III

An Expanded Financial Structure for Ordinary Dividends

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Abstract

This paper develops a dividend financial structure and a dividend formula based on the contribution principle in a complete expanded format. Objectives are suggested for the design of a dividend system that (1) fits an environment demanding carefully defined and demonstrable equity among dividend classes, (2) is understandable and controllable by management, and (3) has continuity from year to year.

The structure and formula are derived from the most generalized equation of equilibrium, and each dividend class is treated as an ongoing microcosm of page 5 of the annual statement. Modified investment-year method (IYM) interest credits, including policy loan effects, and an objective approach to charging all company expenses in the dividend formula are given special attention. Formulated deficit and surplus in each dividend class are an explicit part of the structure. A predetermined period for amortizing issue expenses is recognized, as are specific profit charges aimed at the defined surplus goals and consistent with company surplus policy.

The similarity of dividend funds to generally accepted accounting principles (GAAP) reserves is evident during the issue-expense amortization period. Formulas for the emergence of profit on the statutory reserve basis and on the internal-management-adjusted reserve basis are developed.

I. Introduction

There is substantial current interest on the part of regulators, consumers, actuaries, and others in the techniques used for the distribution of surplus. Because of the current debate relative to the use of new-money or portfolio rates, public attention has tended to focus on the investment methods used for determining the contribution to surplus; however, that is but one aspect of surplus distribution worthy of attention. There is also current concern about dividend illustrations and their comparability among companies, as well as concern that companies that prepare illustrations on one basis may switch their dividend formulas to another basis at a future time when circumstances are different. A number of criteria are desirable for a proper financial structure for dividends—one that demonstrates equity in the distribution of surplus across all years of issue and all policy forms.

II. Objectives

For an approach to have merit, it should attempt to meet the following objectives:

1. The dividend formula should be related to the Analysis of Operations by Lines of Business (page 5 of the NAIC annual statement) in the United States, so that all items of this analysis move through the formula in such a way that the dividend for a particular policy from a year-of-issue, age-at-issue, pension or nonpension class either can be a considered a microcosm of page 5 or can be reconciled to this or some other company statement. This will provide a discipline in dividend work that may prove valuable in dealing with regulators and the public.
2. The dividend formula should permit a demonstration of equity across all classes by reason of its derivation from actual experience and the reconciliation suggested in the preceding paragraph. Coefficients or factors in the dividend formula should tend to be uniform for all classes to

the extent feasible, and the actuary should have specific reasons for departures from uniformity.

3. The actuary should be aware of the disposition of company expenses, and all expenses should either be recognized in the dividend formula or be specifically excluded, so that no company expenses are "lost" in the process. The expense charges in the dividend formula should be based on a well-defined analysis and allocation procedure. Direct costing procedures should be followed to the extent possible, since approximate or inconsistent allocations may break down as the volume of business increases or as the mix of business changes.
4. The charges for amortization of issue expenses should be explicit. Profit charges in the formula should be explicitly defined and should be combined with funds from the amortization of issue expenses to determine whether or not company surplus objectives can be met.
5. Profits arising from nonparticipating benefits and from subsidiary operations should be credited in dividends in a defined manner.
6. The dividend formula and related structure should also allow company management to understand the relationship to the annual statement for current and long-range financial planning. It should be feasible to relate new expenditures under consideration directly to the changes that would result in the associated charges in each dividend. While these facilities commonly have been available in group insurance and group pension lines, they have not been available explicitly in ordinary lines.
7. The structure should have a dynamic capability of showing how changes in the future will affect dividends in all classes. It would replace existing asset share procedures. There should also be a monitoring capability to ensure fit to actual experience. The structure should allow reasonably easy maintenance and changes of dividend scales for in-force policies both during and after the period of amortization of acquisition expenses.
8. Any smoothing of dividends should preserve actuarial equivalences and be reflected in the surplus accounts in each class of the dividend structure.
9. Investment income factors should include specific formulation of any investment year method (IYM), policy loan, and federal income tax (FIT) ingredients according to an appropriate marginal formula. If IYM effects are included, a definite historical and continuing structure of investment-year funds and IYM rates should be established; such funds should be consistent with deficits and surplus inherent in the dividend formula.
10. While the structure should be related to statutory reserves, it should also provide for analysis on an internal-management basis, similar to GAAP, that reflects going-concern principles.

Expanding on the traditional three-factor contribution formula, which explicitly involves interest, mortality, and expenses, I propose a structure in which issue expenses are separated from renewal maintenance expenses, and are amortized by reference to a special "surplus" component in the dividend formula. The normal three-factor structure makes no explicit provisions for such items as amortization of initial expenses, earnings on surplus funds, contributions to surplus, federal income tax, profit or loss on surrenders or lapses, and earnings on ancillary nonparticipating benefits. Actuaries, however, typically have found ways of incorporating some or all of these factors into the traditional three-factor formula (e.g., subtracting federal income tax from investment income before apportioning it, varying expense charges in the dividend to make provision for issue-expense amortization, and so forth).

The proposed surplus component would encompass the desired amortization of issue expenses, some earnings on the excess of invested assets over funds intrinsic to the dividend structure, the desired contribution to surplus (or profit), the contribution to surplus by nonparticipating ancillary benefits, and the profit or loss on surrender or lapse.

In view of the many different forms that federal income taxation may take, depending on the financial results and corporate structure of the company, I propose keeping this factor partially separate.

These considerations suggest a "five-factor" formula based on (1) investment experience and reserve increase, (2) mortality experience, (3) renewal or maintenance expenses, (4) federal income tax, and (5) return of, and contribution to, surplus. This last factor consists of five components: (*a*) amortization of issue expenses, (*b*) profit (contribution to surplus), (*c*) profit (or loss) on surrender or lapse, (*d*) profit from nonparticipating benefits, and (*e*) apportionable net investment income (after FIT) on the difference between total invested assets and the funds intrinsic to the dividend structure. (This may be positive, negative, or zero.)

Consider the effects of the surplus factor on company surplus: The surplus (including the mandatory securities valuation reserve [MSVR] for these purposes) of a mutual life insurance company at the end of any year (assuming no unusual transactions during the year) would be equal to the following:

The surplus at the end of the previous year

+ the specific contribution to surplus included in the dividend formula
+ investment earnings (after FIT) on the surplus funds and non-interest-bearing liabilities to the extent not apportioned in dividends

+ amortization of prior years' acquisition costs charged against policies during the year
– issue expenses and commissions on policies written during the year
± variations between actual company experience and that reflected in dividends
± capital gains (losses) not reflected in the IYM structure, changes in nonadmitted assets, and other such items outside of net income.

III. Definitions

n = Policy duration (policy year for π, P, E, G, R);
V_n = Policy reserve (statutory basis);
$f_n V_n$ = Dividend fund (asset share);
$(f_n-1)V_n$ = S_n = Statutory surplus inherent in dividend structure;
π_n = Gross premium in policy year n;
P_n = Net premium in policy year n;
F_n = Death benefit (including any terminal dividends and any appropriate factor such as $\pi_n/2$ for any "unaccrued premium" includible in amount payable on death);
C_n = Cash value (including any terminal dividend);
D_n = Annual dividend;
$(TD)_n$ = Terminal dividend;
m = 0, 0.5, 1, respectively, for full, pro rata, or zero dividend payable in year of death;
i' = Portfolio rate, IYM rate, or combination thereof (before FIT), including policy loan ingredient;
(FIT) = Federal income tax on Fraser marginal formula (Phase I company) with coefficients m^I, m^A, m^{NP}, m^P;
$(FIT)_{CR}$ = FIT credit for required interest on mean reserve involving m^{NP} *or* m^P;
i'' = $(1 - m^I)i' - m^A = i'$ after FIT investment income and asset components;
q'_{n-1} = Mortality rate applicable in policy year n;
w'_{n-1} = Termination rate applicable at end of policy year n;
E_n = $E'_n + E''_n$ = Premium taxes and nonissue expenses and commissions in policy year n;
E'_n = Portion of E_n incurred at the time the annual premium is paid;
E''_n = Balance of E_n, incurred at the middle of the policy year on policies in force at year $n-1$;
E_0 = Issue expenses, commissions, and certain home office, agency manager, general agent, and related allowances and costs for selling and supervision in excess of an amount allocated to E'_1 for purposes of smoothing gradation of E'_1 into E'_2 (see note below on CRVM reserve basis);
G_n = Profits from nonparticipating benefits and from corporate subsidiaries allocated at n;
R_n = Any investment income (after FIT) on excess of ordinary invested assets over funds intrinsic to the dividend financial structure apportioned at n;
B_n = Charge at n for amortization of issue expenses E_0 for $n \le k$ and charge for profit for $n > k$;
(ΔD_n) = Smoothing adjustment of D_n made at $n \le k' \le k$;

$$_np' = {}_{n-1}p'(1 - q'_{n-1} - w'_{n-1}) = \prod_{t=1}^{n}(1 - q'_{t-1} - w'_{t-1}),$$

where $_0p' = 1$;

$$a_{\overline{k-n|}} = \sum_{t=1}^{k-n} ({}_np')^{-1}\, {}_{n+t-1}p'(v'')^t.$$

Note on E_0: In the formulas, V_n is assumed to be on the net level premium (NLP) reserve basis. Where V_n is on the Commissioners Reserve Valuation Method (CRVM) basis, the issue-expense term E_0 is to be replaced in all formulas by the term

$$E_0 - (P_2 - P_1),$$

and the premium-related non-issue-expense term in the first year, E'_1, is to be replaced in all formulas by the term

$$E'_1 + (P_2 - P_1).$$

In other words, issue expenses on each policy, and in aggregate, are reduced by $(P_2 - P_1)$; first-year premium-related non-issue expenses are increased by $(P_2 - P_1)$ and charged against the increased first-year loading; and the issue expenses amortized by B_n are $E_0 - (P_2 - P_1)$. The formulas are otherwise unchanged.

IV. Generalized Formulas for the Financial Structure and for Dividends

The formulas for the financial structure and for the generalized dividend scale are based on the generalized equation of equilibrium underlying the Analysis of Operations by Lines of Business. The formulas are developed in this section, leaving the discussions of each factor to later sections.

For annual premium cash-value life insurance, the generalized equation of equilibrium is as follows for each policy-form, age-at-issue, and year-of-issue class:

$$\begin{aligned} f_nV_n = \; & (P_n + f_{n-1}V_{n-1})(1+i') \\ & + (\pi_n - P_n)(1+i') - [E'_n(1+i') + E''_n(1+i'/2)] \\ & - q'_{n-1}[F_n(1+i'/2) - f_nV_n] \\ & - w'_{n-1}(C_n - f_nV_n) \\ & - (FIT) \\ & + G_n + R_n \\ & - D_n[1 - mq'_{n-1} + (i'/2)q'_{n-1}(1-m)], \end{aligned} \tag{1}$$

where $f_0V_0 = -E_0$.

The function f_n must be derived and defined so as to control surplus (or deficit) design. It drives the generalized dividend design and controls the timing of the release-from-risk mechanism of the financial structure. (Indeed, the generalized dividend formula need not refer to reserves at all, since the dividend fund could be designed as a function independent of V_n, such as a natural reserve.)

In the dividend structure, f_n is determined so as to (1) amortize issue expense in k years; (2) smooth dividends over k' years to remove jumps arising from irregular commission rates and commission-related marketing and field management compensation and expense reimbursements; (3) provide for necessary adjustments (e.g., on economatic types of contract, to ensure early dividends large enough to provide one-year term dividend additions); and (4) provide for surplus development $n > k$ so that net income after dividends is adequate to meet company surplus objectives. In the design of f_n, actuarial equivalence is always preserved by accounting for the effects of interest, mortality, and terminations.

Formula (1) can be solved for the dividend by setting $f_nV_n = V_n + S_n$, factoring the FIT m^I and m^A components into i', and omitting the higher-order term $(i'/2)q'_{n-1}(1-m)D_n$, as follows:

$$\begin{aligned} (1 - mq'_{n-1})D_n = \; & \\ & (P_n + V_{n-1})(1+i'') - V_n \\ & + (\pi_n - P_n)(1+i'') - [E'_n(1+i'') + E''_n(1+i''/2)] \\ & - q'_{n-1}[F_n(1+i''/2) - V_n] \\ & - w'_{n-1}(C_n - V_n) \\ & + (FIT)_{CR} \\ & + G_n + R_n \\ & - [S_n[1 - q'_{n-1} - w'_{n-1}) - S_{n-1})(1+i'')]. \end{aligned} \tag{2}$$

Formula for Surplus Intrinsic to the Dividend Financial Structure

Now, let B_n be the charge for amortization of issue expenses and for profit, and (ΔD_n) the smoothing factor with present value at issue equal to zero. These factors are discussed in detail in Section V. Then

$$S_n = \frac{B_n - (1 - mq'_{n-1})(\Delta D_n) + (1+i'')S_{n-1}}{1 - q'_{n-1} - w'_{n-1}}, \qquad S_0 = -E_0. \tag{3}$$

Formula for the Generalized Dividend

Introducing formula (3) into formula (2), the generalized dividend formula emerges:

$$\begin{aligned} D_n(1 - mq'_{n-1}) = \; & \\ & (P_n + V_{n-1})(1+i'') - V_n \\ & + (\pi_n - P_n)(1+i'') - [E'_n(1+i'') + E''_n(1+i''/2)] \\ & - q'_{n-1}[F_n(1+i''/2) - V_n] \\ & + (FIT)_{CR} \\ & - w'_{n-1}(C_n - V_n) \\ & + G_n + R_n \\ & - B_n + (1 - mq'_{n-1})(\Delta D_n). \end{aligned} \tag{4}$$

Formula (4) is the basic generalized dividend formula, and formula (3) is the formula for surplus in the dividend financial structure. It is desirable to extend the software to produce S_n along with D_n, not only for design purposes but also to meld the new dividend scale to earlier dividend scales, which are likely to have a somewhat different shape. This will be discussed later.

