QFI ADV Model Solutions
Fall 2017

1. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2g) Demonstrate and understanding of and be able to apply the concept of Duration Times Spread (DTS).

Sources:

Ben-Dor et al, Quantitative Credit Portfolio Management, Section 3.2.4

Commentary on Question:

This question tests the candidates understanding of duration times spread concepts, approaches to validating the DTS concepts.

Solution:

(a) Identify two key hypotheses supporting DTS as a primary measure of credit exposure.

Commentary on Question:

The candidates did below average on this section. Most candidates identified the relationship between spread volatility and spread, but many failed to identify that the value from relationship being applicable over a wide range of bond characteristics

Spread volatility is a linear function of spread

Relationship of spread volatility to spread remains constant over a wide range of bond characteristics

1. Continued

(b) Outline the steps that develop a theoretical basis for DTS based on a simple variant of the Merton Model.

Commentary on Question:
The candidates did below average on this section.

A non-analytical model solution is provided below, however, candidates could also receive full credit for providing an analytical based outline.

A significant number of candidates did not attempt this section or provided only summary of the Merton model.

Use zero coupon bond to model debt portion of corporate structure

Express the spread of the bond and the volatility of the spread in terms of a common set of terms that determine creditworthiness

Develop analytical relationship between spread and spread volatility

Confirm that analytical relationship is close to linear

(c) Critique your teams' hypothesis in light of the research results. Provide support for your conclusions.

Commentary on Question:
Candidates did as expected on this section.

Most candidates were able to identify that sectors 1-7 do not have a non-linear component and draw some, but, not all, of the appropriate conclusions regarding α and β

All sectors show $\alpha = 0$,

The hypothesis that $\alpha=0$ cannot be rejected.

Sectors 1 – 7 show no demonstration of non-linear component, candidate needs to reference the reason through commentary on the t-stat result.

Sectors 1 – 5, 7 show statistically significant β

Assumption of linear factor of (β) 10% does seem to hold

1. Continued

(d) Explain how spread volatility reacts for bonds from sector 8 in both high and low spread environments.

Commentary on Question:
Candidates did above average on this section.

Most candidates correctly identified that Sector 8 shows γ "positive and statistically significant" and that this would mean higher volatility for wide spreads. The most common weakness in candidate responses was in not identifying the impact for tight spreads.

Sector 8 shows γ positive and statistically significant

Volatility lower than in the linear case for tight spreads

Volatility higher for wide spreads.

2. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Infrastructure

Learning Outcomes:

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

(6e) Demonstrate an understanding of infrastructure investments.

Sources:

QFIA-126-16: Infrastructure as an Asset Class

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

Commentary on Question:

This question tested a candidate's understanding of infrastructure as an asset class and the role it plays as an alternative asset in a total portfolio context. Most candidates could list high-level facts about infrastructure, but had difficulty incorporating these facts into an applied context.

Solution:

(a) Describe other ways an investor can obtain exposure to infrastructure assets.

Commentary on Question:

Most candidates did well and were able to directly recall the various ways to get exposure to infrastructure assets.

2. Continued

- Listed infrastructure funds (closed or open-ended)
- Direct or co-investment in listed/unlisted infrastructure companies
- Shareholders of infrastructure companies (i.e. sectors of equity market)
- Infrastructure bonds (corporate bonds or PPP)
- Fund-of-funds, ETFs, passive funds, and derivatives built around listed infrastructure indices

(b) Explain why infrastructure exposure may be unable to reduce total portfolio risk for ABC.

Commentary on Question:
Most candidates did poorly on this question. A common mistake was to list the risks of infrastructure, but with no explanation as to how this affects total portfolio risk.

An approach to receive maximum points would be to provide several arguments within the following categories:

1) *Explain how the exposures in part a) behave in a total portfolio context (particularly with ABC's existing asset exposure or with ABC being a pension fund)*

2) *Explain the findings of the Inderst reading, particularly on the topics of diversification.*

   Some examples of suitable answers are below. Note that the list is not exhaustive and other reasonable answers were accepted.

- Listed infrastructure indices show high correlation with general stock market indices, and ABC has a high allocation to equities

- Unlisted infrastructure funds seem to show low correlations with other asset classes, but this research is based solely on Australian data

- The data used for the Australian unlisted funds is also of questionable quality (small and incomplete sample of funds, data gathered from different sources, appraisal based valuations, etc.).

- In particular, appraisal-based valuation tends to underestimate volatility and correlations with listed instruments and overstate their diversification potential.

- ABC is based in the US, and there is no data for US unlisted funds

2. Continued

- Cash-flow profiles of infrastructure companies suggest diversification potential against equities, but it isn't clear that investors capture this when investing in infrastructure funds

- There are substantial swings in correlation over time, and most notably, correlations with other asset classes increased during the financial crisis

- In particular, correlations increased with equities (which ABC has exposure to) and direct property

- Sampling effect on correlation coefficients (e.g. from different data frequencies or different valuation methods) may have an impact

- Diversification might be better considered across underlying economic drivers and market risk factors, rather than common financial-asset classes such as equities, bonds, etc.

- In particular regard to infrastructure equity or infrastructure bonds, ABC may already have some of this exposure in its existing asset mix

- Annualized volatility of listed infrastructure is quite high, and if the investment to infrastructure is funded by liquidating low volatility assets (e.g. bonds), this could result in a net increase in portfolio volatility despite "diversification"

(c) Describe how an infrastructure fund might invest across the infrastructure asset class in order to maximize portfolio diversification.

Commentary on Question:
Most candidates did well and were able to list methods to increase diversification within the infrastructure asset class. Candidates could have done better and received more points by describing how these methods increase diversification or why they would be particularly effective for ABC.

- Infrastructure sub-sectors (or economic vs social infrastructure)
- Due to low correlation across sectors (e.g. utility vs transport) driven by different economic exposures/risk factors

- Geography (regional diversification, e.g. European or emerging markets)
- Minimizes things like disaster/political risk, and also ABC is based in the US and currently only has US-based investments

2. Continued

- Development stages (greenfield, brownfield, secondary stage, etc.)
- Due to impact on time it takes to fully invest and to distribute capital

- Fund vintages
- Due to J-curve, cyclicality of markets, timing, etc.

(d) Recommend which of the three investments is most likely to satisfy the goals for the pension fund's 5 million investment.

Commentary on Question:
Most candidates did well on this question. Almost all candidates missed the requirement for further due diligence.

Evaluation of three potential investments by risk, return, and degree of investor control

- The seed investment is an investment in an early stage company with no proven "track record" or history of revenues. The level of risk is high.
- ABC has a high allocation to equity, and the level of seed investment risk seems to be too high to diversify ABC's portfolio risk.
- Because ABC is likely to be the first outside investor, the possibility of taking an active role in the company, possibly as outside board members, is high.
- ABC might not want to take an active role in the seed company.

- The buyout fund targets one specific industry, which might not provide a good portfolio diversification given ABC's high allocation of equity.
- Infrastructure funds may not outperform buyout fund from IRR perspective, but it shows more stable returns/lower volatility over the vintage years.

- The infrastructure fund seems to be more consistent with ABC's investment objectives of portfolio diversification without taking an active role in the invested company.
- Further due diligence is needed to decide which one to invest. The items include underlying sectors, historical return consistency, experience of fund managers, etc.

3. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2k) Demonstrate an understanding of measuring and marking-to-market counterparty credit risk in credit derivatives.

Sources:

Measuring the marking counterparty risk, QFIA-104-13 Chapter 9

Commentary on Question:

This question tested candidates' understanding of measuring and marking-to-market counterparty credit risk associated with credit derivatives and basic concepts of credit risk modelling.

Solution:

(a) Identify and explain if the following creates right-way or wrong-way exposures to the buyer:

(i) An oil producer selling oil in a swap

(ii) A company writing put options on its own stock

Commentary on Question:

The candidates performed above average on this section. Full credits were granted to candidates who correctly identified right-way/wrong-way exposure and supported their answer with reasonable explanations.

3. Continued

Right-way/wrong-way exposures are exposures that are positively/negatively correlated with the credit quality of the counterparty.

An oil producer selling oil in a swap creates **right-way exposures** for the buyer as the exposures have lower expected credit losses associated with them than would be the case without correlation.

A company writing put options on its own stock creates **wrong way exposures** for the buyer as the exposures have higher expected credit losses associated with them than would be the case without correlation.

(b) In a discussion regarding counterparty risks, your colleague makes the following statements.

