# Claim Reserve Run-Out Studies: The Method and Its Application to Long-Term Accident and Health Product Reserve Adequacy Test <br> By Xianmei Tang, Kyle Miller and Eric Thomas 

> "There are neither two suns in the sky, nor two sovereigns over the people."

- Confucius
"If valuation actuaries are tired of having over 50 regulatory authorities to please, they need only to remember one thing: reserves ought to be adequate!"
- An Anonymous Actuary


## Background

When talking about valuation in the United States, people have a feeling that it is all about government prescription. To regulatory authorities, the insurance company is acting like a naughty first-grader who resists doing homework with the excuse of not getting enough instructions. Then, the government/teacher comes up with a clear solution: use this interest rate, use that mortality table, apply such a method, and then just subtract one value from another. The valuation requirement appears crystal clear for most insurance products.

For long-term accident and health (A\&H) products, e.g., individual disability income (IDI), however, it is not the case. For example, the standard valuation law (SVL) has such words as "the commissioner shall promulgate a regulation containing the minimum standards applicable to the valuation of health [disability, sickness and accident] plans," which effectively says nothing about the reserving. For claim reserves or disabled life reserves (DLR), the Health Reserve Model Regulation states "... assumptions regarding claim termination rates for the period less than two (2) years from the date of disablement may be based on the insurer's experience, if such experience is considered credible ...," which effectively leaves the assumption up to the company. As a matter of fact, the prescribed morbidity assumptions (e.g., 85

CIDA for IDI), whether incidence or claim termination, ironically have proved not conservative relative to actual industry experience. So, at the end of the day, what really matters is the adequacy requirement.

There are a range of techniques to assess a company's reserve adequacy, such as gross premium valuation (GPV), cash flow testing (CFT), rule of thumb (or "educated" guess, "actuarial" judgment), etc. Just like those standard statistical methods used to set up reserves, e.g., the chain ladder and BornhuetterFerguson methods in automobile insurance, the claim reserve run-out study, as a statistical tool, has been commonly used to test appropriateness of reserves for some short-term health product lines. It has also been introduced to long-term accident and health products such as IDI, but there are some confusing and complicated issues that users tend to muddle through as addressed later.

Regardless of the technique, doing the test is only half the issue. The real question is what the company must do if the reserve proves inadequate. GPV and CFT typically place a deficiency reserve that may flip the sign of income from positive to negative (e.g., Line 3 of Exhibit 6 of the National Association of Insurance Commissioners (NAIC) blue book). The claim reserve run-out study typically uncovers the neces-
sity to change valuation assumptions and strengthen reserves accordingly, which may wipe out the capital (e.g., Exhibit 5A). Due to the long-term benefit period of these products, what matters to claim reserve run-out is termination rates. As a result, companies typically slow down claim termination to secure reserve adequacy.

In this article, we demonstrate the use of the claim reserve run-out test for the long-term A\&H products. First we present the method of this test, focused on the calculation of reserve margin with claim reserve run-out triangles based on reserve valuation date and claim development period. Then we present a numerical illustration to show interpretations of testing results. And finally we discuss the limitations of this technique in applying to long-term product lines.

## The Magical Claim Reserve Run-out Triangle

For an open claim, the adequate reserve at any time point is expected to cover the future claim payments over the lifetime of the claim (i.e., run-out). For a given observation period before the claim runs out, the starting reserve is expected to cover the claim payments during the observation period and the ending reserve. This can be expressed by a recursive formula shown as Equation 1.

Equation 1

$$
\begin{aligned}
V_{t}^{(m)} & =\sum_{k=0}^{n-m}\left[B_{k} * \frac{1-\frac{d_{t+k}}{2}}{(1+i)^{k+0.5}} * \prod_{j=0}^{k}\left(1-d_{t+j-1}\right)\right]+V_{t+n-m}^{(n)} * \frac{\prod_{k=0}^{n-m-1}\left(1-d_{t+k}\right)}{(1+i)^{n}} \\
V & =\text { Tabular reserve } \\
B_{k} & =\text { Benefitor claim payment in period } k \\
d & =\text { Claim decrement,i.e., termination (including recovery and death for IDI) } \\
t & =\text { Claim duration since incurrence at beginning of study period } \\
n & =\text { Study period, e.g., } 5 \text { for annual data from year } 2003 \text { to } 2008 \\
m & =\text { Valuation date point indicator } \\
m & =0,1,2, \ldots, n .(\text { e.g., } m=0 \text { for valuation date } 2003 ; m=5 \text { for valuation date } 2008) \\
i & =\text { Valuation interest rate }
\end{aligned}
$$

Correspondingly, with a claim pool where the number of claims at duration $t$ is $J^{(t)}$, the total reserve for claims at duration $t$ in a given observation period $n$ can be expressed as Equation 2.

