
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Thursday, April 27, 2017

Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
 - a) The morning session consists of 9 questions numbered 1 through 9.
 - b) The afternoon session consists of 6 questions numbered 10 through 15.

The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION***
Morning Session

- 1.** (8 points) You work for QRS Life Insurance Company. A liquidity risk manager at QRS has drafted a strategic framework to build stress-testing scenarios for QRS's cashable term deposit block of business in the Canadian retail and commercial market.

The cashable term deposit product has maturity terms ranging from 1 to 5 years. A term deposit may be withdrawn any time before maturity with a reduced interest rate paid to the depositor. The assets backing the liabilities of the term deposits are Canadian and US government and corporate bonds.

Your manager has requested that you review the framework drafted by the liquidity risk manager, using the ideas from Chapter 3 "Scenario Analysis and Stress Testing", from the book *Liquidity Risk Measurement and Management: A Practitioner's Guide to Global Best Practices* by Leonard Matz and Peter Neu.

One section of the framework reads as follows:

"Considerations for creating and evaluating deterministic scenarios:

- Synchronize the ordinary course of business scenario with the budget, so as to avoid the need to explain why business volume or rate forecasts are inconsistent.
- Evaluate hypothetical liquidity scenarios at the 30-day and the 5-year points.
- The scenarios should primarily be focused on changes in the S&P500 index.
- A given scenario can be reflective of two or more different stress levels."

- (a) (2 points) Explain omissions or concerns with the framework section provided.

You continue discussions with the liquidity manager.

- (b) (2 points) Critique each of the following comments from the liquidity manager.
- (i) "A financial crisis in Asia would be far away and very different from our company's liquidity risks"
 - (ii) "We should include future cash flows from management actions"
 - (iii) "We can use changes in the insurance company's rating to define stress level".

1. Continued

You are now reviewing stand-by liquidity and stress scenarios.

- (c) *(1 point)* Outline a framework that could be used to forecast the quantity of stand-by liquidity available.

You are asked to review the cashable term deposit block and develop two stress scenarios to assess any liquidity risks. The QRS chief risk officer requires all tested scenarios to be supported by detailed reasoning and justification. He has asked for a brief memo from you on these two scenarios.

- (d) *(3 points)* Outline your response.

2. (8 points) Bob and Joe are colleagues at ABC Company and were just assigned to work for you in hedge funds. Their first assignment is to invest \$100,000 (net of transaction costs) in mutual funds over their first month.

Bob invests all of his money in the ABC S&P Index Mutual Fund. The fund's strategy is to mimic the overall stock market.

Joe likes to trade funds as he feels necessary and is trying to compare his progress with Bob's over their first month.

Using a simple consumption-based model that assumes rational investors and a constant discount rate to determine the expected average return on ABC S&P Index Mutual Fund, Joe determines that Bob should be getting returns just 0.1% better than the risk-free rate. He is astonished to find out that Bob was earning rates nearly 4% above the risk-free rate.

- (a) (2 points) Identify and describe the "puzzle" or "puzzles" Joe could be facing.

You next consider having Bob and Joe work on a day trading strategy. You are aware of the following four details:

- For several weeks prior, Bob has been reviewing private information on several computer stocks.
- Bob reviews gains and losses on an hourly basis.
- Joe is very comfortable with airline stocks, but is otherwise uncomfortable with all other stock market sectors.
- Everyone that Joe has talked to recently has made profits from day trading.

- (b) (1 point) Describe a belief or preference from the field of behavioral finance that could impact their actions in day trading for each of the four details above.

You decide to implement the day trading strategy. After only an hour of trading, Bob has lost \$500 and Joe has earned \$1000. Given $\alpha = 1/2$, their utility functions are as follows:

$$U(x, z) = \begin{cases} (x+z)^\alpha & \text{for } x \geq -z \\ -K_i * [-x+z]^\alpha & \text{for } x < -z, \text{ where} \end{cases}$$

- K_i is a multiplicative factor unique to each trader i
- x = current trade opportunity's profits/(losses)
- z = prior hour's profits/(losses)

- (c) (2 points) Describe how each of $U(x, z)$, K_i , x and z helps to explain the "puzzles".

2. Continued

The next trade opportunity is for computer stocks for which, Bob and Joe have known but different values of K_i either 1.75 or 2.75.