- Collateral agreements can eliminate all counterparty risks.
- Liquidity puts reduce credit exposures by altering the effective maturities of trades.

Explain whether you agree or disagree with him.

Commentary on Question:
The candidates performed above average on this section. Full credits were granted to candidates who provided a reasonable opinion and supported their opinion with reasonable explanations.

Disagree. Collateral agreements do not eliminate all counterparty risks.
Exposures may exist below the thresholds;
Market movements can increase the exposure between the time of the last collateral exchange and the time when default is determined and the trades are closed out.
The collateral received/posted depreciations/appreciates in value during the close-out period.

Agree.
Liquidity puts give the parties the right to settle and terminate trades on pre-specified future dates.
Liquidity puts may reduce credit exposure by shortening the effective maturities of trades.

3. Continued

(c) Calculate the net exposure to the counterparty.

Commentary on Question:
The candidates performed below average on this section. Most of candidates knew how to calculate the net exposure but failed to realize the net exposure should be floored to zero and therefore lost partial marks.

Netting		Margin		Net Exposure
Trade 1	1.0m	Collateral 1	0.5m	0.5
Trade 2	-0.8m	Collateral 2	-0.3m	0
Trade 3	2.1m	Collateral 3	0.6m	1.5
Trade 4	-0.9m	Collateral 4	0.8m	0
Trade 5	1.3m	Collateral 5	-0.9m	2.2
				4.2

Total net exposure = 4.2

(d) Describe in four steps how to determine the market value of default risks at the t-th future time period when only one of the two counterparties has a credit exposure. (Ignore the effect of correlation between exposure, defaults, and interest rates.)

Commentary on Question:
The candidates performed as expected on this section. In general, candidates demonstrated reasonable understanding on the concept of credit risk modelling.

1) Calculate EE*(t), the risk-neutral expected exposure for period t, that is, the average of exposures weighted by their risk-neutral probabilities, over all possible scenarios. This is the risk-neutral expected market value that would be lost with default during that period with no recovery, given the effect of all applicable netting collateral and other credit enhancements.
2) Calculate the risk-neutral likelihood of default L*(t) during the period and the risk-neutral mean fraction of exposure lost in the event of default.
3) Obtain C(t), the price of a default-free zero coupon bond of maturity t.
4) Calculate V(t)= EE*(t) x L*(t) x C(t)

(e) Calculate:

(i) the total present value of default losses to B due to first default by A

(ii) the credit adjustment to B

3. Continued

Commentary on Question:
The candidates performed as expected on this section. In general, candidates demonstrated reasonable understanding on calculating default losses in part (i). Very few candidates were able to justify the negative credit adjustment to B in part (ii).

Time	PV(X)	PV(Y)	Loss rate (bps)	PVLR=PV(Y)*loss rate (comp A exposure)	PVLP=PV(x)*loss rate (comp B exposure)
1	38,300	40,500	24.6	99.6	94.2
2	42,000	45,000	23.5	105.8	98.7
3	36,000	39,500	22.6	89.3	81.4
4	22,000	25,500	21.5	54.8	47.3
			Total	**349.5**	321.6
			Credit Adjustment	**27.9**	-27.9

(ii)-27.9

4. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

(1m) Calculate the hedge ratio for a call option given the dependency of the Black-Scholes volatility on the underlying.

(1p) Identify several stylized empirical facts about smiles in a variety of options markets.

(7c) Demonstrate understanding risk management and dynamic hedging for existing GMXB and it embedded options – including:
   (i) Hedgeable components including equity, interest rate, volatility and cross Greeks
   (ii) Partially Hedgeable or Unhedgeable components include policyholder behavior, mortality and lapse, basis risk, counterparty exposure, foreign bonds and equities, correlation and operation failures
   (iii) Static vs. dynamic hedging

Sources:

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Ch 6

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Ch 7
QFIA-116-13: The Impact of Stochastic Volatility on Pricing, Hedging and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities

Commentary on Question:

This question tests candidates' understanding of volatility modeling and its applications to guaranteed living benefit riders in variable annuities. Overall, the candidates performed below average for this question, especially for part (b), (c) and (d).

Solution:

(a) Describe the Hedge Portfolio I including its impact on the company's hedge profit/loss.

4. Continued

Commentary on Question:
The candidate performed as expected on this section. Many were able to identify the nature of this hedge strategy, but few were able to fully describe the impact of this hedge strategy on the company's hedge profit/loss

Hedge portfolio I is a Black-Scholes delta hedge. The hedge accounts for changes in the price of the underlying, but does not account for varying stock volatility. Under this strategy, the company's expected profit should not be affected, but is exposed to high risks since changes in volatility are not hedged.

(b) Identify potential problems with Hedge Portfolio II.

Commentary on Question:
The candidate performed below average on this section. Many were able to identify the nature of this hedge strategy, but few were able to correctly identify the issues of this strategy – the inappropriate vega measure – and its impact on the company's hedge profit/loss.

Hedge portfolio II is a Black-Scholes delta-vega hedge. It accounts for changes in the price of the underlying and for varying volatility. However, the suggested vega overestimates the change in the value of the variable annuity contract caused by a change in volatility, because it assumes constant volatility. In reality, a change in volatility would be short-term and have a smaller effect on the option value. This exposes the company to higher risks.

(c) Recommend improvements to Hedge Portfolio II.

Commentary on Question:
The candidate performed below average on this section. Some were able to correctly propose using modified vega to improve the strategy. Others also proposed using Heston model, but the proposal was just for vega without corresponding changes in the delta measure, thus the proposal wasn't valid as a whole, as compared to using the modified vega. Credit was given when a valid whole proposal was given.

In Portfolio II, the vega hedging ratio needs to be modified. It should reflect the fact that not all the maturities react the same way to a change in today's volatility. The derivative of the contract value $\psi_t$with respect to σshould be replaced by a sum of similar derivatives taken at different times τ between time *t* and maturity. Each derivative should be weighted by a function of $(\tau - t)$.

4. Continued

(d) Calculate the GMAB delta immediately after the contract issue.

Commentary on Question:
The candidate performed poorly on this section. A few candidates were able to correctly calculate the Black put option delta and correctly take into account of the inforce that was exposed to GMAB risk. However, due to the volatility assumption, the Black put option delta should be augmented by additional vega term. Partial credit was given for any steps completed correctly.

GMAB is a European put option

Pre-decrement $\Delta = BlackPutDelta + BlackVegas * \frac{\partial \sigma_{t,T}^{implied}}{\partial S_t}$

$BlackPutDelta = N(d1) - 1$

$$d1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + 0.5 * \sigma_{t,T}^{implied} * \sigma_{t,T}^{implied}\right) * T}{\sigma_{t,T}^{implied}\sqrt{T}}$$

$BlackVegas = S_t * \sqrt{T} * N'(d_1) = S_t * \sqrt{T} * \frac{e^{-d_1^2/2}}{\sqrt{2\pi}}$

At contract issue t = 0:

T = 5

K = S_0

$\sigma_{0,T}^{implied} = 0.2 - 0.1 * T * ln\frac{K}{S_0} = 0.2$

$\frac{\partial \sigma_{t,T}^{implied}}{\partial S_t} = 0.1 * \frac{T}{S_t} = \frac{0.5}{S_0}$

r = 2%

$$d1 = \frac{0 + (2\% + 0.5 * 0.2 * 0.2) * 5}{0.2\sqrt{5}} = 0.447$$

4. Continued

N(d1) = 0.672

$$BlackVegas = S_0 * \sqrt{5} * \frac{e^{-\frac{0.447^2}{2}}}{\sqrt{2\pi}} = S_0 * 0.807$$

Pre-decrement $\Delta = 0.672 - 1 + S_0 * 0.807 * \frac{0.5}{S_0} = 0.075$

With assumed decrement, GMAB is only applicable to 70% * $(1\text{-}3\%)^5$ = 60.11% of the sales. Thus

Final Δ = 60.11% * 0.075 = 0.0458

(e) Determine if the implied volatility assumption is consistent with the empirically observed shape of smiles for short and long equity option maturities.

Commentary on Question:
The candidate performed as expected on this section. Many were able to reach the correct conclusion, but were short of correctly demonstrating or supporting their conclusions. Partial credits were given for any steps completed correctly. (Note that proper demonstration, not necessarily as shown in the table below, is required for full credit).