## Equation 2

$$
\begin{aligned}
\sum_{m=0}^{n}\left(\sum_{j=1}^{J^{(m)}} V^{(m), j}{ }_{t}\right)= & \sum_{m=0}^{n}\left\{\sum_{k=0}^{n-m}\left[\sum_{j=1}^{J^{(m)}}\left(B^{(j)}{ }_{k} * \frac{1-\frac{d^{(j)}{ }_{t+k}}{2}}{(1+i)^{k+0.5}} * \prod_{j=0}^{k}\left(1-d^{(j)}{ }_{t+j-1}\right)\right]\right)\right] \\
& +\sum_{m=0}^{n}\left[\sum_{j=1}^{J^{(m)}}\left[V^{(n), j}{ }_{t+n-m} * \frac{\prod_{k=0}^{n-m-1}\left(1-d^{(j)}{ }_{t+k}\right)}{(1+i)^{n}}\right]\right]
\end{aligned}
$$

Define $V_{t}=\sum_{m=0}^{n-1}\left(\sum_{j=1}^{J^{(m)}} V^{(m), j}{ }_{t}\right)$, i.e., to exclude the ending reserve from the total reserve, then we have Equation 3.

## Equation 3

$$
\begin{aligned}
V_{t} & =\sum_{m=0}^{n-1}\left\{\sum_{k=0}^{n-m-1}\left[\sum_{j=1}^{J^{(m)}}\left(B^{(j)}{ }_{k} * \frac{1-\frac{d^{(j)}{ }_{t+k}}{2}}{(1+i)^{k+0.5}} * \prod_{j=0}^{k}\left(1-d^{(j)}{ }_{t+j-1}\right)\right]\right\}\right. \\
& +\sum_{m=0}^{n-1}\left[\sum_{j=1}^{J^{(m)}}\left[V^{(n), j}{ }_{t+n-m} * \frac{\prod_{k=0}^{n-m-1}\left(1-d^{(j)}{ }_{t+k}\right)}{(1+i)^{n}}\right)\right]
\end{aligned}
$$

Suppose each starting reserve $V^{(\mathrm{m}) \cdot j}{ }_{t}$ and its corresponding ending reserve $V^{(\mathrm{n}), j}{ }_{t+n+m}$ are valued with the same assumptions. Then, the difference of the actual claim termination between experience and valuation assumptions during that period determines if the starting reserve is adequate. For example, if the actual claim termination is slower, then more claim payments are made during the period than those assumed by valuation. Therefore, the starting reserve proves to be inadequate. In other words, if the starting reserve appears inadequate, it indicates that the assumed claim termination rate by valuation is not conservative compared with the actual claim termination experience during the observation period. This is the foundation of how a claim run-out study determines reserve adequacy.

In reality, choosing an observation period raises reliability and credibility issues. If the observation period is too short, many claims won't reach run-out status; therefore, the study won't show a full story. If the observation period is too long, the tail data may be insufficient. Ideally, the run-out study observation period is set to be the maximum coverage period for short-term coverage (e.g., two years). For long-term coverage:

- The benefit period can be lifetime, and it may take over 30 years for a claim to run out. Therefore, the observation period is typically set at five to 10 years for annual data, and three to five years for quarterly data.
- For the assumed claim duration, note that most claims end before 10 years. Therefore, people usually assign a conservative overall assumption for durations 11 and later while tracking the detailed experience only in the earlier claim durations. However, this practice may cause a reserve inadequacy as mentioned later.

Let's get back to the method. With claim experience, we test if the starting reserve supported the future claim payments and ending reserve. If yes, it proves the valuation claim termination rate $d$ is appropriate in the aggregate. If not, it proves that the termination assumption is not supported by experience. For this purpose, we define:
(a) Reserve Margin (MG ) as the difference between the starting reserve and the present value of future claim payments and ending reserve;
(b) $M G \%$ as the ratio of Reserve Margin to starting reserve;
(c) $\widetilde{V}^{(m, t)}{ }_{t+m}$ as the total experience reserves at valuation date $m$ for the open claims at duration $t$ as of beginning observation period;
(d) $\widetilde{B}^{(m, t)}{ }_{k}$ as the total claim payments in the $k$ - $t h$ year since the beginning observation date for those open claims at duration $t$ as of valuation date $m$.

Suppose reserves for claim duration $T$ and later are appropriate. Then the reserve margin for claim durations before $T$ can be derived from a recursive formula shown as Equation 4.

For example, let $T=11$. Then Equation 4 can be rewritten as durational formulas (see Rewritten Equation 4), which can be tabulated as triangles.

