Small Talk
September 2015
Issue 44

Liability Durations for Premium Paying Products

By James Ward

Asset/liability management (ALM) has become an ever-increasing discipline within the actuarial profession. The development of sophisticated insurance products in an increasingly complex economy has necessitated an ever evolving analytical framework to measure and monitor risks being born by insurance companies. This evolution has gotten the attention of state insurance departments as well as rating agencies that have a growing interest in monitoring company ALM programs. And while smaller insurance companies are not on the forefront of product innovations driving the increased focus on ALM, they are getting swept up in the requirement to develop and enhance their ALM programs to the satisfaction of regulators and rating agencies.

While ALM covers a broad set of risks, interest rate risk is generally the most commonly addressed and monitored risk within implemented ALM disciplines primarily because it is one of the most identifiable and applicable risks facing insurance companies. Plus there exist metrics that provide (at least on the surface) quantifiable analytics that are practical to implement. The most common of these metrics is duration.

Duration is a measurement of a change in market value (price) for a change in interest rates. Most of the theoretical development and practical application of duration as a metric revolves around fixed income securities. As such, calculating the 'A' part of a duration-based ALM strategy is fairly straight forward.

Despite the increased prevalence of ALM programs, the theoretical and practical approaches to calculating duration for liabilities are undeveloped. In other words, the 'L' of ALM can be more of a challenge. The rest of this article will discuss practical approaches to calculating liability durations.

For purposes of this article, the formula used to calculate (effective) duration will be

$$D^{Eff} = (P_{-i} – P_{+i}) / (2 \times \Delta i \times P_0)$$

where,

P_0 is the average present value of liability cash flows at base interest rates,

P_{-i} is the average present value of liability cash flows with interest rates shocked down,

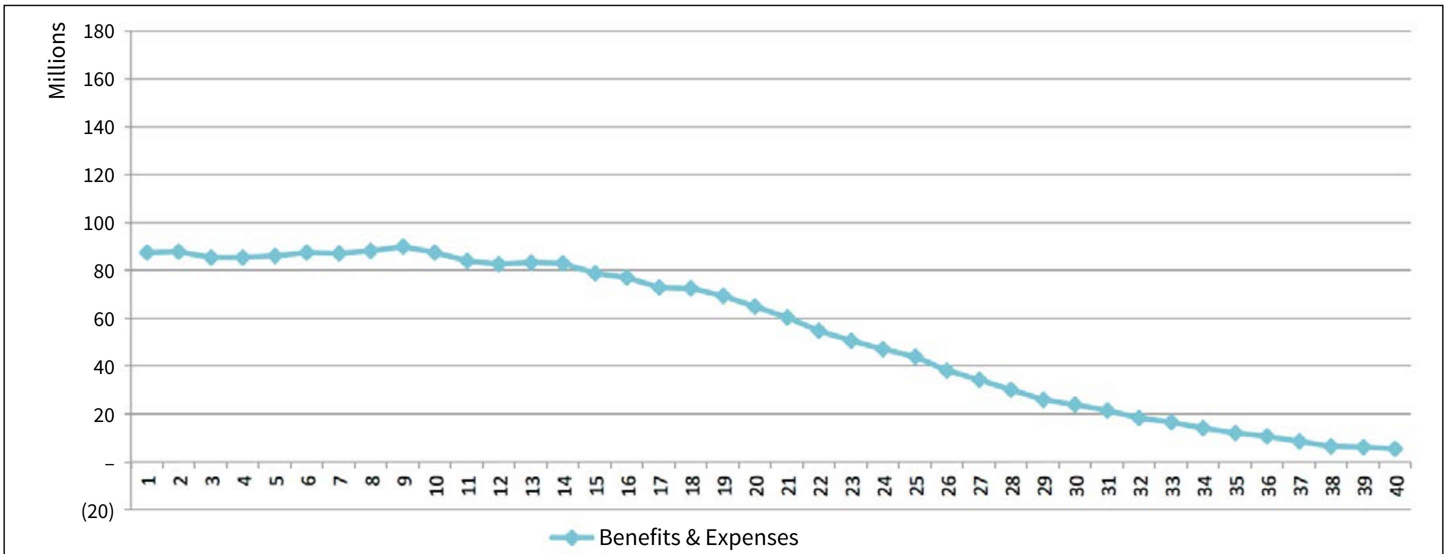
P_{+i} is the average present value of liability cash flows with interest rates shocked up, and

Δ i is the amount of the interest rate shock.

