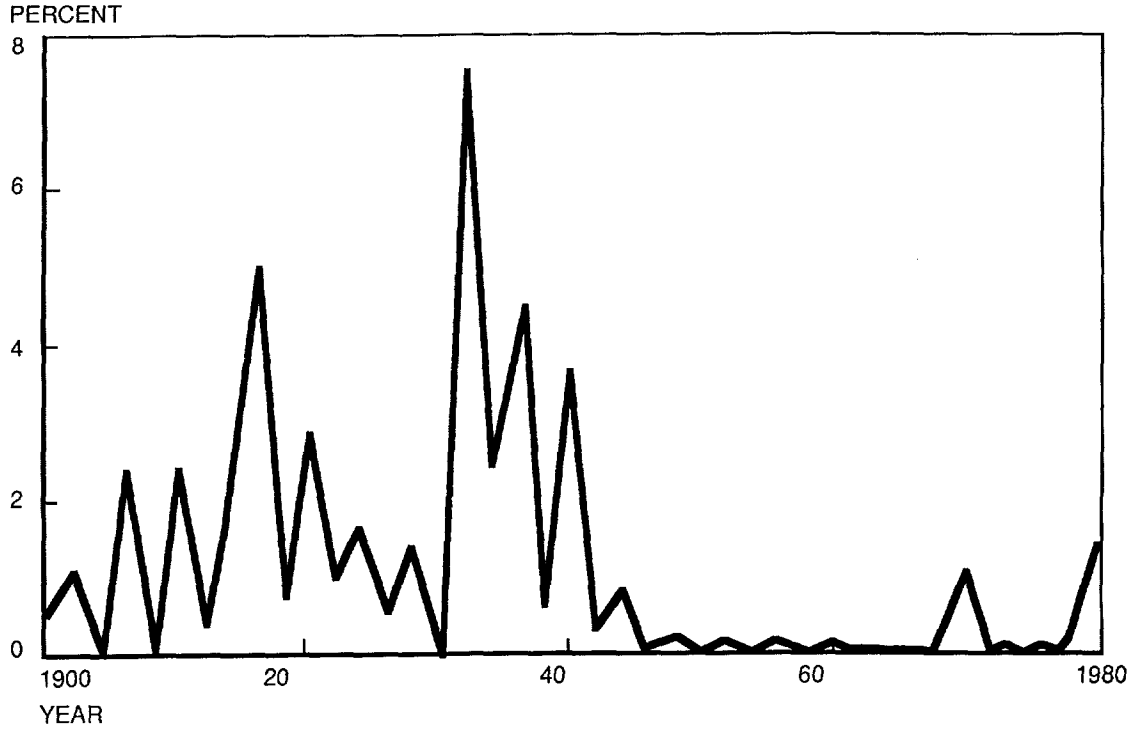
Related to this we have studied defaults based on number rather than amount. Although most investors are more interested in larger losses on large amount issues, if you're trying to assess the credit of a particular company and the probability of whether it's going to go into default, the amount is not going to help identify your risk. So, we're looking at the frequency as opposed to the severity.

Lastly, the most important thing to keep in mind is that we usually look at losses on bond default from the time of issue, that is, from the par value of the bond. However, the time you buy a bond is going to affect what your loss really is. In the study that the C-1 Risk Task Force did, those bonds that are going to go into default during the following year are identified as of January 1. By that time the market has usually discounted the value sufficiently so that you've experienced a significant amount of loss already. On the average, the bond would be about 60 on January 1, as opposed to 40 at the end of the month of default. So, two-thirds of your loss has happened before the bond goes into default, and you can avoid being pointed at as having had a loss due to default by selling it and still have a pretty significant loss. So, timing is everything in investing.

Now I'm going to review some aggregate results to give you a historical perspective on bond default experience. We have studies that were prepared by W. Braddock Hickman under the auspices of the National Bureau of Economic Research since 1900, with experience through 1943. The National Bureau updated this same study; the work was done by Thomas Atkinson for the next 20 years, from 1944 thru 1965. Since 1965 there have been several experience reviews prepared, and what we used in our research were the results as summarized by Professor Edward Altman. The following charts will give you an idea of what our findings were. I think that they're pretty interesting. You can see that the default rates on straight bonds during the early part of the century were volatile and a lot higher than they have been since 1940 (See Chart 1).

Chart 2 shows the level of default since 1940. These represent all outstanding, straight bonds. Since 1940 you can see that the level of default has been very, very low. And then if you like to look at the numbers, which sometimes talk to you a little bit better, you can see in Table 1 in 10-year intervals what the average default rates were on

**DEFAULT RATES**  
Straight Bonds

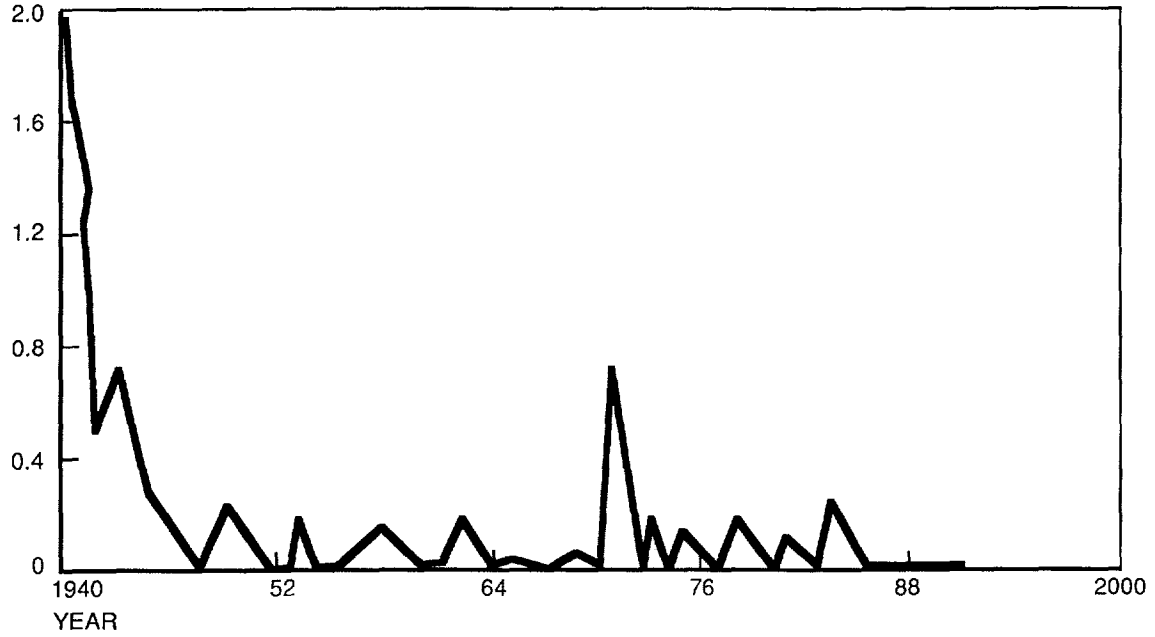


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FUNDING FOR INVESTMENT RISKS  
CHART 1

# DEFAULT RATES Straight Bonds

RATE – percent



3006

PANEL DISCUSSION  
CHART 2

## FUNDING FOR INVESTMENT RISKS

these bonds. The average default rates since 1945 were very low compared to what they were in the first part of the century.

TABLE 1

Default Rates

Period	Average	Standard Deviation
1900/1909	0.890%	0.667%
1910/1919	2.012	1.241
1920/1929	0.952	0.403
1930/1939	3.200	1.843
1940/1949	0.425	0.550
1950/1959	0.041	0.048
1960/1969	0.028	0.044
1970/1979	0.126	0.178
1980/1985	0.120	0.082
1900/1944	1.650	1.472
1945/1985	0.078	0.110

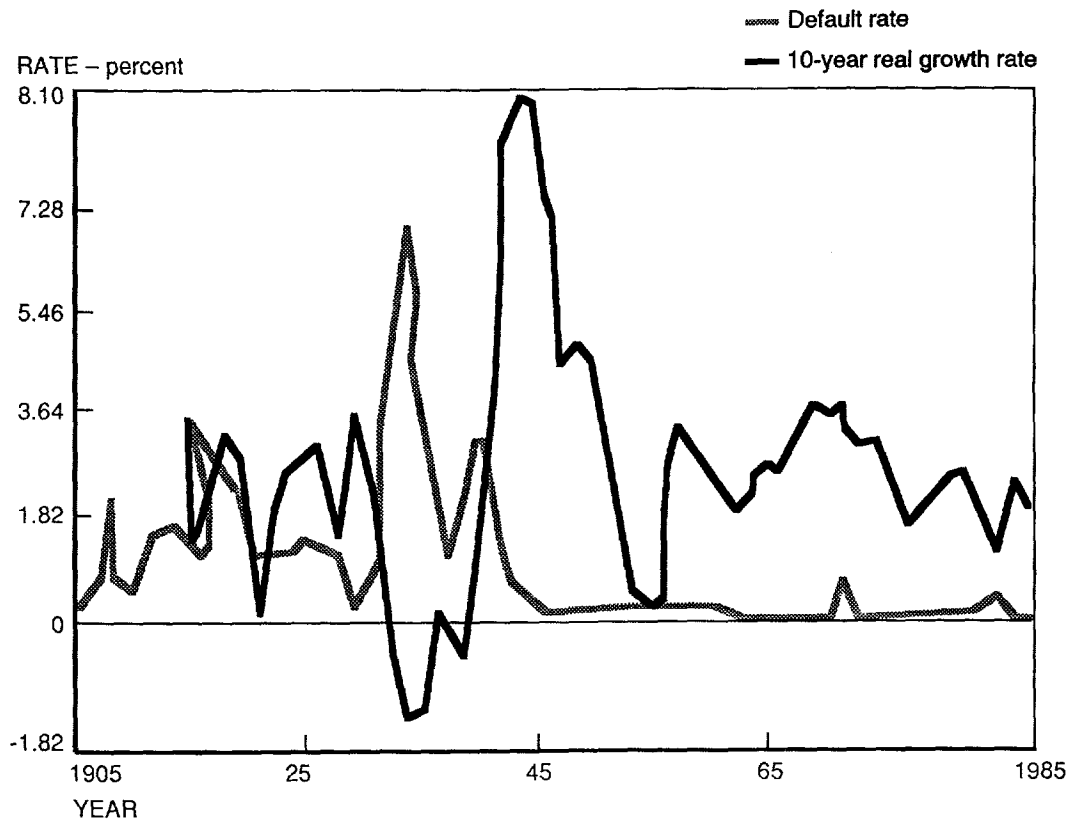
Now this is intended to be an overview, and we are trying to get some historical perspective, but you can't ignore the differences between the data that we have available from these different reports. First of all, there were many fewer defaults during the Atkinson period (1944-65) than there were during the previous period. The Hickman data included 1,200 companies defaulting, whereas the Atkinson data only included 120. The amount of bonds outstanding continued to increase, but the default rate in the later period was very low. The definition of default in the Hickman and Atkinson studies included those issues in which the issuer forced an exchange. The Altman data doesn't consider these transactions to be defaults.

There have been some subsequent studies to identify the effect of the change in definition. Paul Asquith et al found that about 25% of the amount of defaults were excluded in the most recent study that would have been included in the prior study. This appears consistent with the data that we had from Hickman and Atkinson; about 23% of their data included defaults due to forced exchanges.

Well, you might ask why this drastic decrease in default rates occurred, and I wish I could answer you. It's hard to establish a definite reason, but I would like to point out some information with regard to economic measurements. Chart 3 shows the default rate compared with the 10-year real growth rate. Chart 4 considers the default rates compared to the standard deviation in the economic growth rate. I think that this at least suggests that when you have a growing economy and it's relatively stable, aggregate default rates go down.

Let's go on to review some information relating to bond quality. There's increasing interest in assessing the risk of bond issues lately because there are so many more available. In 1970, for example, there were \$116 billion of corporate bonds outstanding,

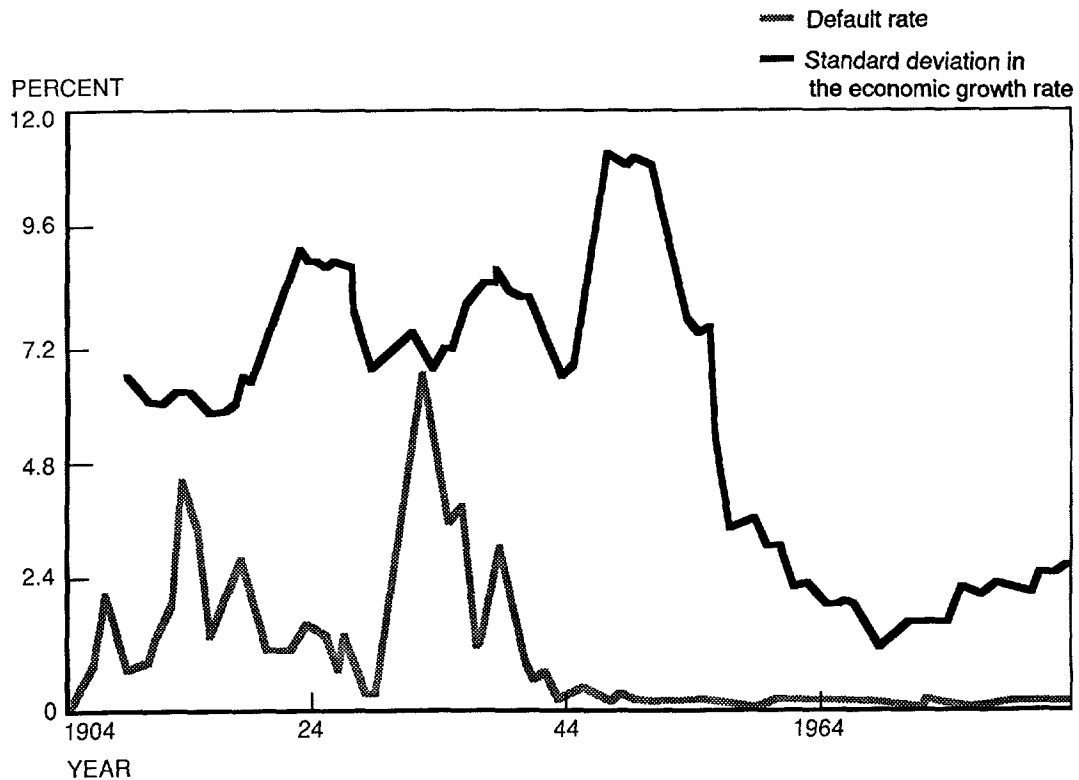
# DEFAULT RATE AND REAL GROWTH RATE



3008

PANEL DISCUSSION  
CHART 3

# DEFAULT RATES VS. STANDARD DEVIATION IN ECONOMIC GROWTH



3009

FUNDING FOR INVESTMENT RISKS  
CHART 4

## PANEL DISCUSSION

and by 1987 this figure was \$650 billion. I've used here the agency rating systems for classification of the bonds into investment grade and below investment grade. There have been more issues of below investment grade issues in the 1980s, and Chart 5 shows these issues as a percentage of total bonds outstanding. Toward the end of 1980 the solid line, or the junk percentage, shows that there is a greater percentage of lower grade bonds being issued now than there have been for a long time.

