Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities

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1. Introduction

There appears to be universal agreement amongst economists and actuaries about the substantial financial benefits from payout (or immediate) annuity contracts, but the public and press have yet to embrace this risk-management instrument. Furthermore, a consensus has yet to emerge about the optimal age at which to annuitize, as well as the optimal design of the ideal payout annuity. Indeed, the global trend away from defined benefit (DB) and towards defined contribution (DC) pension plans in conjunction with exceptionally low levels of voluntary annuitization cry out for a new way—or revisiting old ways—of thinking about the provision of lifetime retirement income.

This paper explores the financial risk-and-return properties of a *concept product* called an advanced-life delayed annuity (ALDA), which is a variant of a *pure* deferred annuity contract that is linked to, and adjusted for, consumer price inflation. Reduced to its essence, our product would be acquired at a young age—and small premiums would be paid over a long period of time—but the ALDA would only begin paying an inflation-adjusted life-contingent income at the advanced age of 80, 85 or even 90. The product would contain zero cash value, no survival or estate benefits and could not be commuted for cash at any age. Of course, these stringent design requirements might be impossible to attain given the current regulatory environment, But, in theory, these features combined with standard actuarial, interest and (possibly) lapsation discounting would reduce the ongoing premium for this insurance to mere cents on the dollar. ALDA is a close relative of a DB pension and intended for those who don't have one.

From a slightly different perspective, this type of product is akin to buying car, home or heath insurance with a large deductible, which is also the optimal strategy—and common practice—when dealing with catastrophic risk. By analogy, the ALDA longevity insurance would only kick-in once the longevity risk became substantial and financially unsupportable. Indeed, the *raison d'être* of life-contingent annuities is the acquisition of mortality credits, which at advanced ages are substantial and unavailable from any competing asset class. During the early years of retirement—when most pension decisions are made—the magnitude of these credits is quite small once survivor benefits, insurance fees and antiselection (i.e., annuitant versus population) costs are included. In contrast, the ALDA would entitle the holder to insurance against the risk of outliving assets, but only when the assets actually run the risk of being depleted, which is later in life.

The intellectual origins of this idea can be traced to a 25-year-old article by Stephenson (*JRI*, 1978), and has doubtlessly been toyed with, contemplated and possibly even designed by many pension actuaries ever since. Currently, they are simply unavailable. Stephenson criticized existing annuity products in the marketplace and argued in favor of adopting designs that contain high ratios of "protection to investment." He developed a concept called the *index of protection* and demonstrated that properly designed deferred annuities could provide greater inflation-protected value to consumers. This paper will argue that the fairly low actuarial premium for providing this longevity insurance, together with some well-known behavioral economic factoids, make a compelling case for offering (and perhaps even imposing the purchase of) ALDAs in all DC pension plans, as a substitute for a DB pension.

In a microeconomic modeling framework, the ALDA would transform the consumer choice and asset-allocation problem from a stochastic date of death to a deterministic one in which the terminal horizon becomes the payment commencement date. From a practical point of view, retirees would no longer have to worry about outliving their assets. They would be secure in the knowledge that, if and when they reach an extreme age, their longevity insurance would begin. (In fact, this might create interesting incentive effects in their own right.)

The remainder of the paper will be organized as follows. Section 2 will provide a brief review of the extant academic literature on the gains from annuitization and discuss some of the empirical evidence regarding the low levels and behavioral aversion to voluntary annuitization. Section 3 is the theoretical core of the paper, which describes the financial, economic and actuarial properties, as well as the different design possibilities for the ALDA contract. Section 4 discusses some related annuity products that have recently been made available to the public and describes an attempt by the author to get ALDA introduced to Canada. Finally, section 5 concludes the paper with some general comments.

