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The Value of the Firm: The Option Adjusted Value of Distributable Earnings

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Abstract

The goal is to maximize the value of the insurance firm. The first step is to develop the right objective function to measure the firm's value. The second step is to use the objective function to identify and quantify the risks to which the firm is exposed. The third step is to use the objective function to analyze proposed strategies to maximize the value of the firm relative to risk, i.e. either maximize value given a fixed risk or minimize risk given a fixed value. Such strategies are to be applied to the pricing of new business, the management of in force business and the acquisition/divestiture of blocks of business or entire companies. The challenge is more difficult when the firm is exposed to multiple stochastic risks, many having embedded options and not all of which are independent. The fourth step is to use the objective function to provide management information about the performance of the firm during each time period and to allocate capital to future and existing projects. It would be ideal if the external financial reporting of the firm's performance could be presented on this basis. In this way the owners of the firm would know its value and income for a given period and be able to assess the impact of management's actions on that value.

This paper defines such an objective function, demonstrates its properties and shows how the different steps above can be carried out. Since the current accounting environment less accurately quantifies the value and performance of the firm over time, the paper first presents background on the goals and evolution of current accounting systems for life insurance companies, culminating with the environment after the enactment of Financial Accounting Standard 115 and prior to the resolution of the issue of the market value of liabilities.

An overview is provided of several methods of adapting the current accounting structure to accommodate a market value of liabilities. Two of these approaches are examined in some detail. Second, a brief, intuitive presentation on option pricing for assets is given and used to provide the foundation for several potential market values of liabilities. Clarifications between the application of market value concepts to accounting and asset/liability management are given. Limitations of the applicability of these solutions to the issues described above are identified. Third, the proposed objective function is then motivated, defined and explored. Fourth, several concrete examples are provided that demonstrate the capabilities of the tool. Alternative ways of evaluating results are demonstrated and their relationships are shown. Fifth, various general applications of the tool are given that complete and augment the goals stated in the first paragraph.

Although the focus of the paper is on interest rate risk, extensions to other stochastic risks are indicated.

Current Accounting Environment

Introduction

The terms 'market value of liabilities', 'market value of surplus' and 'economic surplus' are used in similar but not necessarily identical ways when referencing the issues of market value accounting and asset/liability management. Because of this ambiguity and the attention being given to market value accounting as an alternative solution to a decreasingly credible accounting system, this section presents some general concepts regarding accounting systems used for the insurance industry, a brief history of the evolution of certain GAAP accounting concepts, the market value accounting issue and proposed solutions together with issues that each raise. Insights into these issues relative to asset/liability management are presented in the next section.

Accounting systems

Accounting systems quantify the financial state of a firm via the balance sheet, what the firm owns (assets) and the claims on what is owned (liabilities and surplus), and the financial progress of the firm via the income statement, how those values change from one accounting period to the next. 'Surplus' represents both the amount of the firm's assets owned by shareholders and the amount of funds in excess of the liabilities that are available to cover obligations of the firm should the provision for liabilities be inadequate. Accounting systems employ a system of standardized rules so that the financial results of two firms are comparable and that the financial results of firms are comparable over time. Inherent in these rules must be a basis for the estimation of the values of assets and liabilities.

The transactions of a life insurance company consist of activities such as collecting premiums, investment income and investment maturities and paying claims, surrender values, commissions, expenses, taxes and shareholder dividends, if the company is a stock life insurance company. Such transactions must be recorded on the general ledger of the firm. In this sense, the accounting system is transaction driven. In an insurance company the products or services provided by the firm result in transactions over many accounting periods. Its liabilities are referred to as long-duration contracts. Utilizing the valuation basis for assets, the reserving system for liabilities and the transactions for a given accounting period, accounting systems allocate profit or loss to each accounting period.

In the USA there are two principal bases for accounting systems for insurance firms: these are referred to as statutory and GAAP. Statutory accounting is based on statutory accounting principles (SAP) and GAAP accounting is based on generally accepted accounting principles. The legal bodies responsible for SAP are the state insurance departments and, indirectly, the NAIC. The legal bodies responsible for GAAP are the Securities Exchange Commission (SEC), the Financial Accounting Standards Board (FASB), and the American Institute of Certified Public Accountants (AICPA).

Statutory accounting

Statutory accounting principles emphasize solvency, i.e. the ability of the firm to provide for its policyholder obligations. SAP is referred to as being 'balance sheet' oriented. Thus it tends to overstate liabilities and understate surplus and income. Key examples of this conservatism can be seen in the valuation of liabilities through conservative mortality/morbidity and interest assumptions and in the treatment of acquisition costs, which are immediately expensed or charged against income. Other examples of conservatism are the rules for deficiency reserves, the interest maintenance reserve for interest-related capital gains and losses, the asset valuation reserve and the requirement of conservative risk based capital requirements. The values of many assets are 'not admitted', so that the assets of the firm are understated. The rules for asset valuation do not always ensure conservatism as many investments are carried at amortized cost rather than market value. Market values may be either higher or lower than amortized cost. The use of book values for assets was consistent with the accounting tradition of holding assets at historic cost and the fact that insurance enterprises typically have held securities to maturity (in light of the long duration nature of their liabilities) and did not engage in active trading. Statutory accounting has a liquidation orientation for valuing the company. Revenue is annualized premiums plus earned investment income. SAP applies to both stock and mutual life insurance companies.

SAP can result in counterintuitive financial performance for insurance companies. For example, an insurance company experiencing rapid growth will report a pattern of lower statutory earnings than it would under normal growth.

An insurance company experiencing a decline in new business or no new business will show higher statutory earnings than it would under normal growth. The use of statutory accounting poses problems for those needing to analyze the performance of stock life insurance companies.

GAAP accounting: a brief history of DAC, FAS 60, FAS 97 and FAS 115

The problems posed by statutory accounting were addressed in 1973 when GAAP reporting principles were required for stock life insurance companies. The goal was to account for insurance companies as 'going concerns' with the emphasis on a realistic income statement. All assets were admitted, liabilities were valued on a 'conservatively realistic' basis and acquisition costs, both internal and external, that were primarily related to and varied directly with the production of new business were not immediately expensed, but instead were capitalized and amortized over the life of the block of business. As a result of this approach the 'matching principle' was fully implemented in GAAP accounting. These issues are treated in the *Audits* of *Stock Life Insurance Companies,* first published in 1972 and in *Statement* of *Financial Accounting Standard Number 60* (FAS 60), published in 1982. In 1974 Ernst and Young, then Ernst and Ernst, published the text GAAP: Stock Life Companies [37].

Originally, GAAP accounting only applied to stock life insurance companies. In the past, mutual insurance companies contended that for them GAAP was SAP and declared that their statutory statements were in compliance with GAAP. FAS 120, *Accounting and Reporting by Mutual Life Insurance Enterprises and by Insurance Enterprises for Certain Long-Duration Participating Contracts,* issued in January, 1995, extends the requirements of FAS 60, FAS 97 and FAS 115 to mutual life insurance companies effective for fiscal years beginning after December 15, 1995. For insurance contracts not covered by FAS 120, the AICPA's *Statement* of *Position 95-1, Accounting for Certain Insurance Activities* of *Mutual Life Insurance Enterprises, will* apply for fiscal years beginning after December 15, 1995.

The hallmarks of FAS 60 were:

- insurance contracts were (mainly) long duration contracts; revenue was defined to be earned investment income and premium, where premium is recognized in proportion to performance under the contract;
- capitalized acquisition costs (also known as 'deferred acquisition costs' or DAC) were to be amortized in proportion to premium revenue over the life of the block;
- liabilities were valued using all relevant applicable assumptions where such assumptions were chosen as 'best estimate' plus a provision for adverse deviation;
- those assumptions, once chosen, were 'locked in' (the lock-in principle), i.e. could not be changed unless severe adverse experience developed in the future.

With this definition of revenue, DAC was amortized in proportion to premium income. Profits would emerge as a level percent of premium revenue, plus a portion due to release from risk which would occur as actual experience emerged more favorably than that assumed in the reserve basis and from any deviation of actual experience from expected. With this reserving system for liabilities profits would emerge over the entire life of the block of business.

The DAC was termed recoverable at issue if there were sufficient future revenues in the business to amortize the DAC after providing for all future benefits and expenses. Subsequent to issue, if experience deteriorated to the extent that based on then current best estimates of future experience the net GAAP liability, i.e. the GAAP benefit reserve less the DAC, together with future premiums would not provide for future benefits and expenses, then a state of 'loss recognition' was said to exist. This meant that the business would have to be revalued on the revised, new best estimates of future experience. The changing of the assumptions was termed 'unlocking'. In this event, the DAC was reduced by the deficit. If the deficit was larger than the amount of outstanding DAC, then additional benefit reserves were established. The amount of write down of DAC plus the amount of any extra benefit reserves established was an immediate charge to GAAP earnings. If future experience exactly equalled the new assumptions, then no further GAAP losses (or gains) would be reported.

The mechanism by which a loss recognition situation is determined is the computation of a gross premium valuation (GPV). For a block of business a GPV is the present value of all future policyholder benefits and company expenses less the present value of future premiums where the present value is computed at the net earned rate of the assets supporting the block. If the GPV is less than or equal to the net GAAP liability, then no loss recognition exists. If the GPV exceeds the net GAAP liability, then a loss recognition situation exists and the magnitude of the loss is that excess.

FAS 60 worked well for many years, but the mechanism was stressed by two events that occurred in the 1980s. First, some insurance companies applied the letter of the law of FAS 60 when preparing GAAP financial statements for single premium deferred annuities. Often, little provision for adverse deviation was present, thus the majority of profit was released in proportion to premium which meant released in the year of issue. This approach 'front-ended' earnings. The concern raised by this was heightened by the fact that one of the companies using that practice became insolvent. Second, universal life, with its flexible premiums, became a major life insurance product. Since premiums were flexible, their use as the basis of revenue would create volatility in the reported earnings of insurance companies selling significant amounts of these products. There was a disconnect between the flow of premium and the actual source of earnings of the insurance company for this type of product.

In 1987 the Financial Accounting Standards Board adopted Financial Accounting Standard Number 97 (FAS 97) which dealt with these issues. FAS 97 was effective in 1988 and applied to deferred annuities and universal life policies in particular. It did not apply, in general, to traditional fixed premium, fixed benefit life insurance. The hallmarks of FAS 97 were:

- a new definition of revenue;
- a new method for amortizing DAC;
- a new principle called 'unlocking';
- defining the GAAP benefit reserve to be the account value.

Revenue was defined to be the sum of interest earned and the various loads and fees charged against the product, e.g. pure mortality charges, policy fees, premium loads, administrative charges, front end loads and surrender charges. DAC would be amortized by a level percent of 'estimated gross profits', i.e. the periodic revenues above less expenses associated with those revenues, e.g. pure mortality costs, general expenses and interest credited. The GAAP benefit reserve was the account value. The assumptions underlying the estimated gross profits were to be best estimates with no provision for adverse deviation.

If there were any material deviation of actual experience from assumed, then the company would be required to 'unlock' its assumptions by adopting new best estimates from that date going forward and recompute the amortization of DAC since the inception of the block. This resulted in a new amortization percentage for amortization of DAC. The difference between the prior period DAC and the recalculation of the prior period DAC resulting from the application of the new amortization percentage to the historic actual estimated gross profits would be an immediate charge or credit to earnings; and the resulting DAC balance would be amortized by that new percentage of the revised future estimated gross profits. FAS 97 excluded realized gains and losses from the definition of estimated gross profits.

In 1991 the AICPA promulgated Practice Bulletin Number 8 (PB 8) which declared that for products accounted for under FAS 97 realized gains and losses were part of a product's investment returns and their inclusion in estimated gross profits should be considered if it would materially impact DAC amortization. The motivation was that if an insurance company realized a significant interest related capital gain, then that gain front ended the excess of the coupon on the sold security over that from a similar security that could be purchased with the proceeds. If the security had not been sold, then that excess investment income would have been included in future estimated gross profits and resulted in amortization of DAC. If no action were taken, then the realized capital gain would go through earnings, but there would be no change in the DAC. Therefore, equality of treatment (holding the original security with its higher coupon versus realizing the capital gain and owning a security with a lower coupon) motivated the inclusion of the realized capital gain in estimated gross profits and in the amortization of DAC. of course, there should be symmetrical treatment for interest related realized capital losses. Realized losses result in reduced amortization or, possibly, negative amortization of DAC. There is an 'income statement geography' issue as the realized capital gain is reported in net income and the amortization of DAC due to the gain is reported in operating income.

By itself, FAS 97 exposed insurance company earnings to new levels of volatility due to DAC amortization mechanics, the unlocking provision and the use of best estimate assumptions. PB 8 created even more potential volatility as companies realized capital gains in 1991 through 1993 and then capital losses in 1994 and 1995.

The issue of a fully market valued balance sheet (and therefore income statement), had been in discussion for some time during the late 1980s and 1990s. The impetus for this issue may have been heightened due to the insolvencies in the savings and loan industry during the 1980s where it was believed that if financial statements had been prepared on a market value basis, then the financial problems would not have become so severe. When FASB was discussing market value accounting issues many from the insurance industry argued that it was important that both sides of the balance sheet be marked-to-market or else significant swings in surplus could occur from having assets marked-to-market but liabilities accounted for at book. For many industries this is not a severe problem as they do not have liabilities which are interest sensitive or as long a duration as those of insurance companies.

In 1993 FASB issued FAS 115, to be effective in 1994 and optionally at year end 1993, which generally called for marking assets to market, but did nothing to change the financial reporting of liabilities. FAS 115 requires a company to segregate its applicable assets (FAS 115 does not apply to all assets, e.g. commercial mortgages and real estate are excluded) into three accounts or classes: the held-to-maturity (HTM) account; the available-for-sale (AFS) account; and the trading (T) account. Securities in the HTM account are valued at amortized cost, but, effectively, they may never be traded. Securities in the AFS account are valued at market with the change in market value being directly added to equity, or surplus, i.e. not going through the income statement. ('Below the line' is the phrase used for this type of treatment by the insurance industry.) Securities in the T account are valued at market with the change in market value reflected in the income statement. For changes due to the AFS account and the T account there are corresponding GAAP deferred tax offsets.

FAS 115, by marking the assets to market but leaving the accounting for liabilities at book, created even further potential for volatility in both insurance company earnings and surplus values. In addition, it made comparability of results for two insurance companies more difficult. For example, consider three companies with identical assets and liabilities, but the first puts all its assets in the HTM account, the second all into the AFS account and the third into the T account. If all three companies are managed in the same manner, then the reported financial results could be extremely different for each company due to changes in the interest rate environment, although the underlying financial transactions of all three companies are identical.

In December of 1993 the SEC realized the potential impact in financial reporting for financial institutions due to FAS 115. As a result, the SEC issued instructions that insurance companies should make two adjustments to their financial reporting process. The first was that for assets supporting FAS 97 liabilities and held in the AFS or T accounts each company should compute the unrealized capital gain or loss of those assets and then determine the change in DAC that would have occurred due to PB 8 if those assets would have been sold. This change in DAC would be offset against the surplus adjustment for AFS account unrealized gains and losses and against income for the T account unrealized gains and losses. For each of these there would be corresponding GAAP deferred tax adjustments. This, essentially, created a new type of DAC on the balance sheet. It is sometimes referred to an 'imaginary DAC' or 'virtual DAC', although the former term is a more accurate description.

The second adjustment called for the insurance company to compute a gross premium valuation for each of its lines where FAS 60 or FAS 97 applied and where assets were held in either the AFS account or the T account. This gross premium valuation would be computed as if the FAS 115 applicable assets in the supporting portfolios were sold (thereby realizing all the unrealized capital gains and losses) and new assets purchased. If there would be any loss recognition resulting from the GPV computed using a net earned rate reflecting the hypothetical asset structure after the restructure, then a reduction in 'DAC' and, if necessary, an increase in a liability for such unrealized or imaginary loss recognition would be established.

The changes caused by PB 8, FAS 115 and the SEC instructions created a higher probability of negative amortization of DAC, both real and imaginary. There was not any significant accounting literature in this area; but it was a reasonable position to allow both positive and negative amortization as long as three conditions were satisfied. First, the amount of DAC should not become negative. Second, the amount of DAC at any time should not exceed the amount originally capitalized on the then in force business accumulated with interest. Third, the DAC should be recoverable. Volatility in surplus would be dampened by the creation of imaginary DAC and the taking of unrealized loss recognition. Thus was the state of GAAP accounting in 1994.

Contemporaneous with these events members of the insurance industry had been attempting to create a basis for adjusting GAAP accounting to reflect the market value of liabilities. It is now possible to give an outline of some of the proposed solutions.

Proposed solutions for market values of liabilities

There are five major proposals for addressing the problems created by FAS 115's marking the assets to market but leaving the liabilities at book. They are treated in turn from the simplest to the most complex. The first three proposals are included here for completeness of the discussion, with the primary focus being on the last two proposals.

The first proposal is that nothing further needs to be done. This position asserts that the SEC's creation of imaginary DAC and unrealized loss recognition provides sufficient relief from the volatility of surplus created by FAS 115 and that further action is unnecessary given the difficulty of determining a basis for a market value of liabilities. As events stand in 1995, this approach is not likely to be adequate.

The second proposal is for the creation of a GAAP analogue to the interest maintenance reserve (IMR) in statutory accounting. Much of the logic that supports the IMR in statutory accounting is applicable in a GAAP environment. A GAAP IMR would remove the problem that when an interest related capital gain is realized, the gain net of taxes flows through net income in the current period. The future earnings from that gain might be needed to support the product line. The proposed treatment would not allow the gain to be released immediately, but would add it to the IMR and then release it over the time-to-maturity of the original asset that was sold. The change in the IMR due to the gain and the subsequent amortization of that portion of the IMR due to the gain would be an element of operating income, thus eliminating the 'geographic' difficulty in the income statement. This would also apply to realized interest related capital losses and the GAAP IMR would be allowed to become negative.

Implementing this approach would eliminate the need for PB 8 adjustments, i.e., reflecting realized capital gains and losses in the revenue stream for the amortization of DAC, and for reflecting unrealized capital gains and losses in imaginary DAC. For the latter situation one would create an imaginary GAAP IMR that would offset the change in unrealized capital gains and losses. It would actually simplify the existing situation.

The third proposal, made by Richard S. Robertson, would be to determine a method for valuing the GAAP liability and the DAC using interest rates current as of the date of the valuation and not the rates in place when the liability was established. This method is described more fully by Dicke (1993). This proposal remains the closest in spirit to the current GAAP accounting for life insurance companies. It is transaction based with a reserve system that allocates earnings over the lifetime of the block. Note that this method, however, does not reflect the value of embedded options, e.g. guaranteed surrender values, interest rate guarantees, loan provisions, flexible premium provisions or fixed/variable transfer provisions, in the value of the liability. Thus it is not exactly comparable to the market value of assets which does reflect the impact of embedded options. A further challenge is that although a theoretical basis for the adjustments can be described, resulting applicable calculations are not easily describable.

The fourth proposal is to define a market value of liabilities based upon an analogy to assets, i.e. value the liability as if it were a fixed income security by using option pricing techniques. This method has the advantage of reflecting the presence of embedded options in the value of the liability. This requires many new assumptions to be made in valuing the liability than are extant in the current GAAP accounting environment. Since the market value of the liability is similar to a gross premium valuation (which is comparable to a realistic value of the net GAAP liability) it would imply that DAC and its offshoots would be discarded. Credit for the DAC is embedded in this market value of the liability and its amortization is implicit. This method has several difficulties. First, the market value of surplus so obtained does not represent an estimate of the intrinsic or fair value of the insurance enterprise. It will not necessarily provide a value of what a willing buyer would pay a willing seller in an arms-length transaction. Second, although the market value of the liability is a gross premium reserve which provides for the strain of new business, this effect might or might not be completely realized depending on the spread used to discount the liabilities. The question about spreads is part of a larger issue regarding the provision, if any, for adverse deviation. Third, there is no unambiguous definition of the market value of a liability. As will be seen, this stems from the lack of a clear choice in the spread used to discount the liabilities. A more complete discussion on this issue is provided later.

In each of the four methods discussed above the value of surplus is obtained by subtracting the value of liabilities from the value of assets, although the values would now be market values. The fifth method is to define the market value of surplus directly as the value of the firm based on what a willing buyer would pay a willing seller (see Dicke, 1993). This value is computed using classical actuarial appraisal techniques. In this scheme, the market value of liabilities is the market value of assets less the market value of surplus. This is a significant shift in the accounting architecture. Profits no longer emerge over time; but the present value of profits emerge when the business is written. (This could be mitigated, as in FAS 60, by the use of conservatively realistic assumptions instead of best estimates.) This approach would remove the need for FAS 60, FAS 97, PB 8, FAS 115 and imaginary DAC and unrealized loss recognition. An approach based on what a willing buyer would pay a willing seller automatically puts all the assets on a mark-to-market basis. Like the fourth method, many new assumptions are needed. The difficulty in the fourth method stemming from lack of clear choice of a spread by which the liability cash flows are to be discounted has an analogue here. The question is what discount rate should be used in the appraisal. Also, this approach does not reflect the value of the embedded options in the liabilities. From the above one can conclude that the current GAAP accounting environment is subject to ambiguity and inconsistency with regard to the underlying economics of transactions, can result in significant volatility of earnings and/or surplus, is losing the goals of company comparability and comparability over time, is not capturing all the risk exposures of the company and so is not reflecting what has really transpired and the impacts of company management actions on the firm. In the sense of Khun (1970) the stage is set for a 'paradigm shift' in the accounting environment,

The Market Value of Liabilities

Introduction

This section is divided into three parts. The first part describes the computational architecture for applying the theory of option pricing to estimating the market value of assets and then specializes it to liabilities. It will be seen that the only 'free variable' in this architecture is the spread at which the asset or liability cash flows are to be discounted. The second part of the section examines accounting consequences from the adoption of a number of potential choices for the spread. The third part considers certain approaches to asset/liability management that result from the market value of liabilities. Cautions and limitations to these approaches are then identified and explored.

Option pricing architecture

Fixed income securities contain a variety of guarantees and embedded options which create interest rate risk. Examples are: guarantees of performance; bond put and call options; sinking fund acceleration provisions and/or call provisions; and prepayment features in mortgages and mortgage derivatives, e.g. mortgage pass-throughs and collateral mortgage obligations (CMO). In general, the principle of no riskless arbitrage requires that it is impossible for an investor to make an investment with zero net outlay which has a positive probability of positive return now or in the future.