V. The Net Surplus Charge Factor $B_n - (1 - m\,q'_{n-1})(\Delta D_n)$

Formula (3) can be rewritten as

$$(f_n - 1)V_n = \frac{B_n - (1 - m\,q'_{n-1})(\Delta D_n) + (1 + i'')(f_{n-1} - 1)V_{n-1}}{1 - q'_{n-1} - w'_{n-1}}. \quad (3)$$

As noted earlier, the function f_n, which is the ratio of the dividend fund (asset share) to the reserve, drives the dividend financial structure and determines the incidence of recovery of issue expenses and the accumulation of surplus. Formula (3a) shows that the factor B_n, the charge for amortization of issue expenses and for profit, adjusted by the smoothing factor (ΔD_n), determines f_n. The factor $B_n - (1 - m\,q'_{n-1})(\Delta D_n)$, therefore, is the essence of the dividend formula design, and it is desirable to consider the details of this factor thoroughly.

Suppose we require that issue expenses be amortized over k years and find that dividend smoothing is needed over k' years, where $k' \le k$. The issue-expense amortization period, k, can be related to some criterion such as half the expectancy of life of the policy-form, issue-age, and issue-year class, and might turn out to be something like this:

Issue Age (x)	Issue-Expense Amortization Period (k)
Less than 45...........	20
46-55...................	$(65 - x)$
56 and over...........	10

Dividends must be smoothed to remove the effects of irregular renewal commission rates and related linear expenses, to establish a proper level of early dividends on economatic policies, and so on. The need for smoothing usually disappears after ten years or so; thus $k' \le 10 \le k$.

Factor B_n

Since the recovery of issue expenses is the prime objective in the years immediately after issue, B_n must be dedicated to such recovery until it is completed and thereafter can be designed to provide profit for building surplus. The simplest and most practicable design of B_n appears to be as follows:

$$B_n = \frac{E_0}{a_{\overline{k}|}} \quad \text{for } n \le k \text{ (issue-expense amortization)}$$

$$= g_n V_n + \frac{E_0 r_n}{a_{\overline{k}|}} \quad \text{for } n > k \text{ (profit),}$$

where

$$a_{\overline{k}|} = \sum_{t=1}^{k} (v'')^t \,{}_{t-1}p';$$

$${}_tp' = {}_{t-1}p'\,(1 - q'_{t-1} - w'_{t-1}) = \prod_{s=1}^{t} (1 - q'_{s-1} - w'_{s-1});$$

$$v'' = \frac{1}{1 + i''};$$

g_n = A function defined to develop surplus consistent with company surplus policy and objectives;

r_n = A, function that equals unity at $n = k + 1$ and decreases rapidly thereafter.

The minimum requirement for B_n at durations $n > k$ is needed to ensure smoothness of dividends at durations immediately following k.

This simple flat design of B_n for $n \le k$ *is* not unique. For instance, issue expenses might be amortized at a decreasing rate by duration:

$$B_n = \frac{E_0}{a''_{\overline{k}|}}\,\lambda^{n-1},$$

where $0 < \lambda \le 1$ is a constant and

$$a''_{\overline{k}|} = \sum_{t=1}^{k} (v'')^t \,{}_{t-1}p'\lambda^{t-1}.$$

It is likely, however that early dividends will turn out to be negative unless λ is close to unity. If $\lambda > 1$, early dividends are likely to be excessive; the amortization period, k, would have to be extended; and surplus development would be inadequate.

The profit charge $g_n V_n$ for $n > k$ is related to the reserve to recognize the asset risk as paramount. More generally, the charge would take the form $g_n V_n + h_n F_n$, where h_n would represent claim risk.

Company statutory surplus, including MSVR, can be regarded as consisting of two parts: (*a*) a portion representing the extent to which capacity is already utilized and (*b*) the balance, which represents further strength and serves as a source of company vitality. (See *RSA*, III, No. 1 [1977], 27-32.)

In many companies, the first part of statutory surplus is related primarily to (1) the potential loss of both asset values and income resulting from defaults in a serious recession with inflation and disintermediation, and from common stocks in the general account; (2) recession-related claim increases in the disability line; and (3) surplus losses from investment income rates lower than valuation interest rates on products supported by assets with shorter maturities than the liabilities, and from new-money rates higher than investment income rates on products with asset maturities longer than the liabilities. This part also contains provision for adverse variation in claims, epidemics, earthquakes, and other probabilistic events not related to the economic environment. The size of this portion may depend largely on the company's attitude as to the worst economic environment that should be contemplated without governmental bailout.

The predominance of the economic environment in surplus policy naturally leads to a profit charge B_n $(n > k)$ that is based primarily on reserves.

Factor (ΔD_n)

The smoothing factor (ΔD_n) is superposed on the basic amortization schedule on an actuarial-equivalence principle $n \le k' \le k$, so that

$$\sum_{t=1}^{k'} (v'')^t {}_{t-1}p'(1 - mq'_{t-1})(\Delta D_t) = 0.$$

(ΔD_n) can be determined to produce dividends increasing smoothly to D_k by constant first or second differences.

Because of the definitions of B_n and (ΔD_n), we have

$$\sum_{t=1}^{k} (v'')^t {}_{t-1}p'[B_t - (1 - mq'_{t-1})(\Delta D_t)] = E_0;$$

thus, the net amortization amount in year $n \le k$ is $[B_n - (1 - mq'_{n-1})(\Delta D_n)]$, with $(\Delta D_n) = 0$ for $n > k'$.

Surplus S_n

The values of S_n are as follows:

$S_0 = -E_0$;

$$S_n = -\left[\frac{E_0}{a_{\overline{k}|}} a_{\overline{k-n}|} + \sum_{t=1}^{k'-n} (v'')^t \frac{{}_{n+t-1}p'}{{}_np'}(1 - mq'_{n+t-1})(\Delta D_{n+t})\right] \quad \text{for } n \le k'$$

$$= -\frac{E_0}{a_{\overline{k}|}} a_{\overline{k-n}|} \quad \text{for } k' \le n \le k$$

$S_k = 0$;

$$S_n = \frac{g_n V_n + (1 + i'')S_{n-1}}{1 - q'_{n-1} - w'_{n-1}} \quad \text{for } n > k.$$

To preserve equity across all classes, it is desirable that the $B_n - (1 - mq'_{n-1})(\Delta D_n)$ design apply uniformly across all dividend classes, taking account of different values of E_0 in different years of issue. The design should be appropriately different in NLP and CRVM policies and in those with different valuation interest rates, so that surplus development will be consistent with risk.

It is desirable to provide an ongoing monitoring capability to ensure that actual amortization and surplus development conform to dividend design expectations. When significant differences develop, B_n for future years should be changed to adjust for such discrepancies on a level basis by duration k.

It is evident that the aggregate

$$[B_n - (1 - mq'_{n-1})(\Delta D_n)]$$

for all in-force policies must be in excess of aggregate E_0 on new business, plus any deferred development expenses, if there is to be a positive net income in aggregate within the dividend financial structure.

VI. Comparison with Three-Factor Dividend Formula

The traditional three-factor contribution formula combines the many factors of the generalized dividend formula into three main sources of earnings: excess interest earnings, mortality savings, and earnings from excess of loading over expenses. It is possible to reduce formula (4) to these three sources plus remaining factors. Assume, for simplicity, that the net premium reserve is in curtate form with interest rate i and mortality rate q_{n-1} and with deaths occurring at the end of the policy year. Then

$$V_n = (P_n + V_{n-1})(1 + i) - q_{n-1}[F_n - (TD)_n - V_n].$$

Substituting this in the first term of formula (4) produces

$$(1 - mq'_{n-1})D_n$$
$$= [(i'' - i)(P_n + V_{n-1}) + (FIT)_{CR} + R_n]$$
$$+ [(q_{n-1} - q'_{n-1})\{[F_n - (TD)_n](1 + i''/2) - V_n\}]$$
$$+ (\pi_n - P_n)(1 + i'') - [E'_n(1 + i'') + E''_n(1 + i''/2)]$$
$$- B_n + (\Delta D_n)(1 - mq'_{n-1}) + G_n\}$$
$$- [w'_{n-1}(C_n - V_n)]$$
$$- \{(TD)_n [q'_{n-1} - (i''/2)(q_{n-1} - q'_{n-1})] + (i''/2)q_{n-1} F_n\}.$$

This formula demonstrates the difficulties presented in forcing a complex set of explicit functions into a simple implicit three-factor format. The simplification can cause a ragged development of surplus from class to class even though some order can be introduced by a complex series of asset share tests. The generalized formula, on the other hand, incorporates asset share surplus as part of the basic design and generally appears compatible with the current environment, which demands a structure with demonstrable equity.

VII. Investment Income Rates (i', i'')

While taking no authoritative posture about portfolio as opposed to investment-year method procedures, I will assume a modified IYM procedure that defines i' in formulas (1) and (2) as follows:

$$i' = (1 - \beta)[\alpha i^{IBYM} + (1 - \alpha)i^P] + \beta i^L,$$

where

- α = A constant across all classes, $0 \leq \alpha < 1$;
- β = Ratio of loans to fV for a block of calendar years of issues;
- i^{IBYM} = IYM rate for the block of calendar years of issues, excluding policy loans (*I*nvestment-*B*lock-of-*Y*ears *M*ethod);
- i^P = Portfolio rate for all blocks of issues in an overall class (e.g., nonpension) excluding policy loans;
- i^L = Policy loan interest rate for the block of issues.

In the model, α is a very important design constant. A spectrum of scenarios of future new-money interest rates, policy loan borrowing rates, and related termination rates will illustrate the range of effects of various choices of α on $[\alpha i^{IBYM} + (1 - \alpha)i^P]$ and help select the value of α. In this testing, starting at $\alpha = ½$ may prove efficient.

A value of α in excess of two-thirds is probably unacceptable because of the availability of guaranteed cash and loan values and the danger of replacement of old policies. (Only in group annuities with 100 percent pass-through and investment antiselection charges based on market values on withdrawals could α appropriately be near unity.) Of course, $\alpha = 0$ results in a portfolio rate adjusted for policy loan effects. The α factor also provides stability in classes such as pension trust policies, where poor persistency causes shorter liability cash flow.

A firm structure for IBYM rates requires a history (or reconstruction) of IYM rates by investment year, including investment rollover effects, which, for discussion purposes, are assumed to be on a constant index basis. Also required is a history (or reconstruction) of the buildup of policy loans in each issue-year block. Determination of the IBYM rate involves the partitioning of aggregate f_nV_n for the block of issues into positive and negative asset-increase "slabs" representing the increase (decrease) in f_nV_n in each past calendar year, with the increase (decrease) in loans deducted. Values of f_nV_n can be determined from historical valuation reserve summaries by utilizing aggregate S_n approximated by iterative reference to formula (3) or by other procedures. of course, S_n is negative for $n \leq k$ but positive thereafter. The aggregate f_nV_n (including loans) increases for many years, then decreases as the result of deaths and terminations, but eventually begins to increase again as aggregate S_n builds up. This was also observed in the paper "Investment Generation Revisited" by Matz and Peters (*TSA*, XXIX [1977], 345-68).

The asset slabs in f_nV_n (less loans) are applied as weights (positive and negative) to the constant index IYM rates in each calendar year to derive the unadjusted IBYM rate for the block. The portfolio rate then is produced by weighting the IBYM rates by the aggregate f_nV_n (less loans) for each block. An adjusted IBYM (less loans) rate for each block is determined by applying the α adjustment factors. Then the loans are introduced at the policy loan rate (less expense) weighted by the ratio of loans to aggregate f_nV_n (including loans). Finally, the adjusted IBYM rate after marginal FIT rates for investment income and assets is calculated:

$$i' = (1 - m^I)i' - m^A.$$

These marginal FIT rates are discussed in Section VIII.

Even under recent conditions of substantial fluctuations in yields on new money, the spread in unadjusted IBYM rates before loans from oldest to newest nonpension issues might be reduced to about two-thirds by α-weighting and loan effects, and to about 40-50 percent by FIT effects. It is evident, therefore, that the controlled use of IYM procedures does not involve such a wide spread of dividend interest credits (after FIT) as might be anticipated.

This type of structure facilitates the determination of the effects of changing future investment yields. For instance, if current new-money rates decrease, but remain above portfolio rates, the portfolio rates will continue to increase because of the rollover of old investments. The α-averaging thus dampens the effect of varying new-money rates, and FIT adds further dampening.

The suggested method for determining adjusted IBYM rates provides complete and demonstrable control of i'' under changing investment conditions. It is superior to techniques involving "risk charges," which are mathematically arcane and can be subjective because of market pressures.