2. Gains from Annuitization

The industry, as well as scholars in the field, are well aware of—and continue to puzzle over—the extremely low levels of voluntary annuitization exhibited among elderly retirees. From a theoretical perspective, this phenomenon is inconsistent with a standard Modigliani life-cycle model of savings and consumption, as described by Yaari (1965). In a life-cycle model with no bequest motives, Yaari (1965) demonstrated that all consumers hold actuarial notes as opposed to liquid assets. This implies that when given the chance, retirees should convert their liquid assets into life annuities that provide longevity insurance and protection against outliving one's money. The rationale behind Yaari's results is that returns from actuarial notes (life annuities) dominate all other assets, since the *living* inherit the assets of the *deceased*. Moreover, at older ages, the higher probability of dying increases the relative return from actuarial notes, conditional on survival.

A simple example should help illustrate the benefits from annuitization. Suppose there is a 20 percent chance that a 95-year-old female will die in the next year. If 1,000 such females enter into a one-year life annuity agreement by investing $100 each in a pool yielding 5 percent, the funds will grow to $105,000 by the end of the year. Of the initial 1,000 females, 800 are expected to survive, with rather small variance around the expected value, leaving $105,000/800 = $131.25 per survivor. This is a net (expected) return of 31.25 percent. This far exceeds the risk-free return of 5 percent (or perhaps any risky return) because the annuitants have seceded control of assets in the event of death.

Algebraically, if r denotes the risk-free interest rate per period (say, the T-bill rate), and if p is the probability of survival per period, then the return for the survivors from the one-period annuity is expected to be $(1 + r)/p - 1 > r$. The following table illustrates the value of these so-called mortality credits at various ages.

Age	Mortality Credits (b.p.)
55	35
60	52
65	83
70	138
75	237
80	414
85	725
90	1256

Table 1: The Investment Benefits from Annuitization. Assuming 40/60 male/female split for Annuity 2000 Table under 6% interest

This risk-sharing principle is the concept underlying all immediate annuities and all pension plans for that matter. In practice, the agreement is made over a series of periods, as opposed to just one. The mechanics, however, remain the same, and the survivors derive a higher return compared to placing their funds in a conventional (non-mortality-contingent) asset.

While the example we have provided assumes that r is fixed, in theory, the exact same principle applies with a variable investment return as well. In fact, the returns might be even higher. For example, the 1,000 females can invest their $100 in a stock mutual fund that earns the random return R. They do not know in advance what the fund/pool will earn. At the end of the year the annuitants will learn (or realize) their investment returns, and then split the gains among the surviving pool. Moreover, in the event the investment earns a negative return—and loses money—the participants will share in the losses as well, but the effect will be mitigated by the mortality credits. Algebraically, the return will be $(1 + R)/p - 1 > R$. This concept is the foundation of a *variable* immediate annuity.

In practice, most insurance companies go one step further than the above example and actually *guarantee* that the annuitant will receive the mortality credit enhancements, even if the mortality experience of the participants is better than expected. In other words, in the above-mentioned example for fixed annuities, with an expected 20 percent mortality rate, the insurance company would guarantee that all survivors receive 31.25 percent on their money, regardless of whether or not 20 percent of the group died during the year. We refer the reader to Poterba (1997) for a history of the development of this interesting product in the United States.

Nevertheless, despite the highly appealing arguments in favor of annuitization, there is little evidence that retirees are voluntarily embracing this arrangement. Modigliani (1986), Friedman and Warshawsky (1990), Mirer (1994), Poterba and Wise (1996) and Brown (1999, 2001), among others, have pointed out that very few people consciously choose to annuitize their marketable wealth. In the comprehensive Health and Retirement Survey (HRS) conducted in the United States, only 1.57 percent of the HRS respondents reported annuity income. Likewise, only 8.0 percent of respondents with a DC pension plan selected an annuity payout. The North American-based Society of Actuaries and LIMRA, as reported in Sondergeld (1997), conducted a study which shows that only 0.3 percent of variable annuity contracts were annuitized during the 1992-1994 period. According to the National Association of Variable Annuities, of the $909 billion invested in variable annuities, only 2 percent were annuitized.