The valuation or pricing of fixed income securities is based on arbitrage pricing theory and utilizes the concepts of no riskless arbitrage (the law of one price), complete markets and risk neutral valuation. Sources on these topics can be found in Cox *et al.* (1979), Cox and Rubinstein (1985), Jarrow (1988) and Pedersen *et al.* (1989). The theoretical assumptions required for arbitrage pricing to hold include: information is freely available; borrowing and lending take place at the same interest rate; the market continuously trades with no transaction costs, no taxes, and no restrictions on short sales; investors are price takers, acting rationally based on all available information and preferring more wealth to less wealth; and markets are complete. A complete market implies that all combinations of securities are available and are perfectly divisible within the market.

The concept of arbitrage pricing implies that two securities, A and B, having the same cash flows in all possible future states of the world, must have the same price. If the price of A, for example, were greater than the price of B, then an investor could sell A, purchase B, use the future cash flows from B to meet the obligation to the buyer of A and pocket the difference in initial price as riskless profit. The foundation for assuming no riskless arbitrage is that if such discrepancies in prices were significant, then they would be observed and trading would commence to take advantage of the difference. Once commenced, however, trading would result in the convergence of the prices of A and B. The result of efficient market trading ultimately drives away discrepancies in price.

Suppose B is a 'basket' of securities whose net cash flows in all possible future states of the world equals those of A. B is said to be a replicating portfolio for A. Thus the arbitrage-free price of A must equal the price of B, which equals the sum of the prices of the individual securities. For this to be true in general it is required that the market be complete, i.e. any asset can be represented by the sum of individual assets from some basic group that spans the entire set of future outcomes. (As an analogy consider this special set of assets as similar to the basis of a vector space, i.e. a set of vectors that both span the space and are independent.) In this case A is said to be priced (or valued) consistently relative to B, As replicating portfolio. Note that if A is valued consistently relative to B and B is valued consistently relative to C, then A is valued consistently relative to C.

A further consequence of no riskless arbitrage and the other assumptions is that the values obtained for securities must be independent of individual investors' preferences. This is not to say that investors do not have different views of the future states of the world or have the same aversion to risk, it says that the price obtained is the same no matter what those views and levels of aversion are. Since all preferences are equally valid and lead to the same price, then it makes sense to choose the preference in which the computations are the easiest. The simplest frame of reference is that of an investor who is risk-neutral, i.e. not risk averse and not a risk seeker. For a risk-neutral investor the value of the security is the expected present value over all paths of its future cash flows discounted at the risk free rate.

To value a security with fixed cash flows requires a set of fixed income securities free from default and without embedded options which are traded in a market that is active, robust in volume, liquid with ease of trading at narrow bid/ask spreads and covering a large range of maturities. From this ideal set it is possible to infer the prices of zero-coupon bonds. Since any security's cash flows can be considered the sum of a series of zero-coupon bonds, then it is possible to use the law of one price to compute the price of the given security as the sum of the prices of the respective amounts of zero-coupon bonds from the ideal set. The security is priced consistently relative to the collection of zero-coupon bonds. This ideal set of reference securities is the discount bills and coupon notes and bonds issued by the United States Treasury. From this set of Treasury securities it is possible to infer the prices and corresponding yields of hypothetical Treasury zero-coupon bonds. The set of interest rates corresponding to these zero-coupon bonds is referred to as the Treasury spot rates and constitutes what is known as the term structure.

Market forces will eliminate arbitrage opportunities from the market for Treasury securities. Thus it is possible to value a security with fixed cash flows consistently relative to the implied Treasury zero-coupon bonds. Because of risk-neutral valuation the value of the security is the present value of the security's cash flows discounted at these risk-free rates. Since there is no arbitrage opportunity among the Treasury zeros, then there are no arbitrage opportunities among several securities each valued consistently relative to the Treasuries.

Barring unusual circumstances, the price obtained in this way for a security with fixed cash flows is larger than the market price. This is because the market demands a premium for assuming default and liquidity risks as well as other risks. To adjust for this, a spread is added to the Treasury spot rates such that the resulting price equals the market price. This spread, called the spread-to-Treasuries, represents the market's expected incremental return over investing in Treasury securities. It is the reward for taking on risk. In this manner, two securities with fixed cash flows can be compared. If an investor has a desired target for the spread-to-Treasuries then it is possible to compute the price necessary to obtain that incremental return.

The cash flows of a fixed income security that are not fixed, but depend only on the level of interest rates (path independent) or the particular sequence of interest rates over time (path dependent) are called contingent cash flows. Even if the default and liquidity aspects of two such securities are the same it may not be possible to compare them by examining their prices, nominal coupons or yields due to the presence of the embedded options. The goal is to value them in a manner that removes the impact of the embedded options.

The process of determining the spread on this type of security is based on solving for the spread which equates average price of the security to the market price. This average price equals the probability weighted net present values of the security's cash flows over a large number of potential future interest rate paths. The present values are computed using the one period future risk-free rates for each path plus a spread. The paths must have the property that they correctly reprice the Treasury zero-coupon bonds at the date of valuation. Paths satisfying the necessary conditions are said to be arbitrage free at the date of valuation. The resulting spread obtained from solving the algorithm above represents the spread to be earned net of the impact of embedded options. Thus one can compare two securities on the same basis, net of the impact of any embedded options. In this case the spread is called the option adjusted spread (OAS).

The details of how such paths are generated are beyond the scope of this paper. The following provide references for the interested reader: Black *et al.* (1990), Hull (1993), Ho and Lee (1986), Heath *et al.* (1990), Jacob *et al.* (1987), Miller (1990, 1991, 1992), Pedersen *et al.* (1989) and Tilley (1992).

The following describes the mathematics of the process.

Definitions:

Let i_o denote the initial term structure of Treasury spot rates.

Let p be the index for paths (p ranging from 1 to P).

Let t be the index for the time period (t ranging from 1 to N).

Let j be a general index.

Let prb_p be the probability of path p.

Let $r_{p,t}$ be the one period future rate for path p, time t.

Let $ACF_{p,t}$ be the asset cash flow for path p, time t.

Let OAS be the option adjusted spread.

Let $MVA(i_o)$ be the market value or price of the asset.

$$MVA(i_o) = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=1}^{N} \left[ACF_{p,t} / \prod_{j=0}^{t-1} (1 + r_{p,j} + OAS) \right] \right\}. \quad (1)$$

Following Reitano (1991), for a security with fixed cash flows under the assumption of a flat term structure, i.e. i is constant for all maturities, let $P(i)$ be the function that assigns to each value $i \geq 0$ the value of the future cash flows. The rate i can be specified in any system of units. Assume $P(i)$ is twice differentiable with a continuous second derivative. The modified duration, $D(i)$ is defined as

$$D(i) = -\frac{dP}{di} \bigg/ P(i). \quad (2)$$

The convexity function, $C(i)$, is defined to be:

$$C(i) = -\frac{d^2P}{di^2} \bigg/ P(i). \quad (3)$$

Using first and second-order Taylor series expansions, the following two equations are approximations to the value of $P(i)$ resulting from a small shift in rates from i_0 to $i = i_0 + \Delta i$:

$$P(i)/P(i_0) = 1 - D(i_0)\Delta i \quad (4)$$

$$P(i)/P(i_0) = 1 - D(i_0)\Delta i + \tfrac{1}{2}C(i_0)(\Delta i)^2 \quad (5)$$

These equations can be generalized to the case of a non-flat term structure where Δi becomes a parallel shift to the term structure i_0.

For assets with contingent cash flows it is possible to calculate an option adjusted duration and an option adjusted convexity. Following Fabozzi (1994) these are called effective duration (OAD) and effective convexity (OAC), respectively. In this paper they will be referred to as just duration and convexity for simplicity and are computed assuming a parallel shift in the term structure (the implied Treasury spot curve).

Let Δi be a small, positive change in the level of the term structure of interest rates from i_0 to $i_0 + \Delta i$. Let $MVA(i_0 + \Delta i)$ be the market value that results from shifting the initial term structure upward by the amount Δi and valuing the resulting cash flows. The computation uses the value of OAS computed from the original term structure. Let $MVA(i_0 - \Delta i)$ be defined similarly. The following definitions can be made:

$$OAD = -\frac{MVA(i_0 + \Delta i) - MVA(i_0 - \Delta i)}{2 * \Delta i * MVA(i_0)}, \quad (6)$$

$$OAC = -\frac{MVA(i_0 + \Delta i) - 2 * MVA(i_0) + MVA(i_0 - \Delta i)}{(\Delta i)^2 * MVA(i_0)} \quad (7)$$

The equations preceding these hold for OAD and OAC in place of D and C, respectively.

Insurance liabilities may be analyzed in a manner similar to fixed income securities by substituting liability cash flows for asset cash flows in equation (1). In this case one needs a cash flow model that describes the liability cash flows in terms of management and policyholder behavior along a set of arbitrage free interest rate paths.

Nearly all insurance liabilities grant guarantees and options that expose the company to interest rate risk. The list includes, but is not limited to:

- single premium and flexible premium deferred annuities;
- immediate annuities;
- guaranteed interest contracts;
- terminal funded annuities; universal life;
- non-participating and participating ordinary life;
- disability income and long term care.

The features that create risk include:

- cash surrender at book value;
- minimum crediting rate guarantees;
- flexible premium or 'dump in' provisions with or without 'window' limitations;

- bailout provisions;
- return of premium provisions;
- partial withdrawals both with and without penalty;
- benefit responsive options in institutional pension products;
- fixed account/variable account transfer options;
- policy loans, both regular and wash loans.

These expose the insurance company to reinvestment and disintermediation interest rate risks.

Define the following additional terms.

Let $LCF_{p,t}$ be the liability cash flow for path p, time t.

Let LS (liability spread) be the spread chosen for discounting liabilities.

Let MVL be the market value of liabilities at the given spread.

$$MVL(i_0) = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=1}^{N} \left[LCF_{p,t} / \prod_{j=0}^{t-1} (1 + r_{p,j} + LS) \right] \right\}. \quad (8)$$

As with assets one can compute duration and convexity, D_L and C_L respectively, for liabilities. The formulae for them are similar to the above with $LCF_{p,t}$ in place of $ACF_{p,t}$. The subscripts A and S (instead of L) are later used for assets and surplus.

The market values of liabilities

Unlike many fixed income securities and common stock, insurance liabilities have no corresponding secondary market on which they trade. As a result there is no market price with which an option pricing model can be used to determine a liability's spread. One is forced to arbitrarily choose a spread for discounting liability cash flows. In this section several choices for the spread will be considered. The implications of each choice with regard to market value accounting will be examined. The notation i_0 denotes the initial term structure. Note that assets and liabilities are valued consistently using the same interest rate paths.

$$LS = 0$$

The MVL resulting from discounting the liability cash flows at the risk-free rate, i.e. a spread equal to zero, would be a conservative estimate of the amount of funds that the insurance company should hold such that those funds together with future premiums and investment income would mature the obligations.

$$MVL(i_0) = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=1}^{N} \left[LCF_{p,t} / \prod_{j=0}^{t-1} (1 + r_{p,j}) \right] \right\}. \quad (9)$$

Advantages of this definition include the following. It is a simple and unambiguous choice that is not subject to manipulation. It is responsive to changes in market interest rates and reflects the value of the embedded options. There is less volatility over time as this choice of spread is always zero, and the spread may change over time for other choices.

Disadvantages are that it is overly conservative, may result in materially understating surplus and earnings and does not relate the underlying value of the assets to the liabilities as does, for example, the choice for the spread defined next.

$$LS = OAS$$

The OAS is the option-adjusted spread-to-Treasuries of the asset portfolio supporting the liabilities. The principal use of this application would be as an option-adjusted gross premium valuation if, as is usually the case, expenses are included in the cash flows.

$$MVL(i_0) = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=1}^{N} \left[LCF_{p,t} / \prod_{j=0}^{t-1} (1 + r_{p,j} + OAS) \right] \right\}. \quad (10)$$

The advantages of this choice are:

- it is explainable;
- it relates the MVL to the character of the asset portfolio that supports the liabilities;
- it is responsive to changes in market interest rates and reflects the value of the embedded options;
- it should be relatively free of manipulation;
- it would provide a more realistic value;
- and it is useful for loss recognition purposes.

Disadvantages are:

- it requires the computation of an OAS for the asset portfolio, which may contain securities for which it is difficult to obtain a reliable value and, hence, exposes itself to subjectivity and manipulation;
- it can expose the MVL to volatility if the OAS of the asset portfolio changes dramatically due to restructuring or other causes;
- it could understate the MVL (and so overstate surplus and income) if the size of the OAS is large due to taking

significant credit risks or duration risks in the asset portfolio. Some might argue that it is too generous.

LS = Default Spread (DS)

DS is the default spread that can be assigned to the insurance company. It may be based upon debt ratings of itself or its parent, the claims paying ability/financial strength ratings assigned by ratings agencies or a combination of the two.

$$\text{MVL}(i_o) = \sum_{p=1}^{P} \text{prb}_p * \left\{ \sum_{t=1}^{N} \left[\text{LCF}_{p,t} \Big/ \prod_{j=0}^{t-1} (1 + r_{p,j} + \text{DS}) \right] \right\}. \quad (11)$$

The advantages of this method are:

- it values liabilities in the manner most similar to assets, i.e. where the spread reflects the default costs;
- it is responsive to changes in market interest rates and reflects the impact of embedded options;
- it is understandable in that it relates to the risk of the insurance company.

The disadvantages are:

- debt ratings and financial strength ratings are not consistent nor is either consistent across rating agencies;
- there is a potential problem in relating the value of DS to the debt/financial strength rating; DS has no provision for liquidity as the OAS does in assets;
- it is contrary to accounting for assets in that the borrower must carry the debt on the balance sheet at book, not market;
- it is not likely to be viewed as having any relation to the amount the insurance company should hold to mature its obligations;
- it has the property that lower ratings lead to higher surplus, a counter intuitive result.

Consider two insurance companies with identical liabilities. Company A has its entire assets invested in Treasury securities and is cash matched to its liabilities. Company B has an equal market value of securities that are invested in call protected C rated corporate bonds whose expected cash flows are matched to the liabilities. Because the quality of B's bond portfolio, B is viewed as riskier than A. B, accordingly, has been assigned a lower credit rating. This lower credit rating translates into a higher DS for company B than for company A. Therefore, $\text{MVA}_A = \text{MVA}_B$ and $\text{MVL}_A > \text{MVL}_B$. Let MVS be the market value of surplus and be defined by MVS = MVA – MVL, then $\text{MVS}_A < \text{MVS}_B$.

This result is not incorrect given the definition of DSs and a truly economically based balance sheet. This is due to the fact that the excess surplus in B reflects the positive value of B's put option in the event of insolvency which places the assets and liabilities in the hands of the state guarantee associations and/or ultimately back to the policyholders. This result, even if economically rational, does not seem to be an acceptable basis for an accounting system.

LS = cost of funds spread (COF)

COF is the spread-to-Treasuries that discounts the future liability cash flows back to the initial cash flow of the block at issue. COF is then assumed fixed for all time. If market interest rates change causing embedded options in the liabilities to become more valuable, then the MVL will rise. COF can be thought of as the cost of funds, i.e. the cost to the insurance company of acquiring the business from the policyholders.

$$\text{MVL}(i_o) = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=1}^{N} \left[\text{LCF}_{p,t} \Big/ \prod_{j=0}^{t-1} (1 + r_{p,j} + \text{COF}) \right] \right\}. \quad (12)$$

Its advantages are that it is responsive to changes in market interest rates and reflects the value of embedded options in the liabilities and provides an innovative method to manage profitability and interest rate risk.

Disadvantages are:

- one must determine the COF for each block of business, which may pose problems for companies with large existing blocks;
- it is not related to the amount that the insurance company needs to hold to support its liabilities;
- it reflects a limited view of profitability;
- it creates the situation where for two companies with otherwise identical circumstances, the one with the higher acquisition costs could have the greater MVS.

This last result reduces the method's utility as a basis for market value accounting, and is somewhat similar to the situation for the default spread. Assume insurance companies A and B have identical assets, liabilities and management strategies for the business. Assume that company A has higher acquisition costs than company B. This means that the initial net cash flow for A's block was smaller than for B's block. Since future benefits and expenses are the same, then $\text{LS}_A > \text{LS}_B$ as the liability cash flows have to discount to a smaller number for A than for B. Thus $\text{MVL}_A < \text{MVL}_B$, and so $\text{MVS}_A > \text{MVS}_B$.

Asset/liability management applications of the market value of liabilities

Up to this point the discussion involving the market value of liabilities has centered on its use as a basis for a market value accounting. The concepts underlying the market valuation of liabilities originally arose from efforts to improve asset/liability management. These concepts and their applications to asset/liability management are presented here. Cautions and limitations to these applications are presented in the next part of this section. The concepts are applied, in a different context, see later.

The term market value analysis is used here to represent the totality of various devices by which a firm measures and/or controls interest rate risk. The 1980s witnessed a major innovation among insurance companies in the measurement of interest rate risk and techniques to control it. The innovation was to treat the insurance liability as a fixed income security and to apply fixed income security analysis and management techniques, including option pricing theory, to the assets and liabilities of the firm. In particular the concepts of market values, duration, convexity and immunization were applied to the assets and liabilities of insurance companies. Progress was made in bringing these concepts into the design and pricing of insurance liabilities. Further, the theory of duration and convexity was generalized beyond parallel shifts in the yield curve (Reitano, 1991a; Ho, 1990). Further, Reitano has extended most of the classical work into the very general domain of arbitrary movements in the term structure. For simplicity of exposition, examples in this section will be stated from the perspective of parallel shifts in the term structure.

From the perspective of funds management the risk posed by reinvestment and disintermediation is that the insurance enterprise does not have the funds available to pay its obligations without incurring a loss. This is what is meant by interest rate risk. The most conservative approach would be to cash match the assets with the liabilities. Cash matching would typically employ the use of assets with no embedded options and little or no default risk. Reinvestment risk would be diminished by assuming a conservative reinvestment rate. Market risk would be eliminated by holding the securities to maturity. Mathematical algorithms can be used to choose the portfolio that cash matches with least cost.

There are two problems with this. It is extremely costly in terms of sacrifice in return and it may be difficult or impossible if the liability and/or asset cash flows are not fixed but have embedded options.

Another approach is to relax the cash match criteria and manage the market values so that a change in the term structure of interest rates results in changes in market values such that the market value of assets remains larger than the market value of liabilities. If the market value of assets exceeds the market value of liabilities, then the insurance enterprise can liquidate assets necessary to pay its obligations and the remaining market value of assets still exceeds the remaining market value of liabilities. One wants the following equation to be valid at any time t and remain valid for reasonable shifts in the term structure:

$$ACF_t + MVA(i_t) \geq LCF_t + MVL(i_t). \qquad (13)$$

The exposure of a portfolio of liabilities and its supporting assets to interest rate risk is often displayed via market value diagrams. A typical example is shown in Figure 1.

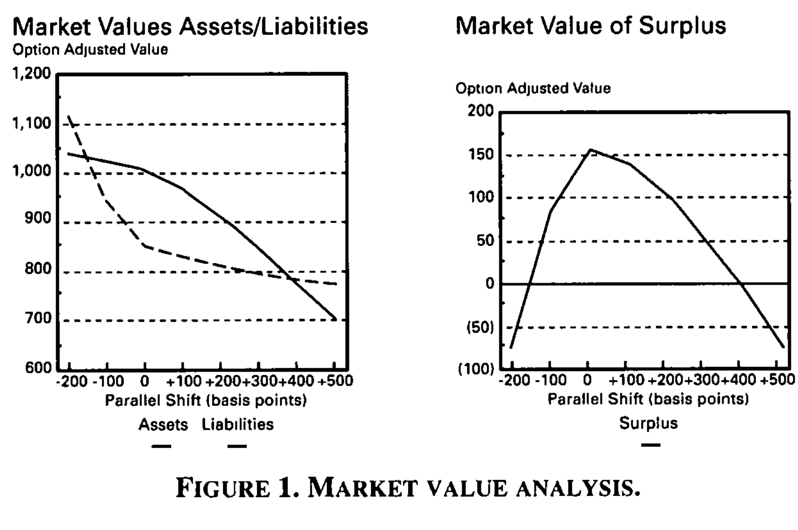

FIGURE 1. MARKET VALUE ANALYSIS.

Simply put, the degree of interest rate risk is smaller the larger the value of MVS for a wider range of parallel shifts in the term structure. This quantification is really a relative measure because, as noted in the prior section, there is freedom in the choice of the definition of the spread to be used for discounting the liabilities. Different choices of spread result in different values of MVL and MVS. Once a basis is chosen, however, one can analyze the situation and take management action. The term economic surplus is sometimes used to denote this market value of surplus, and is used to distinguish it from accounting surplus.

Immunization theory for fixed income portfolios provides that under suitable conditions and restrictions it is possible to immunize the value of the portfolio and its rate of return from changes in interest rates that occur over an investment horizon (Bierwag *et al.*, 1953, Bierwag, 1987). More precisely, if a portfolio of securities is chosen such that its duration equals the investor's time horizon, then the portfolio is immunized so that the annual realized rate of return can never fall below the initial yield to maturity at which the securities were purchased. Some of the restrictions are that the yield curve is flat, the change in rate is a parallel shift occurring instantaneously with no further changes over the horizon, no external cash in flows or out flows, investment cash flows can be reinvested at the same rate as that earned by the portfolio and the original securities have no embedded options and have positive convexity. Bierwag (1987) defines the notion of a duration window for which the above results hold. The fact that the rate of return on the portfolio is the yield at the initial point in time can be seen from Babcock's formula (1984). Market and reinvestment risk are balanced around that duration window.

This work can be extended to insurance enterprises by treating liabilities as fixed income obligations. As a special case assume that at time t, $S_t = A_t - L_t \geq 0$, $D_A = D_L(L/A)$ and $C_A > C_L$. Let S'_t denote the value of surplus immediately after an instantaneous parallel shift to the term structure. Use similar notation for A'_t and L'_t. Then $S'_t > S_t$. Using equation (5)

$$\begin{aligned} S'_t = A'_t - L'_t &= A_t[1 - D_A\Delta i + \tfrac{1}{2}C_A(\Delta i)^2] \\ &\quad - L_t[1 - D_L\Delta i + \tfrac{1}{2}C_L(\Delta i)^2] \\ &= A_t - L_t + [L_tD_L - A_tD_A]\Delta i \\ &\quad + \tfrac{1}{2}[A_tC_A - L_t C_L](\Delta i)^2 > A_t - L_t \\ &= S_t. \end{aligned}$$

In the second line of the equation above the first bracketed expression is zero because the condition on the durations of A and L implies that $AD_A = LD_L$. In the second bracketed expression the fact that $S_t = A_t - L_t \geq 0$ and $C_A > C_L$ justify the inequality.

Note that $SD_S = A_tD_A - L_tD_L$. This case says that if $D_S = 0$ and $C_A > C_L$, then the dollar amount of surplus is instantaneously immunized against small parallel shifts in the term structure.