- (d) (1 point) Explain which value of K_i more likely belongs to Bob and which more likely belongs to Joe.

Finally, you need to evaluate the expected utility for Bob and Joe given the following probabilities for the current trade's profit:

- $p(\$100) = 0.75$
- $p(-\$100) = 0.25$

- (e) (2 points) Calculate the expected utility for each of Bob and Joe for the current trade opportunity.

3. (7 points)

- (a) (1 point) List and explain the three requirements of a successful performance attribution algorithm.

You manage an investment portfolio with initial investment of 100 million. The return was 20% for the first six months of the year. After the first six months the portfolio receives an additional investment of 100 million. In the last six months of the year the return was minus 5%.

- (b) (2 points) Calculate each of the following for the portfolio:
- (i) the one-year money-weighted rate of return, and
 - (ii) the one-year time-weighted rate of return

Your performance is measured against a multi-currency benchmark.

Your analytics platform generated the following information for the portfolio:

Global Outperformance Summary (bps)	
FX Allocation and Hedging	X
Local Market Allocation	-8.7
Local Market Management	19.1

FX Outperformance Breakdown							
Currency	FX Rate Begin	FX Rate End	Avg Depo Rate(bps)	FX + Depo Rate(bps)	Excess over Base Currency Depo (bps)	Portfolio Weight	Benchmark Weight
USD	1.0000	1.0000	0.23	2.1	0	45.6	45.6
EUR	1.3028	1.2701	0.28	-242.1	-244.1	23.1	28.1
JPY	0.0115	0.0119	0.12	321.5	319.4	25.5	20.5
GBP	1.5661	1.5369	0.57	-181.5	-183.5	5.8	5.8
Total						100	100

3. Continued

You assume that the net exposure of the portfolio doesn't fluctuate over the attribution period used by the model. You also assume that the interaction effect between FX and local return is zero.

- (c) (3 points) Estimate each of the following:
 - (i) the outperformance contribution of GBP
 - (ii) the outperformance contribution of JPY
 - (iii) the total FX outperformance contribution
- (d) (1 point) Assess whether your FX views reflected in portfolio weights were correct.

4. (5 points) Bank ABC offered the following loan terms to Bob:

- Loan amount is 200,000
- Interest on the loan is 5% per annum
- Repayment is a lump sum of all interest plus principal at the end of two years

Assume the following:

- The discount rate is 5% per annum
- Probability of default in any given year is 3%, given survival to the beginning of that year
- Default only occurs at the end of a year
- 100% of any principal and interest is lost in the event of a default

(a) (2 points) Calculate the present value of expected loss at time 0.

(b) (2 points) Calculate the present value of unexpected loss at time 0.

During the 2008 financial crisis, a record number of home loans entered default. When a mortgage defaults, the lender will seize the property and try to sell it on the market.

(c) (1 point) Discuss the relationship between probably of default (PD) and loss given default (LGD) with respect to a residential mortgage loan.

5. (6 points) You have been asked to investigate the volatility of the instantaneous forward rate modeled by a G2++ model:

$$\begin{aligned}r(t) &= x(t) + y(t) + \varphi(t) \\ dx(t) &= -ax(t) dt + \sigma dW_1(t) \\ dy(t) &= -by(t) dt + \eta dW_2(t) \\ dW_1(t)dW_2(t) &= \rho dt\end{aligned}$$

The market discount function is $P^M(t, T) = A(t, T) \exp\{-B(a, t, T)x(t) - B(b, t, T)y(t)\}$

where $B(a, t, T) = \frac{1 - e^{-z(T-t)}}{z}$

- (a) (1 point) Describe the advantages of G2++ over the one-factor Hull-White model.
- (b) (1 point) Write down the formula for the instantaneous forward rate at time t for maturity time T in terms of A and B .
- (c) (3 points) Derive the absolute volatility of the instantaneous forward rate.
- (d) (1 point) Explain the relationship between the correlation ρ and the possibility of achieving a humped shape of the volatility structure derived in (c).

6. (6 points) Your company is concerned about its credit exposure to the communications and technology sector and is looking to hedge the counterparty risk to this sector.

(a) (1 point) Identify and describe three counterparty credit risk mitigants.