Thus, in the face of poor empirical evidence, various theories have been proposed to salvage this aspect of the life-cycle hypothesis and to justify the low demand for longevity insurance. For example, in one of the earlier papers on this puzzle, Kotlikoff and Spivak (1981) argued that family-risk pooling may be preferred to public annuity markets, especially given the presence of adverse selection and transaction costs. Indeed, a married couple functions as a mini-annuity market, as elaborated by Brown and Poterba (2000). Friedman and Warshawsky (1990) showed that average yields on individual life annuities during the late 1970s and early 1980s were lower than plausible alternative investments. The reduced yield was largely attributed to actuarial loads and profits, which have declined over time, according to recent work by Mitchell, Poterba, Warshawsky and Brown (MPWB, 1999). In a different vein, Kotlikoff and Summers (1981) argued that intergenerational transfers accounted for the vast majority of U.S. savings and therefore bequest motives *solve* the puzzle. Bernheim (1991) and Hurd (1989) echoed this view. In other words, individuals do not annuitize wealth simply because they want to bequeath assets.

Bernheim (1991) further argues that large pre-existing annuities in the form of Social Security and government pensions might serve as an additional deterrent to voluntary annuitization. In a distinct line of reasoning, Yagi and Nishigaki (1993) argue that the actual design of annuities impedes full annuitization. One cannot obtain a life annuity that provides arbitrary payments contingent on survival, which is dictated by Yaari's (1965) model. They must be either fixed (in nominal or real terms) or variable (linked to an index). This constraint forces consumers to hold both marketable wealth and annuities.

In summary, many explanations exist for why people do *not* annuitize further wealth. Although these justifications have explanatory power, they fail to provide financial advice on optimal product design as well as normative strategies for the elderly. Furthermore, they cannot account for the casual observation that most people shun life annuities simply because they want to maintain control of their assets.

This paper takes the approach that consumers will remain reluctant to annuitize a large lump sum at retirement (regardless of *if-and-when* academics manage to solve the so-called puzzle.) What is needed is to realize that a sudden irreversible transaction will never be popular. The only alternative is slow annuitization. In its simple form, our product would allow individuals to voluntarily acquire a lifetime payout annuity in small increments over long periods of pre-retirement saving. ALDA could be offered as an additional rider on existing saving and insurance products or could be sold as a stand-alone product. The critical factor would be to take the edge off a daunting and irreversible annuitization decision.

The next section describes the pricing mechanics of this product.

3. Pricing ALDA

We start by letting $a_x(r \mid T)$ denote the *factor* for a—real \$1 per annum until time T—life contingent annuity that is issued and purchased at age x under a real pricing interest rate of r. The income flow is adjusted for inflation, and, therefore, in nominal terms, the annuity will initially pay \$1 per annum and then increase by the realized rate of the consumer price index (CPI). With slight abuse of standard actuarial notation—and a survival probability denoted by $({}_s p_x)$—we assume that:

$$a_x(r \mid T) := \int_0^T e^{-rs}({}_s p_x)ds, \tag{1}$$

where the annuity income is paid; inflation is accrued; and the interest rate is compounded in continuous time. Without any loss of generality, we will suppress the symbol $T = \infty$ and use $a_x(r)$ when we are dealing with a *lifetime* annuity that pays until death. Later we will expand our notation to account for the possibility of *lapsation*, which will impact the ongoing premium payment. In this paper and the subsequent numerical examples, the ALDA commencement age will range from *x=65* to *x=85* while the ALDA purchase age will range from *y=35* to *y=45.*

It is pretty straightforward to show that the net single premium (NSP) at age $y < x$ for a \$1 per annum ALDA benefit is the annuity factor in equation (1) discounted for the probability of survival and the time value of money (TVM). Mathematically we have that:

$$\text{NSP} = e^{-r(x-y)}\, a_x(r)({}_{x-y} p_y), \tag{2}$$

where the first term captures the *(x-y)* years of interest, the second term represents the annuity factor which commences at age x, and the third term is the conditional probability that someone currently aged y will survive for *(x-y)* years. Note that the real (after inflation) interest rate r is used in two places in equation (2). The first is to discount a single cash flow prior to the annuity commencement date—which covers the next *(x-y)* years—and the second is to price the annuity and discount the repeated cash flows that occur after age x. Thus, in practice one could envision using slightly different interest rates during the deferral period versus the payout period. In this case, we would use the notation r_1 and r_2 and position them in the appropriate place within equation (2). In fact, one could go a step further and use a real yield curve r_t—implied perhaps from Real Return Government Bonds—as opposed to a single interest rate, which would conform to capital market pricing techniques.