For a second special case assume that at time t, $S_t > 0$, $D_A = D_L$ and $C_A > C_L$. Then the ratio of surplus to assets is immunized at time t, i.e. $r'_t = S'_t/A'_t > r_t = S_t/A_t$. Again using equation (5)

$$\begin{aligned} r'_t = S'_t/A'_t &= [A'_t - L'_t]/A'_t \\ &= 1 - L'_t/A'_t \\ &= 1 - L_t/A_t \frac{1 - D_L\Delta i + \tfrac{1}{2}C_L(\Delta i)^2}{1 - D_A\Delta i + \tfrac{1}{2}C_A(\Delta i)^2} > 1 - L_t/A_t \\ &= r_t. \end{aligned}$$

By the conditions given, the expression in brackets above is less than 1.

Reitano (1991a,b) generalizes the results of Bierwag *et al.* (1983) and Bierwag (1987) to non flat term structures with non parallel shifts in the term structure. Instead of Bierwags duration window, Reitano defines an immunization boundary. Reitano proves that the immunization boundary gives rise to a minimum annualized return $i(k)$ on investment over every investment period $[0, k]$ for which a yield vector exists so that $P(i)$ is immunized at time k. This can be applied to S of an insurance enterprise. Reitano also derives the two results given above in this more general setting. The minimum annualized return is, of course, more complex in this general setting.

A hybrid between cash matching and full immunization is that of cash matching for liability cash flows out to a certain number of months and immunization for the remaining liability cash flows. This situation can be relaxed even further to allow for a degree of active management through contingent immunization (Bierwag, 1987). In this case the investor stipulates the degree of risk tolerance at the start of the investment. If errors develop over the period of the plan, the investor can move the current duration closer to the remaining time horizon of the plan. If r is the initially promised rate of return and x the maximal tolerable loss, then $r_f = r - x$ is the floor to the rate of return, i.e. the minimally acceptable rate of return. The maximum tolerable loss, x, is the safety margin. Over time the closer the projected return is to r_f, the greater the danger that the realized rate may fall below r_f. If the projected rate falls to r_f, then strict immunization over the remaining planning horizon is triggered. Essentially, the investor is willing to trade off the safety margin x for the potential of obtaining excess returns.

A different application of the market value of liabilities concept was made by Griffin (1990). His paper uses the cost of funds method coupled with the techniques from option pricing theory to analyze a liability and set targets for asset performance. This approach solves for the required spread on assets (RSA). It is computed as follows. First, create a set of arbitrage-free interest rate paths based on the current Treasury term structure. Second, using the features built into the proposed liability (e.g. crediting rate strategy, book value surrenders, surrender charge design, interest rate guarantees, bail outs, or return of premium features) calculate the present value of the liability cash flows along each path at a tentative value of the RSA. Third, compute the weighted average present values (weighted according to the path probabilities). Fourth, if that average equals the initial net cash flow, then the estimated RSA is the RSA. If not, iterate the process to find the RSA which produces an average equal to the net initial cash flow. This part of the RSA represents the amount the insurance enterprise must pay to acquire the business. Thus the RSA includes the cost of funds for the insurance enterprise. (The cost of funds is the amount COF defined earlier.) To this RSA must be added amounts for the credit risk, investment expenses and the profit target. In this case the profit target is expressed in basis points. The final value of RSA is the spread that the asset portfolio must earn for the insurance enterprise to achieve its desired profit level.

Over time, the relative performance of the asset and liability portfolios will not track with what was anticipated at issue. The RSA can then be recalculated by using the then current market value of assets as the amount to which the future liability cash flows must be discounted. The new RSA, adjusted for the effect of any expenses, can then be subtracted from the spread-to-Treasuries actually being earned on the assets to determine the excess spread available to the insurance enterprise.

A slightly different market value approach is taken by Ho *et al.* (1992). In this case the quantity COF (the cost of funds spread defined earlier) is determined for a new block of business. It is held fixed for all future time periods. On this basis, the market value of liabilities is computed at each future point in time according to equation (12). Based on the liability cash flows and the market values of liabilities it is possible to calculate the total return on liabilities. This method presupposes equality between the asset value and the liability value, i.e. a zero surplus on a market value basis. The net return to the insurance enterprise equals the total return on assets supporting the block less the total return required by the liabilities.

If at the end of a period the MVA exceeds the MVL, then net return earned by the insurance enterprise reflects that excess. Although the authors do not state this explicitly, that excess would then be considered transferred to surplus, i.e. funds not associated with the product. Other things being equal, if the change in MVL exceeded that for MVA, then that would contribute to a negative total return and would require an infusion from surplus so that MVA = MVL going into the next period. By holding the COF constant for the block of business, the change in value of embedded options in the liabilities is reflected by increases or decreases in the MVL.

The methods describing immunization can be overlayed on these approaches. It should be noted that if one is immunizing the dollar amount of surplus and its return over a long holding period, then the duration of surplus will have to equal that length of time. Thus the market value of surplus will be more volatile over the time period as its duration is large. If one shortens the duration of surplus, then one not only reduces the volatility but also reduces the rate of return locked in by the immunization. One is trading risk for return.

A method of interest that does not directly depend on market values is that of Miller *et al.* (1989). First, a set of interest rate paths is determined. Second, the liability cash flows are projected along each path. The return that must be earned on the assets in order to satisfy the liability cash flows is called the required return (RQ). The realized return (RR) on the assets depends on the value of the assets at the beginning and the end of the period, the cash flows received during the period and the reinvestment income earned on the cash flows received during the period. These amounts will depend on the interest rate environments at each point in time along each interest rate path. The asset cash flows will reflect any embedded options contained in them. Mathematical programming techniques can be used to identify assets which, ideally, will provide a RR in excess of the RQ for each time period. Some compromises, however, may have to be made. For all portfolios that satisfy the required conditions the one with the highest expected realized return would be selected.

Cautions and limitations in market value analysis

There are cautions that one should be aware of when using these forms of market value analysis. For some asset classes, e.g. private placements, residential and/or farm mortgages, commercial mortgages, real estate, defaulted securities and over-the-counter derivatives, there may be considerable uncertainty about the market values. Insurance enterprises tend to hold significant quantities of these assets. Market values are subject to volatility from uncertainty in borrower behavior, policyholder behavior and insurance enterprise (asset and liability management) behavior. The calculated market values may be subject to the nature of the option pricing model used.

Duration and convexity measures are local measures. Their use is valid over small changes in interest rates. Some assets with embedded options are path dependent and the way the change occurs may not be captured in the duration and convexity measures used. It is necessary to maintain the duration and convexity conditions at all times. Even if interest rates do not change the duration and convexity of assets and liabilities will change as time passes. Thus rebalancing will have to be performed frequently, which increases costs. Rebalancing will also be required as there are always new external cash flows entering the picture due to new business and the fact that cash flows from existing assets may not be reinvested at the same rate of return as the asset portfolio itself.

Market values, duration and convexity of liabilities require the ability to accurately predict policyholder behavior. Such prediction is reasonably valid for fixed liabilities, e.g. guaranteed interest contracts (GICs), immediate annuities and terminal funded annuities, but is not as valid for deferred annuities and life insurance. It is important to sensitivity test the policyholder behavior functions used in modeling the liabilities. There is a tendency to micro-manage to the 'numbers' (e.g. market values, durations and convexity) for assets and liabilities. This may be risky unless the policyholder behavior function is known with a high degree of certainty or the results are generally insensitive to policyholder behavior misspecification. Management decisions based on such micro-management may not be robust. Market value analysis uses single-point option pricing, i.e. at the date of valuation only. The asset cash flows are those from existing assets. It does not reflect the reinvestment behavior of the insurance enterprise over time. As such, it presents a serious problem for interest sensitive liabilities using a portfolio crediting strategy.

As seen earlier in this section there is no unambiguous definition of the market value of a liability. There is, therefore, no unambiguous definition of the market value of surplus or economic surplus. Different choices of liability spread (LS) lead to different market values for the same liability. These different market values result in different option adjusted durations for the liability. Thus, the option adjusted duration of a liability is a function of LS. Using choices for LS as described earlier in this paper for a block of single premium deferred annuities results in option adjusted durations which vary by more than a factor of seven (7) from smallest to largest. This stems from the fact that there is no secondary market for insurance liabilities. And with the exception of viatical settlement companies (which buy the policies of terminally ill individuals) there is not likely to be due to underwriting, tax and public policy (insurable interest) issues. Some refer to other insurance companies as buyers of liabilities via assumption reinsurance. It is important to clarify that the assuming company is not just buying the liabilities, it is buying the liabilities and either the supporting assets or cash. The assuming company is entering into the transaction in order to receive the earnings from the net activity of the assets and the liabilities. It is not likely that the market value of surplus as described above would equal what a willing buyer would pay a willing seller for the liabilities and supporting assets.

Market value analysis also does not consider all relevant cash flows. First, if a realized capital gain must be taken to meet a liability cash flow, then tax on the capital gain will need to be paid. But there is no provision in the analysis for this tax. Second, the insurance enterprise is itself subject to income taxes. There is no provision for these cash flows. Third, there is no provision in the analyses for the payment of shareholder dividends. Market value analysis also does not recognize the cost of capital that the insurance enterprise incurs in maintaining statutory reserves, deficiency reserves, interest maintenance reserves, asset valuation reserves and risk based capital. Market value analysis treats the assets and liabilities separately. It does not look at the enterprise as an integrated whole. This may lead to suboptimization in managing interest rate risk. For example, an insurance enterprise may engage in a hedge transaction to mitigate interest rate risk. Because market value analysis does not reflect all relevant transactions and costs it may result in either a costly hedge (buying too much interest rate risk insurance) or an ineffective hedge with regard to the value of the firm.

In summary, this definition of market value of surplus or economic surplus does not equal the value of the firm. Thus the challenges of measuring interest rate risk, managing interest rate risk and pricing new business, blocks of business or entire companies with regard to the value of the firm remain open.

Conclusions

For this method of market valuation of liabilities there are major issues. First, the resulting market value of surplus does not relate to the fair value of the firm from buyer/seller perspective. Second, the definition of the market value of liabilities is inherently ambiguous. Of several not unreasonable choices for the spread to define MVL, none are immune from difficulties. Application of fixed income theory to insurance liabilities and assets results in a much higher level of sophistication in asset/liability management, but there are cautions and limitations in the theory, and the theory is still incomplete with regard to the value of the firm.

The Option Adjusted Value of Distributable Earnings

Introduction

The goal is to define the appropriate objective function, i.e. the proper measurement of the value of the firm and determine how to compute it. The work in this section represents a fusion of finance theory and option pricing theory applied to the insurance enterprise. It is an extension of the author's earlier work (Becker, 1991).

The price of a security

The price of a security reflects its inherent risk. From finance theory the price or value of a security, also known as its intrinsic or fair value, is the risk-adjusted present value of the security's free cash flows. Free cash flows are amounts of money that can be freely transferred to the owner of the security. The owner must be able to dispose of those amounts in any way he or she desires.

If the security is a bond, then in exchange for the purchase price the owner receives the coupon income, any call premium (if the bond is called) and the maturity value. The owner is free to dispose of the income, call premium or maturity value in any way. Any default, liquidity, interest rate risk due to the level and/or volatility of interest rates or other risk is reflected in the price of the bond. Option pricing theory is required for the valuation of bonds with contingent cash flows due to the embedded options. Option pricing theory also provides for consistent valuation of securities on a relative basis.

For a firm, the goal is to maximize the wealth of its shareholders. Copeland and Weston (1988) state this to be the same as maximizing the present value of shareholders' lifetime consumption and no different than maximizing the price per share of stock. Shareholder wealth, or the price of the stock, is the discounted value of after-tax cash flows paid out by the firm. The after-tax cash flows available for consumption are shown to be the same as the stream of dividends paid to shareholders. Shareholder dividends are the free cash flows of common stock. The discount rate is the market-determined rate of return on equity capital (common stock) or opportunity cost of capital for equivalent income streams.

A question arises about capital gains. Shareholders receive both capital gains and dividends from ownership of stock. Why does the above statement refer only to dividends? Copeland and Weston (1988) show that this formulation does include capital gains in that if the firm reinvests funds at the cost of capital that it could have paid as shareholder dividends, then the resulting value of the stock is the same value as if all funds had been distributed as dividends. If the firm in which the security represents ownership is an insurance enterprise, then it is necessary to determine the free cash flows, i.e. shareholder dividends, that can be paid by the insurance enterprise. Unlike other industries, state law regulates the amounts of shareholder dividends that can be paid. This law ties shareholder dividends to statutory accounting. This fact often causes people to turn away from the use of statutory net income as a basis for economic value on the basis that the conservative nature of statutory accounting does not reflect the true economic value or economic reality of the firm. But the fact is that statutory accounting does affect the true economics of the insurance enterprise. Cash flows within the insurance enterprise that are not capable of being paid to shareholders are not free cash flows. So they can not be used either as shareholder dividends or to fund new business projects.

Statutory accounting imposes another restraint. The insurance enterprise is required to hold an asset valuation reserve (AVR). Although not funded from statutory net income, increases in the AVR are charged against surplus and can not be used to pay shareholder dividends or fund new business. If the increase in the AVR reduces surplus below the company's desired level, the ability to pay the statutory net income as a shareholder dividend is diminished. In addition to the limits of statutory accounting, the amount of capital that needs to be held either due to internal required surplus formulae or to satisfy external capital requirements, e.g. rating agencies or a desired level of NAIC risk based capital ratio, will impose a limitation on the ability of the firm to pay shareholder dividends. The insurance enterprise needs to hold such capital and provide shareholders an adequate return on it. In light of the required surplus component, the amount of AVR, described above, that must be managed equals the excess, if positive, of the AVR over the asset default component of required surplus.

The free cash flow or shareholder dividend that the insurance enterprise can pay in a given time period is referred to as distributable earnings and is given by the following formula. For time period t, let DE be the distributable earnings, SNI be statutory net income, AVR be the excess, if positive, of the asset valuation reserve over the asset default component of the required surplus, and RS be the level of required surplus (however determined) that the company wishes to hold. Assume that the level of surplus is at the desired level the insurance enterprise desires to hold.

$$DE_t = SNI_t - \Delta_t AVR - \Delta_t RS. \qquad (14)$$

If the surplus is at a high enough level, then the term involving the AVR can be ignored.

For background purposes three appendices are included. Appendix A provides background on deterministic insurance pricing, especially with regard to the role of required surplus and the impacts of required surplus and taxes. Appendix B provides some additional commentary about the perceived difficulties in using statutory accounting as a basis for value. It is presented in a question and answer format. Appendix C provides deeper insight into the differences between market value analysis and OAVDE analysis and related issues.

Figure 2, following, shows sample free cash flow patterns for several types of securities. The presence of a call option in a bond affects its price relative to a noncallable bond as interest rates change. This is demonstrated graphically in Figure 3.

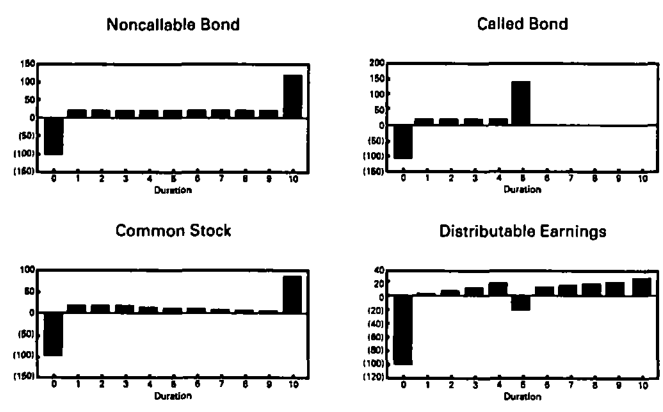

FIGURE 2. FREE CASH FLOW DIAGRAM.

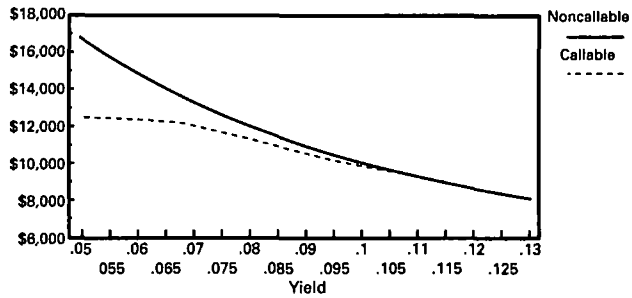

FIGURE 3. BOND PRICES.

Since the assets and liabilities of the insurance enterprise have embedded options, the distributable earnings represent a series of contingent cash flows to the shareholders. As a result, option pricing techniques must be used in order to compute their present value. In this case a set of arbitrage-free paths repricing the initial term structure is obtained and the actual operations of the insurance enterprise are projected out along each path into the future. The asset, liability and distributable earnings cash flows will reflect the actual management of the business from the date of valuation into the future. They involve the nature and behavior of the assets (affected by borrower behavior), the nature and behavior of the liabilities (affected by policyholder behavior and company management behavior, e.g. credited interest rate), results of reinvestment and disinvestment (to cover negative cash flows) and other demands of statutory accounting and required surplus.

Unlike market value analysis, which is single point option pricing, this situation requires multipoint option pricing. This means that at each time interval on each interest rate path one has to be able to purchase or sell securities. These decisions must be made in an arbitrage free environment; thus at each combination of path and time interval one must perform an option pricing exercise from the term structure extant at that point in time. If certain restrictions are placed on disinvestment and range of securities that can be purchased, it is possible to avoid the need for multi-point option pricing, e.g. if future asset purchases are limited to assets without embedded options and disinvestment is accommodated by borrowing.

The weighted average of the present values of distributable earnings discounted along each path is called the option adjusted value of distributable earnings (OAVDE). It is the intrinsic or fair value of the insurance enterprise. Thus, OAVDE is the mark-to-market value of the firm.

If a price is specified, it is possible to solve for the spread that the investor earns at that price. Alternatively, if a spread is specified, the price the investor must pay to earn that spread can be solved for. The following notation will be used.

Let i_0 be the initial term structure.

Let t be an index for time ($t = 0, 1, \ldots, N$).

Let p be an index for the path ($p = 1, \ldots, P$).

Let j be a general index.

Let $r_{p,t}$ be the risk-free future one period rate corresponding to path p at time t.

Let prb_p be the probability of path p.

Let $\text{tr}_{p,t}$ be the federal income tax rate at time t on path p.

Let $\text{DE}_{p,t}$ be the distributable earnings corresponding to path p at time t.

Let OAS be the option adjusted spread for distributable earnings.

The liabilities are supported by assets whose book value equals the sum of statutory reserves, deficiency reserves (if any), interest maintenance reserve required surplus and the excess, if positive, of the AVR over the asset defaul component of required surplus.

$$\text{OAVDE}(i_0, \text{OAS}) =$$
$$\sum_{p=1}^{P} \text{prb}_p * \left\{ \sum_{t=1}^{N} \left[\text{DE}_{p,t} \Big/ \prod_{j=0}^{t-1} (1 + r_{p,j} + \text{OAS}) \right] \right\}. \quad (15)$$

If it is desired to find the OAS for a proposed block of new business, set $\text{OAVDE}(i_0, \text{OAS}) = -\text{DE}_0$, i.e. the initial or 'time 0' distributable earnings for the block. The negative sign is needed as the initial distributable earnings is almost always negative.

It is also possible to solve for an option adjusted yield (OAY) instead of an option adjusted spread. It is sometimes useful to express results in this manner or to compute an OAVDE based on a level discount rate. For example, one choice for OAY is the insurance enterprise's cost of capital. Another choice might be a target rate of return in excess of the cost of capital that the firm wishes to earn.

$$\text{OAVDE}(i_0, \text{OAY}) = \sum_{p=1}^{P} \text{prb}_p * \left\{ \sum_{t=1}^{N} [\text{DE}_{p,t} / (1 + \text{OAY})^t] \right\}. \quad (16)$$

To solve for OAY for a new block set $\text{OAVDE}(i_0, \text{OAY}) = -\text{DE}_0$, as above.

Results can be analyzed on a path-by-path basis. For each path p, it is possible to determine a spread-to-Treasuries, s_p, and a yield, y_p. Statistics about the sets $\{s_p\}$ and $\{y_p\}$ can be determined. These would be the minimum, maximum, the mean, standard deviation, skewness (measuring degree of symmetry) and kurtosis (measuring peakedness and length of the tails of the distribution). Similarly if the spread or yield is fixed, it is possible to examine the distribution of $\{\text{PVDE}(i_0, s_p, p)\}$ or $\{\text{PVDE}(i_0, y_p, p)\}$ where PVDE means the present value of distributable earnings for the term structure i_0 and path p and the discounting is done with a spread-to-Treasury s_p or a constant yield y_p, respectively. OAS and OAY may not equal the means of $\{s_p\}$ and $\{y_p\}$, respectively.

The pattern of distributable earnings shown in Figure 2 includes a second negative quantity. The first negative quantity represents the initial negative distributable earnings amount that is assumed covered by shareholder investment. Unlike other securities where the cash flows are contractual, distributable earnings represent a transfer from the insurance enterprise to the shareholder. If a second negative distributable earnings occurs, there is no contractual ability for the firm to demand it from the shareholders. The firm must cover it from the surplus of the insurance enterprise. But this represents funds which can not be paid to shareholders, so it is a second investment. Such events can happen in a volatile interest environment and they can happen (and do) when interest rates are unchanging. The generalized net present value and generalized internal rate of return algorithms developed by Becker (1988) resolve the problem of evaluating a stand-alone project that requires multiple shareholder investments. This is described more fully in Appendix A on deterministic pricing.

The formulae for computing OAVDE with the generalized net present value algorithm are given below. Additional notation is defined first. PVB stands for present value balance.

Let $\text{PVB}_{p,N}(i_0,i) = \text{DE}_{p,N}$ and $\text{PVB}_{p,t}(i_0,i) = \text{PVB}_{p,t+1}(i_0,i)/(1+r) + \text{DE}_{p,t}$ for $t = N-1, N-2, \ldots, 1$ or 0, if there is an initial distributable earnings amount; where $r = i$ if $\text{PVB}_{p,t+1}(i_0,i) \geq 0$, and $r = (1 - \text{tr}_{p,t})*r_{p,t}$ if $\text{PVB}_{p,t+1}(i_0,i) < 0$.