Chart 6 shows the percentage of bonds outstanding, split between investment and noninvestment grades. This gives you more of a feeling as to how recently, from 1949 to about 1970, a very low percentage of the outstanding bonds were below investment grade. Since then there's been more interest in and a larger percentage of below investment grades, though not anything that we hadn't seen before.

Next, let's take a look at the charts that were just compiled by Moody's for defaults since 1970. Chart 7 shows one-year default rates by bond rating category. I think this is pretty convincing evidence that the rating agencies are able to assess the credit risk of companies and differentiate among different categories of companies. Since 1983 there have been even more gradations of credit and different probabilities of default depending on credit assessment based on the company's financial statements and the on-site reviews.

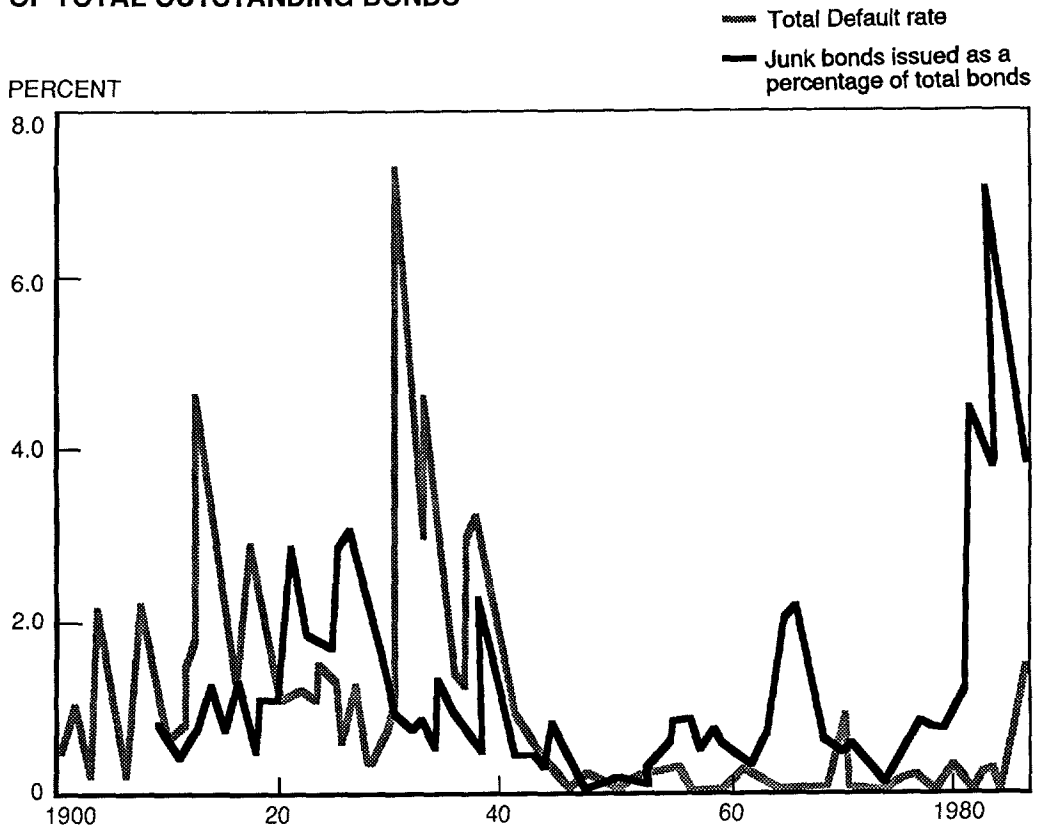
There's been interest in multiple-year defaults, that is, how the incidence of default is going to develop based on the rating at issue. First, the current rating of a bond is the best indication of its probability of default in the short term, not its rating at issue. If you have current information on rating categories of bonds or the rating agencies have seen fit to change their rating, then that really is what you ought to use in trying to assess a probability of default. Chart 8 includes data on multiple-year defaults. It's a cumulative default rate of B-rated bonds versus investment grade issues. And you can see that the investment grade issues look to be increasing fairly steadily while the cumulative default rates of B-rated bonds show that there's a decreasing probability of default as the bond ages.

There's been a lot of interest in what the incidence of default is going to be on lower grade bonds and on different grades of bonds. We don't have a lot of information. This is certainly an area where we could use more research. It does seem interesting to examine the possibility that, if low-grade bonds survive, their rating in the next year will be more likely to go up than the rating of a higher level bond. Perhaps, if lower-rated companies get some money, are doing business efficiently, and are able to service their debt, then they're going to proceed more carefully. We have some information on transition probabilities between rating classes from year to year suggesting this, but it does need more study.

Companies' lives could be considered similar to a survival table with the rating category being something like the issue age and the health of the applicant. The company characteristics would be reassessed based on the rating category each year so that the probability of that bond surviving from year to year really depends mostly on your most current assessment. Companies could be considered to be getting younger, and so there's a difference from mortality tables. Certainly the incidence of bond defaults is not

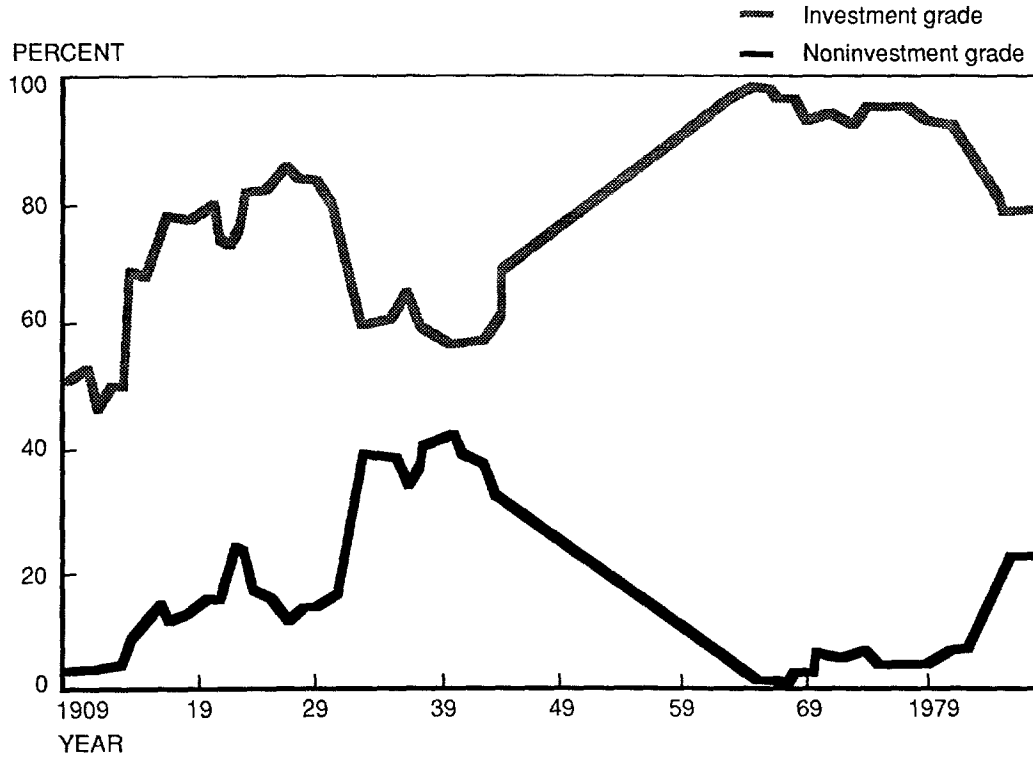


# JUNK BONDS ISSUED AS PERCENTAGE OF TOTAL OUTSTANDING BONDS



FUNDING FOR INVESTMENT RISKS  
CHART 5

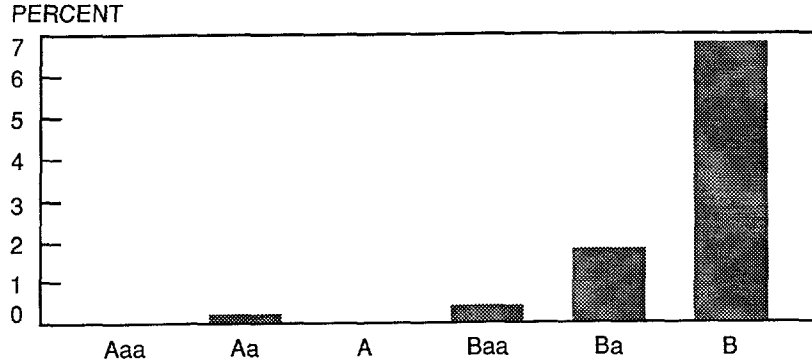
# PERCENTAGE OUTSTANDING BY QUALITY RATING



PANEL DISCUSSION  
CHART 6

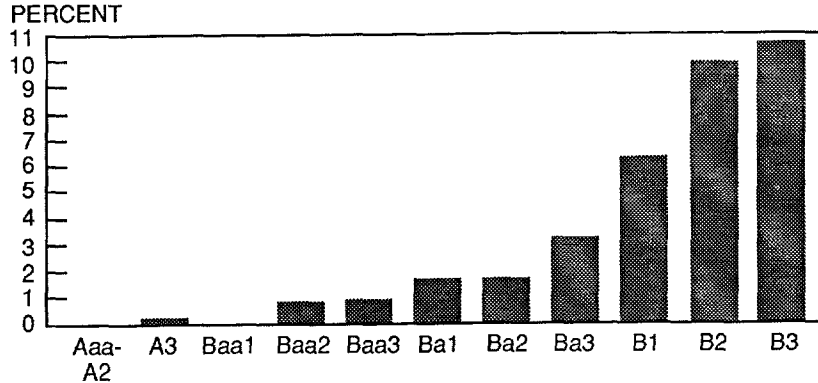
### AVERAGE ONE-YEAR DEFAULT RATES 1970-90

Aaa	0.00%
Aa	0.05%
A	0.01%
Baa	0.18%
Ba	1.66%
B	7.00%



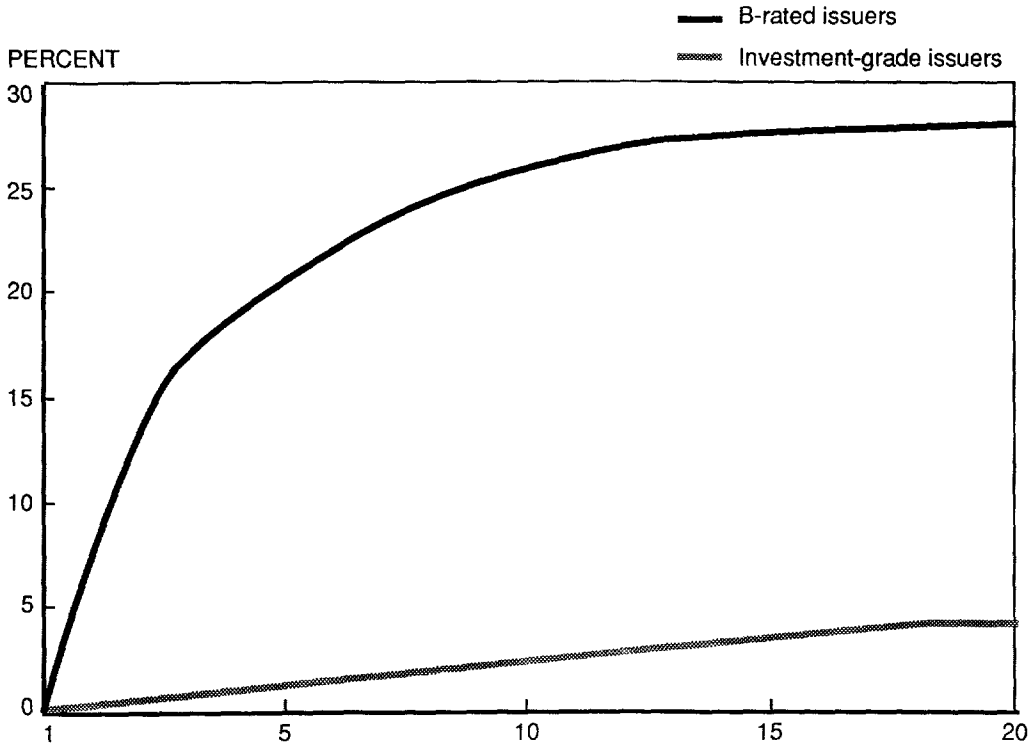
### AVERAGE ONE-YEAR DEFAULT RATES 1983-90

Aaa-A2	0.00%
A3	0.21%
Baa1	0.00%
Baa2	0.46%
Baa3	0.51%
Ba1	1.23%
Ba2	1.25%
Ba3	2.58%
B1	5.92%
B2	9.74%
B3	10.69%



FUNDING FOR INVESTMENT RISKS  
CHART 7

# CUMULATIVE DEFAULT RATES OF B-RATED AND INVESTMENT-GRADE ISSUERS



## FUNDING FOR INVESTMENT RISKS

well known, and the work to date has not explored whether there is a different incidence of default for those lower grade bonds that survive than higher grade bonds that survive.

The last chart I would like to share with you is the standard deviation of default rates (Chart 9). Besides the higher probability of default, the volatility of rates is much greater on lower grade bonds. The pattern of this chart is just about the same as the probability of default.

This suggests the reason why there is as much of a premium as there is on lower grade bonds. There is such a great amount of uncertainty as to what the actual default rate will be in any particular holding period, especially if, as I suggested at the beginning, it is more related to general economic conditions than the particular company. That increased volatility needs to be taken into account in assessing an appropriate price for these bonds.

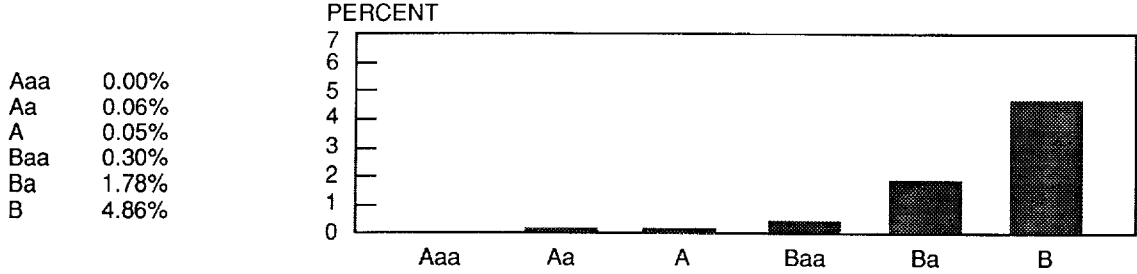
**MR. REITANO:** Dan O'Sullivan is a 1980 Fellow of the Society. He's an actuary with Aetna Life Insurance Company in Hartford where his current responsibilities include reserve valuation research and legislative and regulatory analysis. Recently, these responsibilities have included working on industry task forces on New York Regulations 126 and 128, as well as on the NAIC task force on the MSVR. It's his involvement on the MSVR task force which led to his participation on our panel. Dan will discuss recent changes to the MSVR, as well as what the task force is contemplating for possible future changes.