To provide some numerical intuition for the NSP of our ALDA, we offer the following example under a continuous Gompertz approximation to a discrete mortality table. Note that under a Gompertz law of mortality, the (natural logarithm) of the conditional survival probability is defined equal to:

$$\ln({}_s p_x) = e^{\left(\frac{x-m}{b}\right)\left(1-\frac{s}{b}\right)}, \qquad \textbf{(3)}$$

where *m* and *b* are the "modal" and "scale" parameters of the future lifetime distribution. Under an age assumption of $y = 35, x = 85$ and Gompertz parameters of *m=90* and *b=9.5,* and a real interest rate of $, r = 3.25\%$, we obtain that the net NSP at age 35 for an ALDA that will commence payments at age 85 is $0.731 in current dollars. This *pure* deferred lifetime annuity will pay $1 in real terms each year, commencing at age 85, in exchange for a premium payment of less than $1 today. The $0.731 came about from multiplying the age 85 annuity factor of $a_{85}(0.025) = 6.679$ by the *0.556* probability of survival and then by the *0.1969* time-value-of-money factor.

The following table displays the NSP of a *unisex* annuity purchase age (*y*) and a variety of annuity commencement ages (*x*) under the same Gompertz approximation to mortality.

	Annuity Commencement Age:			
Purchase Age	**x = 70**	**x = 75**	**x = 80**	**x = 85**
y = 35	$3.642	$2.376	$1.412	$0.731
y = 40	$4.294	$2.802	$1.665	$0.861
y = 45	$5.070	$3.308	$1.965	$1.017

Table 2: Theoretical NSP for ALDA
Assuming a 3.25% real (after-inflation) pricing rate.

For reference purposes, the assumed life expectancy at the initial purchase age was 84.7, 84.8 and 84.9 at ages 35, 40 and 45 respectively. Likewise, the implied life expectancy at the annuity commencement date was 87.6, 88.9, 90.7 and 92.9 at ages 70, 75, 80 and 85 respectively. No improvement factors or any other dynamic projection methodologies were used to generate these (illustrative) numbers. The above calculation is trivial from an actuarial point of view since this type of ALDA—i.e., one that is paid by a lump sum up front—is a well known deferred annuity. We now proceed to computing the periodic premium for the ALDA, which involves some subtle assumptions about lapsation behavior.

Payment for ALDA will not be made in one lump sum; rather, the annuitant makes a series of real (after inflation) nonrefundable and noncashable payments between the ages of y and x, which would then entitle him or her to a real $1 per annum for life, commencing at age x. In practice, this would be implemented linking both the periodic premiums and the benefits to the same consumer price index so that all cash flows could be discounted using the same unit of account. We emphasize that the pure actuarial pricing of this product would *not* require any assumptions about future inflation or nominal rates. The *premiums* would be variable in nominal terms, but fixed in real terms. Likewise, the *benefits* would be variable in nominal terms, but fixed in real terms. From a purely economic perspective, the lack of any asset-liability mismatch between the units of account should not require any additional reserves or capital requirements. Of course, the current regulatory environment might impede this theoretical invariance and further increase the cost of the product. A full discussion of these important yet complex issues would take us well beyond the scope of this brief article.