## Rewritten Equation 4

## For Claim Duration 10:

$$
\begin{aligned}
& M G_{10}=\widetilde{V}^{(0,10)}{ }_{10} \\
&-\left(\frac{\widetilde{B}^{(0,10}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(0,10)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(0,10)}{ }_{3}}{(1+i)^{-2.5}}+\frac{\widetilde{B}^{(0,10)}{ }_{4}}{(1+i)^{-3.5}}+\frac{\widetilde{B}^{(0,10)}{ }_{5}}{(1+i)^{-4.5}}\right) \\
&-\widetilde{V}^{\left(5,10{ }_{15}\right.} * 100 \% *(1+i)^{-5} \\
&+\widetilde{V}^{(1,10)}{ }_{10}-\left(\frac{\widetilde{B}^{(1,10)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(1,10)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(1,10)}{ }_{3}}{(1+i)^{-2.5}}+\frac{\widetilde{B}^{(1,10)}{ }_{4}}{(1+i)^{-3.5}}\right) \\
&-\widetilde{V}^{(5,10)}{ }_{14} * 100 \% *(1+i)^{-4} \\
&+\widetilde{V}^{(2,10)}{ }_{10}-\left(\frac{\widetilde{B}^{(2,10)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(2,10)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(2,10)}{ }_{3}}{(1+i)^{-2.5}}\right) \\
&-\widetilde{V}^{(5,10)}{ }_{13} * 100 \% *(1+i)^{-3} \\
&+\widetilde{V}^{\left(3,10{ }_{10}\right.}-\left(\frac{\widetilde{B}^{(3,10)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(3,10)}{ }_{2}}{(1+i)^{-1.5}}\right) \\
&-\widetilde{V}^{(5,10)}{ }_{12} * 100 \% *(1+i)^{-2} \\
&+\widetilde{V}^{(4,10)}{ }_{10}-\left(\frac{\widetilde{B}^{(4,10)}{ }_{1}}{(1+i)^{-0.5}}\right) \\
&-\widetilde{V}^{\left(5,10{ }_{11}\right.} * 100 \% *(1+i)^{-1} \\
& M G \%_{10}= \\
& \widetilde{V}^{(0,10)}{ }_{10}+\widetilde{V}^{(1,10)}{ }_{10}+\widetilde{V}^{(2,10)}{ }_{10}+\widetilde{V}^{(3,10)}{ }_{10}+\widetilde{V}^{(4,10)}{ }_{10}
\end{aligned}
$$

## Equation 4

$$
M G_{t}=\sum_{m=0}^{n-1}\left(\widetilde{V}^{(m, t)}-\sum_{k=1}^{n-m} \widetilde{B}^{(m, t)}{ }_{k} *(1+i)^{-(k-0.5)}-\widetilde{V}^{(n, t)}{ }_{t+n-m} *\left(1-M G \%_{t+n-m}\right) *(1+i)^{-(n-m)}\right)
$$

Where
$M G \%_{t}=\frac{M G_{t+1}}{\sum_{m=0}^{n-1} \widetilde{V}^{(m, t)}}, t<T$
$M G \%_{t}=0, t \geq T$
$\widetilde{V}_{t}^{*}\left(1-M G \%_{t}\right)$ is the expected adequate reserve at duration $t$.

## For Claim Duration 9:

$$
\begin{aligned}
M G_{9}= & \widetilde{V}^{(0,9)}{ }_{9}-\left(\frac{\widetilde{B}^{(0,9)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(0,9)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(0,9)_{3}}}{(1+i)^{-2.5}}+\frac{\widetilde{B}^{(0,9)}{ }_{4}}{(1+i)^{-3.5}}+\frac{\widetilde{B}^{(0,9)_{5}}}{(1+i)^{-4.5}}\right) \\
& -\widetilde{V}^{(5,9)}{ }_{14} * 100 \% *(1+i)^{-5} \\
& +\widetilde{V}^{(1,9)}{ }_{9}-\left(\frac{\widetilde{B}^{(1,9)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(1,9)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(1,9)_{3}}}{(1+i)^{-2.5}}+\frac{\widetilde{B}^{(1,9)}{ }_{4}}{(1+i)^{-3.5}}\right) \\
& -\widetilde{V}^{(5,9)}{ }_{13} * 100 \% *(1+i)^{-4} \\
& +\widetilde{V}^{(2,9)}{ }_{9}-\left(\frac{\widetilde{B}^{(2,9)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(2,9)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(2,9)}{ }_{3}}{(1+i)^{-2.5}}\right) \\
& -\widetilde{V}^{(5,9)}{ }_{12} * 100 \%{ }^{*}(1+i)^{-3} \\
& +\widetilde{V}^{(3,9)}{ }_{9}-\left(\frac{\widetilde{B}^{(3,9)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(3,9)}{ }_{2}}{(1+i)^{-1.5}}\right) \\
& -\widetilde{V}^{(5,9)}{ }_{11} * 100 \% *(1+i)^{-2} \\
& +\widetilde{V}^{(4,9)}{ }_{9}-\left(\frac{\widetilde{B}^{(4,9)}{ }_{1}}{(1+i)^{-0.5}}\right) \\
& -\widetilde{V}^{(5,9)}{ }_{10} *\left(1-M G^{0}{ }_{10}\right) *(1+i)^{-1} \\
M G_{9} & =\frac{\widetilde{V}^{(0,9)}{ }_{9}+\widetilde{V}^{(1,9)}{ }_{9}+\widetilde{V}^{(2,9)}{ }_{9}+\widetilde{V}^{(3,9)}{ }_{9}+\widetilde{V}^{(4,9)}{ }_{9}}{}
\end{aligned}
$$