Once set, α *should not be changed* unless serious corporate conditions require a change, and then only after deliberate management consideration, and perhaps formal action by the board of directors.

If the IYM structure *does not* directly reflect net capital losses on investments arising from exchanges and changes in investment quality and market values, such losses can be amortized over an appropriate period of years and charged ratably against the IBYM rates before loans. The specific procedures adopted will depend on the manner in which capital gains and losses are to be handled under the company surplus policy. Investments valued at market value, such as common stocks, are likely to be handled differently from those valued at cost or amortized value.

Even if the portfolio method ($\alpha = 0$) is adopted for use in the dividend formula, IBYM analysis can be very valuable in understanding the actual investment margins inherent in various blocks of business.

VIII. Federal Income Tax Factors

In his elegant paper "Mathematical Analysis of Phase 1 and Phase 2 of 'The Life Insurance Company Income Tax Act of 1959' "(*TSA*, XIV [1962], 51-117), John C. Fraser reduced the FIT to a linear compound in the following most general form:

$$T = m^{G'}G' + m^{A}A + m^{T}I^{T} + m^{NT}I^{NT} + m^{NP}V^{NP} + m^{P}V^{P} + m^{B'}B' + m^{B''}B'' + m^{D}D + m^{F}F + \text{immaterial constant}.$$

The marginal factors (m) and the independent variables (A, I^{T}, etc.) are carefully defined in his paper. The formula facilitates allocation of the FIT to subclasses by applying the marginal factors to each independent variable of the subclass. This allocation has the characteristic that the sum of the taxes for all subclasses equals the total company tax.

In this paper, I assume Fraser Situation B (i.e., Phase 1) typical of large mutual life insurance companies, where the basis of tax is investment income less \$250,000. I also assume the single-company approach, where the overall company marginal factors are used for allocation of FIT among lines and within lines. The formula is

$$T = m^{A}A + m^{I}I + m^{NP}V^{NP}\ (or\ m^{P}V^{P}) + \text{immaterial constant},$$

where $m^{I}I = M^{T}I^{T} + M^{NT}I^{NT}$. The tax statute defines assets, reserves, investment income, and so on, somewhat differently from values in the NAIC annual statement and in company management statements (e.g., IYM system). I have uniformly disregarded these differences as immaterial among the myriad approximations and alternative choices made in a dividend structure. Hence, I use $i'' = (1 - m^{I})i' - m^{A}$ and $(FIT)_{CR} = m^{NP}$ (or m^{P}) $\times$ (mean reserve in policy year). It is desirable to keep $(FIT)_{CR}$ distinct to reflect the lower interest rate, after FIT, credited to $S_n = f_nV_n - V_n$. If CRVM reserves are used, $(FIT)_{CR}$ is adjusted to the NLP basis by the method of Internal Revenue Code, section 818(c).

IX. Expense Structure

The structure for expense allocation is the most complex aspect of this problem, and, in terms of equity, ranks in importance with the investment income allocation structure. All company expenses, commissions, and premium taxes directly or indirectly attributable to the ordinary line must be included. There are myriad ways of establishing this structure. The following is one that assures a firm relationship among expense accounting, cost accounting, and determination of dividend expense charges, and that enables accountants and actuaries to operate in their appropriate professional spheres. There are three phases in the development:

1. Cost accounting action allocating expenses to insurance functions, such as the LOMA functional categories and corporate overhead, with service functions absorbed on cost accounting principles.
2. Joint accounting and actuarial action categorizing these functional expenses into types of expense, that is, linear, semivariable, essentially fixed, and fixed.
3. Actuarial action grading the base rates from item 2 into issue, maintenance, and development factors of the structure.

A more detailed description of these three phases is as follows:

1. In the accounting action phase, a company-wide functional cost allocation made by each budgetary unit transforms the budget matrix into a matrix of (*a*) function categories within the major and minor insurance lines and the investment area, (*b*) the corporate service areas, and (*c*) the corporate overhead areas, similar to that in the LOMA Functional Cost Study. The corporate service expenses then are allocated to other categories by cost accounting measures. Corporate overhead expenses, enhanced by service expenses, are allocated among major and minor lines and investments. The result is dollar expenses for major and minor insurance lines, the investment line, and corporate overhead by major and minor insurance lines and the investment line.
2. Next, joint actuarial-accounting action assigns each functional insurance expense category from item 1 to a double matrix of (*a*) issue, maintenance, or development types and (*b*) linear, semivariable, essentially fixed, or fixed types. The result is an extensive matrix of home office and field expenses for the various major dividend classifications of ordinary business by insurance function analyzed as to issue, maintenance, and development types, further designated by linear, semivariable, essentially fixed, and fixed attributes. Also, investment expenses are analyzed by type of investment and, in particular, as to policy loan expenses. Corporate overhead expenses allocated to ordinary lines are separately designated. Development expenses of a capital nature, such as EDP development or growth of the field organization, would be amortized over a period, such as five years, rather than expensed, anticipating future improvements in costs to offset the amortization charges.
3. Then, actuarial action rearranges the matrix from item 2 to grading bases, such as premiums, corresponding ordinary life premiums (used to ameliorate percent-of-premium allocated expenses on higher-premium policies), commissions, face amounts, and policies, by consideration of the causes of the expense, ability to absorb the expense, and reasonableness. Commission and premium tax percentages of premiums are then added. The result, after adding commissions and premium taxes, is E_n, E_n', and E_n'' of the dividend formula, expressed as percent of premiums, percent of corresponding ordinary life premium, dollars per $ 1,000 face amount, and dollars per policy.

Once the structure is established and is shown to be reasonable on a continuing basis, it is expected to be continued essentially unchanged for allocations of expenses incurred in subsequent years.

Many decisions must be made in developing the expense structure. For example, issue expenses should reflect all expenses arising linearly as the result of first-year commissions and related manager or general-agent allowances, the cost of medical examinations and inspections, and the cost of issuing policies. There are, however, options for expenses not directly related to issue: the costs of recruiting, training and retaining agents are "housekeeping" expenses to the extent that they provide for replacement of terminating agents, and are development expenses to the extent that they provide field force growth. In general, it makes little sense to charge indirect expenses or overhead expenses to the issue category, only to have to amortize them with "tontining" effects.

Irregularities in renewal commissions, which are charged in E' directly as incurred, can be smoothed easily by design of (ΔD_n). It seems desirable to charge other nonlinear ingredients of E' and E'', relative to grading bases, uniformly over all classifications.

Direct costing principles tolerate reallocation of corporate overhead among classes of pension insurance, nonpension insurance, deferred annuities, and so on, to adjust for market demands. Similarly, fixed expenses within, say, the nonpension class can be reallocated by policy class. However, any concession in one area must be charged to other areas.

Issue expenses, E_0, must be determined retroactively for blocks of policies still within the k periods to reflect the actual issue expenses in earlier years. Also, it is necessary to monitor the actual amortization to test the appropriateness of the annuity function and to make indicated adjustments.

This type of expense allocation structure can be a harsh master because it demands that reduction of expense charges in dividends on current issues be made only if actual expenses are reduced. On the other hand, it relates the level of current dividend scales to actual expenses and directs attention forcibly to expense control, a result unachievable by other techniques in the ordinary lines. As an aid to such control, the software for the dividend structure should incorporate dollar expenses on various grading bases, so that the impact on dividend expense factors of contemplated changes in particular expenditures may be determined.

Credits to E are given for modal loads and policy fees. Profits from nonparticipating benefits (G_n) can be distributed on grading bases used for expenses.

While my preference is to account for all company expenses through product pricing, occasionally a write-off of an unusual expense to surplus is indicated. An example would be an expensive computer system discarded because of ERISA, or some other large expense caused by an "act of God." Another example would be a large capital expenditure for which adequate surplus exists and from which policyholders ought to be shielded.

X. Termination Rates (w'_{n-1})

Termination rates vary by policy form, policy loan rate and level, cash-value level, issue age and duration, and between pension and nonpension business. Termination rates have surprisingly large effects on dividend levels because of the excess of the cash value (including terminal dividend) over the fund ($f_n V_n$), and vice versa, resulting from the development of S_n. It is therefore necessary to determine w'_{n-1} on an elaborate basis. In the absence of studies on a policy-year basis, fairly dependable results can be obtained by observing face-amount "cessations" (face amounts in force at the end of one calendar year but not in force at the end of the next calendar year). With suitable software, these can easily be analyzed, using year-end computer tapes. Crude cessation rates must be adjusted for reporting lags and mortality, graduated, and adjusted to a policy-year basis.

XI. Mortality Rates (q'_{n-1})

Mortality rates, developed separately for nonpension and pension lines, on a select and ultimate basis by issue age, reflecting recent experience, hopefully can apply to all years of issue. Different tables should be used for preferred underwriting classes or for other classes with identifiably different mortality unless such differences are handled by premium differences outside the dividend formula. Excess nonmedical costs are assumed to be offset by underwriting savings. An important objective is to make mortality costs reflect current experience, so that this factor is independent of other factors.

XII. Fitting Structure and Formula to Surplus Goals by the Factor R_n

On the statutory basis, the dividend formula and financial structure, as shown later in Section XIX, (*a*) provide amortization charges $E_0/a_{\overline{k}|}$ for $n \leq k$, (*b*) provide profit charges $g_n V_n$ for $n > k$, and (*c*) recognize issue costs E_0. Within the dividend financial structure, the statutory net income after dividends is the aggregate excess of items *a* and *b* over item *c*, further reduced by any deferred development expenses, such as EDP, in the expense complex. The dividend financial structure also includes a surplus (or deficit) equal to the aggregate S_n.

An important source of investment income not recognized in the structure is investment income (after FIT) on invested assets attributable to ordinary insurance in excess of funds intrinsic to the dividend financial structure (notably, entity surplus, MSVR, the dividend liability, and non-interest-bearing liabilities).

The company surplus policy sets an objective for increase in surplus, reflecting increased surplus needs relative to increases and changes in liabilities and assets during each calendar year. To the extent that net income after dividends (which includes profit emerging from the dividend structure and earnings after FIT on investments not intrinsic to the dividend structure) is inconsistent with surplus objectives, the dividend formula might be enhanced appropriately by a factor R_n. This enhancement might be negative if surplus is deemed inadequate.

One enhancement might be an adjustment $\Delta i'$ (positive or negative) to the adjusted IBYM rates, representing some of such investment earnings. of course, other uniform enhancements could be designed, such as a percentage of dividends in recognition of earnings on the dividend liability, or a percentage of premiums or other grading bases used to allocate corporate overhead expenses if earnings external to the dividend financial structure are deemed to be applied against corporate overhead. Equity principles would appear to be satisfied if the enhancement is applied uniformly across all dividend classes other than issue expenses E_0, which would favor new issues primarily.

It is notable that the generalized dividend formula reflects financial dynamics within the dividend financial structure, except for R_n and G_n. CRVM policies are subject to the same surplus contribution designs as NLP policies. Also, the dividend formula makes no provision for distributing the reserve release accompanying the introduction of CRVM policies, nor is the factor R_n intended as a means of distributing such release. Indeed, company surplus should be higher with the introduction of CRVM policies so as to include the accompanying reserve release.

XIII. Introduction of the Generalized Dividend Scale

Comparisons of dividends on the generalized dividend formula on all inforce policy series with those on the current three-factor formula are likely to show different levels and pitch by duration. The extent of the differences between the old and the new design will necessarily affect the manner in which the transition to the new design will be undertaken. If the old design has been competently developed, with careful attention to equity and to changing economic environments, the differences may not be unduly large. The major difference is likely to be in the pitch of the old and new scales: the specific design of the generalized formula to control amortization of issue expenses and to develop surplus contribution tends to reduce early dividends and increase later dividends. In other words, the new design may tend to shorten the amortization period.

This might lead to a decision not to change to the new scale for policies still in the amortization period until, on an actuarial-equivalence basis, the lower old-scale dividends at later durations offset the higher old-scale dividends at earlier durations. Software developed to compute surplus according to formula (3) can be adapted to determine the crossover duration c:

$$(1-q'_{n-1}-w'_{n-1})S'_n = (D_n^{\text{Old}} - D_n^{New}) + S'_{n-1}(1+i''),$$

where $S'_0 = 0$, $S'_c = 0$.

Such a decision reflects fair dealing with policyholders, who naturally anticipate reasonable continuation of dividend scales illustrated at time of purchase. There also may be a danger of replacement.

For older policies later in the amortization period, this approach probably is not practicable because of the difficulty of estimating issue expenses from inadequate expense records and because of the inapplicability of i'' to years far in the past. Moreover, the greater pitch of the new scale, offsetting IBYM effects, may tend to sustain old-scale dividend levels. If this be so, the simple minimum rule requiring the dividends on the new scale to be no less than those actually paid in the last year of the old scale will be satisfactory, since the new-scale dividends will soon surpass the minimum.

On new policies, the new scale can be introduced immediately. If the ratio of loans to funds has not matured, then an estimate of the mature ratio should be made, taking account of the correlation between disintermediation and current new-money rates; the procedures of the IBYM structure can then be applied. This approach assumes that illustrative dividends on new issues reflect current experience and involve no projection of factors other than loan ratios.