Here i is chosen to be $r_{p,t}$ + OAS if the discounting is to be done as a spread-to-Treasuries or OAY if a level discount rate is desired. This formula may be summarized as:

$$\text{OAVDE}(i_0,i) = \sum_{p=1}^{P} \text{prb}_p * \text{PVB}_{p,1}(i). \quad (17)$$

In formula (17), if the prior PVB term is negative, then the discount rate used is $r = (1 - \text{tr}_{p,t}) * r_{p,t}$, which is consistent with option pricing. This only arises if multiple shareholder investments occur that cause a present value balance to become negative. It is assumed that the insurance enterprise either has other profitable projects or sufficient retained earnings (which could be paid to shareholders) so that the firm can cover a negative PVB. The insurance enterprise will have to earn the amount needed on a pretax basis to cover the negative PVB and the amount of tax due. This is most easily done by tax effecting the risk-free rate. This is equivalent to discounting the negative PVB grossed up to account for the tax due at the risk-free rate.

The use of arbitrage free paths is important for several reasons. First, as already noted, the paths must provide for the consistent relative valuation of security prices at the valuation date and, in general, the same will be required at each point in time along each path. Second, it provides for valuing OAVDE consistently relative to fixed income securities. Third, it provides for consistent relative valuation of OAVDE over time to prior values of OAVDE. Fourth, if non arbitrage free paths are used, it is possible to misidentify an investment or disinvestment strategy as being favorable when it may not really be so because it has actually identified an arbitrage.

The results of the computations may be interpreted as follows. If an investor pays OAVDE for the insurance enterprise, the OAS represents the option adjusted spread-over-Treasuries that he or she expects to earn. OAY is interpreted similarly. Alternatively, if an investor desires a return of OAS over Treasuries or a level OAY, the OAVDE is the fair value or intrinsic value that he or she should pay. This also applies to issuing a new block of business. The OAVDE computation must be adjusted for the associated federal income tax implications if the assumption of a block is being considered.

Evaluation of alternative strategies

A liability, crediting, investment or disinvestment strategy is defined to be a management plan of action with regard to liability design and management, crediting rates, investment (including any hedges) or a disinvestment, respectively. A block of business requires the insurance enterprise to choose a management plan for each of the four items. Define a strategy as a management plan of action that consists of one each of strategies for liability design/management, crediting, investment and disinvestment.

For $s = 1, \ldots, S$, let s represent a distinct strategy. Taken together they represent a universe of possible management plans of action for the block of business. The following describes a method to evaluate these alternative strategies. Let $DE_{p,t,s}$ be the distributable earnings resulting at time t, on path p following strategy s.

Define the resulting OAVDE value obtained for strategy s by:

$$\text{OAVDE}(i_0, \text{OAS}, s) =$$

$$\sum_{p=1}^{P} \text{prb}_p * \left\{ \sum_{t=1}^{N} \left[\text{DE}_{p,t,s} \Big/ \prod_{j=0}^{t-1} (1 + r_{p,j} + \text{OAS}) \right] \right\}. \quad (18)$$

For the generalized net present value algorithm the formula is:

$$\text{OAVDE}(i_0, i, s) = \sum_{p=1}^{P} \text{prb}_p * \text{PVB}_{p,1,s}(i). \quad (19)$$

Recall that i means either discounting at $r_{p,t}$ + OAS or at OAY. For either equation (18) or (19) the OAS or OAY, respectively will depend on s.

To evaluate alternative strategies one can compare the following results for each strategy.

- the OAS, the spreads by path and their distribution, if given the initial distributable earnings amount for new business or price for an existing block;
- the OAY, the yields by path and their distribution, if given the initial distributable earnings amount for new business or price for an existing block;
- the OAVDE, present values of distributive earnings and their distributions, if given a specific spread to the risk-free rates, the cost of capital or other hurdle rate;
- a mean variance diagram of any of the quantities, e.g. mean spread versus the standard deviation of spreads, mean yield versus standard deviation of yield, mean present value of distributable earnings versus the standard deviation of distributable earnings;
- the risk adjusted value for OAVDE computed using an exponential utility function;
- cumulative distribution functions which display the distribution of any of the profit parameters, e.g. spread, yield, present value of distributable earnings.

Examples of these are provided in the next section.

The insurance enterprise as a giant CMO

The difficulty of understanding the use of statutory accounting in computing the fair value of an insurance enterprise may be eased if one thinks of an insurance enterprise as a giant CMO. This analogy can be described in the following manner. A CMO is composed of underlying collateral with associated cash flows and tranche rules which determine how owners of tranches are paid. For an insurance enterprise, the underlying collateral consists of net effects of the asset cash flows and the liability cash flows. The tranche rules consist of statutory accounting constraints, tax accounting constraints and the insurance enterprise's desired level of risk based capital. There is only one tranche and that tranche's cash flows are the distributable earnings. The analogy is not as complex as the actual situation as the tranche rules impact additional cash flows, e.g. federal income tax and shareholder dividends, for which provision must be made. Figure 4, assists in understanding the influence of the actions taken by various parties within the operation of an insurance enterprise. It relates the results of the actions to distributable earnings.

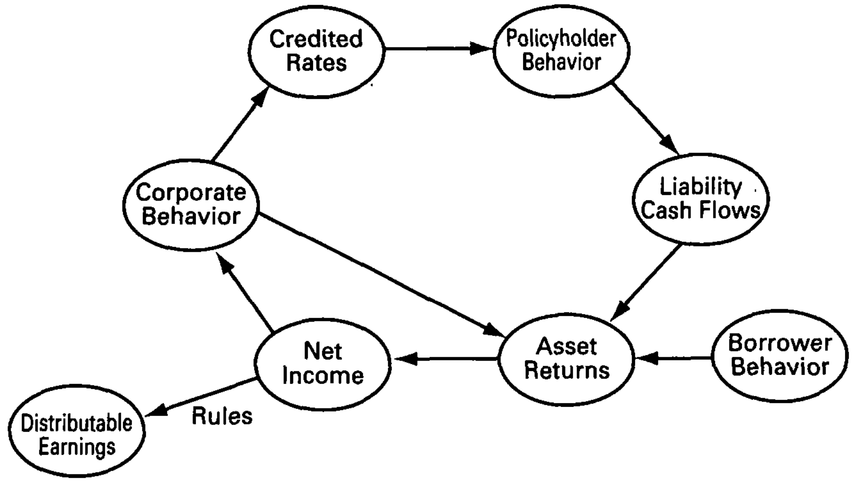

FIGURE 4. INSURANCE COMPANY FLOW DIAGRAM.

Illustrations of OAVDE

Introduction

This section will present five examples analyzed by the OAVDE methodology. In these examples the presentation uses the techniques developed in the previous section.

Evaluation of investment strategies

For simplicity, this example assumes that three of the four components of the strategy have been chosen. Those three are: the liability design, the crediting rate strategy and the disinvestment strategy. The only component allowed to vary is the investment strategy. The goal will be to choose the best investment strategy from a set of five choices. These choices are meant to be illustrative only.

The liability is a single premium deferred annuity. It provides for surrender at book value less a surrender charge that grades off over the first seven years. It has a minimum guaranteed crediting rate of 4%. The interest crediting strategy is the portfolio rate less a spread. The current rate is guaranteed for one year at a time and then reset. The disinvestment strategy is to liquidate assets. The interest margin or spread is set so that with a flat yield curve (deterministic pricing) the internal rate of return on distributable earnings is 15%. This spread represents that needed for the insurance enterprise to recover its investment in the block of annuities, maintenance expenses, commissions, benefits and cost of capital it must hold in doing so at a profit of 15%. (See Appendix A for more information about deterministic pricing.)

The investments will be limited to noncallable, default free bonds. There are no embedded options in the investments. Thus, any resulting financial impacts to the insurance enterprise can only result from the policyholders exercising the option to surrender at book value and any effects of the 4% minimum crediting rate guarantee. Five investment strategies will be examined. These strategies are followed consistently at any point where positive cash flow is to be invested. They are: invest long (IL) in 25 year bonds; invest in assets having a duration at purchase of 4.6 (TDP for target duration of purchase); invest in a ladder (L3/15) of 3 to 15 year bonds in fixed proportion; invest as closely as possible to maintain a portfolio duration of 3 (PD3); and invest as closely as possible to maintain a portfolio duration of 5 (PD5). The March 31, 1994 yield curve was used. The initial distributable earnings is covered by the insurance enterprise. Figure 5 displays the spreads and the OAS that the block of annuities returns to the insurance enterprise. This graph indicates that for each investment strategy the distribution of spreads about the mean is negatively skewed (data farther away from the mean negatively than either positively or equally distributed) with positive kurtosis (data clustered about the mean more so than for a normal distribution, but with longer tails than a normal). That means that for each strategy there is more downside risk than upside potential, that financial results tend to cluster about the mean more often than they would if normally distributed, but large (negative) deviations occur more frequently than would be expected with a normal distribution. This is typical for interest rate risk. This also implies that pricing products with a flat yield curve (deterministic pricing) provides results that are optimistic. The graph shows that the ladder is the best performing strategy. It has the highest OAS and mean spread, the smallest range and the highest minimum over all paths. Investing long has almost the lowest OAS, but has the largest range and the lowest spread of any of the strategies. The OAS differs from the mean spread in that the OAS is the spread that solves the equation over all paths simultaneously. The mean spread is the probability weighted algebraic mean of the spreads of the individual paths.

Table 1 below displays some of the statistical measures that can be computed from the projections. The statistics are computed for the spread-to-Treasuries variable and are shown for the invest long and ladder investment strategies. Similar statistics can be computed for any of the other measures described in this paper. Note that the skewness and kurtosis values are standardized so that values outside the range of ±2 would indicate rejection of the hypothesis that the variable in question is normally distributed.

Figure 6 presents the distributions of yield. Again, the distributions are not normal. The strategy with the best characteristics is again that of the ladder. For the ladder the option adjusted yield is 12%. The yield on a deterministic pricing basis was 15%. Therefore, deterministic pricing overstated the yield by 3%. This is a relative error of 25%. Not all relationships remain constant when comparing spreads and yields. The comments regarding the relationship of the OAS to the mean spread apply analogously to the OAY and the mean yield.

Figures 7 and 8 present the distributions of present values of distributable earnings at the risk-free rates and cost of capital, respectively. For the examples in this section the cost of capital is assumed to be 12%. These results are consistent with the graphs for spreads and yields. The OAVDE at the cost of capital for the ladder is at zero. This is consistent with the OAY of the ladder being 12%.

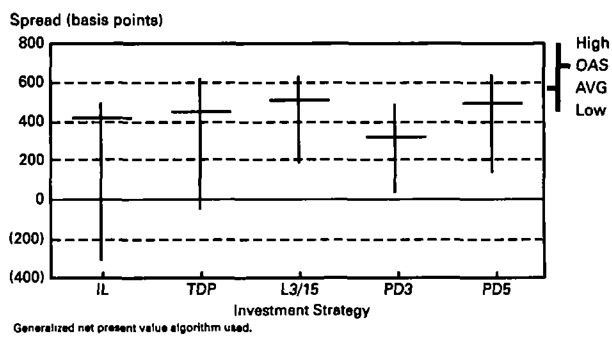

FIGURE 5. OPTION ADJUSTED SPREADS: DISTRIBUTION OF SPREADS.

TABLE 1
STATISTICS FOR SPREAD-TO-TREASURIES (BASIS POINTS) RANDOM VARIABLE

Strategy	OAS	Min	Mean	Max	Std. Dev.	Skewness	Kurtosis
Invest Long	407	−312	412	492	167	−2.92	11.37
Ladder	510	187	512	636	97	−1.37	4.72

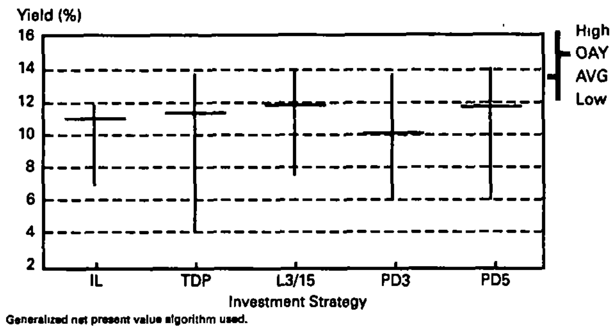

FIGURE 6. OPTION ADJUSTED YIELD: DISTRIBUTION OF YIELD.

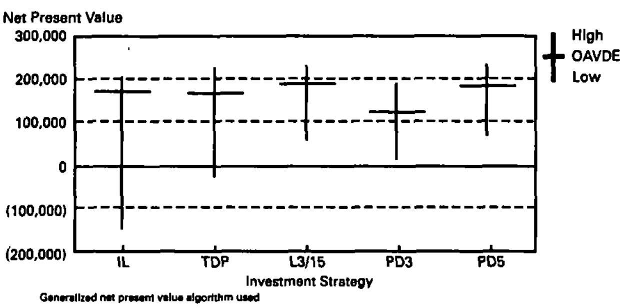

FIGURE 7. OPTION ADJUSTED VALUE OF DISTRIBUTABLE EARNINGS: PRESENT VALUE AT RFR.

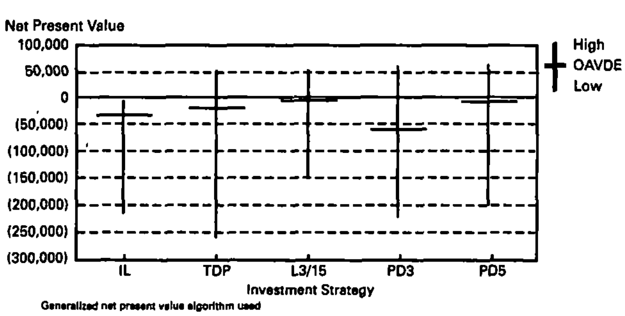

FIGURE 8. OPTION ADJUSTED VALUE OF DISTRIBUTABLE EARNINGS: PRESENT VALUE AT COST OF CAPITAL.

These results can also be analyzed by using a mean/variance diagram. In this case the vertical axis represents the mean of the spread and the horizontal axis the standard deviation of the spread. Figure 9 compares the results of the five strategies in the mean/variance diagram. Again the ladder is the best choice. The invest long strategy has the worst trade off between risk and return. There is some risk in using this diagram. First, the user must make an intuitive trade off between risk and return. Second, the use of variance (standard deviation) as an adequate proxy for downside risk is only valid if the return distribution is normal or if the investor's utility function is quadratic (see Markowitz, 1959).

Quadratic utility functions are difficult to defend and the distribution of results differs substantially from a normal distribution. Third, the method only uses the first two moments of the distribution Caution in using this type of diagram for these problems is warranted.

Another method of comparing many strategies is the concept of risk adjusted value (RAV) described by Cozzolino (1979). The method uses an exponential utility function and requires an estimate of the insurance enterprise's or product manager's aversion to risk. But once done, this method linearly rank orders all of the strategies. This method uses all moments of the distribution; thus no important information is omitted and there is no need to make intuitive trade offs. Figure 10 shows the results for the five investment strategies at two discount rates, the risk-free rate and the cost of capital. No matter which discount basis is chosen, the best strategy is the ladder.

A final method of comparison is to graph the cumulative distribution function of a given statistic for each strategy. One can analyze the percentage of paths for which outcomes are greater than a prespecified comfort level. This method may be preferred if there is difficulty in reaching a consensus about the insurance enterprise's risk aversion factor and/or the insurance enterprise is interested in controlling the behavior near the tails of the distribution. Additional insight can be gained by overlaying the graphs on one another. As an example, Figure 11 shows the cumulative distribution function (CDF) for the yield by path for both the invest long and ladder strategies. The CDF for the ladder strategy everywhere outperforms the CDF for the invest long.

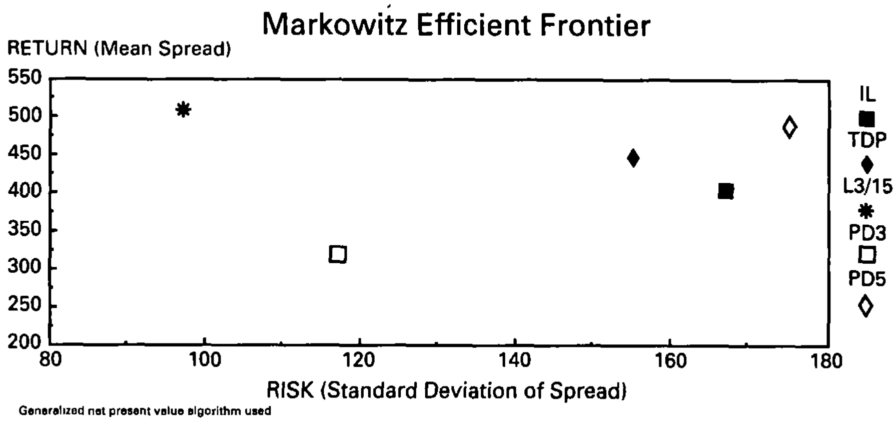

FIGURE 9. RISK/RETURN DIAGRAM: VARIANCE CRITERIA.

Do the results attributable to the ladder strategy depend on the slope and steepness of the initial term structure, which was steeply increasing? Additional tests were done with shallow positively sloped, shallow negatively sloped and steeply negatively sloped term structures. The results are shown in Figure 12. Instead of the raw data, the table has been scaled so that 100 represents a base line for the ladder strategy and other results are multiples of it. The slope and steepness of the term structure is denoted by '+' and '–' signs and the number of repetitions of the sign. The robustness of the ladder strategy is clear. The figure also includes data on the market value of liabilities. These market values were computed on the same basis as the OAVDE values. The spread used for discounting the distributable earnings and the liability cash flows was zero, i.e. discounting was performed at the risk-free rates. These results persisted even for parallel shifts in the term structure.

A test was performed with a slightly different ladder strategy. The new ladder (L2/10) involved bonds with maturities from 2-10 years, instead of the 3-15 years. The results for spreads and yield are shown in Figures 13 and 14, respectively. Differences are not large.

These results are a function of all the assumptions for the liability, the crediting strategy, the disinvestment strategy, the investment strategy and, especially, the policyholder behavior assumption. The universe of assets consisted of noncallable bonds. If the universe of investments had been larger to include callable bonds, sinking funds, mortgage pass-throughs, CMOs, mortgages, etc. then the results could have been entirely different. This example is for illustration only.

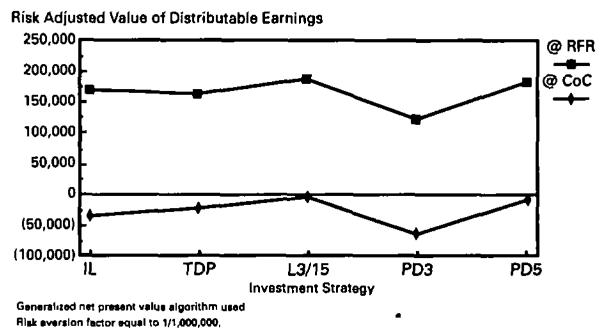

FIGURE 10. RISK ADJUSTED VALUE: RISK CERTAINTY EQUIVALENT.

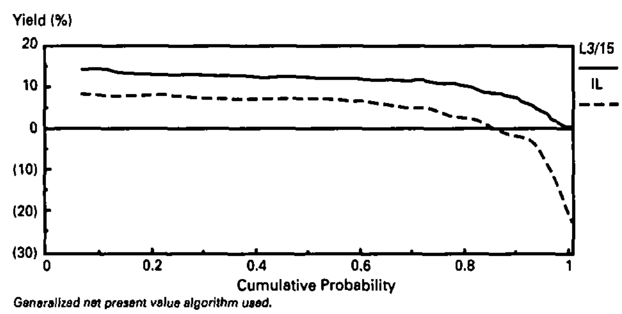

FIGURE 11. CUMULATIVE PROBABILITY DISTRIBUTION YIELD ON DISTRIBUTABLE EARNINGS.

OAVDE and MVL

@ Risk Free Rate

		IL	TDP	L3/15	PD3	PD5
OAVDE	+++	83	82	100	72	96
	+	103	89	113	81	104
	-	215	77	153	164	115
	—	216	39	139	131	99
MVL	+++	104	99	100	99	99
	+	103	99	98	98	99
	-	102	100	101	100	100
	—	103	102	102	103	102

Generalized net present value algorithm used

FIGURE 12. RELATIVE COMPARISON OF INVESTMENT STRATEGIES AND INITIAL YIELD CURVES: OAVDE AND MVL.

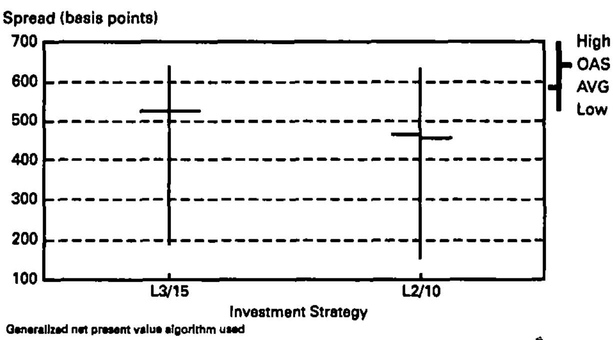

FIGURE 13. OPTION ADJUSTED SPREAD: DISTRIBUTION OF SPREADS.

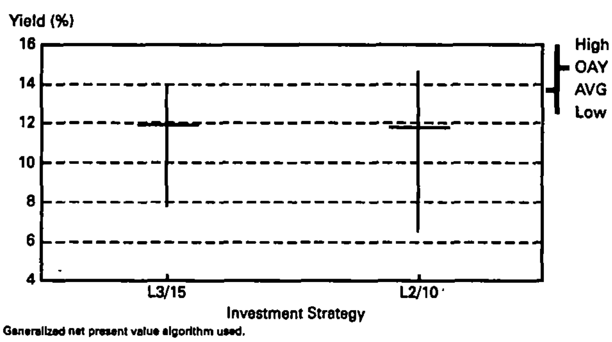

FIGURE 14. OPTION ADJUSTED YIELD: DISTRIBUTION OF YIELDS.

Evaluation of liability design

The second example focuses on liability design. This determines the costs to the insurance enterprise to provide additional liability options. In this example the crediting strategy is the portfolio rate less a spread, the investment strategy is the ladder of 2-10 year bonds (L2/10) and the disinvestment strategy is to liquidate assets. The only strategy allowed to vary is the liability design.

The first liability is the simple single premium deferred annuity with a 4% guaranteed minimum crediting rate. It is denoted in the following figures as 4. The second liability is the above annuity with a return of premium (ROP) feature which places a floor on the cash surrender value equal to the premium paid. This annuity has a slightly higher deterministic spread as a result of having to hold a higher reserve and risk based capital level than the simple annuity. The return of premium feature eliminates taking credit for the full surrender charge during the time the return feature is effective. A return of premium feature is a limited duration option as once the interest credited exceeds the surrender charge there is no value to the option. Note that the extra margin does not represent the full cost of the option; it only represents the cost of holding additional capital. The third liability is the simple annuity above but with a bail out feature (BO) that allows the policyholder to surrender without surrender charge if the credited rate ever drops more than 1% below the initial credited rate. It is also an option with life limited to the surrender charge period. Again the interest margin was adjusted to reflect the fact that if the annuity has a bail out feature, then the reserve must be the account value and the margin must be increased to return the deterministic profit back to 15%. The fourth liability is the simple annuity above but with a temporary interest rate guarantee of 6% for the first 5 years and 4% thereafter (6/4).