**MR. DANIEL E. O'SULLIVAN:** As Bob said, I'm going to be describing some of the recent changes before going into some of the changes that are being contemplated currently. However, before I do even that, I'll be going back and describing the current MSVR structure to make sure we have a common base to work from. By current I mean the MSVR that was used at the end of 1989. After describing that base I'll describe some of the changes that the NAIC has already approved for year-end 1990, and then talk about some of the shortcomings of the current MSVR structure and the direction that the NAIC committee is moving in to address those shortcomings.

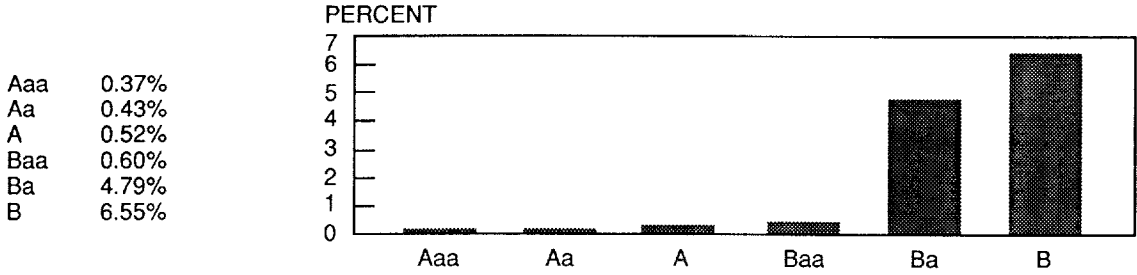
When I talk about those directions you should keep in mind that there's not a consensus among the committee members at this point. What you are hearing is very clearly a work-in-progress report and, if it were to be described by someone else working on it, it might be described somewhat differently. As I go along I'll also try and give some indication as to what the implications are for reserve adequacy testing of some of these factors.

I'll start with the current NAIC rating system as shown in Table 2. This is the yes-no-no system that you've heard so much about. The yes category is essentially all investment grade bonds. There are a few BB- and B-rated bonds that, due to some unusual features or levels of collateralization, do end up as yes bonds, but the category should essentially be thought of as investment grade. The maximum accumulation and annual accumulation factors are shown there for the different ratings of bonds. In addition to those, there's a multiplier that depends upon the level of funding of the MSVR. The

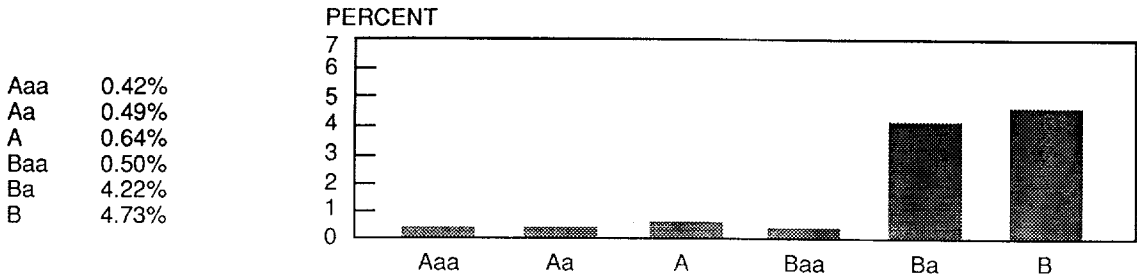
### STANDARD DEVIATION OF ONE-YEAR DEFAULT RATES



### STANDARD DEVIATION OF FIVE-YEAR DEFAULT RATES



### STANDARD DEVIATION OF TEN-YEAR DEFAULT RATES



PANEL DISCUSSION  
CHART 9

## FUNDING FOR INVESTMENT RISKS

annual accumulation is the factor shown times a multiplier which depends on the level of funding of the MSVR at that particular company. If the MSVR is over 75% funded, the multiplier is 1/2. The multiplier increases to the point where if the MSVR is less than 25% funded, the multiplier is three. If the MSVR is not very well funded, you can get some significant increases to it. In addition to this annual accumulation all capital gains and losses are added into the MSVR each year, subject to the maximum shown there and a minimum of zero.

TABLE 2

Current NAIC Rating System

NAIC Designation	Rating Agency Designation	Maximum Accumulation	Annual Accumulation
Yes	AAA, AA, A BBB, BB, B	2%	0.1%
No*	BB, B	10	0.5
No**	CCC and Lower	20	2.0
No	In or Near Default	20	2.0

In terms of reserve adequacy testing, right now Regulation 126 allows use of the MSVR to determine whether the assets are sufficient to mature the liabilities, but only to the extent that the MSVR does not exceed the provision for the cost of default in your projections. For example, if the MSVR is twice what your default cost is, you may only use half of the MSVR in doing your projections.

Earlier this year the NAIC, because of concerns about the range of some of those categories, and concerns generally about insurer solvency, developed a new scale that they'll be using beginning year-end 1990 (Table 3). Instead of four categories there are now six categories. Notice that the yes category has been broken down into Tiers 1 and 2 where all the A ratings are one level, and the BBB, the lowest tier of investment grade bonds, is another level. The annual accumulation factor for all the A- and higher-rated bonds is the same as the yes category was previously. The BBB category now has a 0.2% annual accumulation which is twice what it used to have.

TABLE 3

New NAIC Rating System

NAIC Designation	Rating Agency Designation	Maximum Accumulation	Annual Accumulation
1	AAA, AA, A	1%	0.1%
2	BBB	2	0.2
3	BB	5	0.5
4	B	10	2.0
5	CCC and Lower	20	5.0
6	In or Near Default	20	5.0

## PANEL DISCUSSION

Similarly, the BB- and the B-rated bonds have been broken apart in recognition of the fact that BB bonds have materially different default experience than B-rated bonds. The accumulation factors, you'll note, are quite significant, particularly for the lower rated bonds. The multipliers that I described before of anywhere from 0.5 up to three still apply. While the calculation for the MSVR maximum is done on an aggregate basis, if you were to look at it on a single asset basis and look at a CCC bond, you would be reserving up to 15% of the worth of that bond each year until you got to a more well-funded MSVR position. The NAIC has, however, stipulated that those annual accumulation factors will be phased in over a period of five years on a graded basis. It's not a straight-line phase-in so they'll be moving only a slight amount from the current levels for year-end 1990, and each year that goes by, the increase in the accumulation factor up to the levels shown in this chart will be greater.

The NAIC has also recently signed a contract with Zeta Corporation to help develop and refine the rating system that it has for rating private placements. The NAIC will use on an ongoing basis that system, whatever ratings it gets from the rating agencies, and other judgmental factors to determine into what category a given bond falls. In that process it will have a strict constraint, which it hasn't had in the past, that any bond rated by a public rating agency cannot receive a rating from the NAIC higher than the highest rating agency rating.

That takes us up to where we are currently, and going into the future we're potentially looking at some structural changes to the MSVR. The changes that the NAIC has adopted for the end of 1990 are refinements to the structure that has been used for years. I think the easiest way to talk about the potential structural changes is to talk about them in two distinct pieces. While the MSVR is viewed as a default reserve, it is really acting as more than a default reserve currently. You have credit losses which the MSVR acts as a buffer against. You also have noncredit gains and losses. To see what I mean by that, a simple example is probably easiest.

Assume you have \$100 of assets and liabilities and that your asset is a 10% par bond with three years to maturity, and current interest rates are 12%. Further, assume that when you sell the bond it is the same credit quality as when you bought it. You've experienced no credit-related loss on that bond. However, because of the change in interest rates, you only receive \$95.20 for the bond. That's a capital loss that's going to go into the MSVR, but it's a noncredit loss. One thing that the MSVR does which is very beneficial is to capture that loss and keep the assets and the liabilities on the same basis. Without the MSVR, that loss would generate an operating loss, and you would have to come up with another \$4.80 of surplus in order to get your assets and liabilities balanced again.

Going forward, assume that the contract was being credited the full 10% that the original asset was earning, so we're ignoring expenses and profits. You are then crediting \$10 of interest each year, and you're earning 12% interest on the \$95.20 which is more than enough to meet your contractual payments. What you want to do with that extra income is write off the \$4.80 asset that you, in effect, set up in the MSVR. Even though the MSVR has a minimum of zero, conceptually, for this one transaction, you've set up an asset. The way the MSVR currently operates, that asset is never amortized. It



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is only offset by future annual accumulations or capital gains that are generated from future interest rate movements. This results in an asset that remains in place, and extra earnings in future years.

Some of the other shortcomings in terms of the noncredit gains and losses, are that mortgage loans and Treasuries are not included. Clearly, you wouldn't include Treasuries if you view this as a default reserve, but to the extent that you're capturing the capital gains and losses from movements in interest rates and want to spread those back over time, Treasuries should be included as well. Second, as I was just describing in the example, there's no amortization of the amount in reserve so that you aren't getting the proper incidence of earnings. You're distorting your earnings picture.

Also, the MSVR currently has a maximum and a minimum. That's appropriate for a default reserve; it's not appropriate for this noncredit type of reserve. That reserve should not be bounded, either capped or with a floor. It should depend entirely on the activity that you've had in trading your assets and what kinds of gains and losses you've generated from that. And, finally, given that there's no distinction between credit and noncredit gains and losses, you have no basis for treating the two types of losses separately. By having just one reserve you aren't set up to handle the noncredit gains and losses on a spread basis, as I've described.

On the credit loss side, total credit losses really go well beyond actual defaults. This is one of the issues that we'll be addressing in looking at the MSVR. However, a more significant problem is that there's a major asset category for life insurers -- mortgage loans -- that isn't included. Clearly, as all the recent press has shown, these are subject to default. Also, as mentioned before, you don't have the distinction between the credit and noncredit gains and losses.

We are currently considering two distinct reserves just for the fixed income investments of an insurance company. I am not at this point describing what we've discussed for equity investments. The first reserve would be what we are calling an interest maintenance reserve. It would cover all fixed income investments. There would be no maximum or minimum, and it would amortize any gain or loss in a given year over time. There would be no set annual contribution as there is with the current MSVR. You would merely put in any of the gains or losses on fixed income investments. It's being called the interest maintenance reserve because what you're effectively doing is maintaining your investment income at the level that the original asset was generating. Again, these are noncredit gains and losses, so that's an appropriate treatment. By doing that you're keeping your earned investment income consistent with your interest credited to your liabilities so you aren't generating spurious gains and losses between years.

We picture this reserve being used for reserve adequacy testing, and in fact, to the extent that there is no minimum, that is, this reserve can go negative or be set up as an asset, it would be critical that the reserve be used for reserve adequacy testing purposes. Otherwise the balance sheet could be seriously overstated.

I have a brief illustration, again working off of the three-year bond that I described, of how the amortization would work (Table 4). Having sold the three-year bond, I then

## PANEL DISCUSSION

assume that you repurchase the same bond. This means you've bought a bond at a discount. You're getting the 10% annual coupon, and your interest credited to contract-holders is that same \$10. Having bought the bond at a discount, you're accruing the discount each year. So, only going across the first three columns, your income exceeds your outgo, or what you've credited. You would amortize the \$4.80 asset that you set up in the MSVR in a schedule that exactly mirrors the accrual of discount under the bond. This is a very simple example that implies that the appropriate amortization of this interest maintenance reserve is over the remaining life of the asset that was sold, and it is an escalating amount each year.

TABLE 4

Amortization of Interest Maintenance Reserve -- Illustration

Year	Coupon Income	Accrual of Discount	Interest Credited	Amortization of IMR
1	\$10	\$1.42	\$10	\$1.42
2	10	1.59	10	1.59
3	10	1.79	10	1.79

- Assumption:
- 1) Sell a 10% annual coupon par bond with three years to maturity in a 12% market.
  - 2) Repurchase same bond.

We have talked about, but haven't reached any conclusions regarding other methods of amortization. Some feel that doing this calculation on a seriatim basis for all bonds that have been sold is going to be too onerous, and so we've discussed a straight-line basis, possibly on an aggregate or maybe on a seriatim basis.

The second reserve, reflecting how the MSVR is typically viewed now, is an asset default reserve. There obviously continues to be a need for that. There's considerable disagreement within the group about, theoretically, how this should operate. On a practical basis there's not quite as much disagreement. It would clearly have to include mortgage loans, and the committee overall is looking at any other assets that have a fixed component to them, such as Schedule BA assets. The annual contribution and the maximum would vary by the quality of the asset and, similar to the current MSVR operation, the contribution would vary based on the funding level using the multiplier described earlier. The committee has not yet gotten into a discussion of what numbers are appropriate in terms of maximums or annual contribution levels. One of our hopes is that the asset default study being undertaken by the Society and the ACLI will give us some good data to use in looking at what levels are appropriate.

Another critical element of that asset default reserve is that it recognizes other provisions for default that exist in the company. There are two primary areas where there is currently provision for default. One is in the basic contract reserves where the statutory minimum has an implicit provision for default through the use of a very conservative interest rate in the calculation of those reserves. Second, the valuation of the asset itself,

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using a higher interest rate to discount the cash flows that are projected, has an implicit assumption of default. The lower the quality of the asset, the higher the asset valuation interest rate and therefore the higher the assumption of default. Our concern is that these other areas, where default has been provided for in valuing the assets and the liabilities of the company, not be ignored in setting the level of the MSVR portion of the asset default provision. If they are ignored the requirement would be too onerous.

If one assumes that those other areas where default is provided for make adequate provision for default over time, there would still be some purpose to having this reserve, namely, to account for timing differences. By that I mean that in valuing basic contract reserves there's a provision for default, a level provision for default each year. The actual incidence of default is very volatile. Assume in the first couple of years the portfolio experiences no defaults. If the reserves are rolled forward and release the provision for default to earnings, then you're building in a loss in the future years, if your basic assumption as to your ultimate default experience hasn't changed, even if your original pricing assumption was adequate. What the MSVR would do is capture the amount that gets released out of the basic contract reserves, and then when the default was experienced it would be absorbed by this reserve.

The final point is that the level of protection afforded by the total provision for default should be consistent with the provision for other risks such as mortality and morbidity on a statutory basis. It should be conservative, and it should recognize the volatility of the default risk, but it should not be something that attempts to protect against catastrophic loss.

As I said, I am not going into the equity side at this point, because the committee is much less in agreement on just where that is at the current time. If any of you are interested in either seeing what the Society and the ACLI are doing on the asset default study or even possibly contributing to that study on an ongoing basis, you can contact Warren Luckner at the Society office. He'll give you information on that. While we've had a lot of discussion here on conceptually what we think should happen, ultimately no one's going to agree to it until they see what the numbers are producing, and the only way to get good numbers is to have a good experience study.