The investment area provides you with information on the following derivatives:

Security ID	Counterparty	Security Type	Present Value of Future Cash Flows	Collateral Posted	Collateral Received
IRS001	Company A	Interest Rate Swap	10,000	-	5,000
IRS002	Company A	Interest Rate Swap	150,000	-	125,000
XCS001	Company A	Cross Currency Swap	-75,000	70,000	-
FUT001	Company B	Future	400,000	-	-
XCS002	Company B	Cross Currency Swap	-125,000	-	-
XCS003	Company C	Cross Currency Swap	-50,000	35,000	-
IRS003	Company C	Interest Rate Swap	75,000	-	70,000

Notes:

- Company B is a subsidiary of Company A
- Netting agreements exist across Company A and Company B combined
- There are no netting agreements with Company C

(b) (1 point) Calculate the total counterparty exposure to the above portfolio of derivatives.

6. Continued

After a number of additional derivative contracts have been added to the portfolio the Investment area has hedged the counterparty risk by purchasing a CDS to cover the next three years.

- (c) (1 point) Outline the advantages and disadvantages of using a CDS to hedge counterparty risk of a derivative portfolio.

You are given the following information about the CDS, which has been purchased:

$t =$	1	2	3
Net Exposure	400,000	-150,000	225,000
Time zero likelihood of a default event occurring at time t	3%	5%	10%
Loss given default	20%	X	Y
Zero coupon risk-free bond price	0.95	0.91	0.87

Annual premium = 3554

Upfront premium = 0

Ignore the counterparty risk of the CDS itself.

- (d) (1 point) Evaluate the sensitivity of the premium leg of the CDS to the value of X.
- (e) (2 points) Determine Y by taking into consideration the relationship between the two legs of the CDS.

7. (8 points) You are an asset portfolio manager at a small insurance company that sells term life insurance (including some whole life term products). The Chief Investment Officer (CIO) has asked you to look into a pilot program to diversify the investment portfolio by exploring a substantive infrastructure project. While limited funds are available your CIO is considering either directly investing in the project or investing in a mortgage fund.

- (a) (1 point) Identify and describe (where applicable) stylized economic characteristics that would make the infrastructure project attractive to your company.
- (b) (2 points) Recommend and justify the suitability of investing in the infrastructure project or a mortgage fund from your company's perspective.

Your company is able to purchase at a discount units of the following mortgage funds Alpha, Beta and Gamma. You are also provided the information on the average of the mortgage funds available in the market.

Mortgage Fund Characteristics	Fund Alpha	Fund Beta	Fund Gamma	Average Mortgage Fund
Expected Return	5.7%	6.3%	4.8%	5.5%
Management Fee	0.6%	0.5%	0.9%	1.3%
Loan-to-Value Ratio (LTV Ratio)	73%	85%	76%	75%
Debt-to-Income Ratio (DTI Ratio)	39%	53%	40%	40%
% of Adjustable-Rate	52%	75%	42%	55%
Spread at Origination (SATO)	2.1%	3.6%	1.4%	2.2%

7. Continued

You are concerned with the default risk in the funds and plan to build a mortgage credit risk valuation model to help you assess the default risk in each fund.

- (c) (1 point) Describe two types of models that you can combine to value mortgage credit risk.
- (d) (1 point) Identify and describe the mortgage fund characteristics that can be used as variables in your credit risk model.
- (e) (1 point) Explain why the characteristics identified in (d) are key factors in projecting mortgage defaults.
- (f) (2 points) Recommend and justify a mortgage fund to the CIO that has the lowest default risk, including commentary on the mortgage fund characteristics to support your decision.

8. (5 points)

- (a) (1 point) Explain the advantages of working with an affine term structure model.

You are given the following short rate models:

$$\text{Model 1: } \frac{dr_t}{r_t} = \alpha dt + \sigma dW_t$$

$$\text{Model 2: } dr_t = k(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t, \quad 2k\theta > \sigma^2$$

In both models the initial short rate, r_0 , is a known constant.

- (b) (0.5 points) Identify which of Model 1 and 2 is an affine term structure model.
- (c) (1 point) Determine the expected short rate at time t for Model 1.

To determine the short rate for Model 2, you take the following steps:

Let $m(t) = E[r_t]$.