## For Claim Duration 1:

$$
\begin{aligned}
& M G_{1}=\widetilde{V}^{(0,1)} 1-\left(\frac{\widetilde{B}^{(0,1)}{ }_{1}}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(0,1)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(0,1)}{ }_{3}}{(1+i)^{-2.5}}+\frac{\widetilde{B}^{(0,1)}{ }_{4}}{(1+i)^{-3.5}}+\frac{\widetilde{B}^{(0,1)}{ }_{5}}{(1+i)^{-4.5}}\right) \\
& -\widetilde{V}^{(5,1)}{ }_{6} *\left(1-M G \%_{6}\right) * 100 \% *(1+i)^{-5} \\
& +\widetilde{V}^{(, 1)_{1}}-\left(\frac{\widetilde{B}^{(1,1)} 1}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(1,1)}{ }_{2}}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(1,1)}{ }_{3}}{(1+i)^{-2.5}}+\frac{\widetilde{B}^{(1,1)}{ }_{4}}{(1+i)^{-3.5}}\right) \\
& -\widetilde{V}^{(5,1)}{ }_{5} *\left(1-M G \%_{5}\right) *(1+i)^{-4} \\
& +\widetilde{V}^{(2,1)} 1-\left(\frac{\widetilde{B}^{(2,1)} 1}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(2,1)} 2}{(1+i)^{-1.5}}+\frac{\widetilde{B}^{(2,1)}{ }_{3}}{(1+i)^{-2.5}}\right) \\
& -\widetilde{V}^{(5,1)}{ }_{4} *\left(1-M G \%_{4}\right) *(1+i)^{-3} \\
& +\widetilde{V}^{(3,1)}{ }_{1}-\left(\frac{\widetilde{B}^{(3,1)} 1}{(1+i)^{-0.5}}+\frac{\widetilde{B}^{(3,1)}{ }_{2}}{(1+i)^{-1.5}}\right) \\
& -\widetilde{V}^{(5,1)}{ }_{3} *\left(1-M G \%_{3}\right) *(1+i)^{-2} \\
& +\widetilde{V}^{(4,1)}{ }_{1}-\left(\frac{\widetilde{B}^{(4,1)} 1}{(1+i)^{-0.5}}\right) \\
& -\widetilde{V}^{(5,1)}{ }_{2} *\left(1-M G \%_{2}\right) *(1+i)^{-1} \\
& M G \%_{1}=\frac{M G_{1}}{\widetilde{V}^{(0,1)}+\widetilde{V}^{(0,1)} 1+\widetilde{V}^{(2,1)} 1+\widetilde{V}^{(3,1)} 1+\widetilde{V}^{(4,1)}{ }_{1}}
\end{aligned}
$$

In other words, termination rate assumptions in Equation 3 are tested with real world claim terminations reflected by the actual claim payments. One side of the real world equation is the beginning reserve. The other side is the present values of ending reserve and intermediate claim payments. Within the observation window, the claim payments and ending reserve are what they are, and there is nothing we can do about them.

But the beginning reserve depends on termination rate assumptions during the observation period. An adequate beginning reserve indicates that claims have terminated as fast as the valuation assumption anticipated. If claims terminated more slowly than the rate used in reserving, the beginning reserve won't be adequate, reserve strengthening is required at the valuation date, and termination rate assumptions used in reserving must be adjusted to ensure reserves are adequate in the future.

## A Numerical Illustration

For demonstration purposes, an illustration is presented below (see Table 1 on page 9). Concerning the appropriateness of claim termination rates for durations up to 10 , the illustration tracks tabular reserves and claim payments for all open claims of a sizable block of business at duration 1 to 10 as of yearend from 2003 through 2008, assuming reserves for duration 11 and later are just right. Table 1 shows the experience data according to Equation 4 with reserve margins calculated for individual claim durations. Table 2 and Table 3 summarize the reserve margins for each claim duration and valuation date.