In either event, the NSP must be actuarially amortized over the (x-y) years, contingent on survival. Using our previous notation and assuming no lapsation, the net periodic premium (NPP) for ALDA will be:

$$\text{NPP} = \frac{e^{-r(x-y)}\, a_x(r)({}_{x-y}\,p_y)}{a_y(r \mid x-y)}, \qquad \textbf{(4)}$$

where the numerator is the NSP, and the denominator effectively spreads these payments over the (x-y) years between the initial purchase age and the ALDA commencement period. Equation (4) is, again, a relatively straightforward actuarial method of converting single premiums into periodic life-contingent premiums. Note that the annuity factor in the denominator is subscripted by the purchase age y, while the factor in the numerator is subscripted by the commencement age x. Intuitively, for any given purchase age y, the longer the deferral period (x-y), the greater the annuity factor $a_y(r \mid x-y)$, and the lower the ongoing periodic premium. Similarly, as emphasized in the earlier discussion, it is quite conceivable that the pricing interest rate r in the denominator's factor will differ from (be greater than) the pricing rate in the numerator's factor. This is because a non-flat yield curve in practice will result in different (constant) interest rate approximations, depending on the period that is being discounted. Regardless, they are both real (after inflation) rates.

Here are some examples under the same pricing conditions as we considered in the NSP case. When the initial purchase age is *y=35* and the annuity commencement age is *x=85,* then under an *r=3.25 percent* real interest rate, the net periodic premium (NPP) needed to create a $1 per annum real lifetime annuity is precisely $0.0312 per annum. In other words, a mere three cents on the dollar per annum—paid over a period of 50 years—will generate an income flow of $1 for life. This is a factor of 32 times the ongoing premium. We can scale this quantity up (or down) and declare that, for each $100 of premium per week, month or year, the ALDA will pay a pension of $3,200 per week, month or year. If instead of using ages 35 and 85, we use ages 40 and 80—while retaining the same interest rate of *r=3.25 percent*—the NPP becomes $0.0779, which is a factor of 12.8 times the ongoing premium. Finally, if we increase the interest rate to 4 percent, the NPP becomes $0.061, which is a factor of 16.2. The following table converts the NSP numbers in Table 2 into payout factors that are the reciprocal of the NPP.

	Annuity Commencement Age:				
Purchase Age	**x = 70**	**x = 75**	**x = 80**	**x = 85**	**x = 90**
y = 35	*5.6*	*9.2*	*16.1*	*32.0*	*77.7*
y = 40	*4.3*	*7.2*	*12.8*	*25.7*	*62.6*
y = 45	*3.2*	*5.6*	*10.1*	*20.4*	*49.9*

Table 3: Theoretical ALDA Income Payout Factors:
Retirement Income per Premium Dollar
Assuming a 3.25% real (after-inflation) pricing rate.

Table 3 includes the extreme case in which the commencement age is 90. In this case, a 35 year-old would receive $77.7 real dollars starting at age 90 for each real dollar paid from age 35. Whether or not a 35-year-old would actually persevere and pay premiums for 55 years is debatable, which brings us to the topic of lapsation, which we will return to later.

3.1. Who Takes the Mortality and Interest Rate Risk?

The above description and pricing mechanics are predicated on the ability of the insurance company to guarantee the pricing rate (3.25 percent real, in the above example) and the mortality table. In practice, if the insurance company offering ALDA were to earn less than the pricing rate, and/or experience mortality that was worse than assumed, the company would obviously face the potential of severe losses. This raises the question of whether ALDA should have a participating structure in which a minimal income payout factor would be guaranteed, and then depending on investment performance and mortality experience, the income would be increased.

Indeed, this kind of arrangement—which involves an additional level of risk sharing—is at the heart of some products that have recently been introduced in the North American marketplace. Thus, for example, a commercially viable version of ALDA would guarantee an implicit real rate of *at least* 2 percent applied to the Annuity 2000 mortality table and then, depending on future financial and economic conditions, the benefits could be ratcheted-up (increased) on a periodic basis.

3.2. Lapsation Considerations

While everyone who purchases (or starts) an ALDA likely has the full intention of holding the product to maturity, it is unreasonable to assume that 100 percent of all survivors will continue to pay premiums until the commencement date. In fact, if the product is structured with absolutely no cash value and/or no ability to scale-down the income benefit by reducing premiums, there is a high probability that people will (irrationally) lapse the product. The lapsation phenomena must be taken into account in the original pricing. From a pricing perspective, one can assume the existence of an instantaneous lapse-rate curve—which is a akin to a force of mortality—which determines the probability the contract will be lapsed as a function of the number of years since initiation. This curve will most likely start at a level close to zero and then increase as time evolves, but start to decline again as the ALDA nears the commencement date. The psychological justification would be that, on an aggregate level as individuals "see" the payoff horizon approaching, they are likely to reduce the rate at which they become disillusioned from the product. If we denote the lapse rate curve by l_s we can define the cumulative probability of not lapsing prior to time t, as:

$$L_t = e^{-\int_0^t l_s ds} \qquad \textbf{(5)}$$

This is akin to the probability of survival function which satisfies the property that $L_0 = 1$ and $L_\infty \to 0$ when the integral of the lapse curve l_s goes to infinity and L_∞ will converge to a constant when the "area under the lapse curve" is bounded. From a practical point of view, this implies that the lapse-adjusted NSP will be:

$$\text{NSP}^* = L_{(x-y)} e^{-r(x-y)} a_x(r)({}_{x-y}p_y), \qquad \textbf{(6)}$$

Although it is critical to stress that if the premium is paid in one lump sum (up front), the lapsation factor is irrelevant since the premium becomes a sunk cost. Therefore, the NSP in equation (6) is a mathematical artifact to be used in deriving the lapse-adjusted NPP which we will illustrate shortly. First, though, we must define a lapse-adjusted annuity factor. This factor is denoted and defined by:

$$a_x(r \mid T, l) := \int_0^T e^{-(r+l_s)s} ({}_s p_x) ds \,. \qquad \textbf{(7)}$$

The lapsation curve functions like an interest rate in reducing the initial premium required to fund the annuity. Lapsation will only be relevant in the accumulation (premium) phase of ALDA, and therefore the lapse-adjusted NPP will be:

$$\text{NPP}^* = \frac{L_{(x-y)} e^{-r(x-y)} \, a_x(r) ({}_{x-y} p_y)}{a_y(r \mid x-y, l)} \qquad \textbf{(8)}$$

The lapsation curve will impact the NPP in two partially offsetting ways. It will reduce the numerator by virtue of the smaller number of people who will utilize the product, but it will also reduce the denominator by virtue of the reduced size of the group who will actually cover the actuarial present value of the ALDA benefit. The net effect will be a total reduction in the NPP in equation (8), regardless of the precise shape of the lapsation curve. Indeed, for most reasonable specifications, the premiums will decline quite substantially. To take this concept one step further, a possible specification of the instantaneous lapse rate would be an exponential form similar to:

$$l_s = \lambda_1 e^{-\lambda_2 s}, \qquad \textbf{(9)}$$

where the constants λ_1, λ_2 determine the intensity of lapse as the individual gets closer to age *x*. This specification would allow for a low intensity in early years, a peak half-way through the term and then a gradual decline.

	Annuity Commencement Age:				
Purchase Age	**x = 70**	**x = 75**	**x = 80**	**x = 85**	**x = 90**
y = 35	*11.3*	*20.5*	*39.6*	*87.0*	*233.4*
y = 40	*7.8*	*14.5*	*28.5*	*63.2*	*170.2*
y = 45	*5.3*	*10.2*	*20.3*	*45.4*	*122.7*

Table 4: Lapse-adjusted ALDA Income Payout Factor: Retirement Income per Premium Dollar
Assuming a 3.25% real (after-inflation) pricing rate.

One could envision a wide number of specifications, each leading to their own premiums. For illustrative purposes, the above table takes a simpler approach—to illustrate the impact of even a small lapse rate—and displays the relevant income payout factors assuming a 2 percent lapse rate each year. In other words, the difference between Table 4 and Table 3 includes the assumption that each year 2 percent of the ALDA population ceases to make payments, but for non-mortality-driven reasons. We emphasize again that this is a very crude approximation, and that actual lapsation behavior and intensity in such a product would depend on the number of years remaining until the product commencement date as well as number of health-related factors. Despite the simplicity, a number of interesting facts emerge from Table 4 that are robust to the precise form of the lapsation curve. Income multiples increase by a factor of two to three, but this impact is even further pronounced as the commencement date becomes later. And, despite some of the crude assumptions behind these numbers, this insight is actually quite robust, regardless of the lapse specification.