LIABILITY DURATIONS FOR SINGLE PREMIUM PRODUCTS

For illustrative purposes, the projected cash flows for a hypothetical block of deferred annuities in a payout phase are represented in figure 1.

Figure 1



PV (MM)	Base	Up	Down	Eff. Dur.
Benefits & Expenses	1,455.9	1,449.4	1,462.4	11.0

CONTINUED ON PAGE 12

The graph shows 40 years of projected benefits and expenses. The table below the graph shows the present value of those cash flows discounted at the base interest rates. We move the interest rate scenario up and down by a small increment, just 4 basis points in this case constrained by the low short-term rates in the starting yield curve. If the cash flows are interest sensitive, we would project them again and discount them at those adjusted interest rates to determine the up and down present values. Then using the formula for effective duration where we take the difference between the up and down present values, divided by the product of the base present value times two times the interest rate differential, 4 basis points, we arrive at the result of the effective duration of 11.0 as shown.

As previously stated, duration was initially developed as a metric for fixed income securities. A characteristic of these securities is that the cash flows are all in one direction. The owner of a bond expects to receive coupons and, at the maturity date, the par value of the bond. Using those expected cash flows, one can calculate the duration of the bond. And that metric also works very well for liabilities where the cash flows are all in one direction, such as a block of single premium deferred annuities or a block of payout annuities or structured settlement annuities.

LIABILITY DURATIONS FOR PRODUCTS WITH RENEWAL PREMIUMS

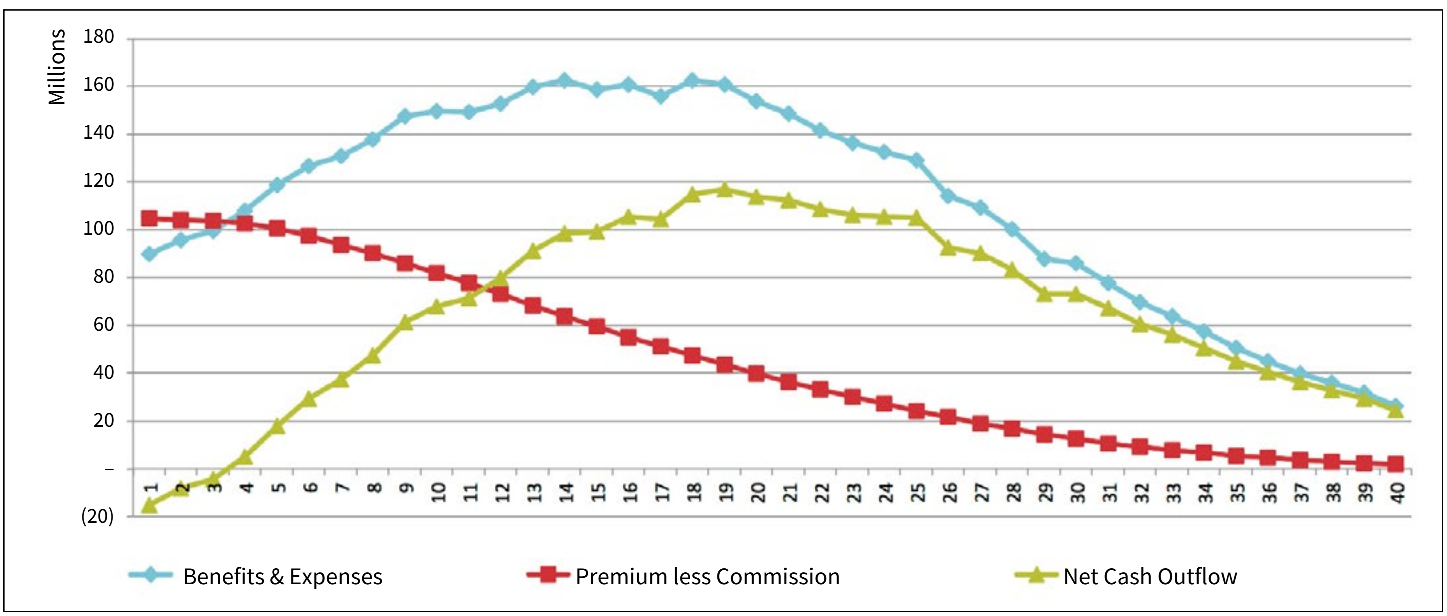
What if a liability block that a company wants to include in their ALM analysis includes renewal premiums? What happens to the duration? Well, that depends on the magnitude and timing of the premiums in relationship to the cash outflows. They may have very little impact, or their impact may be significant. Renewal premiums can reduce the present value of the net cash flows to near zero or even cause it to be negative. And since the present value of the net cash flows is in the denominator of the formula of effective duration, a present value near zero can cause the effective duration to be artificially high. If the present value of the net cash flows is negative, then the formula can produce a meaningless negative result for effective duration.

