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The Frequency of Inversions of the Yield Curve and Historical Data on the Volatility and Level of Interest Rates

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This article presents some observations on inverted yield curves of U.S. Treasury securities and the correlation of volatility of Treasury security interest rates with their absolute level.

The time covered by the study is December 1953 to December 1988. This period provides 421 yield curves covering several business cycles and a range of economic and political adversity.

The phrase "yield curve" is used to mean yield as a function of maturity for yields on three- and six-month discount Treasury securities and coupon Treasury securities of maturity 1 to 30 years. The observations are based on monthly yield data from two sources: Federal Reserve Board publications (FRB) and *The Analytical Record of Yields and Yield Spreads* (SAL) published by Salomon Brothers. Three- and six-month discount rates for FRB data are converted to bond-equivalent yields; Salomon Brothers data are already in that format.

Inverted Yield Curves

The financial impact on interest-sensitive insurance liabilities caused by inverted yield curves will be determined by both the level of the inversion and the duration of the inversion. A yield curve has an "inversion level of *X* percent" if for that yield curve the bond-equivalent yield on a three-month Treasury security exceeds *X* percent of the bond-equivalent yield on a 10-year Treasury security. Table 1 shows both the number of months and the percentage of total months under study that the yield curve was inverted for various inversion levels.

Define an "inverted yield curve" as one whose inversion level equals or exceeds 105 percent. According to this definition, the yield curve was inverted less than 13 percent of the time covered by the study. Note that those months for which the yield curve is inverted are not identical for both sets of data, but they are nearly so. These differences arise from the different ways in which FRB and Salomon Brothers derive their yield curve data.

Table 2 presents information on those months for which the yield curve was inverted. To deal with sporadic inversions, two conventions are adopted. First, a period of inversion lasting less than five months is ignored. Second, periods of inversion separated by no more than six months are grouped together as a single period. During periods of inversion, only those months for which the yield curve is actually inverted are counted.

The yield curve did not remain inverted for the entire period between the listed dates. The yield curve was not inverted for September and October 1969 (period 2). The yield curve was not inverted in the month of July 1974 (period 3). The yield curve was not inverted during the months of May 1980 to October 1980 (period

4). Period 4 was not split into two periods because of the economic environment during period 4.

TABLE 1
NUMBER AND PERCENTAGE OF INVERSIONS FROM 12/53 TO 12/88 BASED ON THREE-MONTH AND TEN-YEAR MATURITIES

Inversion Level	FRB		SAL	
	Number of Months	Percentage	Number of Months	Percentage
>100%	75	17.8%	75	17.8%
>105	53	12.6	54	12.8
>110	29	6.9	30	7.1
>115	20	4.8	14	3.3
>120	10	2.4	5	1.2
>125	5	1.2	0	0.0
>130	3	0.7	0	0.0

TABLE 2
PERIODS OF INVERSION FROM 12/53 TO 12/88 (FRB) BASED ON THREE-MONTH AND TEN-YEAR MATURITIES

Period No.	Number of Months Actually Inverted	Dates (Initial Month to Final Month)
1	5	September 1966 to January 1967
2	5	July 1969 to January 1970
3	15	June 1973 to September 1974
4	27	December 1978 to August 1981

TABLE 2A
PERIODS OF INVERSION FROM 12/53 TO 12/88 (SAL) BASED ON 1-YEAR AND 30-YEAR MATURITIES

Period No.	Number of Months Actually Inverted	Dates (Initial Month to Final Month)
1	9	January 1966 to January 1967
2	17	May 1968 to May 1970
3	11	July 1973 to September 1974
4	28	November 1978 to October 1981

TABLE 2B
PERIODS OF INVERSION FROM 12/53 TO 12/88 (SAL) BASED ON 1-YEAR AND 10-YEAR MATURITIES

Period No.	Number of Months Actually Inverted	Dates (Initial Month to Final Month)
1	5	January 1966 to January 1967
2	10	January 1969 to February 1970
3	15	July 1973 to November 1974
4	29	November 1978 to October 1981

Observations on the Economic and Political Climates

It is worthwhile to examine the eco-nomic and political climates during each of these periods. In June 1966 the FRB raised its reserve requirement from 4 percent to 5 percent. Vietnam War costs escalated sharply by the second quarter accompanied by increased business and personal spending and large federal borrowing. In August 1966 the FRB raised the reserve requirement from 5 percent to 6 percent. Medicare became effective.