XIV. Means for Aggregation

As part of this structure, it is desirable to develop suitable models to enable aggregating such charges and credits as investment income, issue expenses (E_0), maintenance expense (E_n), cost of mortality, and cost of termination, for comparison with the ingredients of pages 5 and 6 of the annual statement. On the one hand, such aggregates establish the appropriateness of the various factors, while, on the other hand, they indicate variations between actual experience and the charges and credits of the dividend structure.

Additionally, the capability for aggregation of S_n is valuable for determination of the appropriateness of the amortization of issue expenses and profit charges in relation to the required buildup of surplus according to the established objectives.

In determination of the original generalized formula, aggregation of dividends is, of course, necessary to ensure that the generalized formula is appropriate overall.

The generalized dividend formula can be adapted to ordinary policy forms other than cash-value life insurance policies; such adaptations are briefly described in the next four sections.

XV. Deferred Annuities

For deferred annuities, it is customary to use a one- or two-factor dividend formula, consisting of a loading factor and an excess-interest factor. For deferred annuities with flexible premiums, the implicit dividend formula could consider only excess interest:

$$D_n = (i'' - \Delta - i)(P_n + V_{n-1}),$$

where Δ represents an expense, risk, and profit charge. The determination of Δ can be made as follows:

Consider formula (2) suitably amended to reflect the characteristics of a flexible retirement annuity (variable premiums, commission allowances, and chargebacks on premium variations, and so forth. If the expression above is substituted in formula (2). an iterative formula for S_n is derived, involving, among other items, a charge equal to Δ on the initial reserve. The size of Δ can be determined to develop a proper level of S_n over a period of, say, ten to twenty years from issue.

Considering the surplus needed to protect against loss from cash surrenders or reduced premiums in a variable inflationary environment, Δ is not insignificant. It will become clear that using an i'' close to the new-money rate involves speculation with company surplus, unless there is careful matching of asset cash flows to the potentially short liability cash flows by actual or notional segmentation of general account assets or by other means (e.g., specialized separate accounts or specialized subsidiaries). As the pattern of funds producing the IBYM interest changes because of disintermediation in the variable inflationary scenario, a trend in adjusted IBYM rates and capital losses will emerge. Based on this trend, the generalized dividend financial structure, enhanced by charges for capital losses, can be used to test the degree of this surplus speculation. A similar approach currently is being used by an NAIC technical advisory committee and an SOA task force in testing minimum valuation reserve bases incorporating dynamic interest rates on guaranteed interest contracts, deferred annuities, and other insurance and annuity contracts against various asset configurations in a variety of upside and downside interest rate scenarios.

XVI. Funds not Involving Life Contingencies, Auxiliary Funds, and the Like

The IBYM funds for these classes can be derived easily from year-end records, and i'' and FIT required-interest credits are directly computed. Expenses are available from the expense matrix.

XVII. Immediate Annuities and Supplementary Contracts Involving Life Contingencies

The IBYM funds and IBYM rates here are valuable for testing the adequacy of the premium rates for these contracts, which usually are nonparticipating.

XVIII. Term Insurance

Although formula (4), suitably elaborated to reflect the cost of conversion, is theoretically applicable to term insurance, it is impractical because the smoothing factor (ΔD_n) must be excessively powerful to produce the simple dividend design needed. The pricing designs by James C. H. Anderson (*TSA*, XI, 357) and Mel Stein (*TSA*, XVII, 235), which essentially utilize formula (1), are far more practical.

XIX. Emergence of Profit

In an earlier paper, "Adjusted Earnings for Mutual Life Insurance Companies" (*TSA*, XXIV, 31), I analyzed the manner in which dividends on various formulas release a mutual life insurance company from risk on participating policies (i.e., control the timing of the emergence of profit). This differs from the use of GAAP reserves as the release-from-risk mechanism of stock companies on nonparticipating policies. It is interesting to update the formulas of that paper, which were based on the traditional three-factor dividend formula, to the generalized dividend formula, to provide deeper understanding of the generalized dividend financial structure on both statutory and going-concern accounting systems. Consistent with the earlier paper, formulas (2), (3), and (4) are reduced to a continuous basis, since we are interested only in principles and the algebraic complications of the curtate format are unnecessary. All items are functions of *n:*

$$D = \pi + \delta''(fV) - \mu'(F - fV) - \omega'(C - fV) - E + G + R + (FIT)_{CR} - \frac{d(fV)}{dn}, \qquad (2')$$

where δ'', μ', ω' are the forces of interest, mortality, and withdrawal, respectively.

$$\delta'' = \delta'(1 - m^I) - m^A;$$

and the factor $(1 - m\mu)$ is omitted for simplicity.

$$D = \pi + \delta''V - \mu'(F - V) - \omega'(C - V) - E + G + R + (FIT)_{CR} - \frac{dV}{dn} - B + (\Delta D). \quad (4')$$

$$\frac{dS}{dn} = \frac{d(f-1)V}{dn} = B - (\Delta D) + (\delta'' + \mu' + \omega')S. \quad (3')$$

Also,

$$\frac{dA}{dn} = \pi + \delta'''V - \mu'''(F - A) - \omega'''(C - A) - E''' + G''' + R + (FIT)_{CR} - D, \quad (5)$$

where A represents assets and triple primes designate actual experience.

On an internal-management-adjusted reserve basis, $f_n'V_n$ is defined to be consistent with GAAP, with a deferred expense credit to the statutory reserve equal to the unamortized issue expenses for $n < k$:

$$f_n'V_n = f_nV_n - \Delta S_n = V_n - \frac{E_0}{a_{\overline{k}|}} a_{\overline{k-n}|}, \quad n \le k'$$

$$= f_nV_n = V_n - \frac{E_0}{a_{\overline{k}|}} a_{\overline{k-n}|}, \quad k' < n \le k \quad (6)$$

$$= V_n, \quad n > k,$$

where ΔS_n is the cumulative effect of (ΔD_n) on S_n:

$$(1 - q'_{n-1} - w'_{n-1})\Delta S_n = -(1 - mq'_{n-1})(\Delta D_n) + (1 + i'')\Delta S_{n-1}.$$

Hence

$$\frac{d\Delta S}{dn} = -(\Delta D) + (\delta'' + \mu' + \omega')\Delta S. \quad (7)$$

Profit on Statutory Basis

In general,

$$\text{Profit} = \frac{dA}{dn} - \frac{dV}{dn}. \quad (8)$$

Using formulas (4′) and (5), this profit turns out to be as follows:

$$\text{Statutory profit} = F(\mu' - \mu''') + C(\omega' - \omega''') + A(\delta''' + \mu''' + \omega''') - (fV)(\delta'' + \mu' + \omega') + (E - E''') + (G''' - G)$$

$$+ \begin{cases} \frac{E_0}{a_{\overline{k}|}}[1 - (\delta'' + \mu' + \omega')\, a_{\overline{k-n}|}] - [(\Delta D) - (\delta'' + \mu' + \omega')\Delta S], & n \le k' \\ \frac{E_0}{a_{\overline{k}|}}[1 - (\delta'' + \mu' + \omega')\, a_{\overline{k-n}|}], & k' < n \le k \\ gV + (\delta'' + \mu' + \omega')S, & n > k. \end{cases} \quad (9)$$

Of course, the aggregate of this statutory profit is offset by the aggregate of $-E_0''$ on new policies being continuously issued.

If actual experience is identical with the dividend basis, the statutory profit within the dividend structure reduces to

Statutory profit

$$= \frac{E_0}{a_{\overline{k}|}} - (\Delta D) + S(\delta'' + \mu' + \omega'), \quad n < k$$

$$= \frac{E_0}{a_{\overline{k}|}}[1 - (\delta'' + \mu' + \omega')\, a_{\overline{k-n}|}] - [(\Delta D) - (\delta'' + \mu' + \omega')\Delta S], \quad n \le k' \quad (10)$$

$$= \frac{E_0}{a_{\overline{k}|}}[1 - (\delta'' + \mu' + \omega')\, a_{\overline{k-n}|}], \quad k' \le n \le k$$

$$= gV + (\delta'' + \mu' + \omega')S, \quad n > k.$$

Formula (10) shows that on a statutory accounting basis (assuming dividend experience and ignoring smoothing effects), after a loss of E_0 at issue, the profit for k years is the issue-expense amortization charge less interest and termination losses on the unamortized issue expenses. After k years, the profit is the profit charge plus interest and survivorship earnings on surplus.

Profit on Internal-Management-Adjusted Reserve Basis

Here,

$$\text{Profit} = \frac{dA}{dn} - \frac{d(f'V)}{dn}.$$

Using formulas (2′), (5), (6), and (7), this profit turns out to be as follows:

Internal-management-basis profit

$$
\begin{aligned}
&= F(\mu' - \mu''') + C(\omega' - \omega''') \\
&\quad + A(\delta''' + \mu''' + \omega''') - (fV)(\delta'' + \mu' + \omega') \\
&\quad + (E - E''') + (G''' - G) \qquad (11) \\
&\quad + \begin{cases} [-(\Delta D) + (\delta'' + \mu' + \omega')\Delta S] & \text{for } n \le k' \\ 0 & \text{for } k' \le n \le k \\ [gV + (\delta'' + \mu' + \omega')S] & \text{for } n > k. \end{cases}
\end{aligned}
$$

Here the loss on policies being continuously issued is the aggregate of $(E_0''' - E_0)$.

If actual experience is identical with the dividend basis, the profit within the dividend structure is as follows, since $A = fV$:

Internal-management-basis profit

$$
\begin{aligned}
&= -(\Delta D) + (\delta'' + \mu' + \omega')\Delta S, && n \le k' \\
&= 0, && k' \le n \le k \qquad (12) \\
&= gV + (\delta'' + \mu' + \omega')S, && n > k.
\end{aligned}
$$

If internal-management-basis adjusted earnings are based on reserves equal to statutory reserves less unamortized issue expenses for years less than k, and to statutory reserves thereafter, the emerging profit (assuming dividend experience and ignoring smoothing effects) is zero for the first k years and is identical with statutory profits thereafter. Thus, statutory profit and adjusted earnings profit differ only as to interest and termination earnings on unamortized issue expenses. Thus, we have the following:

Excess of statutory profit over internal-management-basis profit

$$
\begin{aligned}
&= \frac{E_0}{a_{\overline{k}|}}[1 - a_{\overline{k-n}|}(\delta'' + \mu' + \omega')], && n \le k \qquad (13) \\
&= 0, && n > k.
\end{aligned}
$$

Of course, the aggregate excess for the in-force business is reduced by the aggregate of E_0 on currently issued business.

XX. Comparison with Other Types of Dividend Formulas

The generalized dividend structure and formula are a complete, detailed system for relating ordinary dividends to the company accounts and surplus objectives. The formula is a special form of the generalized equation of equilibrium underlying the risk mechanism.

It is an extension of the three-factor contribution formula. It has become feasible because of the availability of modern computers, while the three-factor contribution formula is a carryover from the era of manual computations. The generalized dividend formula recognizes the design of the profit charge, the dividend fund, and surplus as explicit, objective requirements, while the contribution formula assesses profit charges only as an implicit part of other factors.

Since surplus objectives are explicit in the formula, it bears some resemblance to the prospective asset share approach used by at least one company (see discussion in *TSA*, XXIV, 217) and to the historical fund approach used by at least one other company.

The main attraction of the generalized structure and formula is that it is an evolution of the contribution formula offering many advantages in the current environment and enabling a practical transitional procedure from the current approximate three-factor contribution formula. Importantly, it supplies the actuarial discipline to provide equity across issue-year classes—in particular, with respect to expense and investment income allocations.

XXI. Acknowledgments

I wish to express appreciation for the encouragement and help so generously given by John A. Fibiger, F.S.A., and Harold G. Ingraham, F.S.A., in the development of this paper, and for the dedicated efforts of my other associates at New England Life in reviewing the theoretical and practical considerations involved in this type of generalized dividend financial structure.

Discussion of Preceding Paper

Arthur C. Cragoe

Mr. Cody is to be congratulated for setting forth a new and disciplined approach to surplus distribution. My company is using a slightly different approach in expanding the ideas related to investment income rates. We start with the concept, borrowed from segregated accounts, that when the client directs the type of investment for a portion of his net investable funds, then the investment return of that pool of assets should determine that portion of his "dividend fund" ($f_n v_n$). The client can have more than one directed investment portion. If he does not direct the type of investment for his funds, the balance of his net investable amount for the year shares an overall investment yield which is calculated without including directed investments.

The idea of considering client direction in determining the dividend is not new. The client's choice of plan, face, and amount of premium is reflected in traditional dividend formulas. We are merely recognizing additional client choices. These choices may mean dividends unique to the timing and amount of client-directed investment choices. Because of computers, this refinement is practical today.

A policy loan is only one of several possible directed asset elements, each of which could be recognized in a dividend formula. For example, it is not too difficult to imagine a policy-update procedure under which old and new policyholders may be offered a noncontractual privilege to direct a given percentage of future "dividend fund" increases into money-market investments; another percentage into common stocks; another into a safe, fixed-rate investment (policy loans); and the balance into the unallocated portion of the company's general account.