MR. REITANO: My discussion will be based on a recent paper in the *Journal of Portfolio Management* which was entitled "Non-Parallel Yield Curve Shifts and Durational Leverage." The concept of durational leverage was an outgrowth of an investigation I conducted into a general multivariate duration model. That model was documented in a fair amount of detail in a recent *ARCH* paper and will be the subject of a forthcoming Society seminar.

The purpose of taking a multivariate approach to duration analysis was to be able to understand and quantify the effect of general nonparallel yield curve shifts on a portfolio's value. As you all know, traditional duration analysis assumes parallel yield curve shifts. I'd like to review an example of durational leverage in a fairly simple portfolio, which I think illustrates all of the features of a more general portfolio that you might have back at your home office.

## PANEL DISCUSSION

For assets we have a 10-year 12% coupon bond of \$50 million, and a six-month commercial paper position of \$17.5 million. We also have a single liability payment of \$100 million due in year five, such as a five-year guaranteed investment contract, and Chart 10 is the corresponding cash-flow projection. We don't have options or interest rate sensitivities in the cash flows of this simple portfolio, so it's pretty easy to take a look at it. It's a familiar profile in asset/liability management, and one that is often called a "barbelled" portfolio, or simply, a barbell.

The yield curve that I'm going to use is made up of three pivotal points: a six-month, five-year and 10-year yield point. The values are 7.5%, 9%, and 10% on a semiannual or bond rate basis. For my purposes, I'm going to think of the yield curve as a vector because something that I'll be doing later lends itself very naturally to a vector interpretation. The yield curve graph is again fairly simple (Chart 11). In most applications you would probably have another three or four pivotal points. For example, you might be keying off the Treasuries or some other market observations. In the analysis that we do at the John Hancock, we found that with about nine or so pivotal maturity points, we can do virtually any valuation. Naturally, the assumption is made that yields at other maturities are just interpolated from the given values. My calculations are based on linear interpolation of the bond yields and conversion to spot rates in the usual way.

The traditional price function model reflects a parallel shift assumption as noted earlier.

Yield Curve Shift:

$$(0.75, 0.090, 0.100) \rightarrow (0.75+i, 0.090+i, 0.100+i)$$

Price Function:

$$P(i)$$

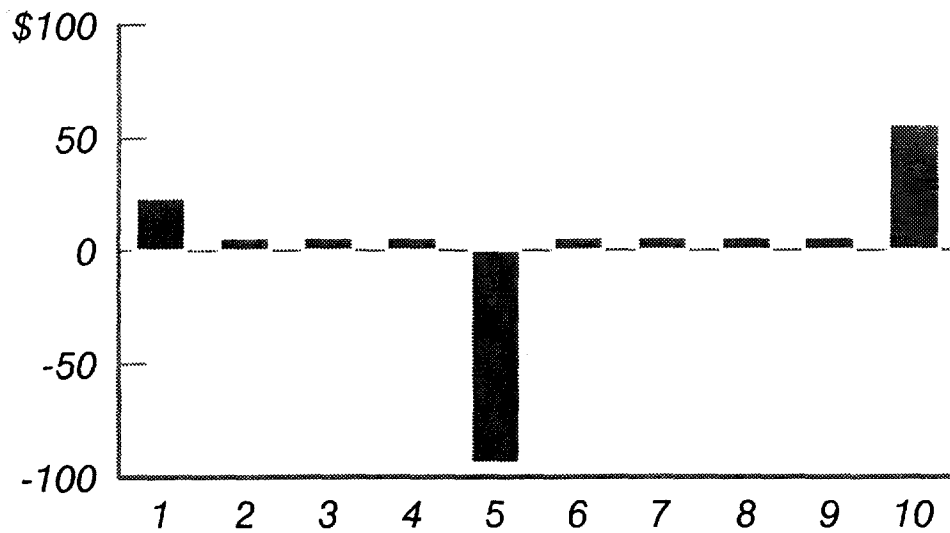
Modified Duration:

$$D = -P'(0)/P(0)$$

The variable in this function equals the amount of parallel displacement. Duration is then defined as minus one times the ratio of the value of the derivative to that of the price function. The negative sign is used to make durations positive on assets like bonds and to be more in line with the Macaulay kind of formula. Usually you can't calculate the derivative exactly because of options in the portfolio or some other kind of interest rate sensitivities in the cash flows. So the typical thing to do is to approximate the derivative using a finite difference formula.

Chart 12 gives an example of a central difference formula for the derivative approximation, but basically it's very simple to implement. You don't even have to think in terms of derivatives. You simply value the portfolio on three yield curves: the given yield curve, the yield curve shifted up by some amount,  $i$ , and one shifted down by the same amount. You then apply this central difference formula or a corresponding forward difference formula that perhaps many of you use instead, and you get the derivative approximation. Typically, the interest shift  $i$ , if set equal to 5-10 basis points, gets you very good approximations to the actual duration.

# Cash Flow

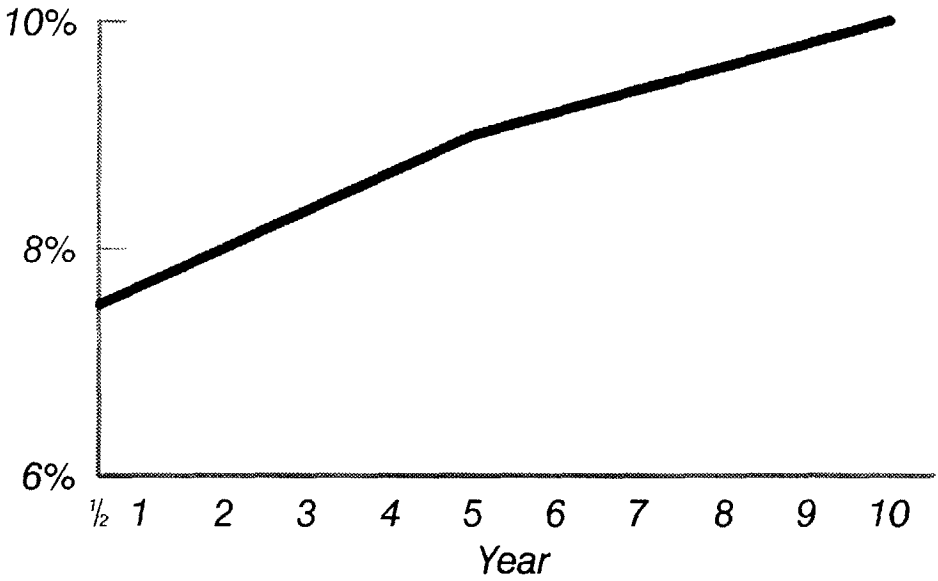


FUNDING FOR INVESTMENT RISKS  
CHART 10

PANEL DISCUSSION

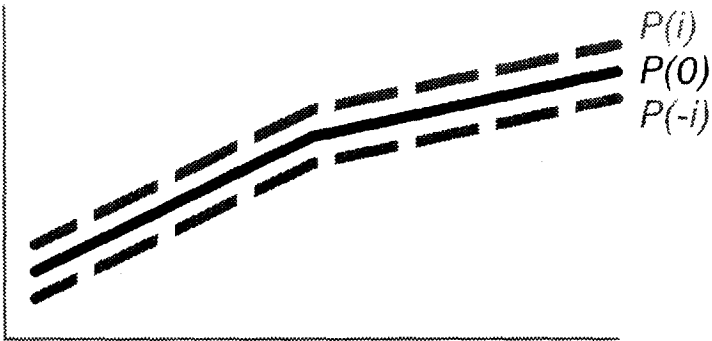
CHART 11

# Yield Curve



# Approximating Duration

$$D \approx \frac{P(-i) - P(i)}{2i P(0)}$$



## PANEL DISCUSSION

If we look back at our examples of assets and liabilities, and use this kind of an approximation, both have a duration of around 4.86, and the resulting surplus position, which has a market value of \$9.28 million, has a duration of 4.85. So, most people would consider this to be a duration-matched portfolio, at least in the classical sense.

The reason that people are interested in duration generally is because duration allows you to approximate what happens to the price when interest rates move. This was very handy in the bond market, for example, where if interest rates shifted by 20 basis points, bond traders could do a really fast mental calculation of what happened to the price of their bond. The approximation formula, again most of you have seen this, gives the new price of the bond or portfolio after we shift the yield curve by  $i$ . The new price is approximately equal to the old price times a discount or an accumulation factor, equal to one minus the product of the duration value and the change in interest rates,  $i$ .

$$\begin{aligned} \text{General: } & P(i) \approx P(0)(1 - Di) \\ \text{Example: } & S(i) \approx 9.28(1 - 4.85i) \end{aligned}$$

For our example of surplus, where we have \$9.28 million for an initial market value and 4.85 for duration, this approximation tells you that if interest rates go up, surplus is going to drop. Conversely, if rates go down, surplus will rise. This formula actually gives you a very good approximation for small changes in the yield curve. After all, this approximation is based on calculus, and that it works well should not be surprising.

For example, if the yield curve shifts up by 50 basis points, the short rate moves up to 8% and so forth, the approximation that we get with this formula for surplus is a market value of about \$9.06 million, or a decrease of about 2.4%.

$$\begin{aligned} & i = 50 \text{ basis points} \\ & (0.75, 0.090, 0.100) \rightarrow (0.80, 0.095, 0.105) \end{aligned}$$

Approximation:	\$9.06 Million	-2.43%
Exact:	\$9.07 Million	-2.24%

If you do the exact calculation on that shifted yield curve, again interpolating and converting to spot rates, you see that this is a pretty good approximation. Surplus didn't drop as much as the duration approximation implied, and a number of you who are ahead of me know that's because of convexity, which I won't talk about. The exact surplus decrease equals 2.24%.

I'd like to next look at what happens to surplus under a nonparallel shift, a relatively small one.

Shift:	(-2 basis points, 17 basis points, -18 basis points)	
	(0.75, 0.090, 0.100) → (0.0748, 0.0917, 0.0982)	
Exact:	\$10.44 Million	+ 12.53%



## FUNDING FOR INVESTMENT RISKS

Under this nonparallel shift, we'll drop the short rate a couple of basis points, increase the five-year by 17 basis points, and decrease the 10-year by 18 basis point. You can see here the pivotal values of the shifted yield curve. This would not be considered a very large shift, but I'll talk about how I measure the size of these shifts a little bit later. However, this shift has a very large effect on surplus. Any of you who work in this field can probably think this through and would find it no surprise that this yield curve shift had a fairly large, positive effect on surplus, and, in particular, it increased surplus about 12.5%.

From the perspective of parallel yield curve shifts, this is an extraordinary kind of yield curve shift because surplus has a duration of a little under five, and typically you don't expect changes of the order of magnitude of 12.5% except with yield curve shifts of 250 basis points or so. This is really an important observation. When you're dealing with nonparallel shifts you find that the way you measure yield curve shifts and define the kinds of yield curve shifts that you need to worry about is very different than when you restrict your attention to only parallel yield curve shifts.

To investigate this result, we need a more general model of yield curve shifts. The model that I use reflects the assumption that each of the pivotal points can potentially move by a different amount.

Yield Curve Shift:

$$(0.075, 0.090, 0.100) \rightarrow (0.075+i, 0.090+j, 0.100+k)$$

Price Function:

$$P(i,j,k)$$

Partial Durations:

$$D_1 = -P_1(0,0,0)/P(0,0,0)$$

$$D_2 = -P_2(0,0,0)/P(0,0,0)$$

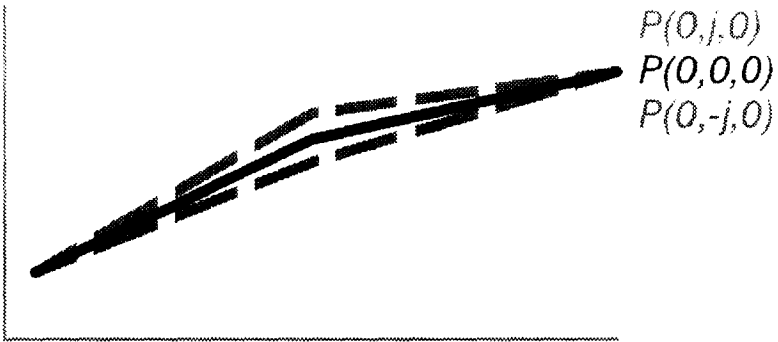
$$D_3 = -P_3(0,0,0)/P(0,0,0)$$

So, I'll model the short rate as moving by some amount,  $i$ , the five-year rate by  $j$ , and the 10-year rate by  $k$ . Correspondingly, I can think of the price function as a function of those three variables, and I'll define what I call partial durations in terms of this function's partial derivatives. Now you may or may not be comfortable with partial derivatives, but the point is that to actually calculate these values is no harder than to calculate the duration measure itself.

For example, to calculate the second partial duration, you use a formula that looks virtually identical to the earlier one for duration (Chart 13). The only difference is that when you shift the yield curve up and down, you only shift the pivotal point that you are interested in. Here, because we're looking at the partial duration associated with the five-year pivotal point, we only bump that value up and down. Again, by interpolation you'll note that all other yields change by various amounts up to the six-month and 10-year pivotal rates, which don't change. This kind of a calculation is actually quite simple with an option pricing model or any other kind of valuation system you have.

# Approximating Partial Durations

$$D_2 \approx \frac{P(0, -j, 0) - P(0, j, 0)}{2j P(0, 0, 0)}$$



## FUNDING FOR INVESTMENT RISKS

Getting back to my example, Table 5 shows the way the duration measures of the assets, liabilities and surplus decompose into these partial durations. The first thing that you'll notice is that the partial durations add up to the duration value. That's no coincidence; that's a theorem. It will turn out, however, that depending on how much you bump the yield curve when you're approximating the various values, that you may find small orders of discrepancy between the duration approximation and the sum of the partial duration approximations. The theorem says that by decreasing the amount that you move the yield curve, you can get these approximations as close as you want.

TABLE 5

Partial Durations

	Assets	Liabilities	Surplus
$D_1$	0.14	-0.45	4.20
$D_2$	0.17	5.31	-35.23
$D_3$	4.55	0	35.88
	4.86	4.86	4.85

The second observation is that the surplus position, while having a duration of only 4.85, has an enormous amount of yield curve sensitivity to the five- and the ten-year yield point. Because they're of opposite signs, the partial durations add up to what looks like a relatively harmless value. However, the actual sensitivity to shifts in the 10-year point alone corresponds to a duration of almost 36, while the sensitivity to the 5-year point is about -35.