3.3 Scaling Down Benefits

If the insurance company is unwilling to price the product using a lapse curve, one could envision an ALDA design in which the premiums could be voluntarily stopped at some age z prior to age x. The benefit would then be reduced accordingly, albeit with the same exact commencement date, in order to avoid antiselection problems. The benefit would be "scaled down" by computing the *ex post* actuarial present value of the premiums at the "lapse age" z and then scaled into the original NSP to arrive at a fractional scaled down ◎ of the originally guaranteed payout factor from Table 2.

$$\eta = \frac{a_y(r \mid z-y)}{e^{-r(x-y)} a_x(r)({}_{x-y}p_y)} \qquad \textbf{(10)}$$

There are a number of self-evident and compelling reasons why this particular incarnation of ALDA would be the most popular from a consumer standpoint, and we envision variants of this design as having the best chance of survival in the marketplace.

In sum, we have described the basic *actuarial chassis* of the ALDA product, which, despite its actuarial simplicity, contains a number of important economic benefits. The main features can be summarized as follows: (a) real inflation-adjusted benefits; (b) an annuity commencement date that is irreversible and well into the retirement years, akin to a deductible on an insurance policy; and (c) slow and prolonged premium payments that counteract the ingrained reluctance of consumers to annuitize in one lump sum.

4. Does This Product Exist Already?

The answer to this question is *yes, but*… Indeed, as mentioned earlier, a number of North American insurance companies are selling variants of ALDA under numerous guises and incarnations. In fact, it seems that some older long-term care (LTC) policies had an element of ALDA as part of their benefit structure. We refer the reader to a recent article in *Best's Review* (February 2004, pg 70-74) for a review of the industry in the payout annuity market. For example, Prudential Financial, GE Life & Annuity as well as Metlife and Principal Financial are just some of the named companies that are in the process of developing, or already offer, a financial vehicle that allows one to acquire lifetime income using a dollar-cost averaging strategy. And, while it is beyond the mandate of this paper to critique the merits and pitfalls of each, it seems the emphasis on real (after-inflation) income has been neglected by most of the current manufacturers. Furthermore, paradoxically, some of the inherent flexibility and choice embedded in these products may detract from the ultimate objective, which is to *encourage annuitization at the lowest possible cost.*

On a more pessimistic note, it seems that industry innovation around retirement income (payout) products has been taking place for decades, but with very few noticeable successes. In the late 1980s, The IDS Life Insurance company in Minneapolis (an American Express company) offered a variant of ALDA called IDS retirement assurance. Under this product, the annuity premiums were paid in one lump sum upon initiation; the deferral or delay period lasted for 30 or 40 years; and the benefit commenced at age 80. This product paid out in nominal terms, included a survivor and/or surrender benefit of premiums paid (without interest) and also included a participating structure linked to mortality credits. The policy statement contained a fairly complicated schedule of mortality credits that would be added to the account upon attaining certain ages. And, despite the differences with the ALDA product described above, this product did in fact come close to achieving the objectives of longevity insurance with a deductible. In fact, the sales literature created by IDS stated quite clearly that "…this product is designed for your later retirement year, and does this at a cost that is far lower than conventional annuities…" Unfortunately, despite the sound theoretical foundations, this product was a commercial failure and the company withdrew sales soon after.

Within the same spirit, in the lead-up to the writing of this paper, the author approached one of the largest insurance companies in Canada with a proposal to develop an ALDA product. The author also volunteered to be the first to purchase the product (at age 35) and assist in the public marketing campaign. Initially, there was much excitement with the concept and the insurance company's actuaries produced the following pricing schedule, which is well within the range of the numbers presented in Table 3. In general, the payout multiples are lower than the numbers obtained using our theoretical model—and justifiably so—although at higher ages the numbers do seem excessively lower than what theory would dictate. But then again, mortality would be guaranteed for a very long (and potentially risky) period of time.