For the last two years we have been keeping track of the increase (or decrease) in the interest-related elements of "dividend funds" for all old and new policies on a time-weighted basis for each policy year. We began using these results in the dividend scale adopted a year ago. The investment-oriented elements are guaranteed cash values, dividend (or coupon) accumulations, supplementary deferred annuity values, paid-up addition values, premium deposits, and loan values. The loan value increase record is the first of several potential directed-asset records. We feel that the liability increase in the cash (or other investment) value, (a "liability slab") is a good approximation to the "asset slab" used by the author, at least for dividend calculation procedures. From one point of view, we have merely adopted a traditional group annuity dividend approach to ordinary insurance and individual annuity dividend calculations.

Of course, we look to the total interest, including an R term, in determining investment-year method (IYM) asset rates, but we apply them to liability slabs. The "slab" is determined over the same time period (one year) as the investment of the assets. In the future, the liability slab wears away because of deaths and surrenders, while the asset slab wears away because of maturities and rollovers. As part of our overall approach to asset-liability management, we hope to monitor, project, and, if necessary, notationally reallocate our emerging IYM asset slabs to approximately match our emerging liability slabs.

Since, for dividends, we are essentially applying IYM-related interest rates to yearly liability pieces and summing these partial results for each year's interest component of the dividend, we feel we can somewhat relax the author's condition on α as being fixed for many years at the same amount and within a narrow range. The first liability slab for old policies is the policy value increase from issue to 1979, the year of transition to yearly IYM asset rates. The asset rate for this starting liability slab is from the remaining portfolio of all assets held in 1979. Someday, future slabs will be important even to these older policies, and we did not want to have an unchanging assumption on either α or the future policy loan mix. Using liability slabs also helps with the problem of using new-money rates on deferred annuities. We do not feel that great theoretical damage would be done by using either the author's method or the pure undirected investment rates in the expense and mortality terms.

In summary, a dividend i' can have different values for the directed (by the client) and the undirected portions of asset value increase. Such recognition in the dividend formula may become more important as more types of noncontractual client-directed asset investment programs come into being. Apparently, this change is capable of being incorporated within the author's excellent new model. Also, the subdivision of liability value into yearly increase pieces is a natural and evolutionary step in the application of IYM asset rates.

Thomas G. Kabele

1. *Cody's Dividend Formula*

Mr. Cody's "generalized dividend formula" has the advantage that it explicitly recognizes (1) select mortality, lapse, and acquisition expense, (2) cash values and terminal dividends, (3) federal income taxes and new-money interest rates, and, most important, (4) management profit goals and management reserve standards. The effect of policy loans (the interest rate and the utilization) is implicit in the choice of interest rate, but even that could be made explicit by adding another term to the dividend formula. I believe that Don Cody's formula is a very fine contribution to actuarial literature.

Appendix A of this discussion summarizes Don Cody's dividend and profit formulas and shows how his "management reserve" is related to GAAP reserves. The following paragraphs discuss some modifications of his formulas.

> *Modification 1:* Use the "management reserve" as the dividend fund, in place of the statutory reserve.

Mr. Cody defines his dividends in terms of a predetermined "dividend fund" and a predetermined profit charge. He gives two different examples of dividend funds:

1. The dividend fund is the *asset share* (fV), is based on the assumptions of the original dividend scale, and contains no explicit profit charge (see Cody's formula [1]).
2. The dividend fund is the *statutory reserve* (V), while the profit charge (B) amortizes initial expenses and provides for a permanent surplus contribution (see Cody's formula [4]).

We could also have a third example:

3. The dividend fund is the *management reserve* ($f'V$), which equals the statutory reserve plus a natural reserve for first-year acquisition expenses while the profit charge (gV) provides only for a permanent surplus contribution (see Sec. V).

Using the relationships given in Mr. Cody's paper, we can show (Theorems 1 and 2 in Appendix A) that all three dividend funds give the same original dividend scale. When the experience factors (particularly interest) change, however, the three different funds give different dividend values. In this situation, which fund produces the most equitable result?

I believe the management reserve will give the fairest result. The management reserve represents funds held for the policyholder and includes no part of company surplus funds held for the benefit of all policyholders. The use of statutory net level reserves and even asset shares may overstate the amount of interest to be attributed to a class of policyholders. Further, the use of statutory reserves results in inconsistencies between blocks that use statutory net level reserves and blocks that use modified statutory reserves.

If the company pays terminal dividends, then the "reserve" for these dividends can be included in management reserves. Policyholders then will receive excess interest on this reserve but not on general surplus funds. They will also be charged for the increase in this "reserve." The use of management reserves as the "dividend fund" will also enable management to obtain internal performance measures as a by-product of dividend valuation.

> *Modification 2:* Establish reasonable profit goals at all durations.

At the early durations, Mr. Cody chooses *zero* to be his profit goal. He compensates for this by amortizing the initial expenses over a short period of time (ten to twenty years).

I do not believe that zero profit is a natural goal for any board of directors. It may be preferable to have reasonable profit goals at all durations and amortize initial expenses over the premium-paying period; the effect on the dividend scale should be minimal.

One possible profit goal would be ten cents per thousand plus twenty-five basis points on the reserve. The reserve charge is akin to the charges made by mutual funds for investment management. The ten cents per thousand covers expense and mortality risks.

For stock companies selling participating business, the profit goal might be the statutory profit limit. In New York this is the greater of fifty cents per thousand or 10 percent of policyholder dividends. In Illinois it is just the 10 percent, and Canada grades the percentage by size of the company from 10 to 2½ percent. For single dividend option participating policies not subject to a maximum profit limit (universal life, indeterminate premium and indeterminate benefit policies), the profit goals may be even higher.

> *Modification 3:* Include excess renewal commissions in the expense reserve.

Mr. Cody includes only first-year acquisition costs in his expense reserve. I have found that this format, which ignores excess renewal commissions, causes discontinuities in the dividend scale (or in GAAP profits). Cody eliminated the discontinuities by using a "smoothing factor," which, in reality, alters his basic reserve. Therefore, I suggest that the expense reserve include explicit factors for excess renewal commissions over the ultimate rate.

> *Modification 4:* Use a modified type 2 GAAP reserve as the dividend fund.

A type 2 GAAP reserve (discussed in Appendix A) is computed using reasonably realistic select and ultimate mortality, lapse, and expense rates, but with the interest factor graded to a conservative ultimate rate. Both the expense and benefit components are computed using the same assumptions, and the expense component includes excess renewal commissions. My "modified" type 2 reserve uses the same assumptions as the type 2 reserve, except that the interest rates are the rates used to calculate cash values.

I believe that the modified type 2 GAAP reserve is a realistic estimate of policyholder funds, and it can be used to calculate dividends for term plans and for modified premium and modified benefit plans. In contrast to a statutory reserve, the type 2 GAAP reserve produces realistic gains and losses for mortality and lapse. Finally, the modified type 2 reserve produces smoothly graded dividend scales without the use of a complicated smoothing factor.

> *Modification 5:* Calculate dividends first and then solve for the dividend fund and the gross premium.

As Mr. Cody has pointed out to me in private conversation, the dividend fund (in his formula [2]) does not have to be a net premium reserve based on a mortality table and interest rate. We could define dividends first (as a straight line or using the reversionary bonus system) and then define the dividend fund to be a type 1 GAAP reserve or, equivalently, his asset share less surplus. The gross premium would be the type 1 net premium plus a profit loading. The dividend fund could be used if assumptions changed from those used in the original asset share calculations.

2. Discipline in Illustrated Dividend Scales

Mr. Cody says that the dividend formula should be related to the Summary of Operations (page 5 of the NAIC Annual Statement) or some other company statement. He says that all expenses should flow through the dividend formula and that the actuary should be aware of the specific charges for the amortization of acquisition expenses (see his items 1, 3, and 4). These are important ways of ensuring discipline in the dividend formula. I would like to suggest others.

1. *GAAP recoverability tests at issue.* The actuary should compare the gross and "net natural premium." The natural premium should be based on asset share assumptions for interest, lapse, select mortality, and expense. Dividends should be considered as a disbursement in the natural premium calculation.

   The comparison of the natural net premium with the gross is better than traditional asset share tests because it is easier for boards of directors to understand and is less volatile than asset shares. It is also helpful if the net premium is split into benefit, expense, and dividend components.
2. *GAAP loss-recognition tests.* Periodically, the actuary should do a prospective gross premium valuation using current assumptions with the current dividend scale to determine the viability of the scale. The test was suggested by Bert Winter of Prudential in *TASA*, Volume L (1948).
3. *Rigorous analysis of the inflation in unit expenses.* Of course, inflation and increasing volume raise total expenses. The inflation in *unit* costs, however, depends greatly on the allocation procedure. Let us consider general maintenance, overhead, and agency administration expenses. If these are allocated on a per-policy basis, unit costs may rise very rapidly, perhaps even faster than the consumer price index (CPI). If they are allocated on a per-thousand basis, unit expenses may actually decline, because the increase in average sizes and the trend toward term may more than offset the rise in the CPI. If they are allocated on a per-dollar-of-premium basis, there may be a slight increase in unit costs, due to the trend toward term.

   The allocation of expenses, particularly on a per-policy basis, may produce a type of assessment cycle. More expense means fewer sales and more lapses of smaller-sized policies, leaving fewer and fewer policies over which to spread expenses.

   Using statistics for the last five to ten years, the actuary should develop historical trends in unit costs, using the actual allocation methods employed in the dividend formulas and asset share tests.

   It is *not* all right to claim that inflation in unit expenses will be offset by increases in the interest rates, especially if the company is using an investment-year method that already reflects the current high rates. Furthermore, there are two roadblocks in the path of increased investment earnings: policy loans and the Phase 1 federal income tax, which is really a highly progressive excise tax on investment income.

3. *Cody on GAAP*

Cody lets management choose the profit charge (part of his B_n) and then defines the following:

1. The management reserve, equal to the statutory reserve less deferred acquisition costs (his eq. [6]).
2. Management surplus, equal to the accumulated profit charge (part of eq. [3]).
3. The asset share, equal to the management reserve plus management surplus (eq. [6]).
4. Dividends (his eqs. [1], [4], and [11]).

He defines his dividend in terms of a fund, and he uses both the asset share (eq. [1]) and the statutory reserve (eq. [4]). We could also use his management reserve as the fund.

He suggests that mutuals do not need GAAP. He suggests that dividends provide the release-from-risk mechanism and that the management reserve can be used to calculate earnings. In fact, in Appendix A (Theorem 4) I prove the remarkable result that, if we use Cody's dividend formula, the dividend fund is exactly equal to a type 1 GAAP reserve and, further, the gross premium is equal to the type 1 net premium.

The dividend fund must be zero before issue, and, at maturity, it must equal the endowment value including any terminal dividend. Cody's management reserve satisfies these requirements, but the result holds even for an otherwise arbitrary fund such as a straight line. My type 1 reserve is based on the realistic assumptions of the original Cody dividend scale, and includes profit charges and dividends with benefits and assumed expenses as disbursements in the reserve calculation.

Therefore, with the proper dividend formula, mutuals do *not* need GAAP, because they do not need what they already have. Many universal life companies are not using the Cody method for GAAP. That is, their GAAP reserve is their "dividend fund," namely, the cash value.

There may be some benefits, however, in calculating the type 1 GAAP reserves. The dividend fund is the total reserve. Calculating the GAAP factors will give us GAAP net premiums and reserves for benefits, expenses, dividends, and profit charges separately. These data are useful for making projections and analyzing earnings by source.

4. *Comment on Profits*

Including the profit charges as a disbursement in the type 1 reserve formula is contrary to the treatment used by stock life insurance companies. My reasons for doing it are the following:

1. If we include the profit charges, then the type 1 reserves are equal to the dividend fund.
2. As far as the policyholder is concerned, the profit charges *are* disbursements, since they reduce his dividends.
3. If actual experience is identical with the dividend basis, then the gain from operations after dividends (but before profit charges) is equal to surplus, plus the built-in profit charge (see Cody's formula [12], Sec. XIX).

The dividend fund is a good liability measure for a mutual company because it is the basis on which we credit excess interest. If the dividend fund is Cody's management reserve, or my type 2 modification, then the fund will also be fairly close to the total surrender value, which is another good liability measure.

If profit charges (B) increase by duration, they give rise to a positive reserve (VS). Ignoring this reserve pushes the profits forward, producing more profit than was actually anticipated in the dividend scale. If profit charges are disbursements in the type 1 reserve, the gain (after dividends but before profit charges) is the current year's profit charge (B). If the profit charges are not disbursements, the gain is

$$ {}_tB + {}_tVS(1 - q' - w') - {}_{t-1}VS(1 + i'). $$

The built-in profit charge enables the actuary to determine which release-from-risk method he wants to employ. A profit charge consisting of deltas for interest, mortality, expense, and/or lapse represents the full-release-from-risk method (see Robert L. Posnak, *GAAP—Stock Life Insurance Companies* [1974], p. 73, and Richard Horn, *TSA*, XXXIII [1971], p. 399). A flat percent-of-premium profit charge produces the natural reserve method, or a zero-release-from-risk method.

If we use a classical three-factor formula with zero lapse rates and ultimate mortality, we can still show (see Appendix A) that the dividend fund will equal the type 1 reserve based on the dividend assumptions, but that reserves will not satisfy the *Audit Guide* definition of a proper GAAP reserve, namely, that the "actuarial assumptions be characterized by conservatism which is reasonable and realistic" (*Audit Guide for Stock Life Insurance Companies*, p. 64).