Figure 15 shows the spreads for the four liability designs. Figure 16 shows the yields for the four liability designs. As in the first example the distributions or spreads and yields are non normal with negative skewness and positive kurtosis. There is a small reduction in OAS for the ROP, but considerable reductions for the BO and 6/4. The same situation holds for OAY.

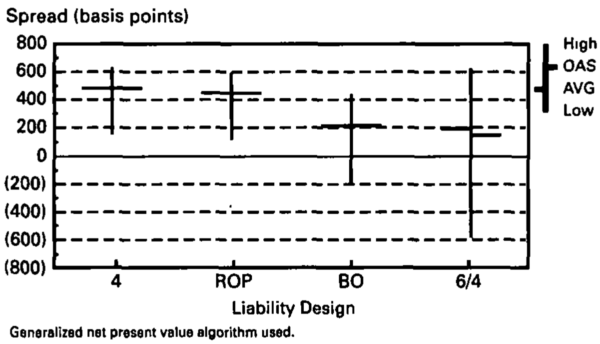

FIGURE 15. COMPARISON OF LIABILITY DESIGNS: SPREAD ANALYSIS.

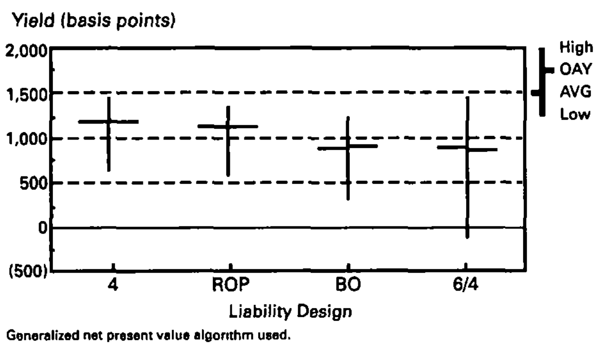

FIGURE 16. COMPARISON OF LIABILITY DESIGNS: YIELD ANALYSIS.

Table 2 shows the loss in yields and spreads that occurs if the extra liability features are added to the base case annuity. Figure 17 shows the risk adjusted values for the four annuity designs. The results of this graph are consistent with those of the graphs for spreads and yields.

TABLE 2.
LOSS OF OAY/OAS ON OAVDE DUE TO LIABILITY OPTIONS

	OAY	*OAS*
4	0	0
ROP	(63)	(42)
6/4	(323)	(338)
BO	(278)	(274)

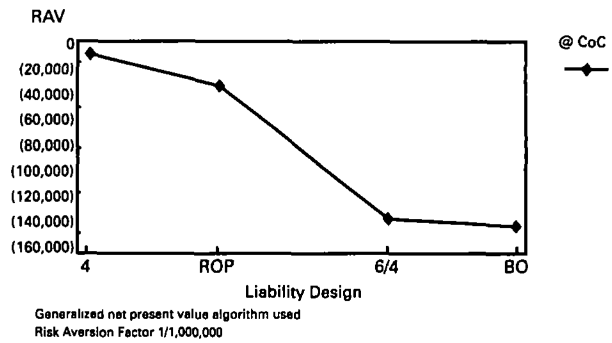

FIGURE 17. COMPARISON OF LIABILITY DESIGNS: RISK ADJUSTED VALUE.

Figure 18 shows the OAVDE results for the four designs if plus and minus 100 basis point shifts in the term structure occur. OAVDE is the fair value of the blocks of annuities. This graph demonstrates their relative value and how much that value can change due to instantaneous shifts in the term structure. The ROP annuity has almost the same characteristics as the base case. If interest rates rise, its value is just slightly higher than for the base case as the cost of the ROP option is reduced and the ROP has a higher margin.

For the BO and 6/4 annuities the OAVDEs, like that for the ROP, rise for the upward shift, but by a much larger amount. This is due to the fact that if rates shocked upwards, the value of the bail out option and the temporary interest rate guarantee would diminish considerably. The bail out, having an additional margin, also elevates the value of OAVDE. The 6/4 annuity does not have any additional margin, but it also carries no extra reserve or required surplus. For a downward shift, however, the values of OAVDE fall materially as each option becomes ever more valuable. Figure 19 shows the market value of liabilities for the four products. Both the distributable earnings for OAVDE and the liability cash flows are discounted at the risk-free rate, i.e. a zero spread-to-Treasuries. The market value results are consistent with what would be expected.

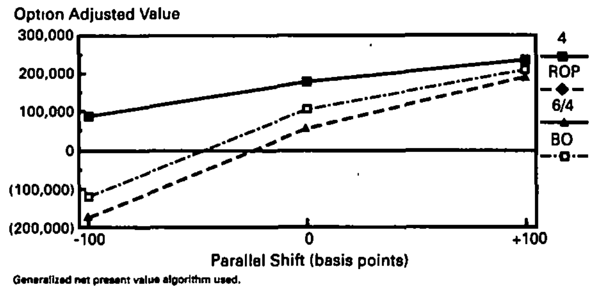

FIGURE 18. COMPARISON OF LIABILITY DESIGNS: OAVDE AT RISK FREE RATE.

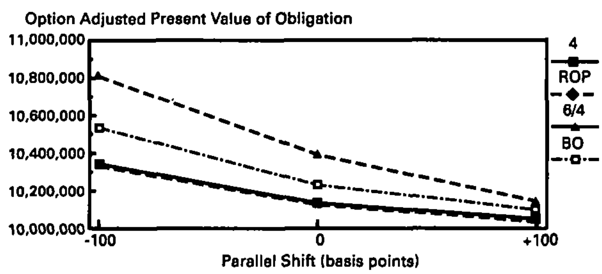

FIGURE 19. COMPARISON OF LIABILITY DESIGNS: MARKET VALUE OF LIABILITIES AT RISK FREE RATES.

Evaluation of hedge strategies

OAVDE analysis can be used to identify an optimal hedging strategy. If there are a number of hedge strategies that have acceptable market values of surplus, the optimal strategy can be found by applying OAVDE analysis to them. One can then choose the best one based upon the criteria described and demonstrated above.

This example consists of a large block of single premium deferred annuities. The annuities are straight forward in design with a 4% minimum interest rate guarantee and no other features. The interest crediting rate follows a portfolio strategy. Disinvestment consists of borrowing short. The assets supporting the block consist of noncallable bonds, sinking funds, mortgages, mortgage passthroughs and CMOs.

Management is considering four courses of action. The base case is to do nothing. Option 1 is to buy interest rate caps as 'interest rate insurance' for the insurance enterprise, i.e. the shareholders. The idea is that since the insurance enterprise sold the put option to the annuity policyholders, it will hedge that risk by buying interest rate caps. The caps are designed with strike levels that will only pay off for a run up in interest rates of 200 to 300 basis points. This is stop-loss interest rate risk insurance. The annuities' prior interest margin is increased by the amount needed to repay the cost of purchasing the caps at the insurance enterprise's cost of capital. If the caps pay off, the cap income goes to the benefit of the shareholders.

Option 2 is to buy a program of interest rate caps as 'interest rate insurance' for the policyholders. The same cap structure is used. The caps and the income from the caps are included in the asset portfolio supporting the annuities. The company takes its regular margin plus an increment to cover the cost of the caps from the portfolio.

Option 3 is to self insure, i.e. raise the margin but do not buy the caps. The extra profit accrues to the shareholders via higher distributable earnings. This is analogous to not buying automobile insurance and putting aside what it would have cost to protect against an accident.

Figure 20 shows the results on OAVDE for the initial term structure and instantaneous parallel shifts of ±100 and ±200 basis points. Here the OAVDE values are computed at the insurance enterprise's cost of capital. Note that a new option pricing projection is performed for each shift in the term structure.

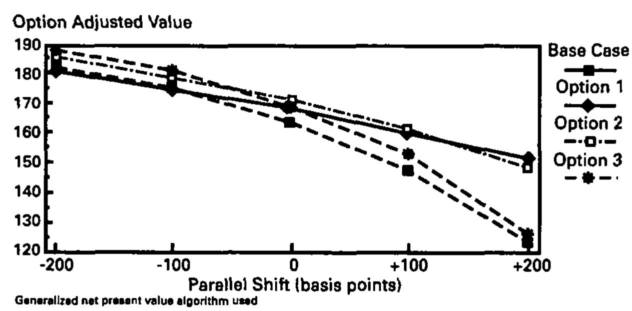

FIGURE 20. PORTFOLIO HEDGE: OAVDE AT COST OF CAPITAL.

As can be seen from Figure 20, the OAVDE of the block declines if interest rates rise and rises if interest rates fall. Option 3, self insure, has greater OAVDE values than the base case as it has a higher margin. Note that all the options have a higher value of OAVDE at the 0 basis point shift. This is due to the fact that the extra margin to cover the caps was deducted for the entire length of the projection and its present value exceeded the cost of purchasing the caps on the valuation date.

Option 2 presents an interesting situation. It would appear that it is better to 'share the wealth' of the caps as the profits in this case exceed those of Option 1 until +200 basis points. It might be thought that the extra income is derived from keeping policyholders with the insurance enterprise and that is the source of the extra OAVDE. In fact, that is not the case. What is happening is that for –200 bp, –100 bp, 0 bp and +100 bp the impact of the caps on the portfolio rate effectively increases the insurance enterprise's margin. To see this consider the 0 basis point shift. The caps are purchased and the margin increased to cover the caps. Now compare the asset portfolios under Option 1 and Option 2 that support the liabilities. The asset portfolio in Option 2 is larger as it contains the book value of the caps. On many of the paths emanating from the initial term structure, i.e. the 0 bp shift, the caps do not pay off as they have high strike levels. On these paths the investment income for Option 2 is less than for Option 1 as the investment income from the caps is negative due to the drop in book value over time. Thus the portfolio rate calculated for Option 2 is less than that for Option 1. In addition, the insurance enterprise takes its margin. So the effective margin for Option 2 is higher than for Option 1.

Once the interest rates have shifted upward enough the caps begin to payoff with sufficient frequency that there is a positive impact in the portfolio rate and the cap income does go to the policyholders. That is why the OAVDE for Option 2 is lower than for Option 1 at +200 basis point shift, but it is less effective for protecting the net worth (OAVDE) of the insurance enterprise. Thus Option 1 is the best hedge strategy against rising interest rates.

Figure 21 shows the results for the market value of surplus where the market value of liabilities is discounted at the risk-free rate. A deterministic appraisal of this block of business at the insurance enterprise's cost of capital produces a value of $185 million. The OAVDE at the cost of capital is $165 million. Thus, if a buyer paid $185 for the block of business, he or she would have overpaid by $20 million if they managed the business using the base case strategy. The impact of the embedded options in assets and the liabilities reduced the value of the deterministic appraisal by the $20 million.

Since OAVDE is the fair value of the insurance enterprise, it is possible to compute a duration and convexity for it. These are the numerical measures which capture the behavior of the 'price curve' of OAVDE from instantaneous parallel shifts in the term structure. Tables 3 and 4 show duration and convexity for both OAVDE and the market value of surplus at the 0 basis point shift and at the +100 basis point shift, respectively. Numerically the goal of the hedge strategy would be to reduce the duration and maximize the convexity of OAVDE. As can be seen, Option 1 achieves both of these in comparison with either the base case or the other options.

Figure 22 displays the distribution of present values of distributable earnings for each of the strategies using the initial term structure. Note the closeness of the ranges of the results for Option 1 and Option 2. Without consideration of the shifts in interest rates one could not discern all the impacts of each of the two strategies. Figure 23 displays the distribution of present values of distributable earnings for Option 1 under the initial term structure and parallel shifts of ±100 and ±200 basis points. The horizontal bar denoting the average value is the OAVDE value. The OAVDE gradually falls as the level of rates rise and the range of the distribution widens as the level of rates rise.

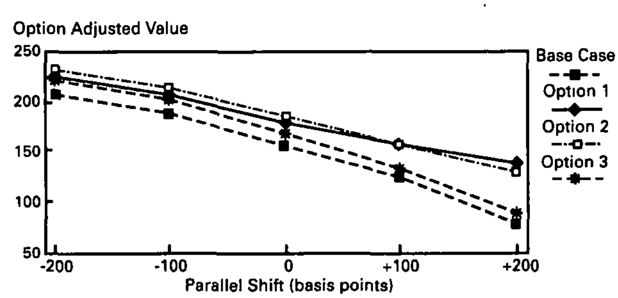

FIGURE 21. PORTFOLIO HEDGE: OAV OF SURPLUS.

TABLE 3.
PORTFOLIO HEDGE AT BASIS POINT SHIFT

	OAVDE (12%)		*OAV of surplus*	
	Duration	*Convexity*	*Duration*	*Convexity*
Base case	8.4	(280.6)	20.3	(113.1)
Option 1	4.6	(63.6)	13.3	191.3
Option 2	5.3	(129.0)	14.5	69.3
Option 3	8.4	(308.2)	19.8	(104.4)

TABLE 4.
PORTFOLIO HEDGE AT +100 BASIS POINT SHIFT

	OAVDE (CoC)		*OAV of surplus*	
	Duration	*Convexity*	*Duration*	*Convexity*
Base case	12.7	(645.6)	24.1	(819.2)
Option 1	4.9	(8.4)	11.9	107.2
Option 2	6.6	(142.6)	14.4	(56.9)
Option 3	12.9	(639.3)	23.8	(857.7)

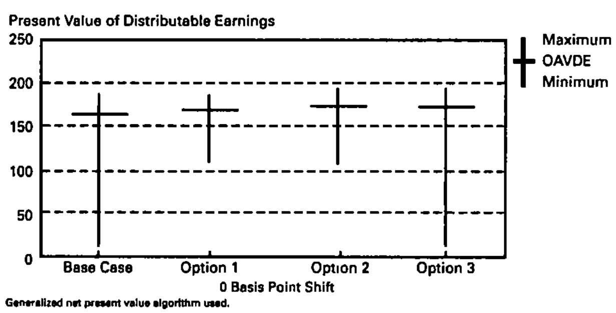

FIGURE 22. PORTFOLIO HEDGE: DISTRIBUTION OF PRESENT VALUE.

In this example the duration and convexity of OAVDE was computed. A situation that can arise with the market value of surplus approach is that atypical values for the duration and convexity for the market value of surplus occur for new blocks of universal life. Depending on the liability design these values can be either extremely positive or even negative. This is often explained away by noting that it is not unusual in the asset market for derivatives to have extremely positive durations or negative durations. This explanation is strained when applied in this case. It is not likely that people would view universal life insurance as being a 'derivative' liability. This situation is not limited to universal life but applies to participating and nonparticipating life, disability income and long term care insurance. It has to do with the liability design (level premium), the level of acquisition costs, and increasing magnitude of benefit payments over time. The application of the market value of surplus as defined does not work well for all liabilities. OAVDE works equally well for all products.

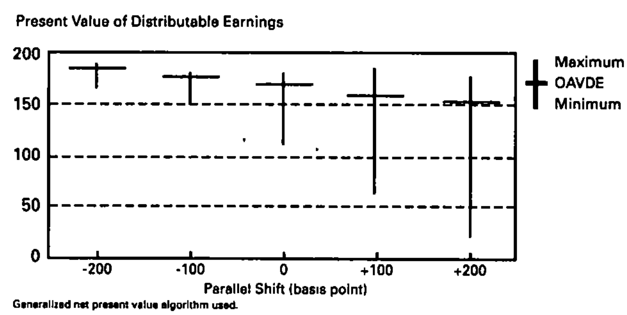

FIGURE 23. PORTFOLIO HEDGE: DISTRIBUTION OF PRESENT VALUES (OPTION 1).

Portfolio restructure

For this example the liabilities consist of a combination of GICs and terminal funded annuities. They are managed together as a single entity. The assets supporting the liabilities consist of noncallable bonds, sinking funds, mortgages and CMOs. Generally, the management strategy has been to cash match for some period and duration match the rest. It is noticed that the existing structure has downside risk to the OAVDE of the block if rates rise. Due to the nature of the liabilities one can not take any management action regarding them. The current structure of the asset portfolio, however, can be changed. Figures 24 and 25 demonstrate how OAVDE can be used to test asset portfolio restructures.

Figure 24 demonstrates how OAVDE was used to validate the results of portfolio restructures. There is a major improvement in the value of the block after the restructure for positive shifts in the term structure. There was, however, a decline for a negative shift. This was to be expected in the trade off. Figure 25 shows the cumulative distribution function for the present value of distributable earnings by path for both before the restructure and after. There is some sacrifice in value at the high end with relatively small probability in return for superior values over the greater range of the paths.

FIGURE 24. PORTFOLIO RESTRUCTURE: OAVDE AT COST OF CAPITAL.

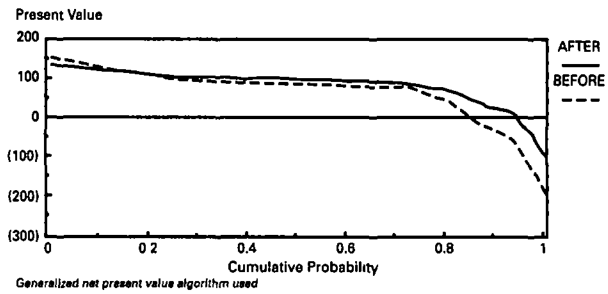

FIGURE 25. PORTFOLIO RESTRUCTURE: CUMULATIVE DISTRIBUTION OF PV OF DISTRIBUTABLE EARNINGS AT +200 BASIC POINT SHIFT.

Hedge trade-offs

This example focuses on the evaluation of trade-offs that can arise from different hedge strategies. The liability is a block of single premium deferred annuities with an initial surrender charge of 7% grading to zero by year eight. The crediting strategy is an asymmetric strategy that is independent of the portfolio earned rate. The strategy follows market interest rates down quickly, but lags market rates on the way up. The baseline investment strategy is to invest in five year noncallable A rated bonds. The disinvestment strategy is to borrow at 100 basis points (bp) over the 90 day Treasury bill. The liability spread for the annuities is 75 bp and the distributable earnings are valued at 300 bp over Treasuries.

The block of $100 million in deposits is received and invested as of August 1, 1995. Figure 26 shows the market values of surplus and the OAVDE values for the block as of September 1, 1995 for the then current yield curve and for parallel shifts of ±100, ±200, +300 and +400 bp. As is seen, there is significant interest rate risk and loss in value of the block from rising rates.

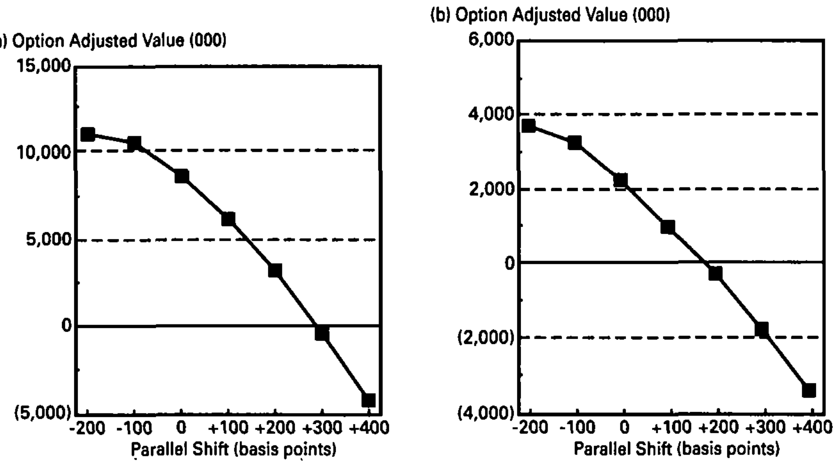

FIGURE 26. OAV OF SURPLUS (a) AND DISTRIBUTABLE EARNINGS (b).

The first alternative (SWAP) is to hedge the risk by implementing a synthetic swap. The insurance enterprise sells a seven year callable bond (no call protection period) and uses the proceeds to purchase a floating rate note that pays 6-month LIBOR. The second alternative (CAP) is to use a 10-year interest rate cap whose notional amount is $95 million and rate index is the five year constant maturity Treasury with a strike level of 8%. The market values of surplus for the base case and the two alternatives are shown in Figure 27; the OAVDE values for the same are shown in Figure 28. An examination of these two figures shows that either hedge significantly improves both the value of the block (OAVDE) and the market value of surplus. For either measure the flattening of the two curves demonstrates the trade-off between reduced performance if rates drop and improved performance if rates increase. The SWAP nicely flattens the OAVDE curve (thus immunizing OAVDE); but the surplus curve begins to fall at the +200 bp shift and above. The CAP results in a flat surplus curve out to + 400 bp; but the OAVDE curve underperforms the SWAP if rates do not rise more than about 125 to 200 bp. If rates rise more than 200 bp, then the CAP materialiy outperforms the SWAP in OAVDE.

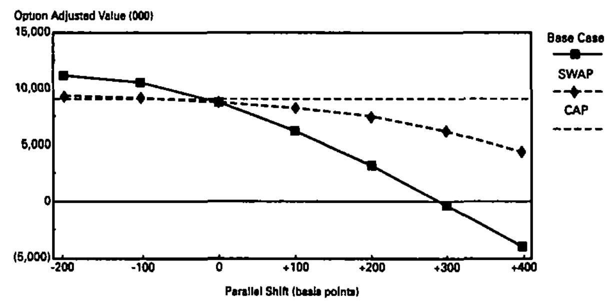

FIGURE 27. OAV OF SURPLUS.

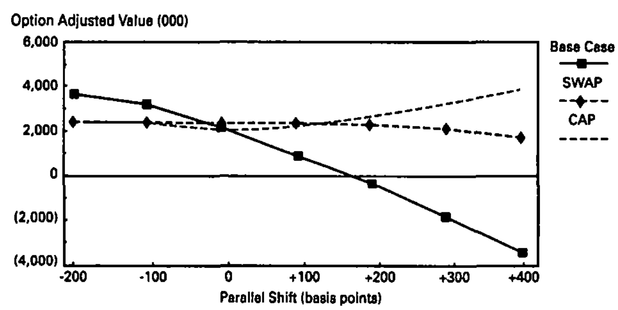

FIGURE 28. OAV OF DISTRIBUTABLE EARNINGS.