Check the assumed function $\sigma_{t,T}^{implied}$ by strike and maturity as below:

Strike K as % of the index price S	Maturity T = 1 year	Maturity T = 5 year
80%S	22.2%	31.2%
90%S	21.1%	25.3%
100%S	20.0%	20.0%
110%S	19.0%	15.2%
120%S	18.2%	10.9%

This shows that the smile is more significant for long maturities than for short maturities, which is opposite to the empirical fact. This indicates that the assumed $\sigma_{t,T}^{implied}$ function isn't properly specified.

(f) Critique the static lapse rate assumption.

Commentary on Question:
The candidate performed above average on this section. Many were able to explain why static lapse rate assumption isn't appropriate.

4. Continued

Since the entire contract includes both GMAB and GLWB, it is inappropriate to assume a fixed lapse rate without taking into account of dynamic policyholder behavior. In general, the industry common practice is to incorporate dynamic lapse component to the base lapse rate such that the more (less) the rider is in the money, the lower (higher) the overall lapse rate is.

5. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2c) Demonstrate an understanding of credit valuation models.

(2f) Demonstrate an understanding of modeling approaches for correlated defaults.

Sources:

Introduction to Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Ch. 1-3 and 6

Commentary on Question:

This question tests the understanding of credit valuation models and various modeling approaches for correlated defaults within the context of different models such as in intensity models and application of the shift algorithm in macroeconomic models.

Solution:

(a) There are four categories of models with different Risk Drivers and Correlations.

Categories of models	**Asset Value Model**	**Macroeconomic Models**	**Actuarial Models**	**Intensity Models**
Model	KMV-Model (Portfolio Manager by KMV)	B	C	Jarrow/Lando/ Turnbull–Model (Kamakura) Duffie/Singleton-Model
Risk Driver	Asset Value Process	Macro-economic Factors	Default Intensity	D
Correlations	A	Implicit by Macroeconomy	Implicit by Sectors	Correlated Intensity

Identify A, B, C, D in the table above.

Commentary on Question:

Candidates performed above average on this question. Most can demonstrate a basic understanding of the different model types and their key characteristics.

A: Asset value factor model or asset correlations captured by factors
B: CreditPortfolioView
C: CreditRisk+
D: Intensity Process or Brownian Motion

5. Continued

(b) Describe how you can introduce dependencies between the default times of the counterparties in a portfolio if intensity models are used.

Commentary on Question:
Candidates performed poorly on this question. Many candidates were able to comment on the usage of stochastic short rate model for modeling intensity and the correlation between Brownian motions. However, the key to model dependencies lies in the use of two or multiple factor model.

The default times are driven by an intensity process modelled using a stochastic differential equation. The sum independent stochastic affine processes with common parameters yields another stochastic affine process. Each obligor's default intensity can be presented by $\lambda i = Xc + Xi$. where Xc and X1,…, XN are independent basic affine processes. Xc is a state variable governing the common aspects of performance while Xi is a state variable for obligor-specific risk or idiosyncratic risk.

(c) Identify for each of the dependence structures A, B, C pictured above the corresponding description (1), (2), (3) above. Justify your answers.

Commentary on Question:
Candidates performed above average on this question. Many were able to successfully distinguish the copulas. Some candidates only commented on the superficial differences between the graphs without demonstrating an understanding of the differences.

Dependence structure A is (2) Gaussian Copula with correlation 0.5
Dependence structure B is (3) Gaussian Copula with correlation 0.8
Dependence structure C is (1) T-Copula with correlation 0.5 and degree of freedom 2

C shows many scattered outliers demonstrating the tail dependency therefore it's a T-copula with a low degree of freedom. It models the tail dependency more effectively than Gaussian copulas.

A and B are both Gaussian copulas because of weak tail dependence. B shows a stronger linear relationship along the x=y axis and therefore is the one with higher correlation.

5. Continued

(d) Describe an easy way to increase extreme portfolio losses when using the t-copula.

Commentary on Question:
Candidates performed as expected on this question. Many candidates commented on changing degrees of freedom but answered the wrong direction.

Decrease the degrees of freedom of the underlying t-distribution. With the degrees of freedom increases, the difference between a Gaussian copula and T-copula is very small.

(e) Describe a three-step algorithm that can generate the conditional migration matrices for different risk segments based on the average migration matrix.

Commentary on Question:
Candidates performed below average on this question. Some candidates were able to explain the high level idea of the shift algorithm and partial credits were awarded for demonstrating an understanding of the procedures in lieu of providing details.

Step 1: Simulate a segment-specific conditional default probability ps for every segment. Can use CPV Macro or CPV Direct for simulation.

Step 2: Calculate the risk index representing the state of the economy seen in light of segment s. Risk index represents the ratio between the unconditional default probability of segment s and segment-specific conditional default probability simulated based on macroeconomic factors.

Step 3: Generate a conditional matrix (mij(s)) for segment s with respect to a scenario: Mij(s) = aij (rs -1) + mij

(f) Interpret the state of economy and potential for downgrades or upgrades seen in light of segments in the CPV model, for the following two cases:

(i) risk index $r_s < 1$

(ii) risk index $r_s = 1$

5. Continued

Commentary on Question:
Candidates performed as expected on this question. Many were able to identify the economic conditions and provide the correct comments. Some candidates incorrectly commented in absolute terms ("won't have upgrades or downgrades") rather than relative terms in comparison to historical average condition.

Risk index $r_s < 1$: The simulation suggests an expansion of the economy. There is a potential for a lower number of downgrades and a higher number of upgrades than historical average.

Risk index $r_s = 1$: This is the average macroeconomic scenario. Potential for downgrades and upgrades are comparable to average conditions.

6. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

(2c) Demonstrate an understanding of credit valuation models.

(2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.

(2i) Demonstrate an understanding an understanding of CDS valuations

(4i) Demonstrate an understanding of the general uses and techniques of stochastics modeling.

(4j) Describe and apply techniques for economic scenario generation.

Sources:

QFIA-122-16: Recent Advances in Credit Risk Modeling

QFIA-124-16: IAA, Stochastic Modeling, Theory and Reality from an Actuarial Perspective, sections I-1 to I-29 and II-1 to II-24

Commentary on Question:

This question tested a candidate's understanding of credit default swaps (CDS) and the bond-CDS basis trading and its applications, including valuation of CDS using Hull White Model. The candidates performed as expected on this question.

Solution:

(a) Compare and contrast structural models and reduced form models for modeling default risk.

Commentary on Question:

The candidates performed as expected on this question. Most of candidates were able to identify the differences between these two models; however, candidates failed to identify that Structural Models and Reduced form Models rely on information from different financial markets (i.e. equity and fixed-income).

6. Continued

- Structural models use information from the equity markets and corporate balance sheets to estimate the likelihood of default
- Reduced form models rely on information in the fixed-income markets (or CDS markets) to infer risks of default.
- Reduced-models models the probability of default using information from to the credit default swaps (CDS) and assets swap spreads, while Structural model can be complex and difficult to calibrate in practice.
- Reduced form models are much more tractable for practical applications
- For both models, the recovery rate is derived endogenously or estimated exogenously.
- Reduced form approaches range from pure default rate models (i.e. Jarrow and ThurnBull) to transition matrix models (i.e. Jarrow, Lando, and TurnBull).
- Reduced form models uses stochastic processes similar to those used in the modeling of the riskless term structure of interest rates to model default.

(b) Describe the steps to value a CDS using the Hull and White approach.

Commentary on Question:
The candidates performed poorly on this question. Most of candidates failed to recall the two-steps of the Hull and White approach.

- The first step is to calculate the risk-neutral default probability at future times from the yields on default-risky bonds and default-free bonds.
- The second step is to calculate the present value of both expected future payoff and expected future payments on the CDS, where the CDS value is the difference between these two present values.
- The CDS spread is the value of the total annual premium payment that makes the CDS value in the second step equal to zero.

(c) Express the CDS payoff (in terms of L, $A(t)$ and R) in the event of default at time $t\ (0 < t < T)$.

Commentary on Question:
The candidates performed as expected on this question. Some candidates failed to use the terms given in the question.

The recovery made on the reference bond in the event of a default at time t ($0 < t < T$) =L [1+A(t)] R. So that the payoff from the CDS if there is a default at time t ($0 < t < T$) =L- L [1+A(t)] R =L[1-R-A(t)R]

6. Continued

(d) Express the present value of future payments on the CDS in terms of *L*, *K*, and $u(T)$, given that there is no default prior to time *T*.