Appendix A

Some Results Concerning Cody's "Internal Management" Dividend Formula

In this appendix relationships are derived concerning the dividend fund, GAAP reserves, the asset share, the gross premium reserve, and Lidstone's remainder function and variation fund. The major results are Theorems 4 and 5.

This appendix uses the following notation, which is consistent with Cody's and the international actuarial notation (*RAIA* and *TASA*, 1947).

Guaranteed values

$_tCV$ = Cash values
DB = Death benefit
$matval$ = Maturity value
$\pi = G$ = Gross premium
m = Maturity duration
n = Premium-paying period
t = Policy year
x = Issue age

Assumptions

$_tE$ = Direct expense (acquisition and direct maintenance)
$_te$ = Nondirect expense (FIT and overhead)
$_ti$ = Interest
q_{t-1} = Mortality
w_{t-1} = Lapse

Management definitions

$_tB$ = Management profit goal
$_tV$ = Reserve or management dividend fund

Computed values

$_tAS = f_tV$ = Asset share
$_tbene$ = Benefits plus assumed expenses and dividends
$_tDiv$ = Annual dividend
$_tLS = {_tGPR}$ = Liability share or prospective gross premium reserve
p_{t-1} = Persistency = $1 - q_{t-1} - w_{t-1}$
P = Net premium
$_tS$ = Surplus goal
TD = Terminal dividends
v = $1/(1 + {_ti})$ = Interest discount

It is assumed that gross and net premiums and direct expenses are paid at the beginning of the policy year and death benefits at the middle, while cash surrenders, nondeferrable expenses, and profit charges are made at the end of the policy year. To simplify the formulas, annual dividends are assumed to be paid at the end of the policy year to all those who pay the annual premium. Terminal dividends, if any, are paid when deaths, surrenders, and matured endowments are paid. Unprimed symbols are used for dividend fund assumptions, and primed symbols for the experience factors used in the original dividend scale.

Definition 1. The *management dividend fund* is *a* sequence of numbers $\{_tV\colon 0 \le t \le m\}$, where $_0V = 0$ and $_mV$ = maturity value.

Examples: The management dividend fund can be defined completely arbitrarily, but the most common examples are the following:

a) $_tV$ = Policy cash values plus terminal dividends (see Robin B. Leckie, *TSA* XXXI [1979], 191).
b) $_tV$ = Modified statutory reserve (CRVM, Ohio, New Jersey, Illinois, Canadian, select and ultimate, etc.).
c) $_tV$ = Statutory net level reserve plus a natural reserve for deferred acquisition expenses (Cody's choice).
d) $_tV$ = Net premium reserve for benefits, deferrable expenses, and terminal dividends (my choice).

Definition 2. The *management profit goal* is a sequence of numbers $\{_tB\colon 1 \le t \le m\}$, where $_tB$ is the profit goal for policy year *t*.

Examples: The profit goals can also be defined completely arbitrarily, but usually they are related to premiums, face amount, reserves, or policyholder dividends. Some examples are as follows:

a) $_tB = 0$, $\quad 1 < t < k$ (*k* usually 10, 15, or 20)
$= 0.005\,{_tV}$, $\quad t > k$ (Cody's choice).
b) $_tB = \$0.10 + 0.005\,{_tV}$, $\quad t > 1$ (my choice).
c) $_tB$ = 10 percent of the policyholder dividend.

Definition 3. The *management surplus* goal, $_tS$, is the accumulation of the profit goals $_tB$ at interest and survivorship (based on the assumptions of the original dividend scale). Thus,

$$_0S = 0, \quad {_tS}(1 - q'_{t-1} - w'_{t-1}) = {_tB} + {_{t-1}S}(1 + {_ti'}).$$

Definition 4. The *management asset share*, $_tAS$ is the sum of the management fund $_tV$ and the management surplus $_tS$.

Definition 5. (Exhibit 12 formula). The *management dividend* is defined by the following formula:

${}_tDiv =$	Dividend =
G	Premium
$+ {}_ti'[{}_{t-1}AS + G - {}_tE' - (DB)q'_{t-1}/2]$	+ Interest
$-(DB)q'_{t-1} - ({}_tCV)w'_{t-1}$	– Deaths and surrenders
$- {}_tE' - {}_te$	– Direct and nondirect expense
$-[{}_tAS(1 - q'_{t-1} - w'_{t-1}) - {}_{t-1}AS]$.	– Increase in assets.

Theorem 1 (page 5 formula, "Equation of Equitable Distribution"). *The management dividend is defined by the following formula:*

${}_tDiv =$	*Dividend =*
G	*Premium*
$+ {}_ti'[{}_{t-1}V + G - {}_tE' - (DB)q'_{t-1}]/2$	*+ Interest*
$-(DB)q'_{t-1} - ({}_tCV)w'_{t-1}$	*– Deaths and surrenders*
$- {}_tE' - {}_te$	*– Direct and nondirect expenses*
$-[{}_tV(1 - q'_{t-1} - w'_{t-1}) - {}_{t-1}V]$	*– Increase in reserves.*
$- {}_tB.$	*– Profit goal.*

Proof.

$$\begin{aligned}{}_{t-1}AS(1 + {}_ti') - {}_tAS(1 - q'_{t-1} - w'_{t-1}) = \\ {}_{t-1}V(1 + i') - {}_tV(1 - q'_{t-1} - w'_{t-1}) + {}_{t-1}S(1 + {}_ti') \\ - {}_tS(1 - q'_{t-1} - w'_{t-1}),\end{aligned}$$

and the last two terms above equal $- {}_tB$.

Note that if there are terminal dividends, we add them to *matval*, *DB*, and *CV*. To avoid clutter, we shall drop most of the subscripts in the rest of the appendix.

Historical note.—Sheppard Homans's original dividend formula was a fund formula that he called the "equation of equitable distribution" (see C. Rietz, *RAIA*, XI [1922], 121). Homans's "fund" was a retrospective reserve, similar to the cash value of universal life insurance.

Definition 6. A *generalized reserve* is a sequence of numbers $\{{}_tV: t = 0, 1, 2, \ldots, m\}$ that satisfies the *equation of equilibrium,*

$${}_tVp = ({}_{t-1}V + P - bene)(1 + i), \quad t = 1, 2, \ldots, m. \quad (1)$$

For a typical guaranteed cost life insurance policy, the discounted value of the current year's benefits and expenses is

$$bene = E + q(DB)v(1 + i/2) + w(CV)v,$$

where the persistency is $p = 1 - q - w$.

For a participating (meaning non-guaranteed-cost) policy, there are three types of reserves, depending on the treatment of dividends (or the nonguaranteed elements).

Type 1: Dividends are included as disbursements in the reserve formula.
Type 2: Dividends are excluded.
Type 3: Dividends are considered as negative gross premiums.

Historical note.—The type 1 and type 2 terminology was introduced by Posnak in *GAAP—Stock Life Insurance Companies*. I have introduced the type 3 terminology. The type 3 method was suggested by various American actuaries in connection with coupon policies (see Lawrence M. Cathies, *RAIA*, II [1913], 17-19).

Equation (1) forms a system of $m + 2$ unknowns (${}_0V$, ${}_1V$, ..., ${}_mV$, P) and m linear equations. Two more equations, or "initial conditions," are needed to solve it. Depending on the initial conditions, there are three kinds of reserves:

Net premium reserve:

$${}_0V = 0; \quad {}_mV = matval. \quad (2a)$$

Asset share or retrospective accumulation:

$$P = G; \quad {}_mV = matval. \quad (2b)$$

Liability share or prospective gross premium reserve:

$$P = G; \quad {}_mV = matval. \quad (2c)$$

The quantity G is usually the gross premium, but actuaries occasionally use a so-called net premium, which, in reality, is an "unloaded" gross premium. If the contract is profitable, we can show that

Asset share > Net premium reserve > Liability share.

Using the definitions, we can prove the following theorem, which I call the Fundamental Theorem of Insurance.

Theorem 1*a*. Fundamental Theorem of Insurance. *The "equation of equilibrium" (1) and the initial conditions (2) determine the reserves and net premium uniquely.*

Proof. Let $D_x = 1$ and $D_{x+t} = D_{x+t-1}(1 - q_{t-1} - w_{t-1})$. Using equation (1), we find that

$$_0VD_x + P\sum_{t=0}^{m-1} D_{x+t} = {}_mV\,D_{x+m} = \sum_{t=0}^{m-1} D_{x+t}\ {}_tbene, \qquad (3)$$

where $_tbene$ is the discounted value of all benefits and assumed expenses: $_tbene = {}_tE = qv(1 + i/2)DB + wv(CV)$. Equation (3) and the initial conditions determine the unique value of the net premium P, and the beginning ($_0V$) and ending ($_mV$) reserves. The equation of equilibrium then gives all other reserves. *End of proof.*

Equation (3) has a different interpretation for each kind of reserve. For net premium reserves, P is the present value of future benefits, expenses, and possibly dividends, divided by the present value of \$1. For the asset share, $_mV$ is the accumulated profit plus the maturity value. For the liability share, the absolute value of $_0V$ is the present value of future profits.

Theorem 2 (statutory source of earnings formula). *Suppose the dividend fund (V) is a generalized reserve with "net" premium P, and is based on assumptions i, q, E, w. Then*

$Div + e + B =$

$(i' - i)[_{t-1}V + P - E - (DB)q/2]$	*Interest gain*
$+ (q - q')[DB(1 + i'/2) - {}_tV]$	*Mortality gain*
$+ (w - w')(CV - {}_tV)$	*Lapse gain*
$+ (1 + i')(E - E')$	*Expense gain*
$+ (1 + i')(G - P).$	*Loading gain*

Proof. By the equation of equilibrium (definition 6),

$$_tV - {}_{t-1}V = i\,{}_{t-1}V + (P - E)(1 + i) - q[DB(1 + i/2) - {}_tV] - w(CV - {}_tV),$$

and by Theorem 1,

$$_tV - {}_{t-1}V = i'({}_{t-1}V) + (G - E')(1 + i') - q'[DB(1 + i'/2) - {}_tV] - w'(CV - {}_tV) - (Div + e + B).$$

Thus

$$Div + e + B = (i' - i)\,{}_{t-1}V + (G - E')(1 + i') - (P - E)(1 + i) + (q - q')(DB - {}_tV) + (iq - i'q')DB/2 - (w - w')(CV - {}_tV).$$

Since $iq - i'q' = (q - q')i' - q(i' - i)$, the theorem is proved.

Theorem 2*a* (GAAP source of earnings formula). *Suppose the dividend fund V is split into benefit and expense portions (V = VB + VE). Then Div + e + B = interest gain + loading gain + benefit gain + expense gain, where*

Interest gain =	
Actual interest	$i'[_{t-1}V + G - E' - (DB)q'/2]$
–Tabular interest.	$- i[_{t-1}V + P - E - (DB)q/2].$
Loading gain =	
Gross – Net.	$G - P.$
Benefit gain =	
Net premium	PB
+ Tabular interest	$+ i[_{t-1}VB + PB - (DB)q/2]$
– Actual benefit	$- [q'(DB) + w'(CV)]$
– Increase in reserve	$- [_tVB(1 - q' - w') - {}_{t-1}VB].$
Expense gain =	
Net premium	PE
+ Tabular interest	$+ i(_{t-1}VE + PE - E)$
– Actual expense	$- E'$
– Increase in reserve.	$- [_tVE(1 - q' - w') - {}_{t-1}VE].$

Proof. The GAAP source of earnings gain is derived from the accounting gain formula (Theorem 1) by two adjustments, which both net to zero:

1. The net premium is subtracted from the gross premium and added to benefits and expenses.
2. The tabular interest is subtracted from actual interest and added to benefits and expenses.

Definition 7 (*type 1 reserve*). Let $_tV'$ and P' be the generalized reserve calculated using the assumptions of the original dividend scale with dividends, profit charges, and overhead expense included as disbursements. Therefore,

$$_0V' = 0, \quad {}_mV' = matval; \qquad (2)$$

$$_tV'(1 - q' - w') = ({}_{t-1}V'' + P''E')(1 + i') - q'(DB)(1 + i'/2) - w'(CV). \qquad (1)$$

Definition 8 (*type 2 reserve*). Let V'' and P'' be the net premium reserve and net premium calculated using the assumptions of the original dividend scale (i', q', W', E'). Dividends, overhead, and profit charges are excluded. That is,

$$V'' = 0, \quad {}_mV'' = matval; \qquad (2)$$

$$ {}_mV''(1 - q' - w') = ({}_{t-1}V'' + P'' - E')(1 + i') - q'(DB)(1 + i'/2) - w'(CV). \qquad (1)$$

Definition 8*a*. Define ${}_tK = {}_tV'' - {}_tV$. The function ${}_tK$ was called the *variation fund* by Lidstone (*JIA*, XLIX [1905], 214). The next theorem was suggested by a result of Lidstone (*JIA*, XXXII [1895], 106, eq. [5]). It is a potentially useful result if both V'' and V are kept on file.

Theorem 3.

$$Div + e + B = {}_tK(1 - q' - w') - {}_{t-1}K(1 + i') + (G - P'')(1 + i').$$

Proof. By definition 8,

$$q'(DB)(1 + i'/2) + w'(CV) = ({}_{t-1}V'' + P'' - E')(1 + i') - {}_tV(1 - q' - w').$$

By Theorem 1,

$$q'(DB)(1 + i'/2) + w'(CV) = ({}_{t-1}V + G - E')(1 + i') - {}_tV(1 - q' - w') - (Div + e + B).$$

The theorem follows.