From a pure risk control perspective the CAP would be the choice as the diminished performance below shifts of less than +125 bp does not seem as significant as the gain from shifts greater than +125 bp. Recall that these graphs are based on instantaneous shifts in the yield curve. The graphs do not indicate the relative likelihood of such shifts occurring; they only indicate the results if they did occur. Empirical studies based on the forty year period from 1955 through 1994 show that the frequency of 200, 300 and 400 bp increases in the 90 day Treasury bill over a period of a year are 12.8%, 4.9% and 2.1%, respectively. The frequencies for the longer maturities are less than these. (The frequencies for similar increases over a 2-year period are 21.1%, 10.9% and 8.1%, respectively; but the insurance enterprise has considerable time to adjust its position.) It should be noted that the frequencies above are only driven to these levels by the experience in 1978 to 1981. In this light it is fair to ask if the reduction in value from the use of the cap is offset by the increased protection afforded by the cap against an event which may occur with a frequency less than 13%.

Examining the shapes of the curves in Figures 27 and 28 one concludes that the SWAP is effective for medium shifts in the yield curve and the CAP is effective at the extreme upward shifts, but penalizes the insurance enterprise for intermediate shifts. Is it possible that one could combine the two hedge concepts and arrive at a better overall result. Consider the hedge SCAP where the SWAP is combined with the above cap at a $25 million notional amount. The results are shown in Figures 29 and 30.

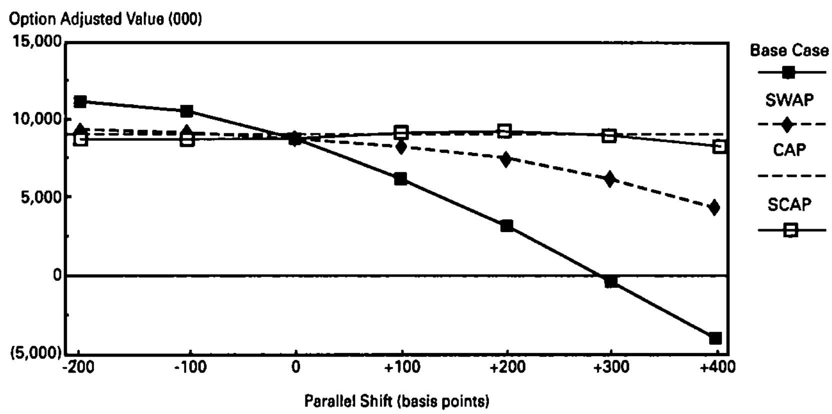

FIGURE 29. OAV OF SURPLUS.

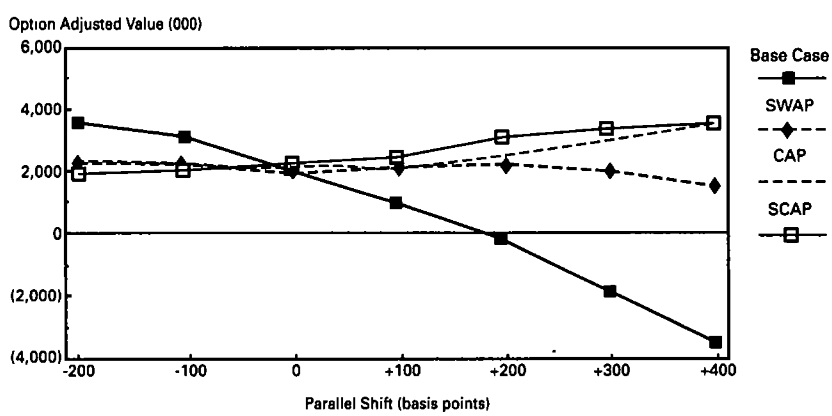

FIGURE 30. OAV OF DISTRIBUTABLE EARNINGS.

SCAP produces a market value of surplus curve almost identical to that of CAP and an OAVDE curve superior to either SWAP or CAP for the initial yield curve and all upward shifts with only a modest concession for downward shifts. Thus SCAP is superior to either of the other two hedges. It is important to note that the superiority of SCAP would not be apparent from examining just the market value of surplus diagrams. In fact, CAP would have been chosen if the only criteria had been surplus. The choice of the proper metric is important. The "OAVDE" metric captures elements of economic reality, i.e. the free cash flows of the firm, that the "market value of surplus" metric does not.

Application of OAVDE

General applications

As seen in the previous section, OAVDE can be used to identify and quantify interest rate risk. It can be used to evaluate different strategies, and so discover strategies that optimize the risk/return posture of the block of business or the company as a whole. There are other risks that are stochastic in nature, e.g asset default, mortality, morbidity, term conversion and lapsation, but are traditionally treated as deterministic. The OAVDE model can be enhanced to reflect the stochastic nature of these other risks. Specific recognition of AIDS or random fluctuations in mortality due to epidemics can be modeled. Also, not all of these risks are independent, e.g. mortality and lapse. It is possible to incorporate these risks as random variables and incorporate the user's perception about the correlation between higher rates of lapse and the resulting higher mortality among the persister group (Becker, 1984; Becker and Kitsos, 1984; Dukes and MacDonald, 1980; Shapiro and Snyder, 1988). This has the added utility of adequately reflecting the fact that if increases in interest rates cause higher interest-sensitive lapses then the model reflects higher mortality among the persisters.

With regard to morbidity in disability income products there is significant historical experience that during recessions disability incidence rates increase and termination rates decrease. The opposite occurs during economic recoveries. If it were possible to correlate economic states with the pattern of change in the interest rate environment, then one could model dynamic incidence and termination rates implied by the change in interest rates.

As regulators and rating agencies become more comfortable with this more sophisticated financial projection methodology it is possible that hedging, whether it be through investment strategy, liability strategy or reinsurance strategy, will allow the insurance enterprise to hold less risk based capital.

A special case of this methodology can be used to determine economically sound risk based capital requirements for interest rate risk. Choose a comfort level, e.g. 95%. As of the date of valuation create a set of arbitrage free interest rate paths. For each path p and each t, $t = 1$ to N, calculate $\mathrm{PVSNI}_{p,t}$, the present value of the statutory net income from $j = 1$ to $j = t$, where the discounting is done at the after-tax risk-free future one period rates along that path. The statutory net incomes are computed on a 'profits released' basis, i.e. they do not include any interest on surplus or retained earnings of the block.

$$\mathrm{PVSNI}_{p,t} = \sum_{j=1}^{t} \left\{ \mathrm{SNI}_{p,j} \Big/ \prod_{k=0}^{j-1} [1 - \mathrm{tr}_{p,k})^{*} r_{p,k}] \right\}. \quad (20)$$

Let RBC_p equal the maximum of the sequence of 0 and the negative of each of the present values above.

$$\mathrm{RBC}_p = \max\{0, -\mathrm{PVSNI}_{p,1}, -\mathrm{PVSNI}_{p,2}, \ldots, -\mathrm{PVSNI}_{p,N}\}. \quad (21)$$

Rank order the RBC_p from lowest to highest. Choose the value of p, p^{95}, such that 95% of the values are less than or equal to RBC_p^{95}. The level of risk based capital as a percent of statutory reserves is equal to the ratio of RBC_p^{95} to the statutory reserve.

$$\mathrm{RBC}_{95} = \mathrm{RBC}_p^{95} / \mathrm{SR}_0. \quad (22)$$

From capital budgeting theory, if a firm has several projects available in which to invest, then the order in which the projects should be pursued is found by ranking the projects in descending order by their net present value or, more accurately, their generalized net present value. Because the OAVDE of a block of business computed using the generalized net present value at the cost of capital is the measure of its fair value or net worth to the shareholders, it is an appropriate device on which to allocate capital to existing lines or to new ventures.

OAVDE analysis can be applied at the insurance enterprise level or at the line of business level. If applied at the enterprise level, the firm can take credit for all offsetting risks that arise from combining all the liabilities and assets. In this way hedging is first performed within the company with any net risk accommodated by either reinsurance or investment vehicles.

Although this would be the most efficient approach, it is more likely that management (and external audiences such as regulators and rating agencies) would want each major product line to be self sufficient. When liabilities with offsetting interest rate risk characteristics are combined to take advantage of the offset there is the risk that the balance of the two liabilities may not be able to be maintained into the future. This additional challenge may make management shy away from relying too much on such combinations.

Another facet of this is the explicit use of the ability to sell new liabilities as a tool in risk management. If one can sell new liabilities at an attractive, or at least acceptable, price, then this can be a useful tool in asset/liability management and shareholder wealth maximization. This should be used as a tactical method not a strategic method. There is the risk of becoming a 'new business junkie' whereby the insurance enterprise must acquire the new liability cash income in order to meet other liability cash expense. The continuing ability to do it at favorable terms may not exist.

Investment strategy and benchmarking

In Section III the process of evaluating strategies was discussed. At its most basic level, if it is possible to identify a robust strategy, then one can simply follow the associated investment strategy. That investment strategy may not be optimal for each path but it will perform well over the spectrum of paths that can occur. The strategy would be revalidated from time to time and adjusted if necessary.

Another approach is to use the strategy to create a model of the business. One can then take the asset portfolio (built from the generic investments used in the model) that results from the application of the investment strategy and determine its associated cash flow characteristics, e.g. cash flow patterns under different interest rate scenarios, duration, partial or key rate durations, convexity and partial convexities. These parameters can serve as the benchmarks for the real asset portfolio. The portfolio manager can then engage in various techniques that will generate the highest return consistent with the benchmarks. It also allows the portfolio manager to take 'bets' if he or she has a belief as to how the future interest rate environment will move. Experience analyses will show if value was added or lost by the tactical bets. As experience develops the model is updated for the actual liability cash flows. This results in an update of the model asset portfolio which is then used to measure the next set of benchmarks.

If the portfolio manager has made bets and deviated from the investment strategy, the actual current portfolio and investment strategy can be put in the model to see if it produces either an OAVDE of greater or lesser value or an OAVDE having more or less volatility than that from the original investment strategy. If the results are superior, the bets have been good; if poorer, then bad and the portfolio can be restructured. This last step closes the loop to ensure that OAVDE has been optimized. Again the strategy should be revalidated from time to time.

The various immunization techniques described earlier can also be applied to the value of OAVDE. Further, the application of mathematical programming as proposed by Miller *et al.* (1989) can be used to identify the investment portion of the strategy that results in the optimal values for the spread and/or yield along each path, which process would also result in a robust value of OAVDE.

Does OAVDE optimization imply a buy-and-hold investment strategy or a total return investment strategy? In and of itself the OAVDE methodology says nothing about this. The person performing the financial projections defines the investment strategy that he or she wishes to use within the OAVDE model. Such strategies may be buy-and-hold based or total return based. Choose the one which optimizes OAVDE.

Option adjusted appraisal values (OAAV)

The investment income on assets in classic appraisal values of insurance enterprises is computed in one of two ways. The first assumes a constant portfolio net earned rate. The second assumes the yield curve at the date of the appraisal remains constant and projects the book yield on the existing asset portfolio into the future. The resulting book yield together with a reinvestment assumption is used to compute the future statutory earnings. These earnings are then discounted at a hurdle rate. More recently, the actual assets are modeled and an explicit reinvestment assumption is made. Many interest rate scenarios are run and the weighted average of the present values of statutory earnings is computed. While the latter is superior to the classical approach, there are still difficulties with the approach.

Let i_0 be given together with a strategy s. The corresponding option adjusted appraisal value for a block of business is given by OAVDE(i_0, i, s) where the OAVDE value is computed at a hurdle rate, i, expressed either as a desired option adjusted spread-to-Treasuries or as an option adjusted yield. Let free surplus equal the excess, if positive, of the market value of total capital and surplus and items in the nature of surplus over the required surplus needed to support the liabilities, their associated assets and the assets backing required surplus. The AVR is treated as an item in the nature of surplus except for the excess, if positive, of the AVR over the asset default component of required surplus. Not admitted assets may or not may not be included in free surplus. If the value of a not admitted asset is realizable and such realization is not reflected in the future distributable earnings, then it should be included in free surplus. If its realization is reflected in future distributable earnings, it should not be counted in free surplus.

The OAAV equals the sum of free surplus, the OAVDE(i_0, i, s) of the existing blocks of business and an estimate of the franchise value, or the value of new business. The OAVDE values are computed using the hurdle rate of the evaluator. The franchise or new business value may be omitted or included. The computation of OAVDE for new business beyond that expected in the next period presents a challenge as the value at issue on a future date will depend on the path taken. Attention should be paid to new business as it is possible that the OAVDE of new business could be negative.

For a stock life insurance company it is possible to compute the return (expressed either as an option adjusted spread-to-Treasuries or option adjusted yield) implied by the market using the market value of the stock as of a given date. It would be the value of i that equates the OAAV with the market value of stock.

Option adjusted value added (OAVA)

For a block of business define its option adjusted value (OAV) at the end of period t after the release of any distributable earnings to be:

$$\text{OAV}_t = \text{OAVDE}_t \,. \tag{23}$$

Define the option adjusted income (OAI) for period t as:

$$\begin{aligned} \text{OAI}_t &= \text{DE}_t + \text{OAVDE}_t - \text{OAVDE}_{t-1} \\ &= \text{DE}_t + \Delta_{t-1}(\text{OAVDE}) \\ &= \text{DE}_t + \Delta_{t-1}(\text{OAV}). \end{aligned} \tag{24}$$

Define the option adjusted value added (OAVA) as:

$$\text{OAVA}_t = \text{OAI}_t - h * \text{OAV}_{t-1}, \tag{25}$$

where h is the insurance enterprise's cost of capital.

If the OAVA is positive for the period, then shareholder value has been added; if negative, then shareholder value has been depleted.

Define the option adjusted total return (OATR) earned by the block as:

$$\text{OATR}_t = \text{OAI}_t / \text{OAV}_{t-1}. \tag{26}$$

If the period over which the OATR is calculated is less than one year, it can be converted into an annualized rate.

Other adjustments can be made to formula (26) to reflect the changing basis of value over the year. Since distributable earnings are received over the course of the year, the following approximates the return for the year:

$$\text{OATR}_t = (2 * \text{OAI}_t)/(\text{OAV}_t + \text{OAV}_{t-1} - \text{OAI}_t). \tag{27}$$

If OAV represents the option adjusted value for all product lines and FS represents the market value of free surplus, i.e. those assets not included in the computation of OAV, the enterprise total return is

$$\text{ETR}_t = (\text{OAI}_t + \text{aftertax total return on } \text{FS}_{t-1})/(\text{OAV}_{t-1} + \text{FS}_{t-1}). \tag{28}$$

The OAVA and OAI can be analyzed in the following way:

- external environment: change in level and slope of yield curve; change in volatility of interest rates;
- internal environment: change in investment and/or disinvestment strategy; change in crediting rate strategy or liability management strategy;
- new business;
- existing business performance; and
- shareholder dividends paid.

Within this framework it is natural to define the fair or market value of surplus, MVS, of the insurance enterprise as:

$$\text{MVS}_t = \text{FS}_t + \text{OAV}_t. \tag{29}$$

Then it is possible to define the fair or market value of liabilities (MVL) as:

$$\text{MVL}_t = \text{MVA}_t - \text{MVS}_t \,.$$

The quantity MVL represents all liabilities of the firm, not just those related to the products manufactured by the firm. The market value of product liabilities would be MVL less the non product liabilities. The fair or market value of income is the change in MVS.

This approach would fully implement the mark-to-market of the insurance enterprise via the appraisal method in a manner that correctly reflects the presence of embedded options in the assets and liabilities and on a basis that is comparable between firms and over time if an appropriate discount rate can be found. While this may seem to be a difficult challenge, there are two reasonable approaches. First, McKinsey and Company (1994) has estimated the cost of capital for insurance enterprises manufacturing life insurance, annuities and reinsurance to be from 10.9% to 11.8%. It would be possible to pick an intermediate rate, e.g. 11.5%, and simply define that rate to be used by all insurance enterprises for external reporting. Second, Childs (1994) presents research that suggests a cost of capital for an AA rated firm of 3% to 4% over the AA long bond yield for insurance enterprises. For an insurance enterprise with higher or lower quality, appropriate adjustments would be made. To stabilize the rate from year to year a moving average of long bond rates could be used. In either case, the methodologies used to estimate the cost of capital could be applied periodically to determine if the chosen cost of capital value remains current or if revision is called for.

Within this framework a fair value accounting basis can be constructed that provides the best information on the performance of the firm on both an absolute basis and in comparison with other firms and pre-determined goals, allows an appraisal of management performance, and serves as framework for making economic decisions. This methodology presents exposure to the key risks of the insurance enterprise and results and risk posture by major product line and the company as a whole in a manner communicable to and understandable by management, shareholders, securities analysts and regulators. The position of the firm, its performance, analysis of performance and sensitivity to risks in the environment can be displayed numerically and graphically using the criteria discussed previously.

In light of the comment in the last paragraph regarding sensitivity to risks please recall that both OAAV and OAV depend on the OAVDE algorithm. The OAVDE value, and so the OAAV, OAV and other values derived from OAV as described above, are the means of probability distributions. As such, they are point estimates and do not reflect the dispersion of results, e.g. the standard deviation or higher moments of the distribution. This is to some extent provided for if one is discounting at the cost of capital or other risk adjusted rate. Alternatively, one could derive risk adjusted values for OAAV and OAV (and other values dependent on OAV) by using techniques presented in Section IV. The benefit of this is that when considering a change in strategy or evaluating the results of a change in strategy the use of a risk adjusted measure might indicate that a result with a lower mean is superior due to a reduction in the dispersion of the distribution; essentially one has traded off an acceptable amount of expected value for a commensurate reduction in uncertainty in that value. Such indication might be missed if a risk unadjusted measure is used.

As an example, consider the use of utility theory. Assume that the insurance enterprise (or product line) has a utility function $U(X)$. One can define the risk adjusted option adjusted value, OAV^{RA}, in the following manner:

$$OAV_t^{RA} = E[U(\{PVDE_t(i_0, p)\}], \qquad (29)$$

where E is the expectation operator applied to the distribution over all paths p.

In the above expression for the present value of distributable earnings the choice of discounting at a constant y or a spread s is omitted; and it is assumed that the present value is computed using the generalized net present value algorithm. Using this definition for OAV^{RA} one can then derive the remaining quantities in this section on a risk adjusted basis. While the challenges of implementing such a fair value basis can not be ignored, it would mean the demise of DAC, imaginary DAC, PB 8, FAS 60, FAS 97, deferred taxes and other accounting issues as they are known today.

In this section it is assumed that all discounting is performed at the firm's cost of capital which is expressed as a level amount. It is also possible to perform the discounting on a pathwise basis using each path's risk-free rates plus a spread, or OAS. In this case the firm's cost of capital or hurdle rate must be expressed as an add-on to the one period risk-free rate. This approach is more consistent with the finance of option pricing; but there is difficulty in determining an appropriate OAS. Also, since the one period risk-free rate can be volatile, the resulting cost of capital would be volatile. It is not generally thought that firms' costs of capital are that volatile.

Other application

This framework is built around the appropriate choice of an objective function.

If circumstances change, all that is required is to ascertain the new objective function for free cash flows. By the use of the appropriate objective function the methodology can be applied to other firms and firms in other legal jurisdictions.

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Appendix A: Determining Pricing

Atkinson (1990) describes the early development of insurance pricing. In the nineteenth century and as late as the 1960s in some cases, gross premiums were calculated by loading or grossing up valuation net premiums for expenses. In 1919 E.E. Cammack developed the equation-based gross premium formula with mortality its only decrement. W.A. Jenkins in 1932, introduced a second decrement for lapse. Deterministic assumptions were made for all experience, including interest rates, which were assumed constant. (This is what is meant by deterministic pricing.) With these methods profits emerged as release from risk. In some cases this approach used more realistic assumptions than valuation assumptions but the formula then included an explicit provision for profit. It might be loading the gross premium by a constant x dollars per thousand or a percentage of premium loading. Only indirectly, if at all, does the basis of the accounting system play a part and there is no relation of the profitability measure to the period by period emergence of earnings under the accounting system.

The next step forward was the change to an accumulation type of formula where premiums less benefits and expenses are accumulated with interest to some point in the future. The value of this accumulation is divided by the number of surviving units at the end of the projection and the quotient is called the asset share. J.E. Hoskins used the following profit objective: the asset share at the end of a given number of years must equal the cash value or reserve plus some margin, e.g. 10% of the cash value. The accumulations were projected using expected, deterministic experience. These models often included a charge for the increase in the statutory reserve on which interest was then credited. By tying the profit target to the reserve at the end of the pricing horizon the accounting system was recognized, but again the emergence of profit did not figure into the premium decision. The year by year change in the asset share was not a key element. Federal income taxes were not included.

In 1959 J.C.H. Anderson recognized the implication of accumulating the profits within the product line up to the end of the pricing horizon, i.e. the asset share model profits as they emerged were retained with the product and not considered available to or released back to surplus until the end of that horizon. Anderson changed the focus to that of profits released where the initial surplus strain of new business is viewed as an investment by surplus and the future accounting profits were repayments to surplus each year. These profits released were not accumulated but discounted. Thus each period's profit did not include any investment income on retained earnings but only on the opening reserve and cash flow during the period. He designated these profits as 'book profits'. They were viewed as a return of both principal invested and interest on this invested amount. Anderson treated the writing of new business as an investment and applied the methods of capital budgeting to determine the return on that investment, where such return could be compared to a predetermined return on investment target. In this manner the accounting system and the emergence of earnings under that system were explicitly recognized. In this methodology interest rates earned by the insurance enterprise remain deterministic (along with all other assumptions), although interest rates are allowed to change each year. Again federal income taxes were excluded.

This ultimately gave rise to an array of profit measures based on book profits besides the return on investment. Examples are: the net present value of book profits; the net present value of book profits divided by the present value of premium (a levelized profit/premium ratio); the net present value of book profits divided by an annuity (a levelized statutory profit per unit in force); and breakeven year, i.e. the first year in which the net present value of book profits turns positive and remains positive. Profit targets were often expressed as a combination of these. In many cases the present values were calculated by discounting the book profits at the pretax asset investment rate earned by the insurance enterprise. In this event the measures expressed the profitability in the product in excess of the return afforded by investable assets. Another choice for the discount rate was the insurance enterprise's cost of capital. If the profit measures were not positive, the insurance product was not earning back its cost of capital. Sometimes a higher target rate was used for the discounting. Here if the measures were zero or positive, the product earned a return at or above the target.

Lee (1979) added two innovations to Anderson's approach. First, he allowed the fund that earned interest to be more general than the statutory reserve. He allowed, in a very general way, for a portion of the book profit not to be released, but accumulated with the product itself. Second, he incorporated federal income tax into the computation. Sondergeld (1982) extended the nature of the fund that was associated with the product to include what has come to be known as required surplus. The total capital required to support the business should include required surplus. If this is not factored into the pricing, the insurance enterprise's return on total capital will be less than anticipated in the pricing. The products will be underpriced relative to the desired return on total capital. This is commented on more fully below.