Commentary on Question:
The candidates performed as expected on this question. Some candidates failed to use the terms given in the question.

The present value of expected future payments on the CDS if there is no default prior to time T = L*K*u(T)

(e) Show that CDS Spread satisfies the following equation:

$$\text{Spread} = \frac{\int_{t=0}^{T}\left[1-R-A(t)*R\right]q(t)v(t)dt}{\int_{t=0}^{T}q(t)*\left\{u(t)+e(t)\right\}dt+\pi(T)*u(T)}$$

Commentary on Question:
The candidates performed as expected on this question. A common mistake was to justify the formula stated in the question rather than deriving from the first principal by demonstrating of valuation of CDS.

- Show and justify the present value of expected future payoff on the CDS = $L\left[\int_{t=0}^{T}[1-R-A(t)*R]q(t)v(t)dt\right]$
- Show and justify the present value of expected future payments on the CDS = $LK[\int_{t=0}^{T}q(t)*\{u(t)+e(t)\}dt+\pi(T)*u(T)$
- Solve for K where the following expression is 0: $L\left[\int_{t=0}^{T}[1-R-A(t)*R]q(t)v(t)dt\right]-LK\left[\int_{t=0}^{T}q(t)*\{u(t)+e(t)\}dt+\pi(T)*u(T)\right]$
- $K = \text{Spread} = \frac{\int_{t=0}^{T}[1-R-A(t)*R]q(t)v(t)dt}{\int_{t=0}^{T}q(t)*\{u(t)+e(t)\}dt+\pi(T)*u(T)}$

(f) Calculate the CDS Spread under the Hull and White model.

Commentary on Question:
The candidates performed above average on this question. Most candidates were able to identify that the A(t) =0 as the buyer of protection will find that the cheapest-to-deliver bond is the one for which the accrued interest is 0.

6. Continued

- A(t) =0 as the buyer of protection will find that the cheapest-to-deliver bond is the one for which the accrued interest is 0.
- $spread = \frac{\int_{t=0}^{10} 0.6q(t)v(t)dt}{\int_{t=0}^{10} q(t)*u(t)dt+\pi(10)*u(10)}$
- $\int_{t=0}^{10} 0.6q(t)v(t)dt$ = 0.6 h/(.01+h)- h/(.01+h)*exp(-(.01+h)*10)=0.0278 Where $h = 10^{-3}$
- Based on that the denominator $\int_{t=0}^{10} q(t) * \{u(t) + e(t)\}dt + \pi(10) * u(10) = 2$ provided, the spread = $0.0278/2$ =0.0139

7. Learning Objectives:

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

(5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.

(5b) Describe how behavioral finance explains the existence of some market anomalies.

(5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

Sources:

QFIA-108-13: Behavioral Finance and Investment Committee Decision Making, by A. Wood, CFA Institute Conference Proceedings, December 2006

QFIA-109-13: A Survey of Behavioral Finance, by Barberis & Thaler

Commentary on Question:

This question tests the concept of behavioral finance with respect to individual and professional investors.

Solution:

(a) Describe a possible behavioral bias with Justin's recommendation.

Commentary on Question:

The candidates performed brilliantly on this section. Almost all candidates received full credit by describing an appropriate behavioral bias.

Justin experienced availability bias when he was making the recommendation. Availability bias occurs when people search their recent memories for relevant information when making a decision. In this case, Justin's investment recommendation may have been biased by his recent positive experiences at DEF Donuts.

(b) Assess Michael's decision to sell his stock according to prospect theory.

Commentary on Question:

The candidates performed as expected on this section. Most candidates determined that Michael should sell his stock and at least partially supported this through prospect theory. Some candidates recommended Michael keep the stock and received no credit. Some candidates received only partial credit by recommending he sell but did not base their arguments fully on prospect theory.

7. Continued

Michael will sell the stock now according to prospect theory. Prospect theory is based upon the utility of gains and losses rather than final wealth positions. Suppose the gains and losses refer to the sale price minus the purchase price. The utility from selling now is v(5). Alternatively, the expected value of waiting and selling next period is ½*[v(0)+v(10)]. Since the value function is concave in the region of gains, the current value of v(5) is greater than the expected value of waiting and so Michael will sell now.

(c) Recommend which equity distribution strategy the CFO should take and justify your answer.

Commentary on Question:
The candidates performed as expected on this section. Most candidates determined that the CFO should utilize a dividend strategy and had reasonable justification for their recommendation. Some candidates recommended the CFO use share buybacks and thus received no points. Some candidates missed points, even if they correctly recommended the dividend strategy, because they incorrectly explained the tax considerations.

The CFO should implement a dividend strategy to release equity back to the shareholders. The shareholders tend to segregate gains and losses and according to prospect theory and their value function, they will get more utility out of dividends than share repurchases. For example, a $2 dividend and $8 capital gain will be more valuable than a $10 capital gain due to the concavity of the value function over gains. Conversely, a $2 dividend and $12 capital loss will be more valuable than a $10 capital loss due to the convexity of the value function over losses. The shareholders are also predominantly tax-sheltered investors and so will not put much weight into the tax advantages of share repurchases over dividends.

(d) Critique the CFO's dividend strategy.

Commentary on Question:
The candidates performed below average on this section. Most candidates agreed with the CFO's dividend strategy but did provide significant justification for their answer. Very few candidates received full credit by explaining the disadvantages of the CFO's dividend strategy.

7. Continued

The CFO's dividend strategy is very common within the industry and a plan for dividend payouts would likely be seen as a positive development for shareholders. However, there are a number of shortfalls with this strategy:

- It is not maximizing firm value or shareholder after-tax wealth. An explicit focus on the asymmetry between dividend increases and decreases will likely compromise long-term value creation for the company and its shareholders.
- The CFO is using her perception of fairness to set the target payout rate which is subjective and likely suboptimal.
- A strategy based a single CFO's notion of fairness will be more susceptible to opposition and unrest amongst key investors and managers.

8. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1p) Identify several stylized empirical facts about smiles in a variety of options markets.

Sources:

Rebonato, R, Volatility Correlation – The Perfect Hedger and the Fox, 2nd Edition, Section 7.4.1, 7.3.2, and 7.5.1

Commentary on Question:

This question tests the candidate's ability to identify stylized facts about smiles.

Solution:

(a) Describe two historical facts which have been observed of interest-rate smiles.

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to describe historical facts about interest-rate smiles.

The following historical facts about interest-rate smiles are described in the syllabus:

- Before, approximately, 1994, interest-rate smiles used to be fundamentally flat across strikes.
- The earliest and most pronounced smiles in interest rates began to be observed in JPY around 1994, when interest rates became substantially lower than in all other major currencies.
- Between 1994 and 1998 the smile surfaces of interest-rate options in other currencies took the lead from JPY and became somewhat (although not as markedly) sloped.
- In this initial period, the shape of the smile used to be monotonically decreasing from low to high strikes. This changed dramatically after the autumn of 1998 (i.e. the aftermath of the Russia/LTCM crisis).
- The magnitude of the smile does not systematically decrease with increasing option maturity; very often it does not appear to decrease at all.

8. Continued

(b) Compare and contrast the behaviors, in normal versus exceptional market conditions, of the level, the skew and the convexity of equity smiles as a function of the maturity of the options.

Commentary on Question:
The candidates performed below average on this section. Most candidates failed to correctly describe the behavior of these measures in exceptional markets.

- In 'normal' market conditions, the 'central' level remains approximately constant, or increases gently as the option expiry increases; in 'exceptional' market conditions, the 'central' level of short-dated options becomes much higher than for longer-dated expiries.
- The skew parameter increases very sharply with decreasing maturity (the more so during 'exceptional' periods), and remains monotonically increasing for all maturities.
- The convexity parameter also increases for long maturities; at the short end of the maturity spectrum, however, its precise behaviour depends on the particular market.
- The shapes of neither the skew nor the convexity curve seem to change as dramatically as the level curve in correspondence with exceptional events.

(c) Compare the shape of FX smiles for mature-market/mature-market currency pairs and the shape of FX smiles for mature-market/emerging-market currency pairs to the shape of volatility smiles of other asset classes.

Commentary on Question:
The candidates performed below average on this section. Only a small number of candidates were able to correctly compare the shape of the smiles.

The shape of the smile for mature-market/mature-market currency pairs is much more variable than in the equity or interest-rate case. The smile can frequently change skewness over time. It can become approximately flat, increase or even reverse its convexity; it can turn into a smirk. Comparatively, the shape of the smile for mature-market/emerging market currency pairs displays an equity-like smile in the direction of emerging-market-currency put/ $call.