The way that we use these partial durations is a natural generalization of the earlier approximation formula. Namely, that we have a separate factor associated with each of the yield curve shifts, reflecting the various partial durations. This formula reduces to the earlier one for parallel shifts because, as I remarked earlier, the partial durations add up to duration. That is, if  $i$ ,  $j$  and  $k$  are the same, giving a nice, neat parallel shift, this formula collapses to the other formula with a bit of algebraic manipulation.

$$\text{General: } P(i, j, k) \approx P(0,0,0) (1 - D_1i - D_2j - D_3k)$$

$$\text{Example: } S(i, j, k) \approx 9.28 (1 - 4.20i + 35.23j - 35.88k)$$

If we apply this formula to that earlier nonparallel shift example, we get very good precision here. It's exact to the number of decimal places that you see. Namely, this model would have predicted the 12.5% change in surplus, given that yield curve shift. The reason that the approximation is so good here is because the actual shift values were very small. Predictably, the first order approximation worked very well.

$$(i, j, k) = (-2 \text{ basis points}, 17 \text{ basis points}, -18 \text{ basis points})$$

$$(0.075, 0.090, 0.100) \rightarrow (0.0748, 0.0917, 0.0982)$$

Approximation:	\$10.44 Million	+ 12.53%
Exact:	\$10.44 Million	+ 12.53%

## PANEL DISCUSSION

This approach gives you one way of thinking about general yield curve shifts, namely, as general shift vectors. However, it might be of some interest to try to translate a general shift back into a comparable parallel yield curve shift, or one that I call the equivalent parallel shift. The basic question is this: If we start with the general yield curve shift,  $(i, j, k)$ , what kind of a parallel shift would be equivalent to it from a durational point of view? The resulting formula is fairly simple, and what you see from the example is that the equivalent parallel shift is a weighted average of  $i, j$  and  $k$ . That is, the coefficients add up to one. This is true in general because the partial durations sum to the duration.

$$\text{General: } i^E = \frac{D_1 i + D_2 j + D_3 k}{D}$$

$$\text{Example: } i^E = 0.87i - 7.27j + 7.40k$$

Note that the weights on  $j$  and  $k$  are pretty large because of the fact that those partial durations were pretty large relative to duration. If we go back to that nonparallel shift that we looked at earlier, and plug it into this formula, we find that a relatively small nonparallel shift is equivalent, from a durational perspective, to a parallel shift of negative 258 basis points. This result is consistent with the observation I made earlier, that because surplus moved by such a large amount, we really expected that in some sense the given shift was comparable to a huge parallel yield curve shift. In fact, that nonparallel yield curve shift is huge from the perspective of durational equivalence.

$$\begin{aligned}(i, j, k) &= (-2 \text{ basis points}, 17 \text{ basis points}, -18 \text{ basis points}) \\ i^E &= 0.87(-.0002) - 7.27(0.0017) + 7.40(-0.0018) \\ &= -0.0258 \\ &= -258 \text{ basis points}\end{aligned}$$

The next question is: How big can this equivalent parallel shift be? That's an obvious question for any of us trying to manage portfolios. Well, the first answer is that it depends on how big the nonparallel shift is. If we double each of the individual shifts, for example doubling  $-2$  to  $-4$  basis points, and so forth, then by definition you can see that the equivalent parallel shift is going to double. So, when we talk about how big an equivalent parallel shift can be, we have to put a restriction on how big of a nonparallel yield curve shift that we want to consider.

The basic answer is then that the equivalent parallel shift can be anywhere in an interval, the size of which depends on the size of the original shift, and the size of  $L$ , or what I call the durational leverage. We'll see that the durational leverage is very easy to calculate. For the size of the original shift, denoted by the absolute value of the  $(i, j, k)$  vector, many of you will recognize this formula as the length of that vector, or the square root of the sum of the components squared. This definition gives a convenient way of calculating and talking about how big a general yield curve shift is if that yield curve shift is not parallel.

## FUNDING FOR INVESTMENT RISKS

$$-L|(i, j, k)| \leq i^E \leq L|(i, j, k)|$$

$$\begin{aligned} L &= \text{Durational Leverage} \\ |(i, j, k)| &= \sqrt{i^2 + j^2 + k^2} \\ &= \text{Length of } (i, j, k) \end{aligned}$$

In our example,  $L$  turns out to be pretty big. That is, the durational leverage in our example turns out to be over 10.

$$\begin{aligned} \text{General: } L &= |(D_1, D_2, D_3)| / D \\ \text{Example: } L &= |(4.20, -35.23, 35.88)| / 4.85 \\ &= 50.46 / 4.85 \\ &= 10.40 \end{aligned}$$

Basically, the durational leverage equals the length of the total duration vector, or the vector whose components equal partial durations, divided by duration. Our portfolio has a leverage value of about 10.4. What this means is that the equivalent parallel shift for this portfolio can be as big as about 10.4 times the length of the nonparallel shift, and as small as minus one times that value. For example, if we're only looking at nonparallel shifts that are a hundred basis points in length, the equivalent parallel shift, which reflects the effect on our portfolio, can be about 10.4 times that, or as big as 1040 basis points, and as small as minus 1040 basis points.

$$\text{Example: } -10.4|(i,j,k)| \leq i^E \leq 10.4|(i,j,k)|$$

Another question that would naturally arise is: what kinds of yield curve shifts give these extremes? In other words, what kind of yield curve shifts will give equivalent parallel shifts that are at the end points of that interval? As it turns out, it's not difficult to define such shifts. Namely, when the yield curve shift vector is proportional to the vector of partial durations, or total duration vector, it will be the worst or best yield curve shift for the portfolio. By worst and best I mean in terms of being either very unfavorable or very favorable. The example that I showed at the beginning of my presentation is such an example. It equalled a relatively small negative percentage of the vector of partial durations, as you can see.

$$\text{When } (i,j,k) \sim (D_1, D_2, D_3)!!$$

$$\begin{aligned} \text{Example: } &(-0.0002, 0.0017, -0.0018) = -0.000049 \times (4.20, -35.23, 35.88) \\ &|(-0.0002, 0.0017, -0.0018)| = 0.00248 \\ &i^E = -0.0258 \\ &= -10.4 \times 0.00248 \end{aligned}$$

This example shift has a length of 25 basis points, and it has an equivalent parallel shift that's exactly equal to 10.4, the leverage value, times its length, times -1, or minus 258 basis points. If we had used a small positive percentage of the total duration vector, the right end point of that interval would have been obtained.

## PANEL DISCUSSION

As it turns out, there are a number of other applications of what I call multivariate duration analysis. Here I've been using a single yield curve for assets and liabilities. More generally, this model reflects the assumption that asset and liability yield curves move together. However, it's natural to be concerned about spread risk, namely, the risk that asset and liability yield curves move differently. For example, you might use a Aaa liability curve and a Baa asset curve, and be concerned about spreads widening. I wrote a paper that's due out in the spring in the *Journal of Portfolio Management* on quantifying that risk. It's a very natural extension of what I have been talking about and demonstrates that you have significant spread leverage as soon as you start moving the yield curves separately. In that case, you can have even larger effects on surplus than when you moved them together.

This approach also extends to convexity and immunization theory. Logically enough, once you start thinking about portfolio sensitivity to different kinds of shifts, you need to reflect convexity. In addition, it turns out that you can develop a whole immunization theory. That is, you can develop a criterion by which you'll be immunized against any specified kind of shift, and you can also develop a criterion under which you'll be immunized against all kinds of yield curve shifts. I've written a number of papers on this issue which will be appearing over the next year or so. Logically, any theory relating to asset/liability management is immediately applicable to hedging as well.

Our next speaker is a 1979 Fellow. Dave Hall is vice president and actuary in the investment division of the Hartford Life Insurance Companies where he directs both the portfolio management and asset/liability management functions. The Hartford Life Companies are comprised of several companies within the ITT Hartford Insurance Group. Life invested assets as of June 30 exceeded \$12 billion, and most are associated with various guaranteed and variable annuity products. Dave also serves as editor for *Risks and Rewards*, the newsletter of the Investment Section, a position he's held since the newsletter's inception in 1988. He was recently elected to the council of the Investment Section. Dave will discuss total return analysis as it relates to asset/liability management.

**MR. DAVID A. HALL:** What do I mean by total return management? Almost all asset classes have two dimensions to return: yield and changes in their market value. For many interest-sensitive products most of us are familiar with the concept of spread management, which is managing a credited rate on liabilities in relation to the emerging asset yields. For some products, however, this doesn't go far enough if the ultimate profitability is to be controlled or optimized. In addition to yield, the management of relative market values is needed. Factors that affect the development of market values are the passage of time, changing market yield levels, embedded options, changes in interest rate volatility, changes in interest rate spreads, be they related to credit or option spread, supply and demand, and a host of other items.

Pricing actuaries are most familiar with and tend to focus on yield, and probably often relay their investment objectives to their investment managers primarily in terms of yield or perhaps yield and duration. Many actuaries are somewhat familiar with the concept of matching durations of assets and liabilities, and there's a growing awareness of the relevance of convexity in asset/liability management, though I sense this is still somewhat

## FUNDING FOR INVESTMENT RISKS

a murky notion for many of us. I plan to address this topic from a perspective of evaluating total return performance of product surplus, defined as assets minus liabilities, on a market value basis over a defined holding period.

For my example I've chosen a portfolio of both immediate annuities and structured settlements. Chart 14 is the projected cash flows of the liabilities. The immediate annuity portion is a life annuity to a male, age 65, with 10 years of payment certain. Overlaid on top of that are cash flow spikes that occur every five years that those of you who are involved in the structured settlement business will find familiar. The effective duration of this string of liability cash flows is about 7.3 years and the convexity measure is .94 which is probably best understood in the context of Chart 15.

Here's how I'm defining present value of liabilities as market interest rates change. Again, we're talking about pretty long duration liabilities, so I'm going to be speaking of interest rate changes at the long end of the yield curve, which are less susceptible to some of the things that Bob spoke about. The duration of 7.3 at zero interest rate change (in the center of Chart 15) means essentially that the slope of the liability return line is about 7.3 at that point. The positive convexity of these can be seen because the liability return line curves up and away from the tangent in both directions. One way of thinking of convexity is that it is the duration of the duration. It's the rate of change in the duration as interest rates change. So, here you can see as rates fall, not only does the value rise, but the slope of that line (the duration) rises as well, and, similarly, as interest rates rise, the curve gets flatter as the duration declines.

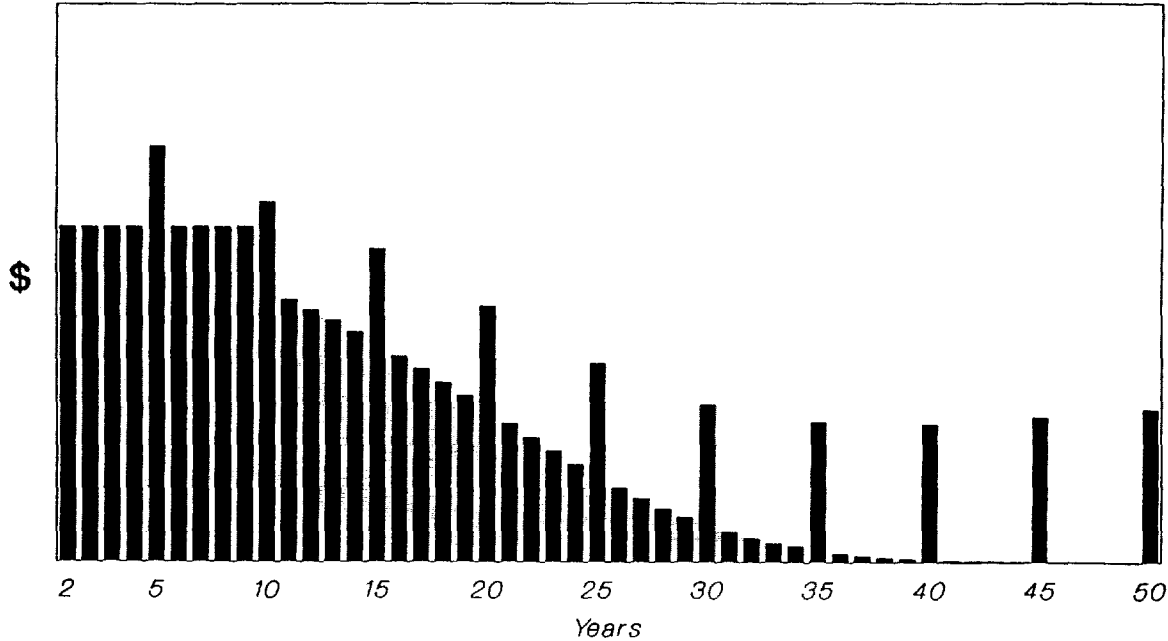
To measure the total return of liabilities I'm choosing a six-month holding period (Chart 16). Conveniently I've chosen this such that no cash flows have occurred in the interim and thus the total return graph of the liabilities looks pretty much like the prior graph moved up for six months of income accrual. I'm choosing to value my liabilities at a spread of 75 basis points over the Treasury curve. At the time I did this I think long Treasuries yielded around 9%.

Now, in order to offset this liability stream I have five assets or packages of assets to consider. The first is a callable Baa bond with a 10.25% coupon maturing in 30 years, callable in five years at a price of 110, or about one plus an annual coupon, a very typical structure for a newly issued, long maturity corporate bond. Second, I have a single A callable bond with a 7% coupon, which matures in 12 years, and it's currently callable at slightly in excess of 103. Third is a package of securities with an 8% coupon GNMA, which amortizes over 30 years and is always prepayable at the mortgageholder's option at par, combined with a 20-year zero coupon Treasury strip. The fourth combination is a couple of AA bonds with 10% coupons maturing in seven and 30 years which are noncallable. And the fifth choice is a 9% coupon bond also rated AA which, although it matures in 30 years, is puttable at the option of the bondholder in seven years.