	Annuity Commencement Age:			
Purchase Age	**70**	**75**	**80**	**85**
35	*5.13 / 4.47*	*8.16 / 6.91*	*13.82 / 11.31*	*25.90 / 20.53*
40	*4.08 / 3.54*	*6.65 / 5.60*	*11.51 / 9.36*	*22.08 / 17.35*
45	*3.15 / 2.73*	*5.30 / 4.44*	*9.42 / 7.61*	*18.54 / 14.45*

Table 5: Actual Payout Factors: Male/Female Income per Premium Dollar
Quoted by large insurance company in Canada (October 2003)
Assuming a 3.25% real (after-inflation) pricing interest rate.

Unfortunately, as the ALDA proposal made its way up the chain of command, it encountered a number of institutional and regulatory obstacles, and finally the initiative was abandoned. The general concerns offered by the company in question can broadly be categorized as follows:

Monthly or weekly premiums. When long-dated annuities are sold, these types of annuities are based on the payment of one single lump-sum premium. In the ALDA case, the (small) premiums would be paid monthly or weekly until the annuity commencement date. This is an administration limitation since most insurance company software systems are not currently set up to handle such a long period of premium collection, or determine the new premium at each year based on the current inflation rates.

Delayed period. The delayed period is the period between the payment of premiums and the commencement of annuity payments. In this case, the annuity payments commence at age 70 to 90, which results in a deferred period of up to 55 years. Currently, the maximum deferred period of any annuity product is 30 years. Most ALDAs are over this limit, and thus very long horizons result in both pricing and administrative issues since the company must track the annuity for quite a long period, and finding matching long-term investment is unlikely.

Inflation indexing. The fact that the annuity in question is an inflation-indexed annuity causes additional complications. For these annuities, the usual deferred (or delay) period accepted is even shorter—10 years. Again, this is due to the availability of matching investments since would be limited to real bonds or taking on the risk component of inflation predictions.

No death benefit. Although this is possible, it means that the annuitant can be paying premiums for up to age x minus one day, pass away and receive nothing. After 40, 45 or 50 years of premium payments, the product provides no death benefit. Most insurance companies do not feel comfortable from a public relations perspective offering such a product, and go so far as to argue that it would have limited popularity in the general marketplace.

In sum, there seem to be a number of institutional and regulatory impediments to offering such long-dated inflation-adjusted products. Furthermore, even if these obstacles can be overcome in an economically viable manner, it remains to be seen whether there is a market for ALDA. Quite likely, a costly and prolonged marketing effort—undertaken by the industry as a whole as opposed to a particular company—will be required to make this concept a commercial success. Corporate patience and long managerial horizons will be necessary, but not sufficient, for success in this market.

5. Conclusion

Despite valiant efforts by finance and insurance professionals to educate the public about the benefits of annuitization, the industry must recognize that few people will consciously choose to hand over a lump sum in exchange for lifetime income when given the choice. Numerous experiments involving "live" money have consistently documented consumer's hyperbolic levels of implied time preference when discounting future needs and cash flows during retirement. This is not to say that all consumers take the money and run when offered the choice to leave a DB pension plan. Rather, when the default *status quo* option is to continue maintaining full control of the funds—as in most DC plans—it is extremely hard to give up such control.

Therefore, in the face of a continuing erosion of traditional DB pension plans with their implicit life annuities, the industry must do more to create, promote and explain viable alternatives. This paper provides another step in that direction by describing the actuarial mechanics of a product called ALDA. In its simple form, ALDA would allow individuals to voluntarily acquire a lifetime payout annuity in small increments over long periods of pre-retirement saving. ALDA could be offered as an additional rider on existing saving and insurance products or could be sold as a stand-alone product. The critical factor would be to take the edge off a daunting and irreversible annuitization decision. Likewise, this article emphasized the importance of framing the discussion in real (after inflation) terms, even though the extent to which the current CPI-U captures the basket of goods demanded by retirees is debatable.

In conclusion, while an introductory (and motivational) article such as this leaves many details to complete, it is hoped that the ensuing dialogue will move the industry away from yet another generation of complex secondary guarantees on variable annuities, and towards a strategy that recognizes the consumer's ingrained reluctance to annuitize.

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