(d) Your manager asks you to develop a model for the dynamics of the S&P 500 equity index using your expertise about equity smiles. He believes that the call option price movement captured by the model should always be in the same direction as the underlying equity movement.

Evaluate your manager's statement.

8. Continued

Commentary on Question:
The candidates performed as expected on this section. Most candidates were able to explain why the statement of the manager was false.

It was shown that in a substantial number of cases (between 7% and 17% of the observations for the S&P 500 index option market in 1994) call prices and the underlying move in opposite directions. The frequent movement of option prices and underlying prices in opposite directions tells us that it might be difficult to replicate both the dynamic of the underlying returns and the dynamic of options' returns using a single model. Hence, the statement of the manager is flawed.

9. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

(2e) Demonstrate an understanding of the term structure of default probability.

Sources:

Introduction to Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Ch. 3

QFIA-124-16: IAA, Stochastic Modeling, Theory and Reality from an Actuarial Perspective

Commentary on Question:

This question tests candidate's understanding of Merton Asset Value Model in the context of credit risk analysis.

Solution:

(a) Describe the position of the debtholder XYZ Life and the equity holders, using relevant derivative instruments and associated parameters.

Commentary on Question:

The candidates performed relatively well on this section. Candidates were generally able to identify the position held by the debtholder and the equity holders. However, many of them were unable to list all the relevant parameters including the underlying asset, the strike price and the maturity.

The debtholder XYZ is shorting to MAB a put option and longing a risk-free bond, or they own the underlying assets and are short a call. The underlying is MAB's asset value, maturing at time 10, and strike of $100 million.

The equity held by the equity holders can be described by a long position on a call option the company's assets. The underlying is MAB's asset value with strike F = $100 million and maturity in 10 years.

(b) Calculate the total value of MAB equity, given the risk-neutral probability of default to be 21.5%.

9. Continued

Commentary on Question:
The candidates performed relatively well on this section. Candidates were generally able to calculate at least d_1 and d_2. Common mistakes include the following: 1. mistakenly identify risk-neutral probability of default as $N(-d_1)$, 2. use formula E =A – D to calculate the equity value but mistakenly caluclate D as the risk-free bond, and 3. Unable to backsolve to get the interest rate.

Risk-neutral probability of default = $Pr^{RN}(A_T<F) = 1 – N(d_2)$
$N(d_2) = 1- 0.215 = 0.785$, $d_2 = 0.79$ approximately ($N(.7852) = .79$) and ($N(.7823) = .78$, $d_2 = 0.784$ approx.)
$d_2 = (\ln(A/F) + (r-\sigma^2_A/2)T)/(\sigma^2_A\sqrt{T})$, $A_0 = F = 100$, $\sigma_A = 10\%$, $T = 10$
Backsolve to get r = 3%
based on a), MAB's equity can be viewed as a call option with F = 100 and $A_T = 10$, T = 10
$d_1 = (\ln(A/F) + (r+\sigma^2_A/2)T)/(\sigma^2_A\sqrt{T})$, $A_0 = F = 100$, $\sigma_A = 10\%$, $T = 10$, $r = 3\%$
$d_1 = 1.107$
or $d_1 = d_2 +\sigma_A\sqrt{T}$
$N(d1) = 0.8665*0.7 + 0.86433*0.3 = 0.86585$
$C_0 = AN(d_1) - Fe^{-rT}N(d_2)$, $C_0 = 28.43$ million

(c) Determine if this call option is currently in the money, at the money, or out of the money.

Commentary on Question:
The candidates performed relatively well on this section. Candidates generally have a good understanding of the ITM concept and were able to identify if the option was in-the-money based on their answer calculated in part b). A Common mistake was to interpret the answer calculated in part b) as per stock equity value rather than the total equity value, i.e. 28.43 dollar per sotck instead of 28.43 million in total.

MAB's total market value of equity = Value of the embedded call option
$C_0 = 28.43$ million from b)
Stock price = \$28.43 million /2 million outstanding shares = \$14.215 per share
Since the call option's strike price K = \$12 which is less than current price, this option is currently in the money

(d) Calculate the volatility of MAB equity.

Commentary on Question:
The candidates performed relatively well on this section. Candidates were generally able to retrieve the formula or derive it from first principle. A Common mistake was to use $N(d_2)$ in the formula, or use a number other than 100 for A.

9. Continued

$\sigma_E/\sigma_A = A/E * N(d_1)$ from reading, formula 3.11 p 169 and II.A-25 ref. 2
d1 = same in b), A = 100, σ_A = 10%, E = Embedded call option price = 28.43 from c)
σ_E = 30.456%

10. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1i) Explain the set up and motivation of the Lognormal Forward LIBOR Model (LFM).

(1j) Describe the calibration of the LFM to Cap and Floor prices.

(1k) Explain the LFM drift terms and their dependence on the calibration and choice of numeraire.

Sources:

Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit, Brigo, D. & Mercurio F, Chapter 6.1 – 6.4

Commentary on Question:

This question tests the understanding of Lognormal Forward LIBOR Model (LFM) and its applications.

Solution:

(a) Describe the advantages of using the LFM model for interest rate cap pricing.

Commentary on Question:

The candidates performed as expected on this section. Most candidates were able to identify LFM is consistent with Black's formula.

Advantages:

a. Easy to calibrate since the market price is quoted based on Black's formula and LFM is consistent with Black's formula.
b. For multiple payoffs, we can take the forward measure for each caplet payoff separately

(b) Describe one shortcoming of using the LFM model when calculating the prices of swaptions.

Commentary on Question:

The candidates performed below average on this section. Many candidates failed to recognize there are dependencies between different future time points and LFM is less useful in this sense.

10. Continued

Consider a swaption with 2 or more tenors, the payoff function depends on the rate and curve at times T1 and T2 and so on. In order to price the swaptions, we need to take the expectation of the payoff function at T1 and T2 and so on. However, we cannot separately evaluate the expectation at T1 and T2. Therefore, we have to consider the correlations between different time points, making LFM less useful in this sense. In this case, LSM should be used. LFM is incompatible with LSM intrinsically.

(c) Derive the value of this contract using the LFM model, in terms of S, τ, $P(0,T_{k-1})$ and $P(0,T_k)$.

Commentary on Question:
The candidates performed below average on this section. Many candidates were able to identify the payoff formula but fail to derive the final answer.

Assume V is the value of this forward rate contract,
$V = P(0,T_k)E_k[F_k(T_{k-1}) - S]\tau$, where E_k denotes expectation with respect to the forward measure Q^k .
Because of the martingale property, $E_k[F_k(T_{k-1})] = F_k(0)$, as $dF_k(t)$ is driftless, according to the formula in the question (it is not a function of μ)
$E_k S = S$, as S is a constant
Therefore,
$V = P(0,T_i)[F_k(0) - S]\tau$

As $F_k(0) = \big(P(0,T_{k-1}) - P(0,T_k)\big)/(P(0,T_k)\tau)$ (the formula is included in the question)

Therefore, $V = P(0,T_{k-1}) - (1 + S\tau)\cdot P(0,T_k)$

(d) Construct a trading strategy which replicates this contract.

Commentary on Question:
The candidates performed below average on this section. Many candidates were able to identify the strategy of buying and selling bonds but fail to provide with correct face amount.

The trading strategy is
1. Buy \$1 ZCB (zero coupon bond) maturity at T_{k-1} and reinvest the proceed at the spot rate $F_k(T_{k-1})$ till T_k.
2. Sell (1+S τ) dollars of ZCB maturity at T_k

10. Continued

The cost and payoff of this trading strategy are:

$Cost = P(0,T_{k-1}) - (1+S\tau)\cdot P(0,T_k)$ (1 pt). This is the same V as in part c.
$Payoff = 1 + F_k(T_{k-1})\tau - (1+S\tau) = (F_k(T_{k-1}) - S)\tau$ = payoff of the contract.

(e) Justify using Black's formula to price caplets.

Commentary on Question:
The candidates performed poorly on this section. Most candidates failed to describe how to justify Black's formula, i.e. using the bond price as the appropriate discount factor.

Historically, it was a convenient pricing mechanism of using Black's formula to price caplets, however, it is not consistent with the price as an expectation under the risk-neutral measure while assuming that the forward rate following a geometric Brownian motion under the risk-neutral measure.