Definition 9 (*Type 3 reserve*). Let ${}_tH - G = v'\,{}_tDiv$ be the effective premium (gross minus discounted dividend). Then, the type 3 reserves ${}_tV'''$ and variable net premiums ${}_tP'''$ are calculated using the assumptions of the original dividend scale, by the formulas

$$ {}_0V''' = 0, \quad {}_mV''' = matval; \qquad (2)$$

Net premiums ${}_tP'''$ are a uniform percentage of effective premiums ${}_tH$: (3)

$$ {}_tV'''(1 - q' - w') = ({}_{t-1}V + {}_tP''' - E')(1 + i') - q'(DB)(1 + i'/2) - w'(CV) - (e + B). \qquad (1)$$

Definition 10. Let VS and PS be the "reserve" and net premium needed to fund the profit charges B. Then

$$ {}_0VS = 0 = {}_mVS;$$

$$ {}_tVS(1 - q' - w') = ({}_{t-1}VS + PS)(1 + i') - {}_tB.$$

If the gross premium is varying, we assume that PS is a uniform percentage of the gross. For type 3 reserves we define $VS3$ and $PS3$ by

$$ {}_0VS3 = 0 = {}_mVS3;$$

$$ {}_tVS3(1 - q' - w') = ({}_{t-1}VS3 + PS3)(1 + i') - {}_tB;$$

PS3 is a uniform percentage of ${}_tH = G - v'\,{}_tDiv$.

For VS and *VS3* we can prove the following:

LEMMA. *If $v'\,{}_tB/G$ and $v'\,{}_tB/{}_tH$ increase (resp. decrease) by duration, then ${}_tVS$ and ${}_tVS3$ are positive (resp. negative).*

Proof. The proof is similar to the proof of Theorem 6.

Theorem 4. *Suppose that dividends are calculated by the Cody formula, and let V be the dividend fund, where ${}_0V = 0$ and ${}_mV = matval$. Let V' and V''' be the type 1 and type 3 reserves, calculated using the assumptions* of *the Cody dividend scale. Then*

$$G = P', \quad V = V', \quad {}_tH = {}_tP''', \quad V = V'''.$$

Proof. By Theorem 1,

$$(B + e + Div) + q'(DB)(1 + i'/2) + w'(CV) = ({}_{t-1}V + G - E')(1 + i') - {}_tV(1 - q' - w').$$

By definition 7,

$$(B + e + Div) + q'(DB)(1 + i'/2) + w'(CV) = ({}_{t-1}V' + P' - E')(1 + i') - {}_tV(1 - q' - w').$$

The theorem then follows by the Fundamental Theorem of Insurance, but we will give another proof.

Let $K = V' - V$. Then

$$(G - P')(1 + i') = {}_{t-1}K(1 + i') - {}_tK(1 - q' - w'). \qquad (4.1)$$

Let $v' = 1/(1 + i')$, $D_x = 1$, $D_{x+s} = D_{x+s-1}(1 - q' - w')$. Then, from (4.1),

$$D_{x+t-1}(G - P') = {}_{t-1}KD_{x+t-1} - {}_tKD_{x+t}. \qquad (4.2)$$

Sum from $t = 1$ to $t = m$. Then, from (4.2),

$$\sum_{t=0}^{m-1} D_{x+t}(G - P') = {}_0KD_x - {}_mKD_{x+m}. \qquad (4.3)$$

The initial conditions are ${}_0V' = {}_0V = 0$ and ${}_mV = {}_mV' =$ *matval*. Thus ${}_0K = {}_mK = 0$, and $G = P'$. (If G varies by duration, P' is a uniform percentage of G.)

Using $G = P'$ and ${}_0K = 0$ or ${}_mK = 0$ in (4.3), it follows that $V = V'$. The proof of the remaining results is similar.

The equation $G = P'$ shows us that the Cody dividend scale *automatically* passes the GAAP recoverability test.

If the profit charges (B) are *not* included as disbursements in the type 1, and type 3, reserve calculations, we can still show that

(i) $G \geq P'$ if B is always nonnegative.
(ii) $V \leq V'$ if $v'B/G$ decreases or is constant by duration.
(iii) $V \geq V'$ if $v'B/G$ increases by duration.

In fact, $G = P' + PS$ and $V = V' + VS$. For type 3, we find $G = P''' + PS3$ and $V = V''' + VS3$. In (iii), the accounting reserve (V') does *not* cover policyholder equity (V), a troublesome result.

Theorem 5. *Suppose that dividends are calculated by the Cody formula. Let LS be the liability share, AS the asset share, and V' the type 1 GAAP reserve. If the profit charge B is nonnegative, then* ${}_tAS > {}_tV' > {}_tLS$.

Proof. The type 1 GAAP reserve satisfies equation (1), while the asset share, and liability share, satisfy (1′). Note that the profit charge is a disbursement in (1) but not in (1′).

$$\begin{aligned} {}_tV'(1 - q' - w') &= ({}_{t-1}V + P' - E')(1 + i') \\ &\quad - q'(DB)(1 + i'/2) - w'(CV) - (Div + e + B). \qquad (1) \end{aligned}$$

$$\begin{aligned} {}_tAS(1 - q' - w') &= ({}_{t-1}AS + G - E')(1 + i') \\ &\quad - q'(DB)(1 + i'/2) - w'(CV) - (Div + e). \qquad (1') \end{aligned}$$

Let $K = AS - V'$. Then we find

$$(G - P')(1 + i') = {}_tK(1 - q' - w') - {}_{t-1}K(1 + i') - {}_tB. \qquad (5.1)$$

Now ${}_0K = {}_0AS - {}_0V' = 0$. Therefore, for every a and b,

$$(G - P')\sum_{s=a}^{b-1} D_{x+s} = {}_bKD_{x+b} - {}_aKD_{x+a} - \sum_{s=a}^{b-1} v'\,{}_sBD_{x+s}. \qquad (5.2)$$

For the Cody dividend scale we have $G = P'$ from Theorem 4. Let $a = 0$, $b = t$. Thus ${}_tK > 0$. The proof for the liability share is similar. We define $K = LS - V'$, $a = t$, $b = m$, and use ${}_mK = 0$.

Suppose the profit charges (B) are *not* included as disbursements in AS, LS, and V'. Then (5.2) becomes

$$(G - P')\sum_{s=a}^{b-1} D_{x+s} = {}_bKD_{x+b} - {}_aKD_{x+a}.$$

By the comment after Theorem 4, $G > P'$, so again we find $AS > V' > LS$. If the profit charges *are* included as disbursements in AS, LS, and V', then $AS = V' = LS$.

If overhead expenses and profit charges are not treated as disbursements in the type 1 reserve, then

$$\sum_{s=t}^{m-1} D_{x+s}(G - P') = \sum_{s=t}^{m-1} D_{x+s} v'\,{}_se + D_{x+t}({}_tV' - {}_tLS).$$

Thus ${}_tV' > {}_tLS$ if the present value of future "loading" exceeds the present value of excluded expenses. In particular, if an overhead expense that increases by duration (such as the Phase 1 income tax) is ignored, the resulting reserve might be inadequate.

Federal Income Tax (FIT)

I recommend that the Phase 1 FIT be handled as a reduction in the interest rate, which is the method suggested by Posnak in *GAAP—Stock Life Insurance Companies* (p. 147). Let

i_b = Before-tax rate;
r = Tax rate (now 46 percent);
h = Fraction of income fully taxable;
c = Current earnings rate;
x = Lower of c and the five-year average rate;
f = Menge ten-for-one factor;
$VTAX$ = Tax reserves; and
V = Dividend fund.

Then the Phase 1 tax is

$$rh[cV - xf(VTAX)].$$

The tax effect is given by reducing the interest rate from i_b to i', where

$$i' = i_b + rh(xf - c)$$

and adding a small additional expense charge,

$$\text{FIT expense} = rhxf(V - VTAX).$$

If *VTAX* is the statutory net level reserve, $V - VTAX$ is probably negative in the early durations and positive in the later. Therefore the FIT expense will give rise to a small positive reserve.

The FIT expense could be ignored, combined with other expenses, or combined with policyholder dividends. I recommend the latter course.

Corollary 1 of Theorem 2 (classical three-factor). *Suppose that* $w = w'$ *and* $E = 0 = e = B$. *Then*

$$Div = (i' - i)[{}_{t-1}V + P - (DB)q/2] + (q - q')[DB(1 + i'/2) - {}_tV] + (1 + i')(G - P - E').$$

Corollary 2 of Theorem 2 (universal life formula). *Suppose that* $V = CV$, *and* $P = G$. *Then*

$$Div = (i' - i)[{}_{t-1}CV + G - E - q(DB)/2] + (q - q')[DB(1 + i'/2) - {}_tCV] + (1 + i')(E - E') - (B + e).$$

Corollary 3 of Theorem 2 (experience refund or two-factor). *Suppose that* $w = w'$, $q = q'$, *and* $E = 0 = e = B$. *Then*

$$Div = (i' - i)[{}_{t-1}V + P - (DB)q/2] + (1 + i')(G - P - E').$$

Corollary 4 of Theorem 2 (Jellicoe one-factor or indeterminate premium method). *Suppose* $w = w'$, $q = q'$, $i = i'$, $E = E'$. *Then*

$$Div = (1 + i')(G - P) - B.$$

Corollary to Theorem 1 (Weck's Metropolitan Fund formula). *Define the dividend fund as*

$${}_tV = 1.075\,{}_tVNL + {}_tVE,$$

where ${}_tVNL$ = *net level statutory reserve and* ${}_tVE$ *is the natural reserve for first-year and renewal acquisition expense. Define the terminal dividends as*

$$TD = {}_tV - {}_tCV.$$

Then

$$Div + e + B = (1 + i')[{}_{t-1}V + G - E' - (DB + TD)q'/2] - q'(DB - {}_tCV) - {}_tV.$$

Proof. By Theorem 1,

$$Div + e + B = (i + i')[{}_{t-1}V + G - E' - (DB + TD)q'/2] - q'(DB + TD - {}_tV) - w'(CV + TD - {}_tV) - {}_tV.$$

Using the definition of terminal dividend, we prove the corollary. (The actual Metropolitan formula is somewhat more complicated and the 1.075 factor is for illustrative purposes. See Allan Lebourveau's discussion in *TSA*, XIX (1967), 252, and Allen Mayerson, Society of Actuaries Study Note, 1958, chap. 12, 'Dividends: Life Companies," p. 65.)

Lidstone's Remainder

The total earnings before payment of dividends, overhead, and profit is ${}_tR = Div + e + B$, which turns out to be Lidstone's "remainder function." In fact, let V and P denote the "normal" reserve and net premium computed using the assumptions i, q, w, and E. Let V′ and P′ denote the "special" reserve and net premium computed using the assumptions i', q', E', w'. Then ${}_0V = {}_0V' = 0$, ${}_mV = {}_mV' = matval$, and

$${}_tV(1 - q - w) = ({}_{t-1}V + P - E)(1 + i) - q(DB)(1 + i'/2) - w(\mathrm{CV}),$$

and

$${}_tV'(1 - q' - w') = ({}_{t-1}V' + P' - E')(1 + i') - q'(DB)(1 + i'/2) - w'(CV).$$

Let $K = V' - V$ be the "variation fund." Then the *remainder function is* defined in one of the following equivalent ways (see Theorem 2 and Theorem 3 with G replaced by $P' = P''$):

i) ${}_tR = ({}_{t-1}V + P' - E')(1 + i') - {}_tV(1 - q' - w') - q'(DB)(1 + i'/2) - w'(CV).$

ii) ${}_tR = (i' - i)[{}_{t-1}V + P - E - (DB)q/2] + (q - q')[DB(1 + i'/2) - {}_tV] + (w - w')(CV - {}_tV) + (1 + i')(P' - P) + (1 + i')(E - E').$

iii) ${}_tR = {}_tK(1 - q' - w') - {}_{t-1}K(1 + i').$

Using the third form of the critical function, we can prove

Theorem 6 (Lidstone, *JIA*, XLIX, 216). *If the discounted remainder* ${}_tF = {}_tR/(1 + i')$ *decreases (resp. is constant) by duration, then the "normal" reserves, V, are less than (resp. equal to) the "special" reserves, V′.*

Proof. To prove Lidstone's theorem, note that all for durations a and b with $b > a$,

$$\sum_{t=a}^{b-1} {}_tF\, D_{x+t} = {}_bK\, D_{x+b} - {}_aK\, D_{x+a},$$

where $D_{x+t} = D_{x+t-1} v^t(1 - q^t - w^t)$ and $v^t = 1/(1 + i^t)$. Note that ${}_tF$ cannot be positive at all durations or negative at all durations, since

$$\sum_{t=0}^{m-1} {}_tF\, D_{x+t} = {}_0K\, D_x - {}_mK\, D_{x+m} = 0.$$

If ${}_tF$ decreases by duration, then, for some u,

$${}_1F > \ldots > F_u > 0 > F_{u+1} > \ldots > F_m.$$

Thus

$$\sum_{t=0}^{b} {}_tF\, D_{x+t} = {}_bK\, D_{x+b} > 0 \quad \text{for } b < u.$$

while

$$-\sum_{a}^{m-1} {}_tF\, D_{x+t} = {}_aK\, D_{x+a} > 0 \quad \text{for } a > u + 1.$$

The proof of the rest of Lidstone's theorem is similar.