So what does one do with these blocks? There are three general approaches that are utilized:

1. Use the net cash flows as they are, and let the duration be extended.
2. Ignore the periods of net cash inflow, that is, if the net cash flow for a period is an inflow to the company, treat the cash flow as zero for that year.
3. Project the liability assuming no renewal premiums are received, which can work for a product such as a flexible premium deferred annuity where renewal premiums are not required.

An example of a liability with renewal premium characteristics would be a mature FPDA block where a significant amount of value has been accumulated and a significant amount of premium continues to be received on an annual basis. Figure 2 represents the cash flows from a hypothetical block such as this.

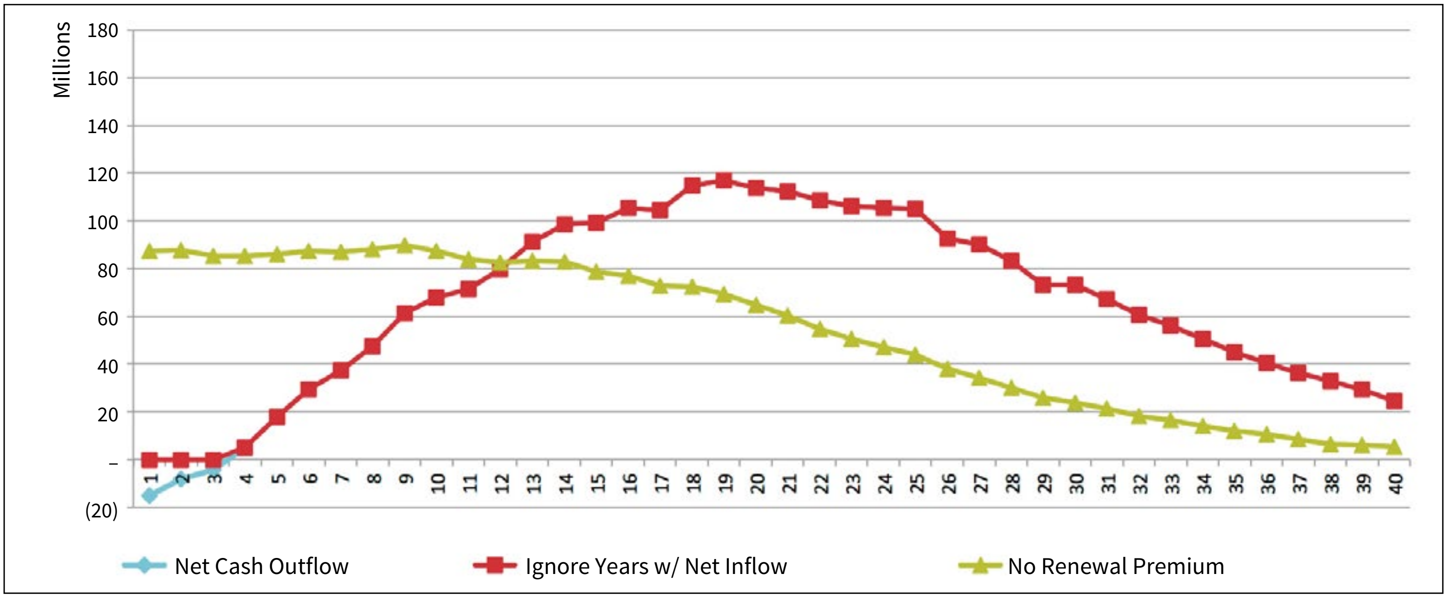
Figure 2



The line that starts highest and finishes near zero represents the projected renewal net premium, that is, premium less commission. The line that peaks highest represents the projected withdrawals and surrenders plus expenses. The line that starts as negative is the net cash outflow.

Figure 3 represents the cash flows used by each of the three approaches.

Figure 3



Note that for this particular example, there are only three years of net cash inflow (negative net cash outflow). So the result of approach two differs only a little from approach one. The third approach (no renewal premium) is based on a separate projection of the liability block assuming no renewal premiums are received. Since this is an FPDA block and premiums are not required for the liability to remain in force, such a projection is possible. Since there are no renewal premiums, the net cash flows occur relatively earlier.