But now it is justifiable under the LFM by using the bond price as the appropriate discount factor instead of a bank account.

Using the bond price as the numeraire transforms the pricing from the risk neutral measure from T_k forward measure.

11. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

(4c) Describe and assess performance measurement methodologies for assets portfolios.

(4f) Calculate and interpret performance attribution metrics for a given asset, portfolio.

(4g) Explain the limitations of attribution techniques

Sources:

QFIA-107-13: Ch 69&70: Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012

Commentary on Question:

Overall, candidates performed below average on this question. This question tests the concepts of performance attribution and its application. It first tests to see if the candidate can list and define sources of performance attribution. Then it progresses to calculate outperformance due to yield curve changes and it challenges the candidates to analyze the outperformance. Finally, it asks the candidate to comment on areas to investigate for a large residual.

Solution:

(a) List and define broad categories of a typical scenario-based return decomposition.

Commentary on Question:

Candidates performed below average on this section. Few candidates were able to correctly list and define the six categories of scenario based return decomposition.

i. Surprise return – the difference of actual cash flows and notional changes from model predictions
ii. Time return – the effect of elapsing time
iii. Yield return – the effect of changes in the yield curve
iv. Implied volatility change – the effect of changes in implied vol. surface
v. Spread return – the effect of changes in the option adjusted spread
vi. Other market return – the effect of change in other market parameters

11. Continued

(b) Calculate the portfolio outperformance due to the change in the yield curve.

Commentary on Question:
Candidates performed below average on this section. Few candidates were able to identify all the formulas needed to calculate outperformance.

Outperformance from average carry [70-7] = $(y^P_{avg} - y^B_{avg}) * \Delta t$
= (1.53-1.48) * 3/12 * 100
= 1.25 bps

Key rate contributions [70-8] = $\sum_j [w_j^P * (y_j - y_{avg}^P) - w_j^B * (y_j - y_{avg}^B)] * \Delta t$
= [20%*(0.56-1.53)-20%*(0.56-1.48)+
36%*(1.11-1.53)-40%*(1.11-1.48) +
44%*(2.32-1.53)-40%*(2.32-1.48)] * 3/12*100
= -0.25 + -0.08 + 0.29

Outperformance from average parallel shift [70-9] = $-(OAD^P - OAD^B) * \Delta y_{avg}$
= - (7.38-8.18) * 0.30 * 100
= 24.0 bps

Outperformance from reshaping [70-10] = $-\sum_j (KRD_j^P - KRD_j^B) * (\Delta y_j - \Delta y_{avg})$
= - [(0.09-0.09)*(0.03-0.31) + (1.46-1.51)*(0.06-0.30) + (5.83-6.58)*(0.69-0.30)]*100
= - [0+1.20-29.25]
= 28.05 bps

Outperformance due to change in yield curve = Carry + Change
= outperformance from average carry + Key rate contributions + Outperformance from average parallel shifts + outperformance from reshaping
= 1.25 + -0.04 + 24.0 + 28.05
= 53.26 bps

(c) Critique the performance of the portfolio manager over this time period, identifying:

(i) what was done well

(ii) what could have been better

Commentary on Question:
The candidates performed poorly on this section. Most candidates failed to recognize the key drivers for outperformance and the inherent risks with the portfolio's position.

11. Continued

(i) What was done well:

- Portfolio manager outperformed the bench mark (i.e. 53.26 is greater than 0.)
- The portfolio manager stayed within 5% of the benchmark weights and, hence, was within the mandate (i.e. yield curve matched weights were all within 5% of benchmark)
- Portfolio manage did well for average parallel shift, reshaping and average carry as performance was positive (i.e. all returns associated with these were positive – 24, 28.05, and 1.25 respectively)
- The tactical position to have a shorter duration (i.e. 7.38 vs 8.18) with the portfolio than the benchmark was a major contributor in his portfolios return above the benchmark.
- Attribution did not just manage yield-curve exposure using just duration. Exposure to re-shaping was also considered.

(ii) What could have done better:

- The overweight to the 1 year and 5 year rate reduced return. Improved performance if he reduced the weight to the 1 and 5 year while maintaining a shorter portfolio duration than the benchmark.
- Portfolio manager did not beat the benchmark with respect to re-shaping, granted it is an immaterial amount (i.e. -0.04 bps)
- Such a big discrepancy in duration (0.8 years) is a very large risk to run so had the yield gone down instead of up, the portfolio manager would have had considerable losses
- The portfolio still had exposure to non-parallel shifts in the yield curve. More KRDs should be included in the attribution

(d) Describe areas that warrant further investigation given the above residual.

Commentary on Question:
Candidates performed as expected on this section. Most candidates recognized that this was a large residual and were able to suggest areas to investigate.

Such a high residual should be promptly investigated; this may mean one of the following four things:

a. Correctness of the total returns. This can be caused by missing or incorrect prices, transaction errors, incorrect corporate actions, etc.
b. Missing or inaccurate analytics produced in the security valuation process
c. Attribution model deficiencies wherein certain contributing factors are not captured by the factor return decomposition
d. Issues with the attribution algorithm

12. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

(7b) Demonstrate understanding of risks associated with guarantee riders including: market, insurance, policyholder behavior, basis, credit, regulatory and accounting.

(7d) Demonstrate understanding of Target Volatility funds and their impact on option costs.

Sources:

QFIA-120-15: Guarantees and Target Volatility Funds

On the Importance of Hedging Dynamic Lapses in Variable Annuities, , Risk and Rewards, 2015 issue 66, pp 12-16.

Commentary on Question:

This question tests the understanding of concepts of dynamic lapse and target volatility funds and ability to apply the theoretical concepts to the product design of variable annuity contract. Overall, the candidates performed as expected on this question.

Solution:

(a) Describe how target volatility funds are designed to achieve their objective.

Commentary on Question:

The candidates performed above average on this question. Most candidates were able to describe what a target volatility fund is, how a target volatility fund achieves its target, or both.

A target volatility fund is an investment fund that targets a certain level of volatility on its returns over a period of time. The fund achieves this target by dynamically rebalancing asset mix to maintain a level of portfolio volatility over time.

(b) Calculate the expected return of this fund based on the above information.

Commentary on Question:

The candidates performed as expected on this question. However, many candidates used a simplified formula, which doesn't take into the account the equity/fixed correlation, to derive the equity allocation. In this case, only partial credit was awarded. Many candidates also rounded to the nearest tenth or hundredth.

12. Continued

Assume the α% of the fund is in fixed income, and (1- α)% is in equities.

$$\sigma_{Fund}^2 = \alpha^2 * \sigma_{Fixed\ income}^2 + (1-\alpha)^2 * \sigma_{Equities}^2 + 2 * \alpha * (1-\alpha) * \sigma_{Fixed\ income} * \sigma_{Equities} * \rho$$

$$0.1^2 = \alpha^2 * 0.02^2 + (1-\alpha)^2 * 0.16^2 + 2 * \alpha * (1-\alpha) * 0.02 * 0.16 * 0.3$$

Solving this equation will yield α = 0.391, 1.655

Discard 1.655 as it implies a short position for equities. So α = 0.391

Finally,

$$r_{Fund} = \alpha * r_{Fixed\ income} + (1-\alpha) * r_{Equities}$$
$$r_{Fund} = 0.391 * 0.04 + (1-0.391) * 0.06$$
$$r_{Fund} = 0.0522$$

(c) Explain how a target volatility fund can be used to design a competitive product.

Commentary on Question:
The candidates performed below average on this question. Many candidates explained what a target volatility fund is in this section but did not relate this back to how it can be used to design a competitive product.

A target volatility fund generally has lower volatility on the returns than a pure equity fund, which, in turn, lowers the cost of the guarantee, so it can be used to design a low-cost variable annuity.

A target volatility fund rebalances more assets into higher return equity in an upmarket scenario, which allows policyholders to participate in the higher returns.

(d) Describe how dynamic lapses will impact the insurance company as the moneyness of the guarantee changes.

Commentary on Question:
The candidates performed as expected on this question. Many candidates accurately explained what a policyholder would do when the guarantee is in the money or out of the money but did not further elaborate on how this will impact the insurance company.

When equities are rising, the embedded guarantees are less in the money, and policyholders will tend to lapse more to avoid paying for the guarantees. Lapses reduce the amount of fees collected by the insurance company.

12. Continued

In a down market scenario, the embedded guarantees are more in the money, and policyholders are not as likely to lapse. The increased persistency (more policies) comes at a time when the guarantees are valuable, thus increasing the expected cost of the guarantee to the insurance company.