The concept of required surplus emerged as the amount of funds that the insurance enterprise must hold in order that the enterprise will remain solvent at some confidence level of possible adverse future experience. The risks became identified as the asset default risk ($C-1$), obligation risk ($C-2$), interest rate risk ($C-3$) and general business risk ($C-4$), the 'C' standing for 'contingency'. (Subsequently, this concept was embraced by the rating agencies and later the National Association of Insurance Commissioners as expressed in their risk based capital requirements.) This meant that not all of an insurance enterprise's capital was available for distribution but that a portion of it must be earmarked to support the liabilities and associated assets.

The more assets an insurance enterprise has in surplus, the lower its return on capital will be as assets backing surplus will not provide a return as large as that of the insurance liabilities. Therefore, it is better to hold as little as is prudent. With prudence necessitating the holding of surplus at an amount equal to at least the required surplus level, this created a drag on the return on capital. In order to provide the shareholders with a target return on all capital employed by the business, the pricing of the individual liabilities had to provide for a return not only on the amount of surplus invested in the liability, but also an extra return such that when it is combined with the aftertax investment income on assets supporting required surplus, the return on the totality equals or exceeds the cost of capital or target return, if higher. Thus the product must earn back its investment and pay rent on the required surplus. The pricing does not provide for the insurance enterprise to ever pay out the required surplus, merely to provide a differential return on it.

Smith (1987) presented an insightful analysis regarding why the average GAAP return on equity that emerges over time might not equal the statutory internal rate of return priced into the product. Becker in a discussion to Smith's paper presented an additional reason for this to occur and identified a solution for it. This is examined below. The method developed by Anderson is based on a policy year approach. Profits are computed assuming the product is issued at the beginning of the year. The formula explicitly assumes that negative cash flows are covered by borrowing at the net earned rate of the insurance enterprise and that the charge for the reserve (or change in reserve) occurs at the end of the year. This means that the shareholder investment occurs at the end of the year, not when the product is issued; and the reserve is only established for those persisting into the subsequent year, which reduces the magnitude of the investment. In reality, insurance enterprises must invest capital and be solvent when the product is issued and must set up reserves for all new business. Without taking these issues into account it is possible to overstate the internal rate of return. For this reason this paper makes special note about the initial distributable earnings at time 0. The earliest form of distributable earnings was given by: statutory book profit, plus after-tax investment income on the opening balance of required surplus, less the change in required surplus. The old mandatory securities valuation reserve (MSVR) was generally ignored. Subsequently, the MSVR was replaced by the AVR and a new reserve, the IMR, was added. IMR effects are automatically included in statutory net income. The excess, if positive, of AVR over the asset default component of required surplus must be considered.

The last issue facing deterministic pricing was the emergence of products whose distributable earnings showed multiple changes in sign. This situation might signify the need for more than one shareholder investment. Historically, most insurance products had an initial investment (negative distributable earnings) followed by positive distributable earnings. More recently product features were added to designs that created multiple sign changes in distributable earnings, e.g. cliff surrender charges, reversionary interest rate bonuses and/or mortality and expense charge give-backs and compensation bonuses. Such sign changes also could occur in the pricing of disability income and long term care products. There is also the possibility that the product is not adequately priced and it has a pattern of alternating and/or late duration negative distributable earnings. The classical algorithms for net present value and internal rate of return are not capable of determining the net present value or internal rate of return for a stand alone project which requires multiple shareholder investments.

The classical net present value and internal rate of return algorithms used in capital budgeting demand a well behaved series of cash flows (one change in algebraic sign) in order to guarantee that both net present value and internal rate of return are economically meaningful. The problem that arises when there is more than one investment, i.e. negative quantity, was originally demonstrated in 1955 by the 'pump project' of Lorie and Savage which had two internal rates of return. An interpretation for this special case was given in 1956 by Solomon. In 1965 Teichroew *et al.* developed a classification scheme that classifies all projects by whether they are simple consumers of capital, investment projects, non users of capital, financing projects, or complex investment projects, mixed projects, which have repeated investment elements. They could not provide a complete analysis for mixed projects. In 1988, Becker provided a general solution to the problem of mixed projects that enables all projects to be classified as either financing or investment and generalizes the classical net present value and internal rate of return algorithms making them economically meaningful in all situations. The application of the generalized net present value to the pump project is shown Figure A.1.

The various deterministic profit measures noted previously can be redefined using the refinements described above.

Appendix B: Objections to Distributable Earnings as a Basis for Fair Value

In this section several objections are presented that people have in using distributable earnings as a basis for fair value estimation.

First, it is common in finance texts to adopt pure cash flows as the basis of the economic reality as accounting earnings have non cash items, e.g. depreciation, in them. The accounting system doesn't necessarily give a real picture of the economic reality. Why should the accounting system be used here? The first part of the statement is true. But most firms do not have limitations placed on them with regard to the payment of free cash flows, i.e. shareholder dividends, similar to insurance enterprises. For them, a pure cash flow basis may be appropriate. But due to the statutory limitation on shareholder dividends from insurance enterprises statutory accounting does impact the economic reality. The payment of income taxes is a result of statute; but that does not make that economic fact of taxes any less real.

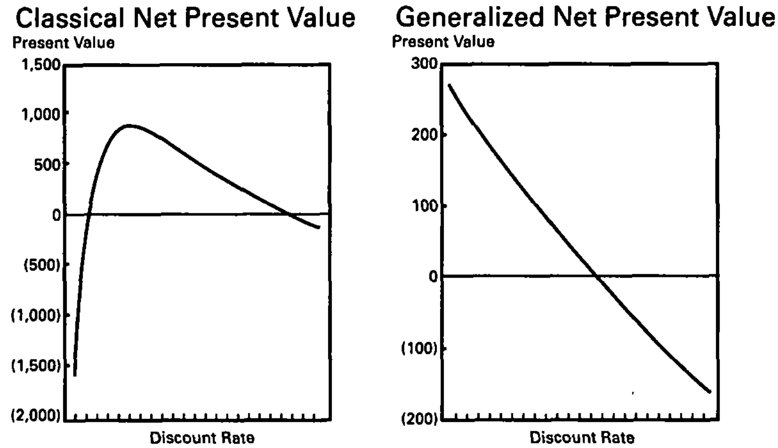

FIGURE A.1. THE PUMP PROJECT.

Second, for a growing company it is known that the statutory net income for the financial reporting period understates the real earnings of the firm. How can that be reconciled? As noted in the response to the first question, state statute limits shareholder dividends, i.e. free cash flows. But OAVDE does recognize the entire value of existing and new business. It does so by recognizing the present value of the future distributable earnings not just the current period's earnings. OAVDE takes into account all the earnings and when they will be available to shareholders. If the insurance enterprise could invest distributable earnings in new business whose generalized internal rate of return substantially exceeded its cost of capital, then the straight forward application of OAVDE analysis would understate the value of the firm as OAVDE would look at the original distributable earnings. This can be remedied by adding the new business to the model for a given number of future years production and computing the resulting OAVDE.

Third, everybody knows statutory accounting is too conservative and doesn't reflect economic reality. That is why GAAP accounting was developed. Why isn't GAAP the right basis? Consider two insurance enterprises, A and B. A has GAAP earnings of $100 million and B has GAAP earnings of $150 million. Both A and B have statutory net income of $75 million. While company B has half again as much GAAP earnings as company A, both of them are limited to the same amount of free cash flows, i.e. shareholder dividends, namely $75 million. Possibly due to AVR and required surplus issues A and B don't have the same final distributable earnings, but one can see that GAAP doesn't change the rules when it comes to paying out shareholder dividends. Also, A and B are both limited to only investing $75 million in new business unless they have free surplus in excess of the AVR and the level of required surplus they need to maintain.

Fourth, the method is tied to free cash flows, i.e. shareholder dividends. What about an investor who is looking for growth stocks? They are expecting appreciation in value, not cash to spend. As noted in the paper, in Chapter 2 of Copeland and Weston the authors address the fact that amounts not paid as shareholder dividends but reinvested in new projects at the firm's cost of capital have the same effect on stock price as if they had been paid out in cash. Also, over long periods of time growth stocks that never pay out dividends will have declines in stock value. As a parallel consider a life insurance policy whose death benefit is a paid up life insurance policy on the life of the beneficiary, whose death benefit is a paid up life insurance policy on the life of its beneficiary. And so on. Somebody at sometime will demand to be paid.

Next, are there not other sources of value that are not recognized by the distributable earnings concept? For example, what about management talent, investment expertise, superior data processing systems, good risk control? The beneficial effects of management talent, investment expertise and good risk control should already have been reflected in the OAVDE amount in terms of such items as expense management and higher productivity, extra investment returns, lesser claim levels. Consider superior data processing systems. The value of this item also should already be reflected in lower operating expenses that are incurred by the insurance enterprise. There may be additional value in that the insurance enterprise could perform third party administration and realize a profit. But if so, the use of the data processing capability should be part of the strategy of the firm. Its value, then, would be reflected in the distributable earnings of the insurance enterprise via the associated revenues less expenses that could be realized from third party administration. If there is added value and it is not in the distributable earnings, then it is not being realized.

Then what if unused capacity exists or if there are undervalued assets? This is somewhat similar to the prior question. If either of these issues exist, it means that the current strategy is not optimal. If the current strategy is the basis of the OAVDE computation, then a better strategy exists which will reflect the unused capacity or undervalued assets. Find it.

Statutory accounting only recognizes 'admitted assets'. GAAP recognizes all assets. Doesn't that mean that statutory accounting isn't the right basis? If a not admitted asset is such that its value will only emerge into statutory income over time, then its value is not realizable now. It will increase the OAVDE. If the not admitted asset's value can be realized now and isn't limited to emergence over time, then its value is already reflected in free surplus. An example of the latter is the value of furniture and equipment. A possible example of the former is agents' debit balances that are not realizable through factoring or securitization.

Another point is that of a prospective buyer of an insurance company knows of actions that could be taken that would add value, then the OAVDE amounts already computed won't reveal that value. Can't the OAVDE value therefore understate the value of the firm? The OAVDE value depends on the strategy being implemented. Under current management the OAVDE value may be all that is realizable to them. The buyer has his or her own strategy. Under that strategy the buyer's OAVDE will be higher assuming both have the same cost of capital. The increase in OAVDE represents the value added by the expertise of the buyer. Merely because the buyer's OAVDE or fair value is higher than that of the current owner/management doesn't mean the buyer should pay it to them! After all, the buyer is the one who will have to deliver on the new strategy. But the new OAVDE can tell the buyer if the asking price is rich or cheap and it puts a ceiling on what the buyer should be willing to pay the seller.

So what happens to OAVDE or the value of the firm if statutory accounting changes? If changes are made in statutory accounting principles or if management changes the level of required surplus deemed prudent, then there will be a discontinuity in OAVDE and in the value of the firm; but, other things being equal, OAVDE will proceed normally under the new rules thereafter. Similar events have occurred with the significant accounting changes in GAAP and SAP and changes in the surplus requirements of rating agencies. These events can also be driven by tax law changes. For example, consider an insurance enterprise whose business is substantially noncancellable disability income. When the federal government adopted the DAC tax, the value of that firm dropped immediately as the firm had no way of passing on the reduction in distributable earnings due to the DAC tax with regard to the then current in force block.

How does OAVDE apply to a defined benefit pension plan? It doesn't. There are two reasons for this. First, there are no shareholder dividends and no need to use surplus to fund growth, both of which are applicable to a stock insurance company and the latter to a mutual company. Second, these plans do not pay federal income tax and are not affected by statutory accounting concepts, e.g. IMR, AVR, reserves or deficiency reserves. In this case the objective function provided by the market value of surplus may be more appropriate to measure the risk posture of the plan. But note that market value analysis is a 'time 0' analysis for the risk posture and does not show how the situation unfolds over time as reinvestments are actually made.

The term 'surplus' is often used with different meanings. How does one keep track of them when analyzing a given situation? The term 'surplus' is defined as the value of assets less the value of liabilities where the values for assets and liabilities are computed under a given set of assumptions. 'Surplus' has three major interpretations: as a redundancy or cushion against adverse experience to the company in meeting its obligations; as a residual or shareholder liquidation value; and as the value of the firm in the sense of what a willing buyer would pay a willing seller. These interpretations are not only different; but the significance they have depends on the particular situation. Consider a firm whose product or service is one for which the entire results of the economic transaction are known within a short period of time. For this firm, the redundancy interpretation of surplus is of small significance as the liability items on the balance sheet are mainly small period accrual items. The residual value interpretation is of large significance as long as the assumptions underlying the accounting system when considered in light of the assets do not produce asset values materially different from market values, i.e. the liquidation must be actually feasible. Next consider a firm for which the results of the economic transaction are only known over a long period of time. In this case the redundancy or cushion interpretation of surplus is meaningful; but the residual value has less meaning as the value of the liabilities will be dominated by the value of the performance obligation arising from the incomplete economic transactions. If there is no liquid and robust market for selling those obligations, then the residual value is hypothetical and vague. In either case, the interpretation of surplus as value of the firm may bear no relation to the actual value of the firm for a going concern. In the case of the firm whose economic transactions are completed quickly, the franchise value of the firm, at least, is ignored. In the other case, not only is the franchise value ignored but also ignored is the value of future distributable earnings arising from the incomplete economic transactions. It is not likely that any set of assumptions, 'market' or 'book', can provide for an accounting system for which all three interpretations of surplus are simultaneously realized in one number. To avoid confusion an accounting system should be designed to focus on one of these interpretations. If the interpretation chosen for surplus is the value of the firm, then the value of liabilities is the balancing item. It may not bear a simple relationship or any fixed relationship to the value of liabilities resulting from the design of an accounting system where surplus is defined to reflect either a redundancy value or a residual value.

Finally, while its true that there is no direct third party market for liabilities, isn't it the case that the market for the purchase of blocks of insurance or entire companies is not that large and so it is hard to infer a discount rate to be used? As noted earlier, one basis that can be used is the insurance enterprise's cost of capital. This can be approximated in either of the manners described in Section V or by deriving a marginal cost of capital. Management might want to use a hurdle rate in excess of the cost of capital. But the market for acquisitions' in a broader sense is not as thin as might be expected. Every day reinsurance transactions take place for both new business and existing blocks of business where the terms of the transactions reflect the price that is reached by a willing buyer and a willing seller. These transactions are not priced on an assessment of the market value of assets less the market value of liabilities, but on the present value of the distributable earnings that result from the interaction of the assets and liabilities, the resulting cash flows and the rules governing their distribution. These present values are evaluated using costs of capital, hurdle rates or profit criteria expressed as return in excess of what can be obtained from other investments.

Appendix C

This appendix elaborates on certain structural and methodological differences between market value analysis and OAVDE analysis, the issue of their ultimate convergence and provides comments on the 'two paradigms' concept articulated by Robert R. Reitano in 'Market Value of Liabilities: Two Paradigms'.

Market value analysis and OAVDE analysis

First, consider what may be termed 'pure' market value analysis. In this approach one compares the market value of existing assets at the date of the valuation to the market value of liabilities at the same date. Alternatively, one could compute the value of the existing assets after a hypothetical restructure or hedge is established; but it still represents the present value of asset cash flows from assets actually held or that could be held at time 0. Note that it does not reflect any investment strategy applicable beyond time 0. Thus it can not be a simulation of the anticipated total performance of the business over time.

The liability and expense cash flows are similarly projected based on time 0 considerations and their market value determined. In this case the interest rate crediting strategy must be constrained to be a function of an index. The index may utilize only exogenous variables, i.e. the future yield curves and, possibly, the prior credited rates. The pure market value analysis approach can not be applied to an interest sensitive liability whose credited rate reflects to any degree the performance of the assets supporting the liability as that performance is not being explicitly or implicitly modeled in the pure approach.

Second, consider a 'modified' market value analysis. In this case the market value of assets is similar to the above. But an actual simulation of the performance of the business is performed which reflects how the business is to be actually managed along the future interest rate paths, i.e. it utilizes all the information in the strategy. From this simulation it is possible to extract the data to determine the liability cash flows and their market value. This method must be used if the actual portfolio earned rate is used in any way in determining the interest crediting rate. The other information generated from the simulation is not needed for the market value analysis. In either case, the market value analysis presents a snapshot of the business at time 0 and does not represent a simulation of the total performance of the business. By design it utilizes single point option pricing, i.e. at time zero only.

OAVDE analysis, on the other hand, reflects the total performance of the business by using a complete simulation of the operation of the business. Thus it utilizes all the information in the strategy and can model all liabilities, including those interest sensitive liabilities whose interest crediting strategy depends on actual asset performance. At each point in time along each path decisions are required about asset investment or disinvestment. Multi-point option pricing is employed to provide relatively consistent security prices needed for investment or disinvestment decisions at each path/time point. As noted in the text, certain simplifying assumptions may be made that do not require the use of option pricing at each path/time point. OAVDE analysis produces the resulting distributable earnings that would emerge along each path/time point based on how the business is to be managed. The path-wise vectors of distributable earnings can then be analyzed using the initial set of arbitrage-free paths to provide the option adjusted value of distributable earnings at time 0.

Convergence of the two methods?

Is it possible that both processes eventually lead to the same result? It is difficult to rationalize how this might occur. First, the objective function for each methodology is entirely different from the other. Second, the issues raised in an earlier section would have to be resolved. For example, cash flows occurring due to capital gains taxes, federal income taxes and shareholder dividends are not considered in market value analysis. Third, based on the description above, there are significant methodological differences between the two approaches.

Is it possible that the market value analysis could be modified to determine the distributable earnings? In the case of the modified market value analysis the additional information, in principle, could be made available to compute the distributable earnings at each path/time point because the modified market value analysis is a simulation using all elements of the strategy. Such a model can be augmented, if necessary, so that any additional items for the determination of distributable earnings are computed. The OAVDE value could be directly determined. In this case, one is simply performing an OAVDE analysis.

In the case of pure market value analysis the additional information for a total simulation is not available in principle. Never-the-less it is possible to define a formula that will approximate a cash flow for distributable earnings. Such a computation is only approximate as the model is not a total simulation and many items required for actual determination of distributable earnings at each path/time point are not available. But even more significant is that interpreting the result of such a computation as a distributable earnings amount is fraudulent in that distributable earnings are only meaningful in the context of an actual simulation of total business performance, i.e. it must utilize the complete strategy. For example, formulae for the distributable earnings cash flows that can and have been derived implicitly from a pure market value analysis assume reinvestment is based on the one period future rate and disinvestment is based on unlimited ability to borrow at the one period future rate because this is all that is available in the pure market value approach. The 'add-on' approach that can be done for modified market value analysis does not work for pure market value analysis.

Two paradigms

For market value analysis the market value of insurance liabilities (MVL) is directly determined. In the case of OAVDE analysis the market value of all liabilities is indirectly determined and is given by the formula MVL = MVA – OAVDE. The market value of insurance liabilities is then found from subtracting the market value of non insurance liabilities from the total. Reitano

(1995) noted above, calls this dichotomy the 'two paradigms'. The principle thesis of his paper is that if there existed separate deep and liquid markets for each of the buying/selling of insurance companies and/or blocks of business and for the buying/selling of individual insurance liabilities by third parties, then any differentials between the two markets would be arbitraged away. If so, then he asks which paradigm would change? His answer and argument are as follows: "In general, we expect it would be the distributable earnings approach. That is, if liability contracts on individuals were as easily and actively traded as are assets currently, we expect that the presence of this market would fundamentally change our notion of 'earnings', to a basis that is more reflective of total returns in the mutual fund or investment banking industries. That is, in this environment it would be entirely inconsistent for statutory accounting to proclaim book value earnings during a period in a market where participants could actually earn, through trading, the change in the asset market value less the change in the liabilities market value, all tax adjusted. This accounting change would then redefine distributable earnings of a firm in terms of future cash flows, which would create implied liability values with the indirect methodology effectively equal to those provided by the direct methodology. In summary, the direct methodology below provides a valuation of liabilities that is more consistent with current asset valuation ..."

There are several observations that are important to fully appreciate this complex issue. First, there is the assumption that statutory book value earnings differing from market value earnings will change. This seems unlikely because statutory book value accounting principles and the resulting earnings are determined by authorities whose mission is unrelated to these market value issues but is related to insuring the protection of policyholders through multiple conservative accounting principles. When the asset/liability management risk is contemplated neither statutory nor GAAP accounting is an adequate basis on which to base the insurance enterprise's balance sheet or income statement. An accounting system based on OAVDE analysis does provide an adequate basis.

It may be that the possible confusion of statutory earnings for the economic earnings under OAVDE and/or the concerns about the underlying use of any existing accounting system (especially statutory accounting) in determination of economic earnings are key mental barriers to understanding OAVDE as an economic accounting basis. These misconceptions often occur because it seems straightforward to conclude that as statutory accounting heavily influences the OAVDE computation, then the economic earnings under OAVDE analysis are the statutory earnings. An earlier section elaborated on the accounting basis and the definition of earnings that emerges from OAVDE and, as shown, the basis is not statutory accounting and the earnings are not statutory earnings. The OAVDE analysis methodology defines the basis. OAVDE does reflect market values, specifically the intrinsic or fair value of the insurance enterprise computed from fundamental financial principles. The intrinsic or fair value itself incorporates the market values of the assets of the insurance enterprise insofar as they result in earnings that may be distributed to shareholders at future times.

Second, the phrase 'all tax adjusted' does not make clear that many tax related issues are not addressed by the current state of the art of market value analysis. In fact, current immunization theory is not comprehensive with regard to taxation. There are unresolved issues here. As noted above, there are significant economic realities that are not reflected in market value analysis.

Third, if there were such a deep and liquid market, then the values of the insurance liabilities would be computed by the direct method from models that reflect only insurance obligation cash flows. As insurance enterprise maintenance expenses, renewal commissions, premium tax, etc. are irrelevant to the owner of the insurance liability, there is no need or reason to reflect them as is done in the models currently used to compute a market value analysis. Thus the direct approach paradigm would change at least to eliminate those unnecessary items.

Fourth, the conclusion asserts that it is the distributable earnings paradigm that changes. The rationale for this position might result from a confusion of statutory accounting and the economic accounting system based on OAVDE. Since OAVDE defines a new basis that, while reflecting certain aspects of statutory accounting is not equivalent to statutory accounting, the rationale is not applicable. The discussion of the first item in this section may also be relevant here.

Fifth, market value analysis considers the value of the liability from the perspective of a liquid and deep direct third party market. The market among insurance companies and blocks of business is based on future earnings from the company or block in question to the acquiring insurance company. Here the market value of liabilities is indirectly inferred from the basic accounting equation. In this case the indirect market value of liabilities is a logical construct. Must that indirect market value equal the third party market value? Is it even the case that a directly computed market value from the perspective of an insurance enterprise must equal the third party market value?