Lidstone called the equality of the "normal" and "special" reserves the equation of equilibrium."

Lidstone showed (using the first two forms of the critical functions) that the introduction of decreasing interest rates, select mortality rates, and decreasing lapse rates *increases* the reserves for permanent plans. In particular, type 2 GAAP reserves will exceed Cody's original management reserve, especially for term plans.

The connection between the critical function and the contribution dividend formula was well known by Lidstone. In fact, his 1905 paper, which introduced the remainder function, was a sequel to this 1895 paper, which discussed Sprague's contribution formula. The above proof of Lidstone's theorem is similar to that given by Lidstone, and by Baillie in *TSA*, III (1951), 75.

Appendix B

Historical Notes

GAAP Reserves

GAAP reserves arose from natural reserves, which were developed by C. O. Shepherd and James E. Hoskins, both of the Travelers Insurance Company (*RAIA*, XV [1926] 6, and *TASA*, XXX [1929], 140). Their reserves used best-estimate assumptions, while for GAAP reserves the assumptions are "reasonable and realistic" with some provision for adverse deviations. According to the American Academy of Actuaries, the provision for adverse deviations should increase the net premiums and reserve.

The use of realistic assumptions dates back to before the invention of natural or GAAP reserves. Select mortality rates were invented by George King and T. B. Sprague (*JIA*, XXI [1879], 246), while lapse rates were investigated by Arthur Hunter of New York Life (*JIA*, XXXVI [1901], 5); Ackland and Bacon (*JIA*, XXXVIII [1904], 539); and George Lidstone (*JIA*, XXXIX [1905], 209). Varying interest rates were invented by Richard Price (1771), and later by R. P. Hardy (*JIA*, XXXI [1894], 325) and D. P. Fackler (*TASA*, IV [1895], 32, 68, 174, 201, and *TICA*, I [1895], 24). They have been used by George Lidstone (1895, 1905), Walter O. Menge (1935, 1965), Shepherd and Hoskins, Charles Connolly (*TSA*, IX [1957], 135), and many others. The inclusion of expense factors in reserves was introduced by Zilmer (1863) and by T. B. Sprague (1870).

For statutory reserves the assumptions are limited by legal requirements, but there have been some welcome moves to make statutory assumptions more realistic, especially in Canada. In the United States there is talk of using a select and ultimate mortality table, and since 1956 companies have become encouraged to use withdrawal rates to compute health reserves (*TSA*, IX [1958], 354). Many companies use lapse rates to calculate settlement option reserves.

Varying Net Premiums

Usually the net premium is a uniform percentage of the gross premium (G), but it is also possible to use a variable net premium. For example, Northwestern Mutual has used a dual net premium approach, where the initial conditions are replaced by ${}_0V = 0$, ${}_mV =$

matval, ${}_{20}V = {}_{20}CV$. There are two net premiums, one for the first twenty years and another after twenty years. The varying net premium approach was invented by R. P. Hardy (*JIA*, XXXI [1894], 325) and later by Guertin (*TASA*, XLV [1944], 329), and is often used with GAAP reserves. Varying net premiums are also the basis of modified statutory reserves (New Jersey, Illinois, and Ohio methods).

Use of Asset Shapes and Gross Premium Reserves

The gross premium reserve technique is as old as the net premium method. In fact, the gross premiums of James Dodson used by the old Equitable, founded in 1762, were equal to the net premium (based on the 3 percent London table) (see also G. Ryan, *JIA*, XXXVIII [1903], 70). In the United Kingdom, gross premium reserves have been popular ever since. In the United States we have used net premium reserves almost exclusively as the result of the efforts of Elizur Wright.

Recently, actuaries have revived the use of gross premium reserves. The minimum cash values for annuities are defined by using an asset share approach. The net premiums are taken as an unloaded gross premium. As specified under the Commissioner's Annuity Reserve Valuation Method (CARVM), the reserves under annuity policies are calculated by a prospective method using the same unloaded gross premium that is used to calculate cash values.

The CARVM also introduced a "triangular approach," based on the proposition that the reserve should cover liabilities adequately not just at maturity, but at intermediate durations as well. If the policyholder lapses at duration t, then in place of an m-year endowment for the maturity value, the policy really is a t-year endowment for the cash value. Put the reserves for the t-year endowment in the tth row of a lower triangular matrix. Then at duration s the reserve is the maximum number in column s.

Bonus Reserves

In Type 1 GAAP reserves, dividends are counted as disbursements. This type of reserve is called a "bonus reserve" method (in the United Kingdom) and was suggested by Coutts (*JIA*, XLII [1908], 161) and even earlier by T. B. Sprague (*JIA*, VII [1857], 61; reprinted in *JIA*, LXII [1931], 96).

The bonus reserve system produces a conservative reserve yet allows the use of realistic assumptions. As pointed out by W. P. Elderton (*JIA*, LXII [1931], 62), the realistic assumptions give a more accurate reserve for participating and nonparticipating products and for deficiency reserves. Actually, the use of conservative interest rates for participating products was really more of a marketing decision than an actuarial one. In America the conservative rate of interest produced dividends that increased by duration, and this is what policyholders wanted (see J. B. Maclean and E. W. Marshall, *Distribution of Surplus*, p. 18). In the United Kingdom the uniform reversionary bonus naturally results in dividends that increase in value. The use of a conservative rate simply produces an implicit reserve for dividends (see Lochhead, *Valuation and Surplus* [1932], pp. 8, 72, 89).

Equation of Equilibrium

The term *equation of equilibrium* has two meanings in the actuarial literature. The term was used by George Lidstone in his paper on the effect of interest and lapse rates on reserves (*JIA*, Vol. XXXIX [1905]). His equation related the "special reserves" to the "normal reserves."

Sheppard Homans, in his 1868 letter to the Massachusetts Insurance Commissioner (reprinted by J. Charles Rietz, *RAIA*, XI [1922], 117), used the term, however, to refer to the equation defining reserves. Lidstone (*JIA*, XXXIX [1905], 213) called the same equation the *fundamental principle*. Other authors use the term *retrospective reserve equation*. The annuity form of Homans's equation of equilibrium was discovered by Demoivre (1725) and Euler (1760) (see T. E. Young, *JIA*, XLII [1908], 189).

Bonus or Dividends

The term *bonus* is used in the United Kingdom to describe the nonguaranteed reversionary additions, and *dividend* to describe cash payments. In the United States, we use only the term *dividend*. Both terms are very old. According to Francis Baily, in *Doctrine* of *Life Annuities and Assurances* (1810), pages 479-84, the term *bonus* was used by the old Equitable, founded in 1762. The Equitable paid its first distribution in 1777 as a 10 percent reduction in premium. Beginning in 1786, the bonus was credited in the form of a paid-up addition (see also Fisher and Young, *Actuarial Practice* *of Life Assurance* [1965], p. 22). The term *dividend* was used by the old Amicable, founded in 1706, to describe both its annual premium reductions and its nonguaranteed variable death benefits, that is, terminal dividends (see Richard Price, *Observations on Reversionary Payments* [1771-1812], pp. 158-91). The term *dividend* was also used by Sheppard Homans in *JIA*, Vol. XI (1863), describing the new Mutual of New York "contribution principle."

Relationship between the Dividend Fund and the Type 1 Reserve

Robert Posnak in *GAAP—Stock Life Insurance Companies* (1974), pages 162, 166, noted:

> If adjusted reserves are computed based on the same interest and mortality assumptions as those underlying dividend scales, the dividend is provided for as a benefit, and the dividend formula uses statutory reserves as a base, the adjusted reserves are very nearly the same as statutory reserves.
>
> When withdrawals, nonforfeiture benefits and expenses are introduced into the calculation of the reserves ... the relationship between statutory and adjusted no longer holds up.

What we have shown is that if we use Cody's scale, and reserve for dividends *and profits*, the relationship still holds. In fact, we get an exact equality, and, further, the gross premium is even equal to the net.

Source of Earnings Formulas

The statutory sources-of-earnings formula was apparently due to Fackler (see Papps, *TASA*, XI [1909]. 53). I found the GAAP sources of earnings formula in papers by Norm Hill and Howard Bolnick of Coopers and Lybrand (1979), and by Samuel Turner of Tillinghast, Nelson and Warren (1979).

The statutory formula gave rise to the 1895-1938 "Gain and Loss Exhibit" (see H. P. Hammond, *Proceedings, NAIC*, 1924, p. 237) and the "interest gain" of the exhibit became the basis of the 1921 Federal Income Tax Act (see E. E. Rhodes, *TASA*, XXIII [1922], 21). Both Fackler and Homans complained that the Gain and Loss Exhibit was unsound; E. E. Rhodes proposed the 1921 act as a pure expedient to replace the 1917 War Excess Profit Tax.

Author's Review of Discussion

Donald D. Cody

The discussions by Mr. Cragoe and Dr. Kabele have extended the theoretical scope and practical applicability of my generalized financial structure and formula for dividends well beyond my original bounds. On reflection, this extension is not surprising because the generalized contribution dividend formula of the paper is an explicit expression of the whole actuarial structure of the insurance and annuity mechanism and should, therefore, encompass all possible conventional and evolutionary arrangements. The structure and formula merely extend to the dividend itself the mathematical precision already characterizing formulas for statutory, natural, GAAP, and gross premium reserves; statutory, nonparticipating, and GAAP premiums; and well-defined investment-year method systems.

Mr. Cragoe has described his company's creative utilization of "liability slabs," similar to my "asset slabs," in determining amounts of investment income arising from investments directed by the policyholder and from the undirected balance. This is a new dimension offered by some companies. He also describes a unique treatment of policy loans as one of the directed investments, so that borrowing policyholders receive credit only for the net loan interest on the loans on each policy. This will appeal to many actuaries as equitable.

Mr. Cragoe has opened up discussion on another of many product designs motivated by the volatile, high inflation, high interest rate, disintermediation-prone economic environment of the foreseeable future. This climate involves both dangers and opportunities. Discussions like Mr. Cragoe's are especially valuable for expanding our actuarial paradigms to allow creative accommodations to a changing market.

Dr. Kabele's discussion is an elegant, comprehensive elaboration and analysis of the generalized dividend financial structure and formula. Since my formulas are a complete and explicit expression of the insurance mechanism, I had appreciated their generality. However, Dr. Kabele's creative analysis has shown how remarkably general they are, indeed well beyond my own expectations. His findings cover a broad spectrum and can be appreciated only by a careful study of his discussion. I will comment only on a few of his findings that appear especially significant to me.

First, Dr. Kabele has rigorously proved that the generalized dividend financial structure and formula, based on a wide variety of dividend funds (reserves), conforms to a (Posnak) type 1 GAAP structure (dividends treated as benefits) where the "net premium" of the "reserve" is the gross premium, thereby assuring automatic recognition of loss and recoverability. In particular, if a mutual company adjusts statutory reserves by adding an asset for unamortized acquisition expenses and uses the generalized dividend formula of the paper, then the adjusted financial statements conform to GAAP in the release-from-risk area. I had suggested this in a previous paper (*TSA*, XXIV [1972], 31-42) and then more strongly in the current paper. It is now rigorously proved and should be an important consideration in any ongoing discussion with the accountants on possible GAAP for mutuals.

Dr. Kabele offers a number of modifications of the dividend funds, which I defined as the asset share (statutory reserve less unamortized acquisition expenses plus accumulated company profits). All of his modifications are reasonable, depending on company objectives and structures for analyzing gains and losses. My own choices were based on several objectives:

1. A desire for just one set of financials, namely, statutory financials.
2. Hard control on the period for amortizing acquisition expenses, with profit charges deferred to subsequent periods for simplicity.
3. Avoidance of formal GAAP financials, which are complex, expensive, arbitrary, and unnecessary.
4. Simplified internal management adjustments to statutory financials with all the advantages of GAAP financials for analysis purposes.

Dr. Kabele's suggestion of using the "management reserve" ($f'V$) as the dividend fund during the period of amortizing acquisition expenses (k) and using the profit charge (B) only thereafter is an excellent one. The management reserve is the statutory reserve less unamortized acquisition expenses defined on the withdrawal, mortality, and interest assumptions at the time of introduction of the dividend scale. Using my formulas, this implies formula (1), solved for D_n, during the period (k) with $f_n = f'_n$ and formulas (3) and (4) thereafter. This approach assures automatic recovery of acquisition expenses by programming the dividend to produce $f_k = f'_k = 1$, rather than updating the amortization charges by monitoring the recovery, as I suggested. In this approach, the smoothing factor would be introduced into formula (1).

Dr. Kabele's suggestion for treating heaped and irregular renewal commissions like acquisition expenses is appealing and can be a desirable substitute for my complex smoothing process.

His scholarly research into the history of dividend financial structures and formulas is fascinating. It is indeed humbling for one's efforts to be associated with those of so many great actuaries of the past hundred years.

Dr. Kabele's discussion has added another full dimension to this paper and deserves to be published as a separate paper, which would be discussed more thoroughly by other actuaries.