Table 1. Illustration of Claim Reserve Run-out Triangles

| Starting Claim Duration (1) | Valuation Date (2) | Starting Valuation Date Point (3) (3) | Ending Valuation Date Point (4) | Ending Claim Duration$\begin{gathered} t+n-1 \\ (5) \\ \hline \end{gathered}$ | Starting Reserve $\widetilde{V}^{(m, t)}{ }_{t}$ <br> (6) | Present Value of Claim Payments $\widetilde{B}^{(m, t)}{ }_{k}$ |  |  |  |  | PV Ending Reserve $\widetilde{V}^{(n, t)}{ }_{t+1}$ $t+n-m$ <br> (12) | Reserve Margin $M G_{t}$ (13) | Reserve Margin \% <br> MG\% $\qquad$ <br> (14) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\mathrm{k}=1$ | 2 | 3 | 4 | 5 |  |  |  |
|  |  |  |  |  |  | (7) | (8) | (9) | (10) | (11) |  |  |  |
| 10 | 2003 | 0 | 5 | 15 | 42,164,219 | 4,778,426 | 4,339,752 | 4,068,224 | 3,623,577 | 3,310,147 | 27,253,421 | $(5,209,327)$ | 0.00\% |
| 10 | 2004 | 1 | 5 | 14 | 41,483,938 | 4,971,352 | 4,710,293 | 4,287,191 | 3,929,000 |  | 29,624,142 | $(6,038,040)$ | 0.00\% |
| 10 | 2005 | 2 | 5 | 13 | 38,449,876 | 4,941,992 | 4,571,597 | 4,078,266 |  |  | 30,594,064 | $(5,736,043)$ | 0.00\% |
| 10 | 2006 | 3 | 5 | 12 | 53,633,803 | 5,853,732 | 5,393,577 |  |  |  | 47,272,912 | $(4,886,418)$ | 0.00\% |
| 10 | 2007 | 4 | 5 | 11 | 57,593,941 | 6,658,733 |  |  |  |  | 52,724,625 | $(1,789,417)$ | 0.00\% |
| 10 | Subtotal: |  |  |  | 233,325,777 | 27,204,234 | 19,015,219 | 12,433,681 | 7,552,577 | 3,310,147 | 187,469,164 | $(23,659,244)$ | -10.14\% |
| 9 | 2003 | 0 | 5 | 14 | 43,420,882 | 4,956,887 | 4,594,807 | 4,738,179 | 3,961,429 | 3,629,281 | 27,802,947 | $(6,262,649)$ | 0.00\% |
| 9 | 2004 | 1 | 5 | 13 | 40,879,319 | 4,990,791 | 4,631,670 | 4,284,533 | 3,969,650 |  | 28,019,967 | $(5,017,292)$ | 0.00\% |
| 9 | 2005 | 2 | 5 | 12 | 56,406,720 | 6,172,573 | 5,618,789 | 5,054,898 |  |  | 45,491,238 | $(5,930,779)$ | 0.00\% |
| 9 | 2006 | 3 | 5 | 11 | 61,905,130 | 7,129,991 | 6,318,597 |  |  |  | 53,392,902 | $(4,936,360)$ | 0.00\% |
| 9 | 2007 | 4 | 5 | 10 | 68,976,460 | 8,131,322 |  |  |  |  | 63,817,107 | $(9,443,026)$ | -10.14\% |
| 9 | Subtotal: |  |  |  | 271,588,511 | 31,381,565 | 21,163,863 | 14,077,610 | 7,931,079 | 3,629,281 | 218,524,160 | $(31,590,106)$ | -11.63\% |
| 8 | 2003 | 0 | 5 | 13 | 44,792,690 | 5,096,730 | 4,593,606 | 4,340,834 | 4,015,495 | 3,720,384 | 26,705,606 | $(3,679,966)$ | 0.00\% |
| 8 | 2004 | 1 | 5 | 12 | 59,143,972 | 6,375,136 | 5,786,028 | 5,265,969 | 4,839,311 |  | 43,746,922 | $(6,869,394)$ | 0.00\% |
| 8 | 2005 | 2 | 5 | 11 | 63,534,411 | 7,150,787 | 6,649,873 | 5,894,528 |  |  | 51,214,380 | $(7,375,157)$ | 0.00\% |
| 8 | 2006 | 3 | 5 | 10 | 73,074,924 | 8,248,897 | 7,576,562 |  |  |  | 63,239,763 | $(12,402,812)$ | -10.14\% |
| 8 | 2007 | 4 | 5 | 9 | 75,259,087 | 8,601,402 |  |  |  |  | 68,271,226 | $(9,554,581)$ | -11.63\% |
| 8 | Subtotal: |  |  |  | 315,805,083 | 35,472,952 | 24,606,069 | 15,501,331 | 8,854,806 | 3,720,384 | 253,177,897 | $(39,881,910)$ | -12.63\% |
| 7 | 2003 | 0 | 5 | 12 | 61,420,817 | 6,546,408 | 5,974,823 | 5,422,706 | 4,935,304 | 4,535,437 | 41,694,841 | $(7,688,703)$ | 0.00\% |
| 7 | 2004 | 1 | 5 | 11 | 67,099,650 | 7,407,954 | 6,765,042 | 6,247,010 | 5,636,563 |  | 49,082,288 | $(8,039,206)$ | 0.00\% |
| 7 | 2005 | 2 | 5 | 10 | 77,523,042 | 8,420,610 | 7,787,820 | 7,164,003 |  |  | 61,953,402 | (14,084,871) | -10.14\% |
| 7 | 2006 | 3 | 5 | 9 | 78,127,431 | 8,861,258 | 7,983,612 |  |  |  | 67,351,078 | $(13,902,529)$ | -11.63\% |
| 7 | 2007 | 4 | 5 | 8 | 78,755,744 | 9,592,099 |  |  |  |  | 70,669,354 | $(10,430,293)$ | -12.63\% |
| 7 | Subtotal: |  |  |  | 362,926,683 | 40,828,330 | 28,511,297 | 18,833,719 | 10,571,867 | 4,535,437 | 290,750,962 | $(54,145,602)$ | -14.92\% |
| 6 | 2003 | 0 | 5 | 11 | 70,899,996 | 8,395,360 | 6,972,220 | 6,359,176 | 5,869,428 | 5,296,916 | 47,092,689 | $(9,085,793)$ | 0.00\% |
| 6 | 2004 | 1 | 5 | 10 | 80,640,052 | 8,651,762 | 7,859,778 | 7,271,622 | 6,778,230 |  | 59,306,398 | $(15,241,409)$ | -10.14\% |
| 6 | 2005 | 2 | 5 | 9 | 84,640,215 | 9,798,467 | 8,435,339 | 7,463,054 |  |  | 64,942,547 | $(13,553,054)$ | -11.63\% |