Let's look at some financial statistics (Table 6). In general, as call protection increases, the yield decreases. I've chosen to exaggerate that somewhat by making the securities with more call features be of a lower quality so that we can magnify the yield difference,

# IMMEDIATE ANNUITIES

## Cash Flow Projection of Sample Liability



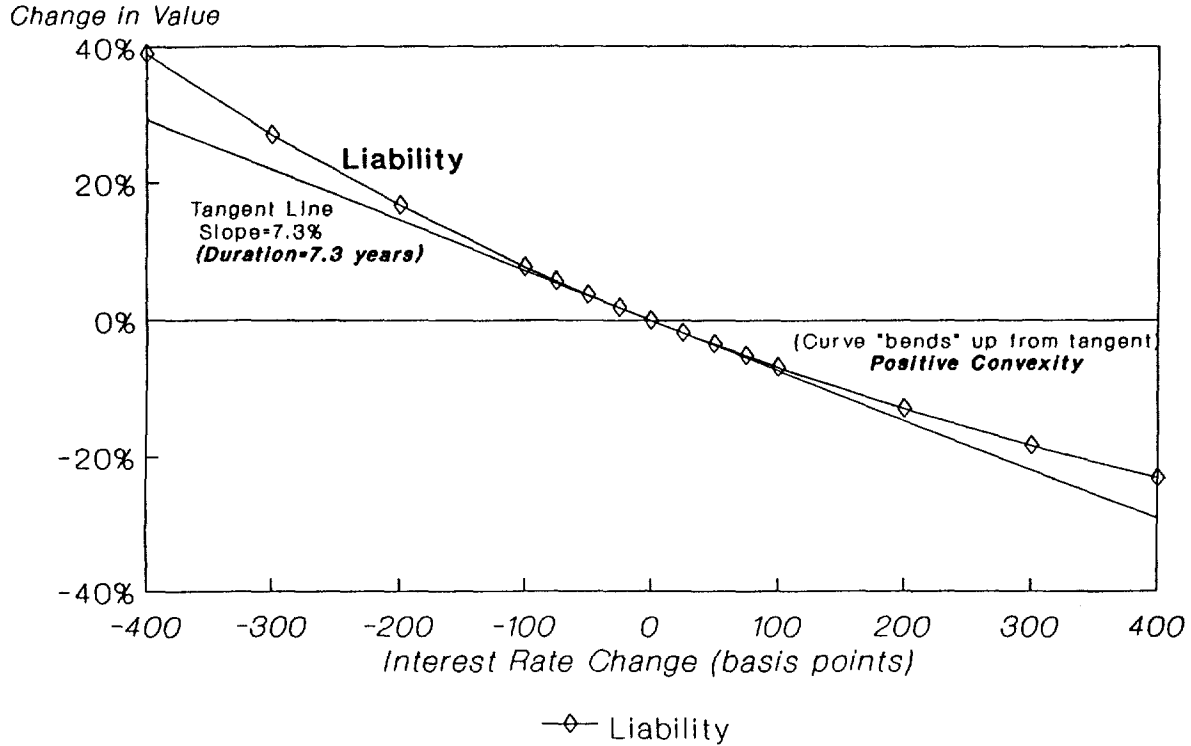
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PANEL DISCUSSION  
CHART 14

Effective Duration = 7.30      Convexity = .94



# Present Value of Liabilities As Market Interest Rates Change

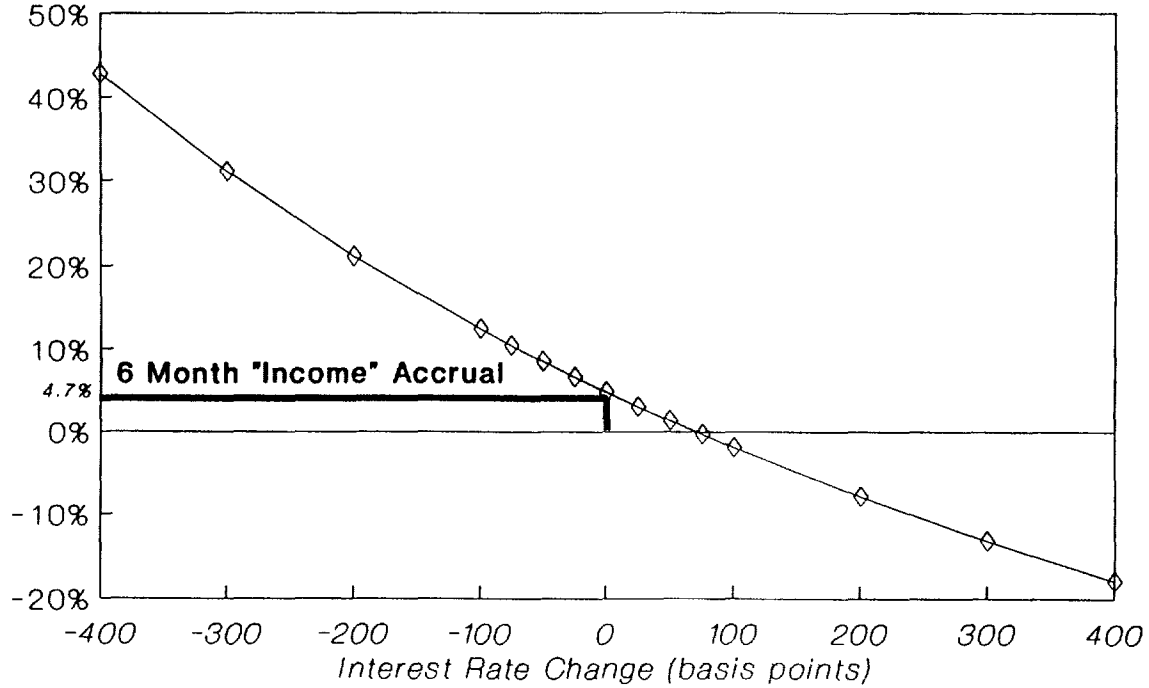


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FUNDING FOR INVESTMENT RISKS  
CHART 15

# Six-Month Holding Period Total Returns

Total Return



—◇— Liability

3036

PANEL DISCUSSION  
CHART 16

## FUNDING FOR INVESTMENT RISKS

ranging from a high yield of 10.25% on the bond which is the current coupon callable in five years to a yield of about 9.20% on the bond which is putable. The durations of all these are pretty closely matched to the duration of the liabilities, and the convexity measures are probably most interestingly understood in relation to the convexity of the liabilities, which was about 0.94.

TABLE 6

Assets Under Consideration  
Financial Performance Features

	Yield	Duration	Convexity
10.25% Callable	10.25%	7.17	-0.15
7% Callable	9.75	7.29	0.51
GNMA/Zero (85/15)	10.07	7.26	0.37
Noncallables (50/50)	9.60	7.30	0.99
Put Bond	9.20	7.31	2.28

Three of these securities therefore have lower convexity. In fact, one is even negative. Negative convexity means that the duration will shorten as interest rates fall and lengthen as rates rise, just the converse of positive convexity. Of course, the put bond has very significant positive convexity that we'll see.

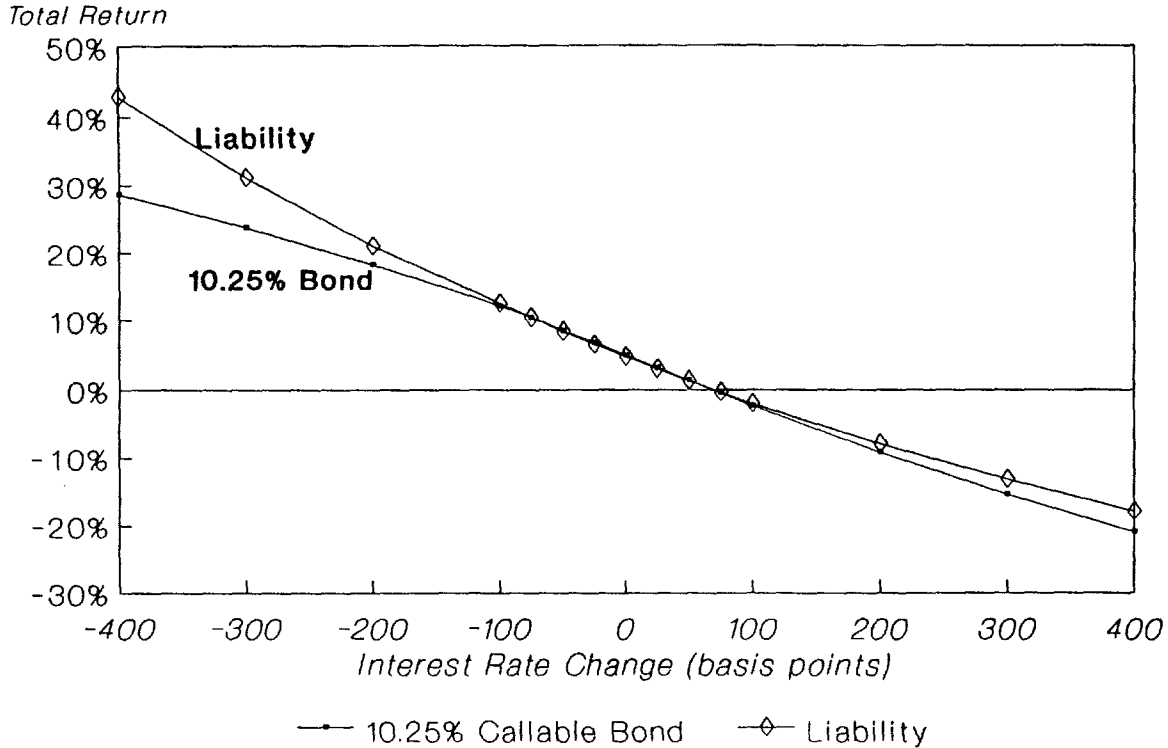
Chart 17 compares my liability total return graph to the first bond, the 10.25% bond callable in five years. You can see that for small interest rate changes those two lines are virtually superimposed on top of each other, and it's only for fairly significant changes that they tend to stray apart. You can see the negative convexity in the asset line as it starts to bend down. In comparison, the liability line continues to curve up. Maybe it is easier to see what's going on if we look at the surplus (the assets minus liabilities), which has a very humped performance (Chart 18). For a zero interest rate change (at the center of the chart) the net return over this six-month holding period is strictly the income advantage of the bond in excess of Treasuries plus 75 basis points (the liability "return"). The bond had about a 50-basis-point yield advantage, and 0.5 of a year's worth of that is about 25 basis points. You can see, though, that as interest rates change in either direction, the relative underperformance of the market value of the asset fairly significantly drags down returns to the extent that for a 200-basis-point drop in rates we've lost, roughly, 3% of relative value. If we thought of it as a 3% decline in the market value of our surplus, it would be somewhat akin to adding an extra 3% acquisition cost onto the product in that scenario.

The second asset is the 7% callable bond which has better call protection implicit in the lower coupon on the bond. Chart 19 shows a similar performance. There is less yield to begin with which breaks down a little slower, but still, as you get outside of plus or minus 50 basis points, you start to break away from the liability return. There is still fairly meaningful degradation of surplus value in either direction.

The combination of the GNMA and the zero coupon really tracks very closely the 7% callable bond (Chart 20). Maybe this tells you that the market is efficient.

# Six-Month Holding Period Total Returns

## 10.25% Callable Bond vs. Liability

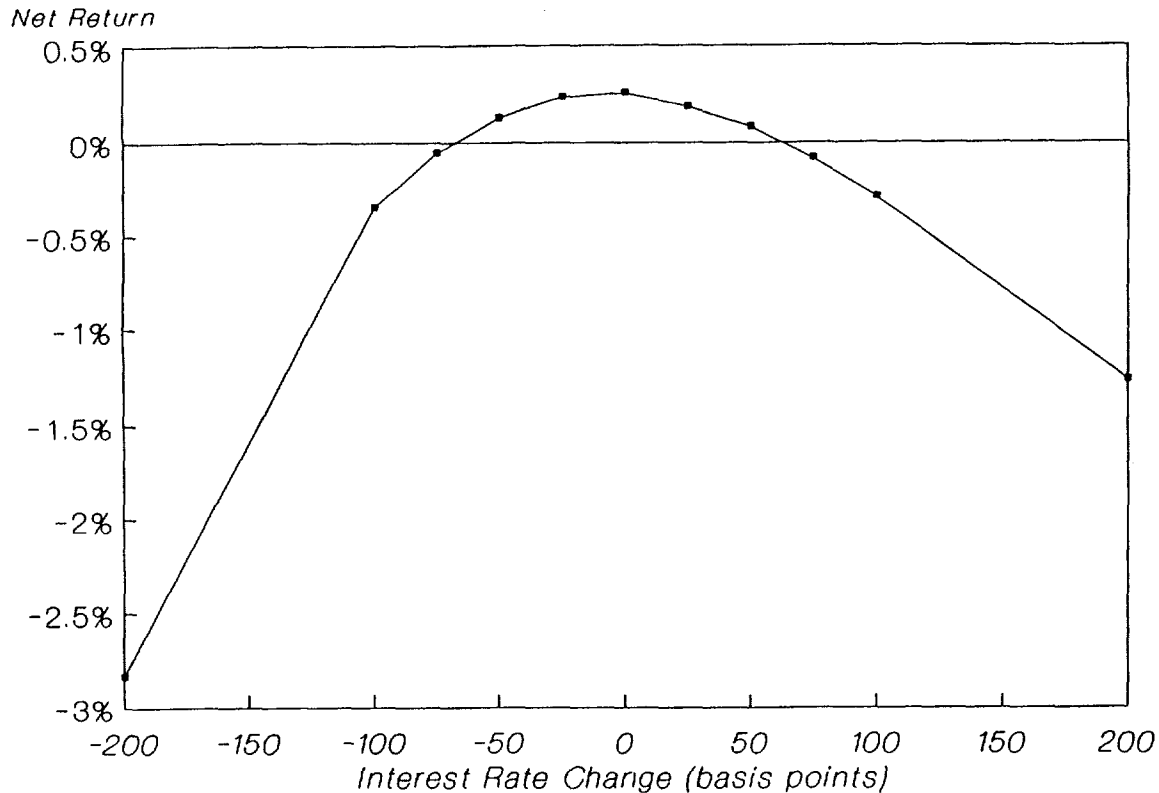


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PANEL DISCUSSION  
CHART 17

# Six-Month Holding Period Total Returns

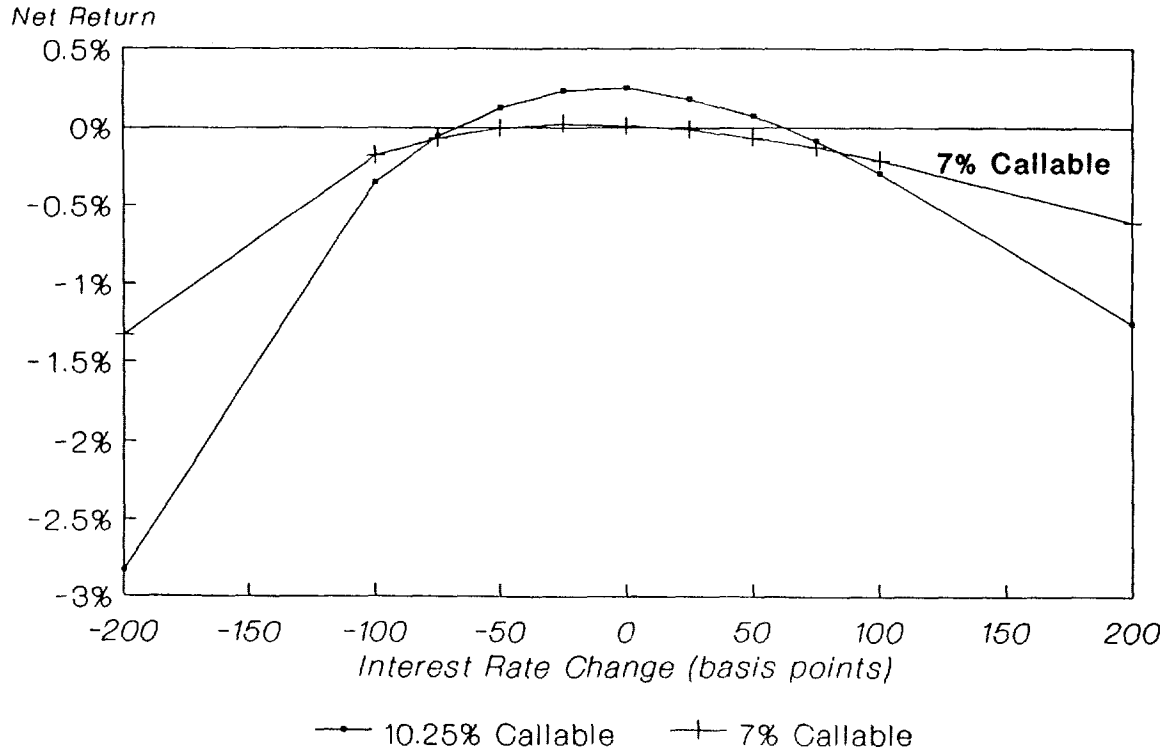
## 10.25% Callable Bond less Liabilities