A period of industrial expansion occurred between the end of 1966 and 1968. The inversion ceased, and the FRB lowered the discount rate. But by the end of 1968 the FRB raised the discount rate from 5.25 percent to 5.5 percent. Defense spending rose to $70 billion from $50 billion in 1966. In February 1969, inflation resurged. In May 1969, the FRB raised the discount rate to 6 percent and increased the reserve requirement by 0.5 percent. The scope of the war was expanded.

In 1973 oil prices tripled due to the Arab oil embargo and the Middle East War occurred.

The late 1970s were characterized by historically high levels of inflation, the deregulation of interest rates, and the introduction of new fixed-income investment opportunities. In late 1978 the FRB attacked inflation by changing its monetary policy from one of managing interest rates to managing the money supply. The money supply was allowed to increase in step only with real growth in the economy. In 1979 oil prices again tripled, U.S. hostages were taken in Iran, and the Federal Government bailed out Chrysler.

It is instructive to consider other maturities in assessing yield curve inversions. Tables 2A and 2B present information for 1-year/30-year and 1-year/10-year maturities, respectively, based on Salomon data. By using the 1-year/30-year definition, the yield curve was inverted 15.4 percent of the possible months. By using the 1-year/10-year definition, the yield curve was inverted 14.0 percent of the possible months. The actual number of months and the periods of inversion for each alternate definition are shown in Tables 2A and 2B.

Under all the definitions:

- Yield curves were inverted a relatively modest percentage of the time;
- The maximum duration of inversion was less than two and one half years, with an average duration of 15 months;
- Essentially the same periods of inversion were identified; and
- Both economic stress and political instability and changes in U.S. policy were present.

With due caution that the future will not necessarily duplicate the past and that the 35-year period is only one possible realization of interest rates, there are several implications from this information. First, lengthy, material inversions of the yield curve seem to be principally associated with political events. Most models of interest-rate movement do not reflect this. Second, the historical data presented earlier may be useful for assessing the "reasonableness" of a set of future interest-rate scenarios. Both the frequency and the duration of inversions should be considered within the scenarios. Third, the interest-rate path generators for option-pricing models should be examined to determine the frequency and duration of inversions that occur. For example, a two-factor path generator using historic volatility for the short and long rates can produce inversions with substan-tially greater frequency than found in the historic data when the initial yield curve is both positively and shallowly sloped. Lattice models that impose yield curve structures should also be considered.

Correlation of Volatility and Level of Interest Rates

Table 3 presents statistical information on yields during periods 1 through 4 based on FRB data. Note that the statistical values are based on only those months for which the yield curve satisfied the definition of inversion. Any months that occurred during each period in which the yield curve was not inverted were excluded.

"Rough" empirical relationships can be discerned from the data within this table. One such relationship is that for a given maturity the standard deviation of the distribution of rates increases with the level of the average of the rates. (This is the case during periods in which the yield curve is inverted. Note that period 2 does not conform for maturities of 3, 5, and 10 years.) This suggests that the volatility of interest rates is higher when the level of rates is higher.

Definitions of Volatility

The concept of volatility of interest rates has to do with the change in the rates. As used in some stochastic option-pricing models, volatility is the standard deviation of the natural logarithm of the ratio of the interest rates (for a given maturity) for successive time periods. This definition can be described algebraically, as follows:

Let i_t and i_{t+1} denote the interest rates for a given maturity in time periods t and $t+1$, respectively. A process often assumed is that interest rates change in the following manner:

$$i_{t+1} = i_t \times e^{m+sZ},$$

where m is the drift in interest rates for period t, s is the standard deviation (volatility) during period t, and Z is a unit normal random variate.

If there were a close and reliable relationship between volatility and the level of the interest rate, then that would be a valuable item of information in the construction of models of interest-rate movement. In fact, many models incorporate a form of this assumption into their schemes. The basic assumption is that volatility is proportional to i or the square root of i, with the constant of proportionality being the standard deviation. Sometimes these assumptions are incorpor-ated to make the process mathematically tractable for solving the partial differential equation of the process.

For a given maturity, consider the natural logarithms of the ratios i_{t+1}/i_t for the collection of 421 yield curves. For each calendar-year's log ratios, calculate their standard deviation (volatility). For each calendar year, calculate the average rate for that year. Then test the null hypothesis that the correlation between the volatility and the average rate is zero at the 5 percent significance level. The purpose is to identify time periods in which the null hypothesis can be rejected; that is, the sample correlation coefficient is significantly non-zero.

Table 4 presents sets of time intervals over which the tests of null hypothesis are made. If the null hypothesis can be rejected at the 5 percent level of significance, then that value of the sample autocorrelation coefficient is shown in the table for the given time period. If there is no entry, then failure to reject the null hypothesis is indicated.

Note the counterintuitive result (negative correlation coefficient) for threemonth securities for 1954 to 1963 and for 1954 to 1968.