| 6 | 2006 | 3 | 5 | 8 | 83,857,941 | 9,822,035 | 9,291,124 |  |  |  | 70,073,357 | $(14,177,893)$ | -12.63\% |
| 6 | 2007 | 4 | 5 | 7 | 106,599,526 | 11,813,058 |  |  |  |  | 93,477,822 | $(12,637,457)$ | -14.92\% |
| 6 | Subtotal: |  |  |  | 426,637,730 | 48,480,682 | 32,558,461 | 21,093,852 | 12,647,658 | 5,296,916 | 334,892,814 | $(64,695,605)$ | -15.16\% |
| 5 | 2003 | 0 | 5 | 10 | 86,872,501 | 9,141,953 | 8,206,063 | 7,373,394 | 6,816,306 | 6,353,289 | 56,758,693 | (13,532,531) | -10.14\% |
| 5 | 2004 | 1 | 5 | 9 | 90,443,364 | 10,097,902 | 9,341,860 | 7,945,410 | 7,031,468 |  | 62,471,838 | $(13,711,591)$ | -11.63\% |
| 5 | 2005 | 2 | 5 | 8 | 89,121,385 | 10,680,922 | 9,356,906 | 8,738,135 |  |  | 67,682,459 | $(15,884,415)$ | -12.63\% |
| 5 | 2006 | 3 | 5 | 7 | 115,308,563 | 11,945,270 | 11,099,254 |  |  |  | 94,190,472 | $(15,978,857)$ | -14.92\% |
| 5 | 2007 | 4 | 5 | 6 | 111,273,624 | 10,354,757 |  |  |  |  | 104,192,574 | $(19,073,533)$ | -15.16\% |
| 5 | Subtotal: |  |  |  | 493,019,437 | 52,220,803 | 38,004,083 | 24,056,939 | 13,847,774 | 6,353,289 | 385,296,035 | $(78,180,927)$ | -15.86\% |
| 4 | 2003 | 0 | 5 | 9 | 97,672,204 | 10,824,647 | 9,504,997 | 8,670,167 | 7,434,687 | 6,581,319 | 59,914,816 | $(5,258,428)$ | 0.00\% |
| 4 | 2004 | 1 | 5 | 8 | 95,342,220 | 11,923,468 | 9,875,728 | 8,673,746 | 8,148,742 |  | 65,054,423 | $(8,333,888)$ | 0.00\% |
| 4 | 2005 | 2 | 5 | 7 | 115,789,668 | 12,402,279 | 11,055,807 | 10,261,140 |  |  | 88,742,327 | $(19,911,492)$ | -14.92\% |
| 4 | 2006 | 3 | 5 | 6 | 120,228,373 | 11,500,227 | 9,705,173 |  |  |  | 105,038,759 | $(19,280,761)$ | -12.63\% |
| 4 | 2007 | 4 | 5 | 5 | 131,741,136 | 12,628,683 |  |  |  |  | 120,214,569 | $(15,085,001)$ | -11.63\% |
| 4 | Subtotal: |  |  |  | 560,773,599 | 59,279,304 | 40,141,705 | 27,605,053 | 15,583,429 | 6,581,319 | 438,964,893 | $(67,869,570)$ | -12.10\% |
| 3 | 2003 | 0 | 5 | 8 | 100,375,332 | 11,906,686 | 11,114,938 | 9,253,815 | 8,186,061 | 7,611,318 | 62,082,600 | $(9,780,085)$ | 0.00\% |
| 3 | 2004 | 1 | 5 | 7 | 120,679,588 | 13,171,317 | 11,587,223 | 10,452,479 | 9,728,896 |  | 86,213,422 | (10,473,749) | 0.00\% |
| 3 | 2005 | 2 | 5 | 6 | 126,846,830 | 13,073,708 | 10,886,953 | 9,593,911 |  |  | 100,470,036 | $(7,177,779)$ | 0.00\% |
| 3 | 2006 | 3 | 5 | 5 | 140,259,432 | 13,880,325 | 12,247,906 |  |  |  | 120,068,668 | $(5,937,467)$ | 0.00\% |
| 3 | 2007 | 4 | 5 | 4 | 140,149,218 | 13,825,240 |  |  |  |  | 132,027,509 | $(21,682,618)$ | -12.10\% |
| 3 | Subtotal: |  |  |  | 628,310,401 | 65,857,275 | 45,837,021 | 29,300,205 | 17,914,957 | 7,611,318 | 500,862,236 | $(55,051,699)$ | -8.76\% |
| 2 | 2003 | 0 | 5 | 7 | 129,604,587 | 15,450,429 | 11,958,793 | 11,166,614 | 9,442,319 | 9,104,165 | 79,648,501 | $(19,049,120)$ | -14.92\% |
| 2 | 2004 | 1 | 5 | 6 | 131,840,967 | 15,830,159 | 12,037,472 | 10,226,089 | 8,779,447 |  | 94,753,103 | $(24,153,722)$ | -15.16\% |
| 2 | 2005 | 2 | 5 | 5 | 147,651,823 | 15,439,527 | 12,529,715 | 10,438,507 |  |  | 107,561,467 | $(15,374,032)$ | -15.86\% |
| 2 | 2006 | 3 | 5 | 4 | 143,063,414 | 15,721,256 | 12,699,241 |  |  |  | 130,696,464 | $(31,871,539)$ | -12.10\% |
| 2 | 2007 | 4 | 5 | 3 | 161,757,873 | 18,565,084 |  |  |  |  | 154,128,847 | $(24,440,616)$ | -8.76\% |
| 2 | Subtotal: |  |  |  | 713,918,664 | 81,006,455 | 49,225,220 | 31,831,210 | 18,221,766 | 9,104,165 | 566,788,381 | $(114,889,028)$ | -16.09\% |
| 1 | 2003 | 0 | 5 | 6 | 73,263,709 | 8,434,521 | 6,406,341 | 5,579,729 | 4,635,777 | 3,979,589 | 43,438,201 | $(5,797,443)$ | -15.16\% |
| 1 | 2004 | 1 | 5 | 5 | 90,007,113 | 12,629,075 | 7,958,976 | 6,794,694 | 5,616,064 |  | 64,870,527 | ( $18,149,117$ ) | -15.86\% |
| 1 | 2005 | 2 | 5 | 4 | 87,777,788 | 11,151,322 | 8,268,516 | 7,324,561 |  |  | 69,188,356 | $(16,528,727)$ | -12.10\% |
| 1 | 2006 | 3 | 5 | 3 | 80,279,847 | 10,775,990 | 8,168,321 |  |  |  | 68,770,555 | $(13,460,600)$ | -8.76\% |
| 1 | 2007 | 4 | 5 | 2 | 75,762,808 | 10,867,262 |  |  |  |  | 66,913,135 | $(12,785,742)$ | -16.09\% |
| 1 |  | Subt | total: |  | 407,091,265 | 53,858,170 | 30,802,154 | 19,698,983 | 10,251,842 | 3,979,589 | 313,180,773 | $(66,721,629)$ | -16.39\% |