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FUNDING FOR INVESTMENT RISKS  
CHART 18

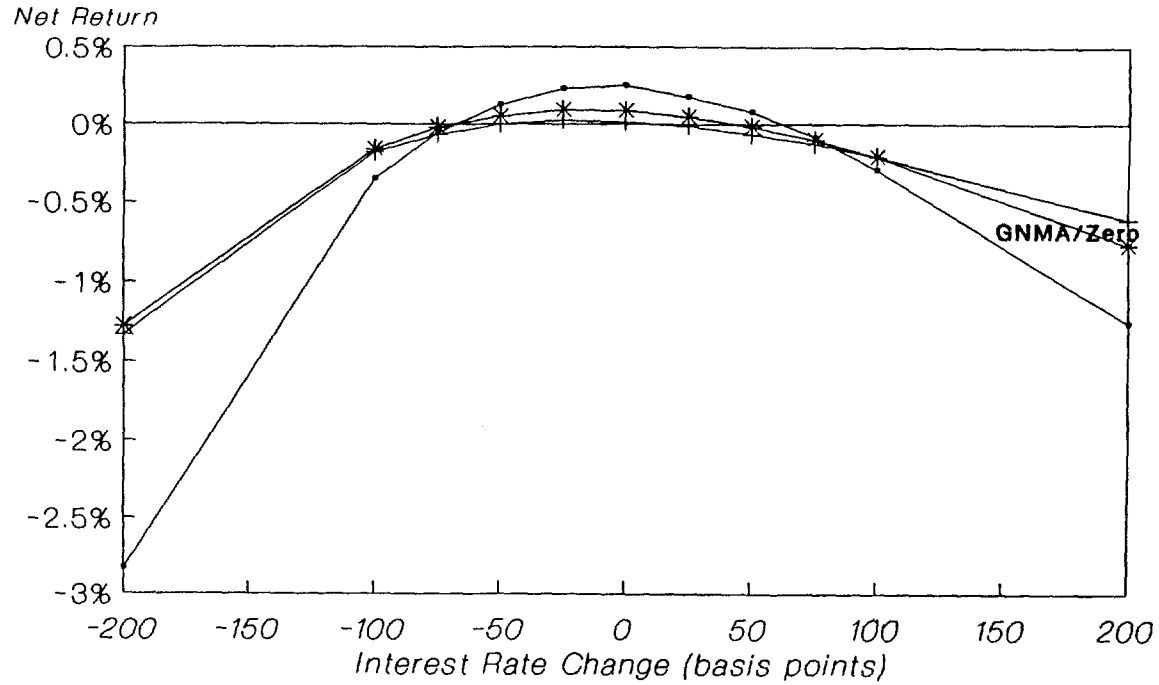
# Six-Month Holding Period Total Returns Assets less Liabilities



3040

PANEL DISCUSSION  
CHART 19

# Six-Month Holding Period Total Returns Assets less Liabilities



—•— 10.25% Callable    —+— 7% Callable    —\*— GNMA/Zero

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FUNDING FOR INVESTMENT RISKS  
CHART 20

## PANEL DISCUSSION

Next is the package of noncallable bonds: one maturing in seven years and one in 30 years (Chart 21). The performance is very similar to the liabilities. If you recall, the convexity measure of this was 0.99 versus 0.94. So, it is marginally more positively convex, and you can see this as it bends very slightly above the liability return line. Performance is relatively very good in all directions for this asset match.

The last asset, which was the put bond that had very positive convexity, slightly underperforms if interest rates don't change (Chart 22). But it doesn't take much to happen before its performance begins to very significantly dominate that of the liability. So, here we can see three different types of assets, all with virtually the same duration, showing fairly significant total return performance differences in just a plus or minus 200-basis-point interest rate change.

You can intuitively mix and match pretty successfully (Chart 23). This line is a 50/50 combination of the callable security with the puttable security. The resulting performance looks fairly close to the noncallable corporate combination. Again, that's telling you the market is somewhat efficient if these are the correct prices.

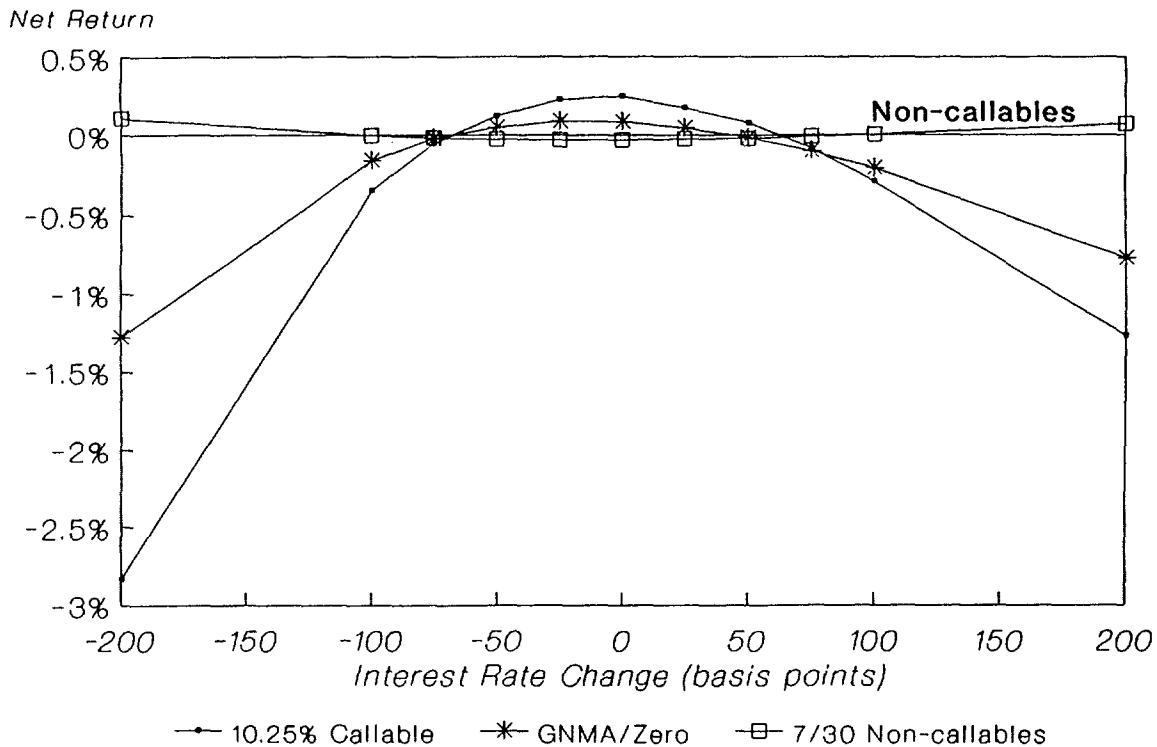
The graphs that we've been looking at thus far show just the differences due to a plus and minus 200-basis-point change. I blew up the next 200 basis points in either direction so you can see that the trends very definitely continue (Chart 24). If we get any very significant market yield changes, owning the callable bond can result in very substantial underperformance. Again, we're only looking at a six-month holding period. That bond isn't callable for another 4.5 years. So, it doesn't require that the bond be called to lose economic value, only that the call feature (that was somewhat out of the money) is now being viewed as very much in the money.

How relevant should all of this be? Let's review the last five years of changes in 10-year Treasury yields (Chart 25). I've chosen to plot the yield on the first day of each calendar quarter. Generally over the last five years, although there's been a lot of day-to-day volatility, Treasury yields have hugged the mid-8% range fairly consistently, getting down into the low sevens at one point in 1986 and getting up above 9.5, pushing ten, at one fateful time in 1987 and then again approaching that level in 1989. This was generally a period where absolute yield levels have not been that variable.

I could have really stacked the odds in my favor if I had plotted the whole decade of the 1980s; the left-hand side of the graph would then have required using a different scale than the right-hand side. However, you can see there are several periods even in the last five years, even though we've generally not thought of yield volatility as being too significant, when we have had remarkable changes in interest rates in a very short period of time. Coming right out of the chute in 1986 we had a decline of more than 150 basis points, then in 1987 we moved more than 200 basis points in a six-month period, and there have been some other pretty significant drops. Next I've plotted a histogram of the quarterly changes in 10-year yields, taking the absolute value of those yield changes (Chart 26). Again, there are a number of quarterly periods here where the yield changes have been fairly dramatic.