Table 4 indicates that periods of 20 years or longer are required for statistically significant non-zero correlation coefficients to emerge for the full range of maturities. Even here only one of the four consecutive 20-year periods shows significantly non-zero sample autocorrelation coefficients. For lesser intervals, that is, 15 years or less, the results support the null hypothesis. For 25-year periods the null hypothesis can be rejected for all maturities during two out of three periods. There is a definite tendency for the non-zero correlation coefficients to consistently emerge across maturities in periods containing very high interest rates. However, note that the magnitude of the sample correlation coefficients diminishes from the 1959–1983 period to the 1964–1988 period.

TABLE 3
INTEREST RATE STATISTICS FOR EACH PERIOD (FRB)

Statistic	3 Month	6 Month	1 Year	3 Year	5 Year	10 Year
			Period 1			
Max	5.58%	6.06%	5.82%	5.79%	5.50%	5.18%
Avg	5.35	5.62	5.38	5.35	5.17	4.95
Min	4.91	4.97	4.75	4.75	4.70	4.58
Std Dev	0.26	0.41	0.37	0.35	0.28	0.22
Range	0.67	1.09	1.07	1.04	0.80	0.60
			Period 2			
Max	8.28%	8.30%	8.17%	8.24%	8.17%	7.80%
Avg	7.69	7.95	7.86	7.72	7.54	7.20
Min	7.30	7.57	7.54	7.29	7.01	6.69
Std Dev	0.41	0.29	0.25	0.39	0.47	0.46
Range	0.98	0.73	0.63	0.95	1.16	1.11
			Period 3			
Max	9.17%	9.40%	9.36%	8.66%	8.63%	8.04%
Avg	8.33	8.44	8.10	7.52	7.43	7.24
Min	7.36	7.22	6.88	6.76	6.69	6.73
Std Dev	0.58	0.64	0.73	0.64	0.63	0.42
Range	1.81	2.18	2.48	1.90	1.94	1.31
			Period 4			
Max	17.61%	17.11%	16.72%	16.00%	15.56%	14.94%
Avg	13.08	13.02	12.69	11.76	11.50	11.28
Min	9.50	9.63	9.57	8.94	8.85	8.91
Std Dev	2.76	2.55	2.25	2.25	2.17	2.00
Range	8.11	7.48	7.15	7.06	6.71	6.03

TABLE 4
SIGNIFICANTLY NON-ZERO SAMPLE CORRELATION COEFFICIENTS

Time Interval	Maturity				
	3 Month	1 Year	3 Year	5 Year	10 Year
10-Year Periods					
1954-1963	−0.67				
1959-1968					
1964-1973					0.71
1969-1978					
1974-1983					0.71
1979-1988	0.67				
15-Year Periods					
1954-1968	−0.60				
1959-1973				0.66	0.65
1964-1978					
1969-1983				0.57	0.55
1974-1988	0.60	0.51			
20-Year Periods					
1954-1973					
1959-1978					
1964-1983	0.50	0.49	0.46	0.52	0.61
1969-1988					
25-Year Periods					
1954-1978					
1959-1983	0.50	0.51	0.49	0.54	0.64
1964-1988	0.46	0.44	0.40	0.45	0.54
30-Year Periods					
1954-1983					0.43
1959-1988	0.45	0.46	0.44	0.49	0.60
35-Year Periods					
1954-1988					0.42

When the magnitude of the statistically significant sample correlation coefficients is considered, only 20 percent to 41 percent of the variation in volatility is accounted for by variation in the level of the rates.

Three additional studies of the relation between volatility and the level of rates were performed:

- The null hypothesis was changed to test for correlation between volatility and the square root of the level of the interest rate.
- The null hypothesis of no correlation between the volatility and the level of rates that occurred one year prior to the associated volatility was examined.
- A nonparametric test of the rank order correlation between the volatility and the level of interest rates was made by using both Spearman's and Kendall's correlation coefficients.

The results were not materially different for these tests. If these latter tests had been significant over more time intervals, then it would have suggested that a stronger relationship existed than indicated by the base test. In that event we would have searched for nonlinear relations between volatility and the level of the rates and/or for significant variable(s) that, when added to the model, would increase the correlation coefficients.

These tests suggest that the two often-used assumptions for volatility are too restrictive. Volatility might have both a deterministic component, that is, a "drift" based on the level of rates, and a stochastic component.

Note that only one definition of volatility has been used here and the tests are designed around that. Other definitions and tests are possible.

Readers are invited to write to the author at his *Directory* address on other empirical results and questions on the topics presented here or on interest rates, yield curves, and term structures.