(e) Discuss how a company can mitigate the impact of dynamic lapses on its VA business.

Commentary on Question:
The candidates performed as expected on this question. Most candidates only listed one way for a company to mitigate the impact of the dynamic lapses.

Company can mitigate the impact by:
1. Changing the product design by adding a surrender charge or market value adjustment upon surrender to discourage policyholders from lapsing.
2. Include dynamic lapses in its hedging program.
3. Enter into a reinsurance agreement.

13. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

Learning Outcomes:

(4a) Understand the concept of a factor model in the context of financial time series.

(4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.

(4i) Demonstrate an understanding of the general uses and techniques of stochastics modeling.

Sources:

QFIA-119-14: Analysis of Financial Time Series, Tsay, 3rd edition, Ch. 9

QFIA-125-16: Market Models: A Guide for Financial Data Analysis, Ch. 6, Principal Component Analysis

QFIA-124-16: IAA, Stochastic Modeling, Theory and Reality from and Actuarial Perspective, sections I-1 to I-29 and II-1 to II-24

Commentary on Question:

This question tests the application of statistical factor model in analyzing zero-coupon yield.

Solution:

(a) Explain how principal component analysis (PCA) can help overcome problems of multicollinearity in the data.

Commentary on Question:

The candidates performed below average on this section.
Though most candidates were able to explain some parts of these problems, few candidates can provide a full list.

- The explanatory variables need to be normalized to have a mean of 0 and variance of 1.
- Instead of regressing dependent variable on explanatory but correlated data, regressing it on principal components under OLS and obtain intercept estimate and slope estimates.
- Then find the principal components and their factor weights.
- Use the factor weights to convert these estimates into coefficients for the original model.

13. Continued

- Because the principal components are perpendicular, i.e. uncorrelated with each other, this method will achieve the orthogonalization of the correlated variables.

(b) Calculate the percentage of variation explained by each of the five components above.

Commentary on Question:
The candidates performed as expected on this section.
The wrong denominator (the sum of the first five eigenvalues given) was used by many candidates for which only partial credit was awarded.

- Percentage explained by component i is $\frac{\lambda_i}{number\ of\ variable}$, in this case, $\lambda_i/10$.
- The percentage of variation explained by each of the five components calculated.

Component	e_1	e_2	e_3	e_4	e_5
% explained	0.8786	0.0843	0.0216	0.0080	0.0054

(c) Recommend the factors that should be included in your model, including an interpretation of each factor.

Commentary on Question:
The candidates performed as expected on this section. The candidate may recommend the first 2 or 3 factors to receive full credit.

If the candidate recommends the first 2 factors,
- The first two components explain about 96.29% of total variability of the data.
- Additional factors do not increase this percentage significantly.
- The first component is the trend component of the yield curve. An upward shift in the first component induces a roughly parallel shift in the yield curve.
- The second component is the tilt component. The factor weights on the second component decrease with maturity. An upward movement in the second component induces a change in the slope of the yield curve.

If the candidate recommends the first 3 factors,
- The first three components explain about 98.45 % of total variability of the data.
- Additional factors do not increase this percentage significantly. Although the third component only explains 2.16% of total variation of the data, it has an intuitive interpretation.

13. Continued

- The first component is the trend component of the yield curve. An upward shift in the first component induces a roughly parallel shift in the yield curve.
- The second component is the tilt component. The factor weights on the second component decrease with maturity. An upward movement in the second component induces a change in the slope of the yield curve.
- The third component is the convexity component. The factor weights are positive for short rates, but decreasing and becoming negative for the medium-term rates, and then increasing and becoming positive again for the longer maturities. The factor weights influences the convexity of the yield curve

(d) Calculate the communality for the 1-year zero-coupon yield based on your recommended model.

Commentary on Question:
The candidates performed as expected on this section.
Most candidates were able to calculate the factor loading for different components, but some of them failed to calculate the communality using the correct formula.

- The factor loading matrix is $\widehat{\boldsymbol{\beta}} = [\sqrt{\lambda_1}\,\hat{\boldsymbol{e}}_1|\sqrt{\lambda_2}\,\hat{\boldsymbol{e}}_2| \ldots \sqrt{\lambda_m}\,\hat{\boldsymbol{e}}_m]$, where m is the number of common factors in the model.
- The communalities are estimated by $\hat{c}_i^2 = \hat{\beta}_{i1}^2 + \hat{\beta}_{i2}^2 + \cdots + \hat{\beta}_{im}^2$.

If the candidate recommended 2 factors in part c),

- The factor loading for the 1st component is
$$\hat{\beta}_{11} = \sqrt{8.7860} \times 0.3024 = 0.8963.$$
- The factor loading for the 2nd component is
$$\hat{\beta}_{12} = \sqrt{0.8434} \times 0.3293 = 0.3024.$$
- The communality for the 1-year yield is
$$\hat{\beta}_{11}^2 + \hat{\beta}_{12}^2 = 0.8963^2 + 0.3024^2 = 0.8948.$$

If the candidate recommended 3 factors in part c),

- The factor loading for the 1st component is
$$\hat{\beta}_{11} = \sqrt{8.7860} \times 0.3024 = 0.8963.$$
- The factor loading for the 2nd component is
$$\hat{\beta}_{12} = \sqrt{0.8434} \times 0.3293 = 0.3024.$$
- The factor loading for the 3rd component is
$$\hat{\beta}_{13} = \sqrt{(0.2161)} \times (-0.6091) = -0.2831.$$
- The communality for the 1-year yield is
$$\hat{\beta}_{11}^2 + \hat{\beta}_{12}^2 + \hat{\beta}_{13}^2 = 0.8963^2 + 0.3024^2 + (-0.2831)^2 = 0.9749.$$

14. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Infrastructure

Learning Outcomes:

(3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.

(3e) Apply liquidity risk models, including modeling cashflow of various types of assets (e.g. indeterminate maturity assets) and liabilities.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

Sources:

105-13 – Report of the Life Liquidity Working Group of the American Academy of Actuaries

106-13 – Liquidity Risk: Measurement and Management – A practitioner's Guide to Global Best, Chapter 3

111-13 – Managing Investment Portfolios, Chapter 8

Commentary on Question:

This question tests the candidate's understanding of liquidity risks and asset portfolio characteristics through an example of a variable annuities product portfolio. The candidates were expected to explain different aspects of the underlying liability and asset portfolio. Well justified calculations and recommendations were required to score maximum points.

Overall the candidates demonstrated good understandings of the tested concepts. Some candidates had trouble with the considerations for deterministic scenario testing.

14. Continued

Solution:

(a) Explain which product features impact the company's net amount at risk associated with the GMDB.

Commentary on Question:
The candidates performed poorly on this section. Most candidates were able to recognize the net amount at risk of the GMDB product and understood its correlation with fund options offering. However, not a lot of elaborations on other product features were observed.

With GMDB, the net amount at risk is directly linked to the evolution of Max (AV - GMDB, 0) overtime.

M & E Fee & Commissions: the amount of fees and commissions deducted from the policyholder's fund has a direct impact on amount of guarantee available in the contract at any given point.

Surrender charge: for VA products, surrenders are paid from the policyholder's fund or the separate account. The decreasing surrender charge structure may encourage more out of money (zero net amount at risk) lapses after 8 years, which likely doesn't impact the company's total net amount at risk.

Partial Withdrawals: withdrawal amount is irrelevant since money is taken from separate account on pro-rata basis for withdrawals. With dollar-for-dollar, withdrawals would make the guarantees deeper in-the-money when market tumbles, hence increasing the amount of benefit claims paid.

Fund Options: more aggressive fund option increases the likelihood of policy getting in-the-money and hence increase the amount of benefit claims paid.

(b) Evaluate the asset portfolio above to support the GMDB liability with respect to credit, diversification and liquidity.

Commentary on Question:
The candidates performed above average on this section. Most candidates provided well justified comments about the asset portfolio.

The portfolio has a good balance of liquid and illiquid assets. However, there is lack of diversification. For instance, the company can add Treasure Notes and reduce the exposure to equity risks while maintaining the liquidity. Also, the portfolio consists of only traditional investments (equity and bonds). There is a chance to add alternative investments to the portfolio to provide for diversification potential.

14. Continued

Liability provision is for 25 years. However, the portfolio appears to have a shorter duration and hence potential duration mismatch.

AA corporate bonds are subject to counterparty credit risks. Also, market value of bonds may drop in inflationary periods.