The results of the three approaches are as follows:

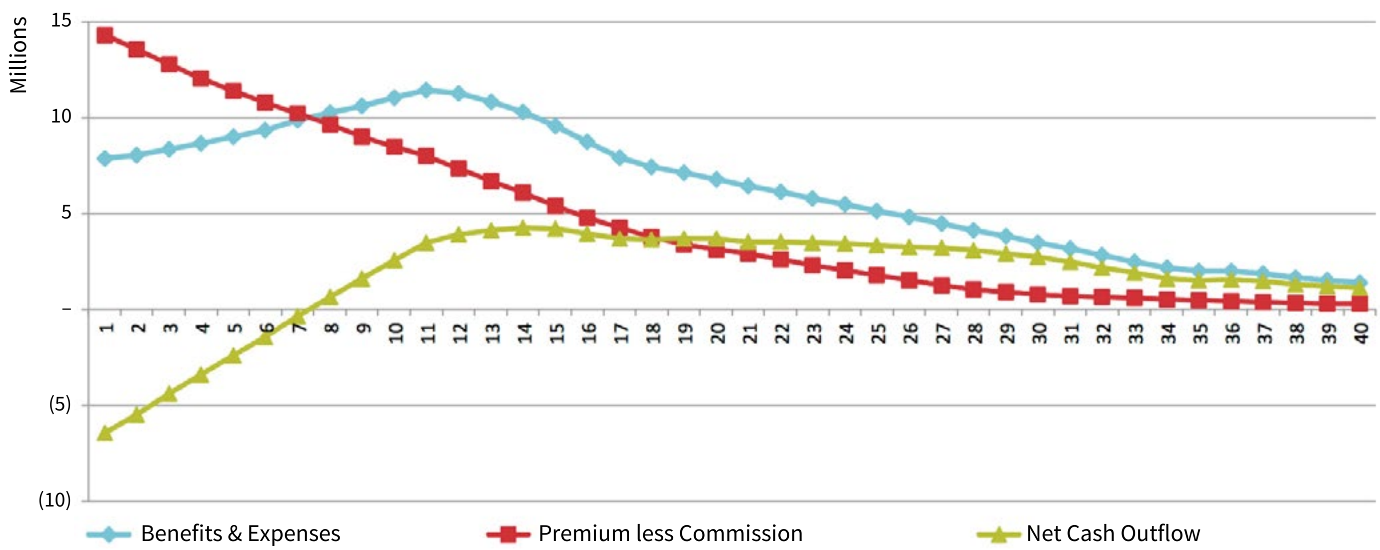
PV (MM)	Base	Up	Down	Eff. Dur.
Net Cash Outflow	1,306.3	1,296.6	1,316.2	18.4
Ignore Net Inflows	1,333.4	1,323.7	1,343.3	18.1
No Renewal Premium[1]	1,455.9	1,449.4	1,462.4	11.0

The duration on the net cash outflow in this example is 18.4. Ignoring the first three years of net inflows reduces the duration only slightly to 18.1. For approach three, since the net cash outflows occur relatively earlier with no assumed renewal premium, the duration is 11 rather than 18.

CONTINUED ON PAGE 14

While some flexible premium products like FPDA's allow the option of projecting cash flows without renewal premium, life insurance liabilities often do not. In looking to apply approaches one and two to a life insurance block, the hypothetical cash flows represented in Fugure 4 are utilized.

Figure 4



PV (MM)	Base	Up	Down	Eff. Dur.
Net Cash Outflow	22.4	22.1	22.7	35.0
Ignore Net Inflows	45.2	44.8	45.5	18.7

These cash flows might be typical of a relatively young whole life block. In this example, the inflows exceed the outflows for the first seven years.

The net cash outflows, represented by the line that is negative at the start of the projection, have a duration of 35.0. If we floor the net cash outflow at zero during the first seven years, the resulting duration is 18.7.

Ignoring years of net inflow does have an impact and reduces the duration. But the reality is that this entails ignoring what might be a significant part of the model with no theatrical justification for doing so.

The third approach is not a viable option for a typical life insurance segment where renewal premiums are required. Even when projecting a model without renewal premium is possible, as in the case with FPDA's, if this is outside of realistic actuarial expectations, is it justifiable?

AN ALTERNATE APPROACH

The two prior approaches that modify the net cash outflows ignore some aspect of the model. Rather than have an approach where something is ignored, an alternate approach is presented here that uses the information in a meaningful way. This approach is as follows:

1. Separate the cash outflows—the benefits and expenses—from the cash inflows.

2. Treat the cash inflows as if they are part of the asset portfolio of the company.

3. Use these separate components to determine a target duration for the assets that back the reserve balance for the liability segment.