(13) $=(6)-[(7)+(8)+(9)+(10)+(11)]-(12)^{*}[1-(14)]$
(14) is derived recurssively for starting claim duration $t$, where
$(14)=(13) /(6)$ for each $t=10,9,8, \ldots, 2,1$; and
(14) $=0$ if $(t+n-m)>10$

Table 2. Sum of Reserve Margins by Claim Duration

| Claim <br> Duration | Current <br> Reserve | Reserve <br> Margin | Expected <br> Reserve | Expected <br> Reserve \% |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 407.09 M | -66.72 M | 473.81 M | $116.39 \%$ |
| 2 | 713.92 M | -114.89 M | 828.81 M | $116.09 \%$ |
| 3 | 628.31 M | -55.05 M | 683.36 M | $108.76 \%$ |
| 4 | 560.77 M | -67.87 M | 628.64 M | $112.10 \%$ |
| 5 | 493.02 M | -78.18 M | 571.20 M | $115.86 \%$ |
| 6 | 426.64 M | -64.70 M | 491.33 M | $115.16 \%$ |
| 7 | 362.93 M | -54.15 M | 417.07 M | $114.92 \%$ |
| 8 | 315.81 M | -39.88 M | 355.69 M | $112.63 \%$ |
| 9 | 271.59 M | -31.59 M | 303.18 M | $111.63 \%$ |
| 10 | 233.33 M | -23.66 M | 256.99 M | $110.14 \%$ |
| Total | $4,413.40 \mathrm{M}$ | -596.69 M | $5,010.08 \mathrm{M}$ | $113.52 \%$ |