- (d) (1 point) Show that $m(t) = r_0 + \int_0^t k(\theta - m(s))ds$
- (e) (1.5 points) Determine an explicit expression for $m(t)$ in terms of θ , t , k , and r_0 .
(Hint: Find and solve a differential equation for $m(t)$.)

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9. (7 points) You have been given data series for both credit spreads and the implied volatility surface. You are working with two different models and have decided to apply principal component analysis (PCA) to the data. Data has been normalized or transformed, where necessary, before the analysis.

You are given the following models:

Model 1: PCA on Credit Spread

$$V = \begin{bmatrix} 1 & L \\ L & 1 \end{bmatrix}, \text{ the matrix of correlations between the variables;}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \text{ is the } 2 \times 2 \text{ diagonal matrix of the eigenvalues of } V;$$

W is the 2×2 matrix of the eigenvectors of V .

Model 2: PCA on Implied Volatility Surface

The data set has been divided into buckets based on moneyness “ m ”:

$m < -1$
$-1 \leq m < -0.5$
$-0.5 \leq m < 0$
$0 \leq m < 0.5$
$0.5 \leq m < 1$
$m > 1$

where:

$$m = \ln \left(\frac{S_t e^{r\tau}}{K} \right)$$

S_t = Index value at time t ;

r = the risk free rate of interest (as given by treasury) ;

τ = time to maturity of options;

K = strike price;

Principal components are calculated separately within each bucket.

For each bucket, λ_1, λ_2 , and λ_3 are the eigenvalues corresponding to the principal components PC1, PC2 and PC3, respectively.

$$\lambda_1 > \lambda_2 > \lambda_3 \dots > \lambda_k.$$

9. Continued

R^2 = Proportion of the total variation explained by the corresponding principal components

PC1: Mean Level

PC2: Slope or Tilt

PC3: Curvature of implied volatility

moneyness bucket	(PC1) R^2	λ_1	(PC2) R^2	λ_2	(PC3) R^2	λ_3	PC1+PC2+PC3 R^2
$m < -1$	50%		40%		10%		100%
$-1 \leq m < -0.5$	51%		27%		12%		89%
$-0.5 \leq m < 0$	45%		24%		19%		88%
$0 \leq m < 0.5$	50%		20%		16%		86%
$0.5 \leq m < 1$	37%		25%		22%		84%
$m > 1$	63%		23%		11%		97%

- (a) (0.5 points) Explain why data may need to be transformed and normalized before the PCA analysis.
- (b) (1.5 points) Express the eigenvalues of V in terms of L for Model 1.
(Hint: The determinant $|V - (I)\lambda| = 0$ where (I) is identity matrix.)

Assume that there are $k = 3$ instruments in the $m < -1$ bucket for Model 2.

- (c) (1 point) Calculate λ_1, λ_2 , and λ_3 for the $m < -1$ bucket for Model 2.
- (d) (0.5 points) Discuss the ability of your PCA results for Model 2 to explain the total variation in implied volatility for the various moneyness buckets.
- (e) (1.5 points) Discuss the relative ability of each of PC1, PC2, and PC3 to explain variation in implied volatility for the various moneyness buckets.

Question 9 is continued on next page.

9. Continued

In addition, you are given the results for the PCA components on short and long maturity buckets.

		$-1 \leq m < -0.5$	$-0.5 \leq m < 0$	$0 \leq m < 0.5$	$0.5 \leq m < 1$
Shorter Maturity ($T < 91$ days)	PC1 R^2	38%	50%	43%	42%
	PC2 R^2	32%	40%	36%	31%
	PC3 R^2	29%	16%	21%	26%
Longer Maturity ($91 < T < 250$ days)	PC1 R^2	72%	56%	38%	41%
	PC2 R^2	16%	28%	36%	32%
	PC3 R^2	12%	16%	25%	27%

- (f) (1 point) Discuss the relative ability of each of PC1, PC2, and PC3 to explain variation in implied volatility for the various moneyness buckets, under the shorter and longer maturity buckets.

You are considering the following models for stochastic volatility of equity returns:

- CIR model
 - Jump CIR++ model
- (g) (1 point) Recommend the model that best reproduces the observed shape of the implied volatility surface across various maturities. Justify your answer.

****END OF EXAMINATION****
Morning Session

USE THIS PAGE FOR YOUR SCRATCH WORK

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