Another way of saying this is that the assets backing reserves have a duration, and the premium inflows (another asset) have a duration, and the combined weighted duration should be compared to the duration of the cash outflows of the liability.

Utilizing the hypothetical life insurance block from above, where the inflows were shown separately from the outflows, the duration for each of those cash flow elements is as follows:

PV (MM)	Base	Up	Down	Eff. Dur.
Benefits & Expenses	165.5	164.7	166.3	11.7
Prem less Comm.	143.1	142.6	143.5	8.0
Net Cash Outflow	22.4	22.1	22.7	35.0

The question trying to be answered in all of this analysis is, "What is the target duration for the assets that support the reserve for this segment?" In this example, the reserve is 55.3 million.

In looking to answer that, it should be noted that the projected premiums, which have already been reduced for any commission payable upon their receipt, are available to cover cash outflows, but they are also a source of profit. As such, the entire premium inflows should not be considered as part of the asset portfolio in this analysis; thus, it is appropriate to reduce the weight of this premium asset as it is used to determine the target duration for the invested assets supporting the reserve. In other words, only enough of the present value of inflows to cover the difference between the present value of the outflows and the reserve is needed.

PV[Outflows] – Reserve = k x PV[Inflows][2]

k = (PV[Outflows] – Reserve) / PV[Inflows]

k= (165.5 – 55.1)/143.1 = 77.15%

In other words, assets backing reserves plus 77.15 percent of the present value of premiums are necessary in order to cover the benefits and expenses of the block.

With this information, it is possible to find the target duration for the assets backing the reserves:

PV[Outflows] x Dur[Outflows] – k x PV[Inflows] x Dur[Inflows] = Reserve x Target Duration

Target Duration = (165.5 x 11.7 – 0.7715 x 143.1 x 8.0) / 55.3 = 19.0

Putting all of the numbers neatly in one chart:

	PV	Eff. Dur.
Outflows (Benefits & Expenses)	165.5	11.7
Inflows (Premium less commission less profit), adj	*110.4*	8.0
Reserve and Target Duration for supporting assets	55.3	**19.0**

CONTINUED ON PAGE 16

Applying this same approach to the FPDA example presented earlier results in a target duration of 18.1. The following table summarizes the results from the various approaches:

	Liability Segment	
Duration Approach	FPDA	Life
Net Cash Outflows	18.4	35.0
Ignore Years with Net Inflows	18.1	18.7
Project without Premium	11.0	n/a
Treat Net Premium as an Asset	18.1	19.0

FINAL CONCLUSIONS AND CONSIDERATIONS

For these two example segments, this alternate approach happens to produce a result similar to that produced by ignoring years with net inflows, yet there are many advantages of this approach. It arrives at a meaningful result without ignoring what might be a material element of the liability model, and it enables the ALM professional to apply a common approach across diverse liability segments.

It should be noted that when calculating the weight of the present value of premiums to include in the asset portfolio, this approach combines present values on a market value basis with reserves on a book value basis. Care should be taken to review results for this approach.

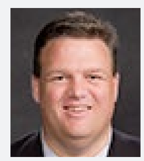
It is not clear how widely this alternate approach is utilized in the industry. The only instance where I have found a similar approach mentioned in literature is the 2007 research report, "Interest Rate Hedging on Traditional Life and Health."[3] As I have shared this approach with industry colleagues, I have found that some are already using this type of an approach, and I have found that others have been looking for a meaningful approach to calculate the duration for liability segments with renewal premiums. My goal in writing this article is to share an idea that can spur collaboration to advance ALM methods available to our industry. ■

ENDNOTES

1 An astute reader will note the graphs, present values, and Duration of the No Renewal Premium FPDA is identical to the earlier graph represented as a hypothetical deferred annuity block in payout phase.

2 This formula correlates to the standard net premium reserve formula: Reserve = PV[Benefits] – k x PV[Premium].

3 "Interest Rate Hedging on Traditional Life and Health," by Craig W. Reynolds and David Wang of Milliman, Inc., is available from the SOA: www.soa.org/research/research-projects/finance-investment/research-interest-rate-hedging-life-hlth.aspx



James T. Ward, FSA, MAAA, is vice president and life valuation actuary at American Fidelity Assurance Company in Oklahoma City, Okla. He can be reached at James.Ward@AmericanFidelity.com.