Catastrophe death claims can create liquidity risk (can't sell assets quick enough if illiquid).

(c) Describe the key considerations in creating deterministic scenarios for stress testing liquidity risk.

Commentary on Question:
The candidates performed as expected on this section. Most candidates provided the right considerations for creating deterministic scenarios for testing. Some candidates provided the stress levels to test or described various scenarios, which is not what the question is looking for.

1. Create and evaluate a wide variety of scenarios.
2. Ordinary course of business scenarios should include seasonal fluctuations.
3. Synchronize ordinary course of business scenario with the budget.
4. Need to evaluate both long-term and short-term scenarios.
5. Stress levels must not be confused.
6. A funding problem can build over months or can become severe in one or two days.
7. Most important, the scenario and its' elements must be relevant to the strategic nature and to the stability/stickiness of the liabilities of the company.

(d) Calculate the cash flow coverage ratio over one month for the stress scenario for:

(i) the reallocation into Real Estate

(ii) the reallocation into Private Equity

Commentary on Question:
The candidates performed brilliantly on this section. Most candidates were able to identity the change in cashflows for the two different asset reallocation strategy. A few candidates had calculation errors.

Current cash flow coverage ratio = (24+50+40)/100 = 1.14

Reallocate 20% of portfolio out of equities means reduce asset cash flows by 20/30 of 24 = reduce asset cash flows by 16 => new amount of cash flows from equities = 8

14. Continued

Asset cash flows with real estate allocation = 8+50+40+8 = 106 => cash flow coverage ratio = 106/100 = 1.06

Asset cash flows with private equity allocation = 8+50+40+0 = 98 => cash flow coverage ratio = 98/100 = 0.98

(e) Recommend any changes to the portfolio allocation given the above analysis.

Commentary on Question:
The candidates performed above average on this section. Most candidates provided justified recommendations. Some candidates didn't recognize the improved portfolio return or better diversification from the reallocation.

Recommend reallocation into real estate for the follow reasons:

- Real estate offers higher expected return than equities and is not much lower than the expected return of private equity.
- Real estate allocation would remain compliant with the required cash flow coverage ratio. While private equity allocation would result in a cash flow coverage ratio below the required limit set by the CRO.
- Real estate has more liquidity than private equity under the stressed scenario.
- Real estate has the potential to offer more diversification than private equity.

Alternatively, recommendation to allocate less than 20% to private equity is also acceptable provided the allocation results in a cash flow coverage ratio meeting the required limit. This allocation strategy could result in a higher portfolio return.

15. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3g) Understand and apply techniques to manage street liquidity risk.

Sources:

Shin, H. S., Reflections on Modern Bank Run – A Case Study of Northern Rock

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Explain how FGH Bank's activities are similar to those that led to the "run on the bank" at Northern Rock.

Commentary on Question:

Candidates performed as below average on this section. Note: Utilizing a large amount of securitization is a similarity, but since it was not a cause that led to the "run on the bank" at Northern Bank, it is not applicable to the answer and did not get any credit.

Main activities at FGH Bank that are similar to those that led to "run on the bank" at Northern Rock are:

A very rapid rate of growth that could result in outstripping of its traditional funding base of retail deposits, like in case of Northern Bank.

FGH Bank's funding of its long-term mortgages with short-term debt. Northern Bank faced a liquidity crisis in stress times because it could not easily reduce its mortgage assets and could not find a creditor to help provide additional financing.

15. Continued

(b) Calculate the impact on leverage assuming a stress scenario where the average Repo haircut increases from 4% to 20%.

Commentary on Question:
Candidates performed above average on this section. The majority of the candidates were able to calculate the impact accurately.

Increases in haircut reduce the maximum permissible leverage the borrower can hold.

The maximum leverage at the 4% haircut is 100/4 = 25.
The maximum leverage at 20% haircut is 100/20 = 5

(c) Explain the risks the CRO should be concerned about under the stress scenario in part b).

Commentary on Question:
Candidates performed as expected on this section. It is given in the question that banks adjust their leverage to reflect the fluctuation in haircut.

The significant decrease in maximum permissible leverage for banks means it either has to sell assets or raise new equity (L = A/E) under the stress scenario.

The deleveraging in the marketplace will force creditors to reduce the size of their balance sheets. As a result, these previously highly leveraged creditors will reduce or stop providing FGH Bank with much needed funding. This pull in funding is a liquidity issue for the bank.

(d) Recommend changes to FGH Bank's activities that decrease its liquidity risk.

Commentary on Question:
Candidates performed as expected on this section. Note that in practice the bank will have to change the duration of its assets & liabilities over a period of time - it cannot be done overnight. Candidates who failed to recognize this did not get full credit.

Going forward the bank should:

Start using long-term debt to finance the bank's assets, and eliminate or substantially reduce use of short-term debt.
Reduce the long-term nature of the assets by reducing the amount of mortgages issued over time.
Seek to sell off a portion of the existing mortgage portfolio.
Seek to reduce leverage. For example, FGH may seek to raise more equity.

15. Continued

(e) Critique the "capital buffer" approach for financial regulation of banks.

Commentary on Question:
Candidates performed below average on this section. "Critique" does not imply that only faults/errors are to be presented; it should also point out what is good. Only a handful of candidates recognized this.

Quantitative restriction on the minimum size of capital buffer is relative to equity

Rationale of capital buffer approach is to maintain solvency

The approach is in the interest of both depositors and creditors

Capital buffer is related to riskiness of the assets

Does not take into consideration that interests of one institution impacts the interests of others

Does not take into consideration the risks posed by mismatch of assets and liabilities

(f) Describe two alternative approaches for liquidity regulations that overcome the shortfalls of the traditional approach of a capital buffer.

Commentary on Question:
Candidates performed as expected on this section. Given below are two alternative approaches. Other suggestions for alternatives were acceptable provided they are well thought out and described.

<u>First:</u>
Regulation that constrains the composition of assets.
Bank can survive a run if it holds more liquid assets
Bank can survive a run if it holds more illiquid liabilities
Creates liquidity buffers -- will help mitigate shocks in liquidity within the markets

<u>Second</u>:
Limit on the raw leverage ratio
Prevents buildup of leverage in the market, but does not take into account asset risk classes.
For creditors, the leverage ratio constraint will reduce the need to hold back lending in down times.
For borrowers, the lower leverage ratio minimizes the impact from an increase in haircut.

16. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1h) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

Sources:

Interest Rate Models – Theory and Practice, 2nd Edition, Brigo, D and Mecurio F, Sections 4.1-4.2

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Justify the use of this model.

Commentary on Question:

Candidates performed as expected on this question, recognizing that de-correlation is needed for complex securities..

Two factor models capture yield curve de-correlation, which is desirable for instruments with exposure to multiple interest rates. One factor models assume perfect yield curve correlation.

16. Continued

(b) Calculate the probability that the short rate at 1-month is negative

Commentary on Question:
Candidates performed brilliantly on this question, mostly calculating the probability perfectly

$$P(r\left(\frac{1}{12}\right) < 0) = \Phi\left(-\frac{0.0025}{sqrt(3e-6)}\right) = \Phi(-1.44) \approx 7\%$$

(c) Discuss the shape of the volatility term structure of instantaneous forward rates with Set 1 parameters in light of the shape that usually prevails in the market

Commentary on Question:
Candidates performed as expected on this question, noting the expected shape of the curve

Term structure of the instantaneous forward curve is usually humped, with a p value of 0.51 no hump is possible. This leads a calculation that is a monotonically decreasing function over time..

(d) Calculate the price of the zero-coupon-bond at 1-month with maturity at 1 year, P(1/12, 1).

Commentary on Question:
Candidates performed as expected on this question, recognizing the correct formula and deriving a numerical result.

$$P\left(\frac{1}{2}, 1\right) = \frac{0.95}{0.99} exp\left\{\frac{1}{2}[5.3e6 - 6.6e6 + 6.8e9] - \frac{0.015(1-e^{-0.9\left(\frac{11}{12}\right)})}{0.9} - \frac{0.008(1-e^{-0.78\left(\frac{11}{12}\right)})}{0.78}\right\} = 0.9457$$

(e) Describe how cap caplet and caplet volatilities are defined in the market model for caps.

Commentary on Question:
Candidates performed below average on this question, mostly omitting an answer.

Caplet volatilities are derived from market prices by inverting the related market formulas or using suitable integrals/averages of compounded forward rates.