Table 3. Reserve Margin for Individual Valuation Date by Claim Duration

| Valuation | Current | Reserve Margin by Claim Duration |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reserve |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Subtotal | \%Reserve |
| 2003 | 750.49 M | -5.80 M | -19.05 M | -9.78 M | $-5.26 \mathrm{M}$ | -13.53 M | -9.09 M | -7.69 M | -3.68 M | -6.26 M | -5.21 M | -85.34 M | -11.37\% |
| 2004 | 817.56 M | -18.15 M | -24.15 M | -10.47 M | -8.33 M | -13.71 M | -15.24 M | -8.04 M | -6.87 M | -5.02 M | -6.04 M | -116.03 M | -14.19\% |
| 2005 | 887.74 M | -16.53 M | -15.37 M | -7.18 M | -19.91 M | -15.88 M | -13.55 M | -14.08 M | -7.38 M | -5.93 M | -5.74 M | -121.56 M | -13.69\% |
| 2006 | 949.74 M | -13.46 M | -31.87 M | -5.94 M | -19.28 M | -15.98 M | -14.18 M | -13.90 M | -12.40 M | -4.94 M | -4.89 M | -136.84 M | -14.41\% |
| 2007 | 1,007.87 M | $-12.79 \mathrm{M}$ | $-24.44 \mathrm{M}$ | -21.68 M | -15.09 M | -19.07 M | $-12.64 \mathrm{M}$ | $-10.43 \mathrm{M}$ | -9.55 M | -9.44 M | $-1.79 \mathrm{M}$ | -136.92 M | -13.59\% |
| Total | 4,413.40 M | -66.72 M | -114.89 M | $-55.05 \mathrm{M}$ | -67.87 M | -78.18 M | -64.70 M | -54.15 M | $-39.88 \mathrm{M}$ | -31.59 M | -23.66 M | -596.69 M | -13.52\% |

In this example, assuming reserves at claim durations 11 and later are alright; claims at duration 10 show that duration 10 reserves have a 10.14 percent deficiency. When determining the reserve margin for claims at duration 9 , the adequate ending reserves must be used. That is, the duration 10 reserves used for calculating duration 9 reserve margin must be adjusted (i.e., increased by 10.14 percent). This produces a reserve margin of 11.63 percent for duration 9 reserves. Then, these margins are used to determine duration 8 reserve margin, etc. (see Table 1). Overall, the necessary reserves for the observation period appear to be at least 113.52 percent of the reserves valued (Table 2). There are variations among the different valuation dates, with the current valuation (i.e., valuation date 2007) having 13.59 percent deficiency (Table 3). All this requires an adjustment to the 2008 valuation termination rate assumption. This results in an immediate reserve strengthening of $\$ 136.92$ million.

## Discussions

Whenever a reserve inadequacy is uncovered (as in the case demonstrated above), companies usually strengthen the reserves by lowering their assumed claim termination rates (CTR), often in the ultimate durations (e.g., 11 and later). The regulators may be immediately pleased seeing the higher reserve; however, despite the new assumptions, reserve inadequacies may re-emerge in the future. This is because the reserve margin concerns merely the difference between starting reserve and ending reserve of an observation period, not the whole reserve itself. By looking at the reserve calculation regime, we see that simply lowering ultimate CTR is a shortsighted way to strengthen reserves. The impact of CTR rate change to reserve is illustrated in Graph 1 for a typical longterm A\&H claim. With the different ways to slow down the CTR, the reserve increase for claims at different durations can be quite different. Therefore, the impact on reserve margins is uncertain.

Another issue is credibility. A company may not have a large claim volume. Even with a sizable total claim volume, there are probably very few claims at a particular duration, especially later durations such as 10 or 11 . As a result, the margin factors used for adjusting the ending reserves in Equation 4 may be based on insufficient experience and therefore inappropriate. If so, the margin, whether positive or negative, and regardless of its magnitude, shouldn't have meaningful implications to reserve adequacy.

To sum up, claim reserve run-out study is an effective way to test reserve adequacy. However, it is valid only if the actuary knows the products and sees the implications of the numbers. We must make sure not only to do the right test, but also to truly understand, interpret and use the testing results properly.

## Graph 1. The Impact of CTR to Reserve Amounts at Different Claim Durations

(Duration 11 and later as Ultimate Durations)


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