Consider a medically standard individual with an annual renewable term policy. In any third party market the market value of that policy must be non-negative, i.e. greater than or equal to zero. Suppose that the policy in question was issued by an insurance company that writes only annual renewable term insurance and that the premium scale is more than adequate at each attained age to cover the expected benefits and associated expenses. In this case a pure market value analysis of the block would result in a negative direct market value of liabilities assuming that future premiums are considered as negative cash flows to be combined with the benefit and expense positive cash flows. There is a contradiction as the third party market value is non-negative but the internally computed direct market value is negative.

A work-around to the contradiction posed by the negative market value of the liability is to separate the liability cash flow stream into two streams: the benefit and expense stream; and the premium stream. Add the present value of the premium stream to the asset side of the basic accounting equation and define the direct market value of the liability as just the present value of only the benefit and expense cash flows, i.e. only the liability cash out-flows. (This technique can be applied to the general case of any liability having renewal premiums. Such renewal premiums must be considered if all the embedded options in a liability are to be valued.) The resulting market value of the liability is now a positive number.

Even if this separation is made it is extremely unlikely that the actual third party market value of the policy would equal the internally computed direct market value based on the insurance company's best estimate of future mortality and expense, the collection of arbitrage free interest rate paths used and any of the choices for spread-to-Treasuries considered earlier. The only way that the internally computed direct market value will equal the third party market value is if the spread-to-Treasuries used in the computation is explicitly solved for so that the computed value equals the third party market value. (This is the same situation that exists with fixed income securities where the market price is used to calibrate the model.) This spread will be volatile and will not equal, other than by chance, any of the types of spreads considered above. This means that, in general, if a real third party market exists, then the internally computed direct market value derived using conventional choices for the spread will not equal the third party market value. (Spreads are important because, as there is no third party market, they are the likely candidates for actual use.)

The argument for the convergence of the two methods was that given a real third party market, then the indirect method market value would change to equal the third party market value, which implicitly assumes that the internally computed direct market value equals the third party market value if the two methods are to converge. But the exposition above demonstrates that this implicit assumption is not necessarily valid.

The scheme of separating the liability cash flows used above creates another layer of complexity in direct market value computations. In the example given let the liability cash flows be so separated. Now consider the computation of the liability spread as defined earlier for the benefit and expense cash flows. All these future liability cash flows are positive. The initial cash flow equals the initial premium less external acquisition costs (commissions and overriding commissions) and internal acquisition costs (underwriting and issue). This initial cash flow is negative for term policies issued today. What spread can be added to the Treasuries such that the present value of the positive liability cash flows equals this negative number? If there are problems such as these, then there are problems with market values and derived duration and convexity computations as well. One might argue that the above example is not conclusive as it involves a pure insurance product, i.e. one without investment characteristics. But a similar example could be constructed using cash value life insurance.

For products with renewal premiums the separation of liability cash flows poses the added problem of adjudicating the appropriate duration of assets, especially in the early durations of a block of business. This is due to the fact that while the duration of liabilities is based on outflows only, the duration of assets will depend on the mix of real assets and the 'virtual' assets representing future premium payments. For a considerable period of time the virtual assets will dominate the real; thus the duration of assets will be dominated by the duration of virtual assets. One could initially invest in assets with significantly different duration characteristics, but the duration of the virtual assets would overwhelm the final computation. This creates problems in deciding how to invest and makes rebalancing decisions more difficult as the actual situation only unfolds over time and prior decisions possibly may have been inappropriate as they were based on assets dominated by virtual assets.

Situations described above also lead to unusual durations for life insurance during the early years. This and the above issues may indicate the strain of extrapolating too far the analogy of insurance liabilities with fixed income securities. OAVDE analysis is not susceptible to the problems here or noted earlier.

Sixth, Michael S. Smith, FSA has shown that the present value of distributable earnings can not be optimized without knowing the investment income earned by the insurance enterprise. This fact and the fact that a pure market value analysis does not provide information on the future investment income (because it is a time 0 computation and not a simulation of future performance) imply that it is unlikely that decisions would both optimize and/or immunize the market value of surplus and optimize and/or immunize OAVDE to the same value. Essentially, the same is true for the modified market value analysis. Recall that for the examples in Section IV the market values of surplus do not equal the OAVDE values.

Seventh, the direct method seems simple and compelling, a straightforward analogy to the market for fixed income securities. But it does so at the expense of implicitly assuming that all insurance liabilities are identical in all ways to bonds, e.g. legal, regulatory, taxation to both the policy owner and company, etc. and that one can ignore the capital requirements of insurance enterprises. The market for fixed income securities provides prices which are used to calibrate the option pricing models. Unless a real deep and liquid third party market for individual insurance liabilities exists which would allow the calibration of the model, it is not possible to test how far the analogy can be meaningfully extended and there can be no unambiguous definition for the market value (or the 'fair' value) of a liability. In short, no market, then no market values.

Applying finance and option pricing theory directly to liabilities is a powerful and useful analogy; but it is not a completely valid analogy for insurance enterprises. The further the analogy is extended the more speculative are the conclusions. What appears very attractive about market value analysis is that it seems to provide an elegantly simple way to decompose liabilities and assets to provide guidance on how to invest for those liabilities to minimize interest rate risk. While market value analysis does provide insight as to the risk posture, it does not provide all the insight into the risk/reward posture of the value of the firm that OAVDE analysis does. This is due to the omission from market value analysis of significant economic variables and their timing that affects the value of the insurance enterprise. Similarly, total return measures based on market value analysis fall short of the mark.

Eighth, this discussion is predicated on the assumption of the existence of the deep and liquid third party market. For a theoretical discussion this is satisfactory when subject to the limitations in market value analyses and the caveat at the beginning of the prior paragraph. But how reasonable is this assumption? The majority of insurance liabilities are sold to individuals. In nearly all cases the owners are the insurers or others who have a financial dependency on the insured; and the products are purchased to meet a particular need which may have little relevance to another individual. What would be the motivation for the widespread trading of such liabilities? These products have favorable federal income tax treatment. This treatment may be lost upon sale. The income tax free status of the death benefit to the beneficiary of a life insurance policy is often lost in transfers for value. State laws and regulations impede the development of such a market due to insurable interest and public policy requirements. Tax qualified products and SEC registered products create additional problems. Deferred annuities, which would have the least problems in terms of a third party market, rarely have embedded options that are not available in newly issued products. New and innovative products from one company are quickly replicated by others.

The investment aspect of most general account insurance liabilities may be characterized as having a put option at book value, a floor crediting rate, perhaps premium flexibility and a floating credited rate not tied to any index, but set at the discretion of the insurance enterprise. Fixed income securities in active markets do not resemble these. Note that there is no similar third party market for certificates of deposit, savings accounts and other non insurance savings vehicles for which there are fewer problems to overcome. Similarly, there is no third party market for mutual fund shares except closed end funds which were designed with that in mind.

While thin, there is a market for the buying and selling of insurance companies and blocks of business. Also, there is a large number of stock life insurance companies for which it is possible to estimate costs of capital. It is then possible to create a reasonable proxy for the cost of capital for stock insurance enterprises overall and this can be used as the benchmark for OAVDE computations for all insurance enterprises.

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Comments on *'The value of the firm: the option-adjusted value of distributable earnings'*

Introduction

I believe that the subject of this conference is important, and I am especially pleased to see the growing collaboration and cross fertilization between professionals in the actuarial and finance/investment fields. About 7 or 8 years ago, when working on my dissertation at the Wharton School, I was invited to spend a summer consulting with the investment management group of a major New York based property-liability insurance company on improving their asset-liability management (with the emphasis on understanding better the liability side of the balance sheet). One of my first requests was to meet with the actuaries in order to see how they were managing and pricing risk and how they were determining the technical reserves and the expected timing of the liability cash flows. I was told that the actuarial and investment side of the company did not communicate; they didn't know each other and didn't speak the same language. I almost hesitate to admit that by the time I left at the end of the summer, I had never met with the actuaries. The two sides of the insurer continued to operate independently. I spent the time applying some financial concepts to some basic actuarial models for generating aggregate claims estimates. While interesting and useful in my dissertation, I remain convinced that the process could have been more valuable and more efficient if the two sides of the business had been able to communicate effectively. I understand that the degree of communications has traditionally been better in the life insurance industry, but it is heartening to attend a conference of the Society of Actuaries and note that the critical issues of finance and actuarial science are coming together.

I very much enjoyed reading through David Becker's article (which is important as I found that I had to go back and read it several times as there is a lot of information contained therein). The paper makes a positive contribution to understanding some very difficult financial management issues and has much to offer as a first step to resolving some important problems that must be addressed by an insurer's management. The issues are adroitly and comprehensively addressed. I would encourage you to look carefully at the paper. Indeed, if for nothing more, the illustrations of how the proposed methodology can be applied to some very specific management problems are, in and of themselves, worth careful analysis.

While I believe that the paper merits careful attention, I do nonetheless believe that it could be improved with some reorganization. Indeed, one of the suggestions that I would make is that Mr. Becker should split the paper into two or three separate papers. Some of the impact of what he is trying to say may be lost by his trying to cover too wide a variety of subjects. In addition, Mr. Becker has also incorporated several brief reviews of the central tenants of financial economics, including option pricing, the concept of arbitrage, and duration/convexity analysis. Possibly some of these have been included under the assumption that the reader is not familiar with these concepts, but I found that they more often interferred with the core development of the paper. Most of these asides seemed too brief to provide any real value to someone not already familiar with the subjects, and the occasional technical flaws serve to distract the reader. I would be interested in knowing whether this discomfort is shared by those with less formal training in financial economics, for I am not certain how to best handle the need for cross-fertilization between the fields of finance and actuarial science, other than to continue the process of educating insurance and actuarial professionals regarding the central tenants of finance (and vice versa).

The paper can be split into four major sections. Mr. Becker provides a good review of the evolution of the accounting treatment of insurance assets and liabilities (and therefore surplus) and the lingering limitations of the accounting treatment; suggests a methodology for estimating the market value of both assets and liabilities (evaluating the strengths and weaknesses of several approaches); extends the suggested approach of valuing assets and liabilities to provide direct estimates of the value investors would place on a firm (the OAVDE noted in the title); and provides some very useful illustrations of how this methodology can be incorporated into an insurance firm's financial management.

I will concentrate my remarks on the last three of these areas. I have little to add to the discussion of the recent developments in both statutory and GAAP/FASB accounting for investment income and changes in liabilities. As a trained economist, I have a fairly strong bias against the use of accounting systems which do not reflect market values. Worse still, the accounting treatment of insurance assets and liabilities provides neither consistency nor transparency. Thus, they are of little use to management, consumers, regulators, or investors. While, as Mr. Becker points out, there have been some improvements, the whole system leaves much to be desired. I am convinced that the only solution is the reflection of actual market values (or the best estimates thereof) along lines such as those outlined in the paper.

Basic Model

Let me turn to the basic formulation of the article. Becker notes that in an arbitrage-free world (and I would add in a world without information or transactions costs), we can price any asset by its risk-neutral value as follows:

$$\mathrm{MVA}(i_0) = \sum_{p=1}^{P} \varphi_p \left[\sum_{t=1}^{N} \frac{\mathrm{ACF}_{p,t}}{\sum_{j=1}^{t-1} (1 + r_{p,j} + \mathrm{OAS}_A)} \right]$$

where

$\mathrm{MVA}(i_0)$ = market value of asset with a given term structure at the time of valuation

i_t = index of term structure at time t ($t = 0$ is the time of valuation)

p = index of probability state (ranging from 1 to P)

φ_p = probability of state p occurring

t = index for time (ranging up to N = final maturity)

j = index for payment (within t)

$\mathrm{ACF}_{p,t}$ = asset cash flow (at t, given state p)

$r_{p,t}$ = riskless interest rate (at t, given state p)

OAS_A = option adjusted spread on asset (adjustment for risk)

A similar equation can be created for the value of liabilities (MVL and OAS_L substituted for MVA and OAS_A). Likewise, Becker creates a similar equation for the value of the firm. However, rather than using the traditional definition of MVE = MVA − MVL, given the legal restrictions of free cash flow, he looks at the value of the firm (or of a book of business) as the market value of the distributable cash flow. In the latter, MVDE or OAVDE and OAS_{DE} are substituted for MVA and OAS_A). I will return to this point later. The nomenclature is a bit complicated, but the idea is clear. Indeed, these various models can all be classified as slightly simplified versions of the Arrow-Debreu model of state-contingent payoffs, only rather than having securities with state-dependent payoffs (one per time/state combination), complex assets are allowed to span the different states.

Basic Model: Information Requirements and Underlying Assumptions

While the above equation is intuitively pleasing and not incorrect for a theoretical point of view, it is important to take a close look at its information requirements and underlying assumptions. In order to directly solve the equation and obtain the risk-neutral estimate of the market value, the analyst must be able to (a) list each and every possible path and the interest rates associated with each path; (b) determine the probability of each path; (c) evaluate the cash flows associated with each point of time on each path; and (d) identify the fixed option adjusted spread that takes account of the risks inherent in the asset. I have to question whether this can be done to a degree that we can feel comfortable with the analysis.

I don't mean just to bring up the standard criticism of the Arrow-Debreu state-contingent models that it is impossible to name all the states. The structure merely requires that we are able to do so in a probabilistic sense. Thus, the problem at hand is not the listing of paths and their probabilities, but the identification of a stochastic process that will generate the paths and probabilities. We would then test these against the riskless government term structure to test if the model fits. There are a number of mathematical techniques for generating these paths and probabilities. The simplest is to assume that volatility = 0 and that the future looks just as today's term structure would predict. This naive model will properly price all government bonds, but it is tautological as it is asked to price the very instruments that were used to construct it (and would assign a zero value to all options on these riskless assets.) A more complex process, using historical or implicit volatilities is needed. Nevertheless, just as the naive model would properly value government bonds, we can create a stochastic process that may be correct today but which is incapable of predicting the future.

If we are to use these kinds of models, we need to be very careful regarding the robustness of the models. We have to ask whether the process being used has been able to solve the valuation problem for a large number of time periods. Moreover, since we are talking about relatively long-term assets and liabilities, we need to test the stability of the process. Is it sensitive to exogenous events (oil shocks, changes in inflationary expectations, changes in Fed policy, judicial rulings on insurer liability, etc.)? If not, the model will add a great deal of mathematical sophistication without adding economic understanding. If we are looking at consensus prices to test the model, does this mean that all market participants have the same consensus estimates of the paths and probabilities? If not, it is more likely that the process will not prove stable. Moreover, as Barr Rosenberg pointed out early in the development of the far simpler capital asset pricing model, when we need to rely on consensus, the consensus itself often leads to the model collapsing on itself.

Even if we are able to accept that the stochastic process is stable (at least within a short period of time), can we really assume that the OAS is constant across all paths and independent of the path? If not, the valuation that we obtain today will have value once the economy starts moving on a specific path or as volatility changes. Also, the problem is far more complicated as we move from the asset to the liability side of the insurer. We are only able to test the OAS on the asset side of the insurer. Here we can argue that we are looking at consensus prices, but this is far more difficult on the liability side, where information is more limited and markets less perfect. The low level of liquidity in secondary insurance markets will severely restrict the ability of the model to gage the liability OAS.

I am certain that one can find an appropriate stochastic process, undertake the needed empirical tests, and make the model work by incorporating a series of ongoing adjustments. I worry, however, that this kind of constant readjustment does not generate confidence. Rather, it is likely to result in the creation of a 'black box' type of model that few understand or appreciate. I recall when starting my career as a banker that we had a black box consisting of a duration-adjusted minimum spread that was used to generate prices on our international lending portfolio. As far as I could tell, none of the account officers understood the purpose of the black box or what it was trying to accomplish. It was just one more bureaucratic step that was needed in the approval process. No one bothered to explain duration or how the spread adjust for duration, and the account officers were unable to incorporate a risk adjustment into the model (which resulted in the bank taking a much higher exposure than was prudent in several high-risk developing countries). I firmly believe that it is not enough for the actuarial staff or investment staff to understand the model, it is critical that it can be explained to other decision makers.

Before going on, I would like to note that while I am skeptical about the implementation of the model due to the instability of the stochastic processes, especially those on the liability side of the insurer, this does not mean that the appropriate application of the model cannot be used to generate a better understanding of the problems that are faces in asset-liability management. This is especially so when the asset and liability portfolios incorporate significant amounts of option like characteristics. The fourth main section of David Becker's paper is an ideal example of how the model can be used as a tool to identify key sensitivities and present a graphical indication of the potential results of a specific strategy. I suspect that actuaries and investment professionals often think in terms of distributions of values rather than in terms of fixed values. The application of the OAVDE model to the decision of the appropriate investment strategy, the appropriate commitment of resources, etc., is welcome. I would like to see the resulting distribution printed rather than just the high-low graph (showing minimum, maximum, average, and OAVDE values), although these are far better than giving a single number.

OAVDE and the Role of Capital Structure

It is important to note that the capital structure of an insurer is just as important as asset risk in determining the risk charge that should be incorporated into the liabilities. It is useful to model the insurer as a financial intermediary. Equity holders invest equity. The insurer then effectively borrows by issuing insurance policies which incorporate an implicit interest rate). Both equity and net premium are invested. The insurer is required to used these assets to pay all losses to the policyholders and all remaining assets belong to the equity holders. Let me look at a simplified version of this structure in Figure 1.

Since the assets are risky, there is some risk that the policyholders will not be paid in full. Thus, their payoff looks like a 45 degree line between the origin and the level of losses. Above this level, policyholders are paid in full. The equity holders have a zero return if assets are worth less than the level of policyholder claims but receives the difference between the asset value of policyholder claims when assets are above this level (the 45 degree line from the face value of policyholder claims). The equity holders, in effect, own an option on the value of the assets with a striking price equal to the face value of the policyholder claim.

As the value of an option increases with volatility, the equity holders have the incentive to increase the overall riskiness of assets. However, if risk is high, policyholders will pay less for their claim (effectively increasing the interest rate paid by the insurer on their premium). Policyholders (or the regulators that represent their interests) will also try to limit any unilateral increase in risk once the policies are written, by insisting that a higher capital cushion is kept. If insurance contracts were riskless (because of some kind of government insurance), the OAS spread on insurance liabilities could be set to zero. Nevertheless, insurance is not riskless, and the implicit spread demanded by the insured should therefore be the number incorporated into the oas analysis. This may have nothing to do with the risk on the assets, as very low-risk assets with very high leverage can increase the risk to the policyholders, while risky assets combined with high levels of capital may result in a low level of policyholder risk. The paper notes the problem with obtaining an estimate of this spread, but I believe that more effort should be directed in this area.

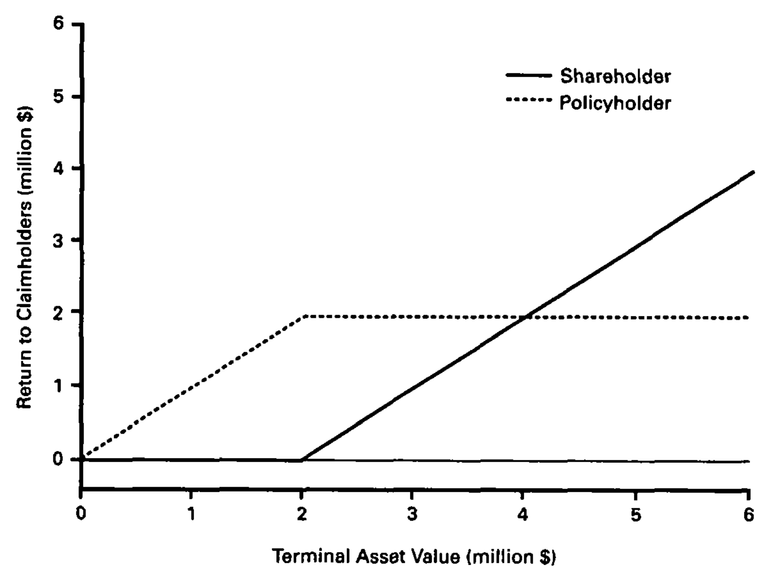

FIGURE 1. THE INSURANCE CONTRACT AS A SET OF OPTIONS. CLAIM AGAINST ASSETS OF INSURER: POLICYHOLDERS VS. SHAREHOLDER.

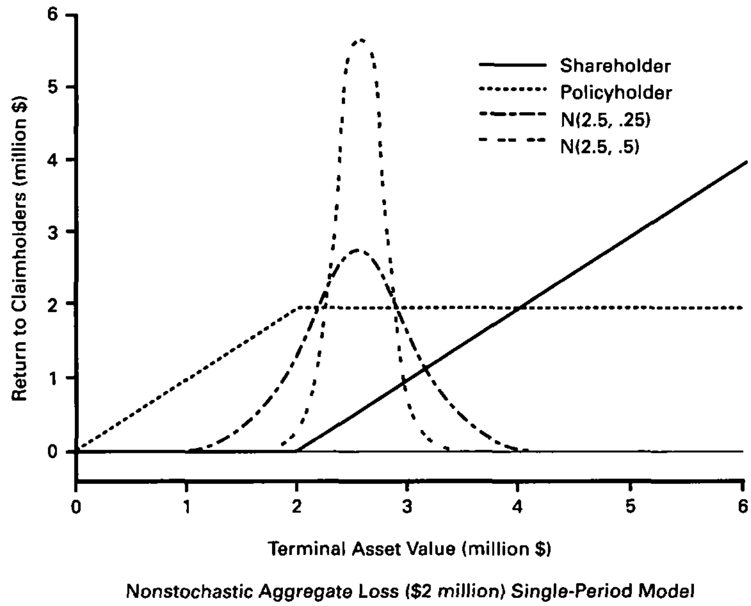

FIGURE 2. THE EFFECT OF CHANGING ASSET VOLATILITY. CLAIM AGAINST ASSETS OF INSURER: POLICYHOLDERS VS. SHAREHOLDER.

Modeling of Behavioral Variables in OAVDE

The final area that I would like to comment on is the importance of incorporating the behavioral variables into the sensitivity analysis. This was briefly noted by Mr. Becker as one of the difficulties in using and OAVDE-like analysis, but I think that more emphasis needs to be placed on this area. One of the greatest problems involved in the pricing of mortgage-backed securities is accounting for prepayment behavior. Insurance policies are full of these kind of behavioral variables (lapse, exercising borrowing options, etc.) that have significant impacts on the value of the business to the insurer. A better appreciation for the impact of these kind of variables needs to be incorporated into the analysis. The OAVDE structure may also be important in determining methods to reduce the volatility of policyholder behavior.

Conclusion

Let me end at this point after congratulating Mr. Becker on his thorough analysis. I believe that the path he is on will continue to lead to important insights into the management of risk.