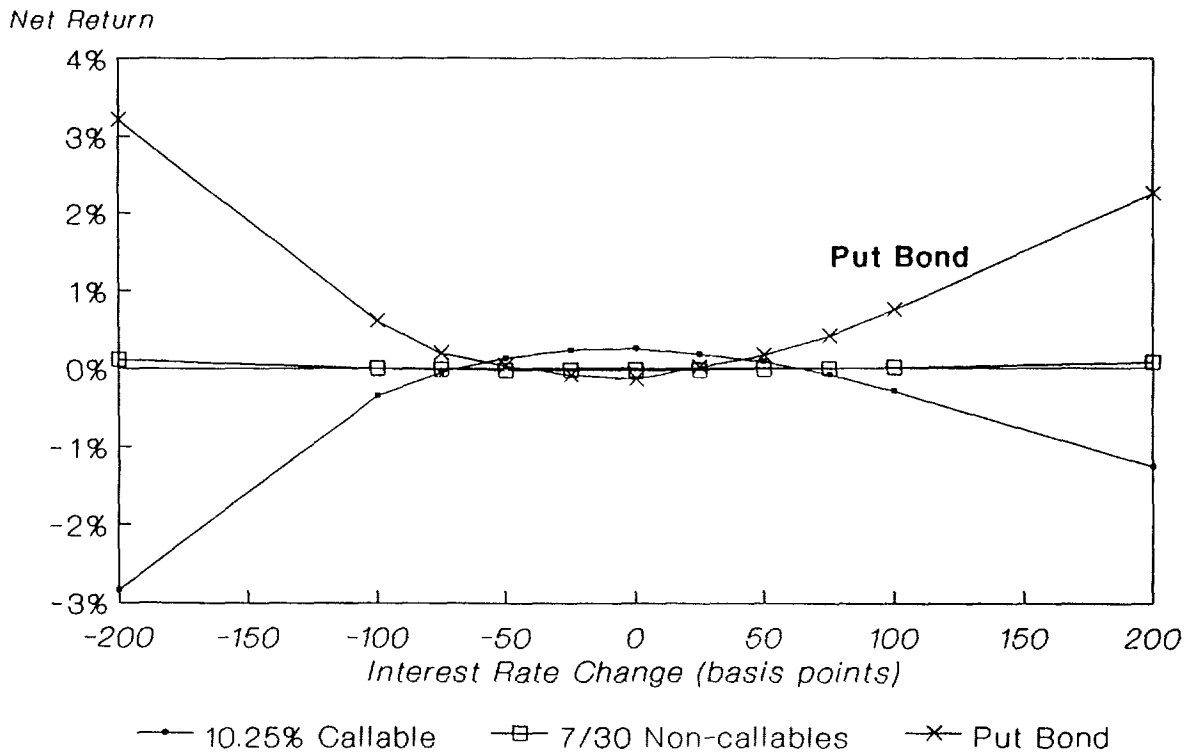


# Six-Month Holding Period Total Returns Assets less Liabilities



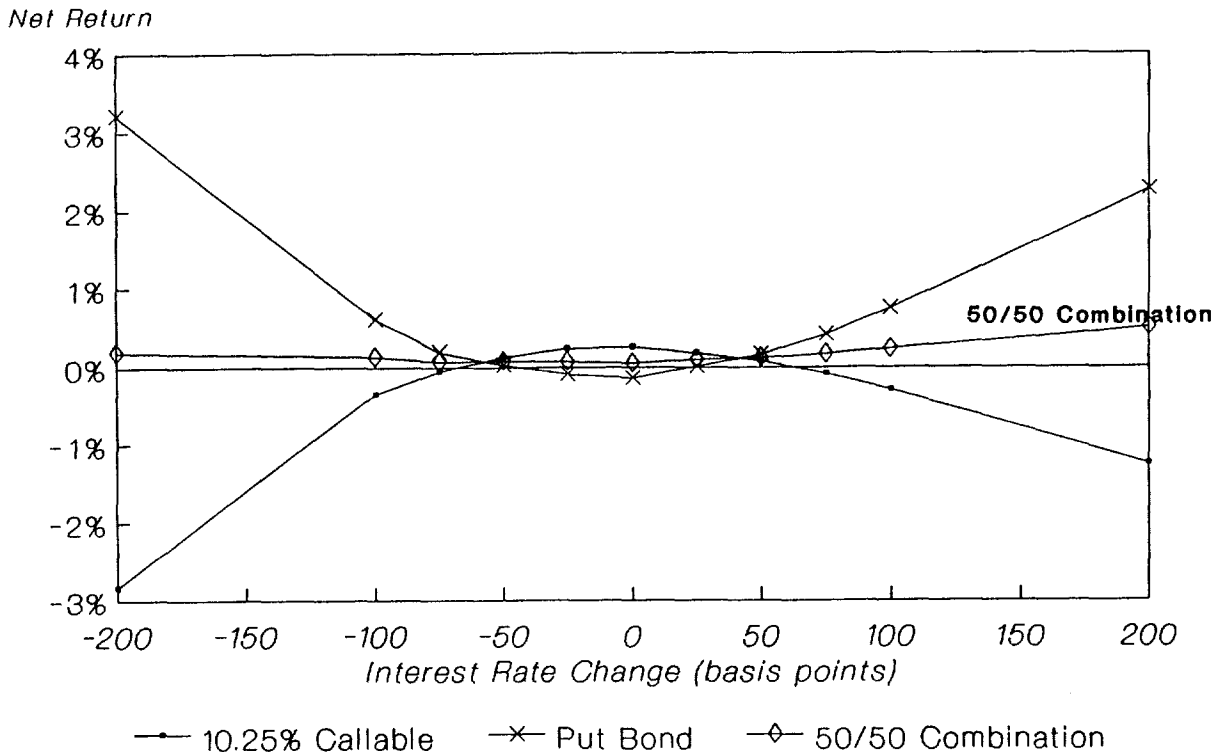
FUNDING FOR INVESTMENT RISKS  
CHART 21

# Six-Month Holding Period Total Returns Assets less Liabilities



PANEL DISCUSSION  
CHART 22

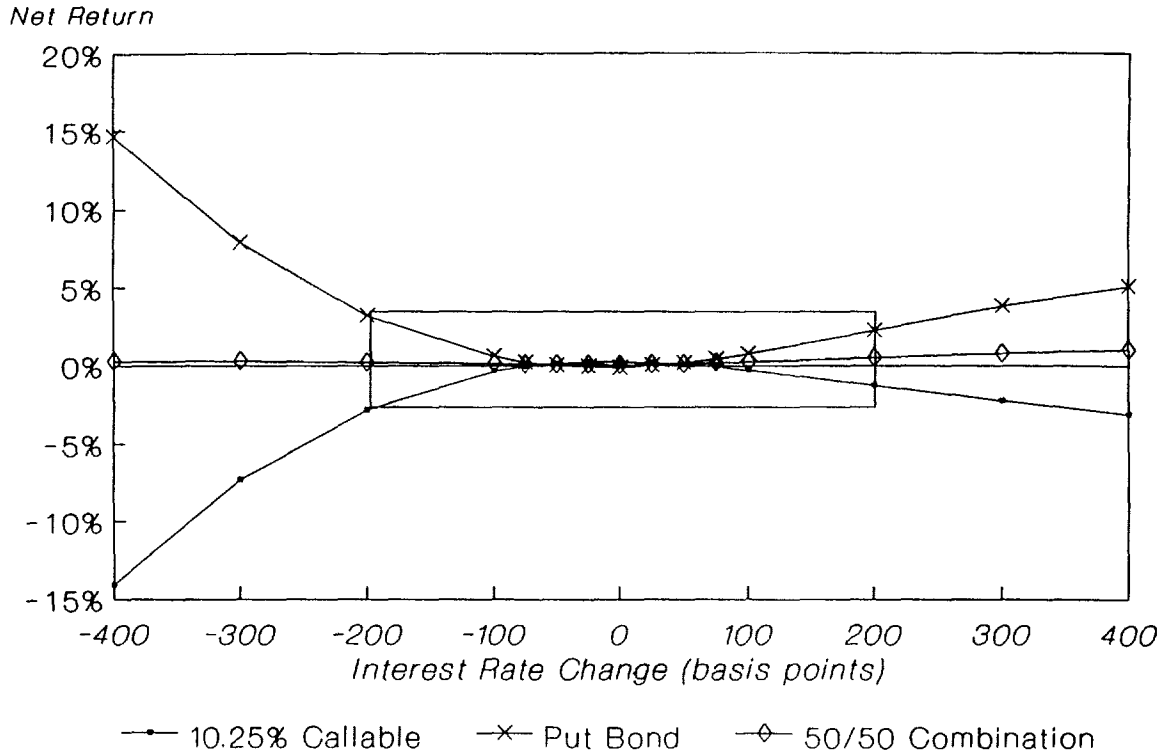
# Six-Month Holding Period Total Returns Assets less Liabilities



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FUNDING FOR INVESTMENT RISKS  
CHART 23

# Six-Month Holding Period Total Returns Assets less Liabilities

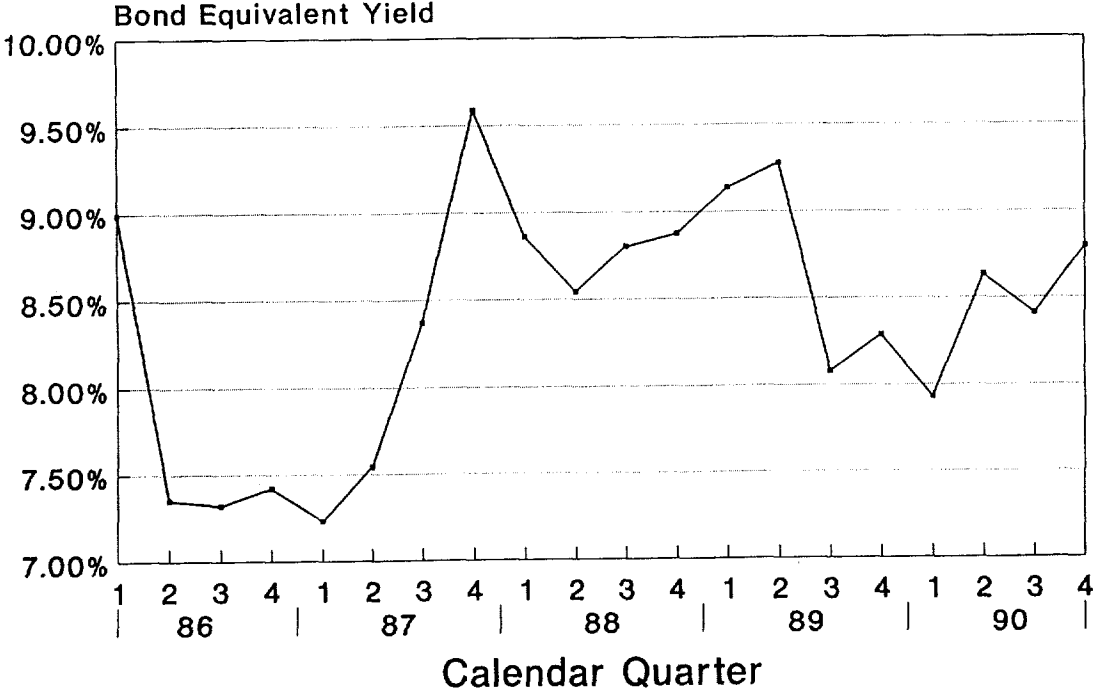


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PANEL DISCUSSION  
CHART 24

# Ten-Year Treasury Yields

Quarterly 1986-1990



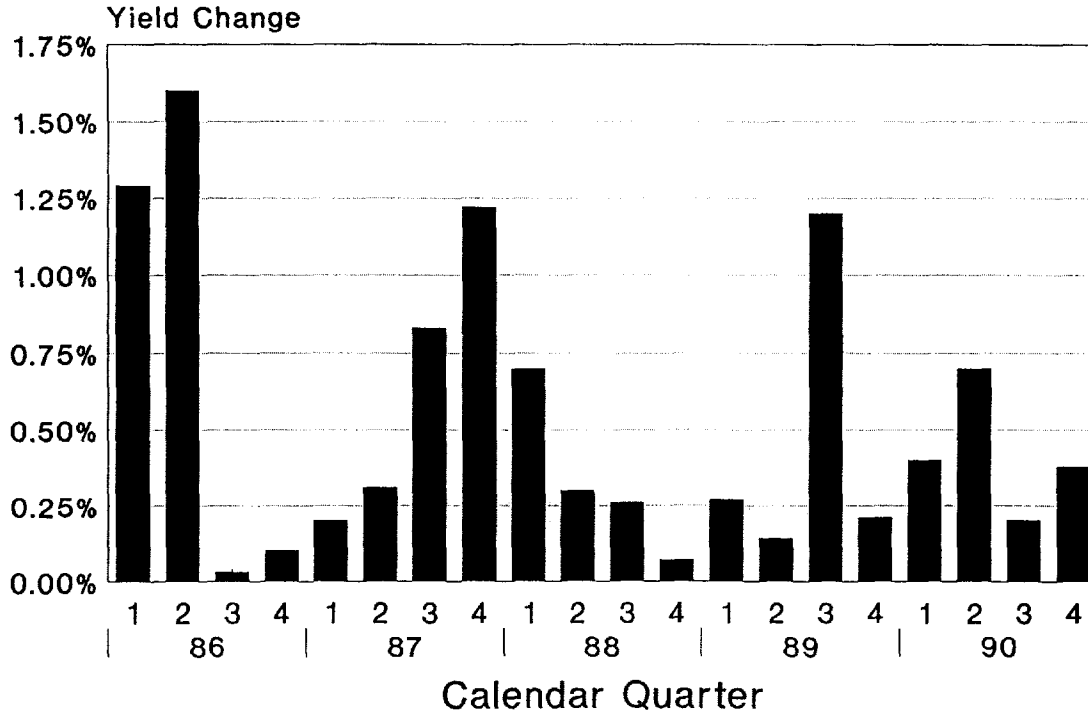
3047

FUNDING FOR INVESTMENT RISKS  
CHART 25

SOURCE: Data from Bloomberg Financial Markets

# Changes in Ten-Year Treasury Yields

## Quarterly 1986-1990



SOURCE: Data from Bloomberg Financial Markets

## FUNDING FOR INVESTMENT RISKS

What can we conclude from all this? For this type of product, the liability cannot be actively managed once it's written. There is no credited interest rate to be adjusted. There is nothing that can be changed. Thus, surplus returns can be optimized only by evaluating and actively managing all aspects of total return, especially for long-duration assets. In addition to a long duration, these types of liabilities typically have high convexities due to their lack of options and their wide cash flow dispersion. Although many long-duration assets lack the same degree of convexity found in these liabilities, it is available. For a given duration, asset convexity can be added in a number of ways. You can add call protection either explicitly by buying bonds with no call features or implicitly by investing in deeply discounted bonds. You can add bonds with puts, as we saw in the chart. You can buy options, in particular, caps, floors and other swap-related options. You can increase the cash flow dispersion, which also increases convexity. You can add some types of principal-only stripped mortgages which have a tendency to shorten in an interest rate decline, which provides positive convexity. So, you can attain the level of convexity that you need with a number of different investment classes.

Why, then, would investment managers buy current coupon callable bonds to back this type of liability? Perhaps their investment objectives were communicated to them only in terms of yield, or only in terms of yield and duration. Perhaps their management has emphasized maximizing current net income, the need to book a high yield today and the inability to wait for change. Perhaps, in order to rebalance and capture a net gain in surplus, it would be required to realize a capital loss, since in a rising interest rate environment you would realize a loss on the assets without the ability to concurrently remark the liability down to a market value. Of course, derivative instruments might be used to rebalance without absorbing this capital loss.

Probably the principal reason is because our industry accounts at book value. I suspect that if our balance sheet and income statement accounting were done on a market value basis, I probably wouldn't have anything to talk about because much of what I'm saying here would be intuitive and well understood.

Let me close with a brief analogy here. Choosing an asset based strictly on yield is somewhat like choosing your spouse based solely on looks. It may be a very gratifying experience for a while, but in the long run a successful marriage is based on the ability of the husband and wife to interact harmoniously as conditions will inevitably change. Similarly, investments must complement liabilities on a total return basis and not just provide attractive, nominal spreads at issue. Do yield-based pricing models need to be modified to reflect total return management if investment requirements for a product are to be projected, priced and communicated in terms of more than just yield targets? And how is this done? That's the open question I leave you with.

Sometimes the optimum way to maximize total return is to give up some yield in return for enhanced convexity.

**MR. B. JOHN MANISTRE:** Mr. O'Sullivan, how similar is the interest maintenance reserve to the scheme that's currently used in Canadian financial reporting for amortizing gains and losses?

## PANEL DISCUSSION

MR. O'SULLIVAN: It is very similar. They have for some years reserved those gains and losses and amortized them on a seriatim basis. They used to have the option of doing it straight line or using the method that I illustrated here. As of two or three years ago, all companies switched to using the scientific, if you will, method that I illustrated, because they found that the shift in earnings by using a straight line method was greater than they wanted to absorb.

MR. MANISTRE: So, it's not too onerous.

MR. O'SULLIVAN: They haven't found it to be too onerous, but that hasn't been completely persuasive to some of the people on the committee yet.

MR. ALAN J. ROUTHENSTEIN: I have a question I guess either for Mr. Hall or Mr. Reitano since I think most of us agree your presentations were related. With regard to traditional actuarial measures in evaluating profitability of products, I have tended to work with different insurance companies in coordinating the types of strategies and theories the two of you were talking about into actuarial pricing. Could you perhaps comment a little bit on how you in your companies tend to incorporate these really investment-side issues into actuarial concepts with regard to profitability?

MR. HALL: That's an interesting question and one which doesn't directly relate to the talk I just gave. What I have discussed here is for annuity products, in particular claim annuity products we sell at Hartford Life, or GIC-type products, either on an individual or group basis. There what we do is relate to our pricing actuaries the types of yields that we think we can achieve for certain asset classes that are as closely duration- and convexity-matched as possible, with the caveat that we don't believe that the credited rate that results should be solely the by-product of a formula that starts with the earned rate and results in a credited rate.

You also have to consider what the competition will bear and adjust your rate accordingly, either up or down. So, to some extent the rate that's to be guaranteed or offered is set in the range of what that first calculation would provide, but also relative to what the competition will bear. Then our objective on the investment side is to manage the assets to provide the necessary return. We have our systems set up so that we're looking at the same types of curves that I have showed you, where we are trying to keep our surplus duration as neutral as possible while adding value through whatever types of active management strategies we can use. We don't buy and hold by any stretch. We do a lot of active management looking at changes in options, but they don't get reflected directly into the pricing equation on the day that the product is sold.

MR. REITANO: At the John Hancock we currently use the duration analysis that I presented primarily for understanding and quantifying the level of income volatility that we see and should expect with our market-value-based internal GAAP system. In particular, my research began by trying to understand how a duration-matched portfolio, which was a barbelled or reverse-barbelled portfolio, could generate a significant amount of GAAP income volatility, since the classical theories indicate otherwise.



## FUNDING FOR INVESTMENT RISKS

The theory I developed has given us a lot of insight as to where our volatility is coming from, by merging statistical analyses of historic yield curve movements and applying that to the current exposure. We've also used it for looking at risk/return characteristics of trading strategies which would lower our volatility. However, volatility is one of those elusive concepts. It is oftentimes difficult to convince people to give up yield or anything that they can put their hands on for something that's a soft dollar concept; for example, we can sleep better at night because there will be less GAAP income volatility. Currently, we don't use this theory directly in pricing.

MR. HALL: At the Hartford, as Bob mentioned, I'm responsible for asset/liability management. I also direct portfolio management. So, the portfolio managers report to me. So that challenge, "Would you trade off yield for better total return?" is one which I have to arm-wrestle myself, which at least means I don't have anybody else to blame if the yield side wins.

MR. ROUTHENSTEIN: Just one comment I'd like to make. One thing that you can do that I've tended to do with my clients is calculate the market value of profit in a block of business, but it's just a matter of what level of assets you are working with. Are you including surplus contributed by corporate, or your statutory surplus strain, or your GAAP deferred acquisition cost (DAC)? By concentrating on the difference between the market value of the liability and the market value of the fund, which in a sense equals the premiums that have come in, less expenses and benefits that have come out, discounted appropriately so that the difference between the two is the market value of your profits, you can coordinate an explanation of the market value of profits with your investment analysis and make it understandable.

