Immediate Annuity Pricing in the Presence of Unobserved Heterogeneity

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Abstract

One of the acknowledged difficulties with pricing immediate annuities is that underwriting the annuitant's life is the exception rather than the rule. In the absence of underwriting, the price paid for a life-contingent annuity is the same for all sales at a given age. This exposes the market (insurance company and potential policyholder alike) to antiselection. The insurance company worries that only the healthiest people choose a life-contingent annuity and therefore adjust mortality accordingly. The potential policyholders worry that they are not being compensated for their relatively poor health and choose not to purchase what would otherwise be a very beneficial product.

This paper develops a model of underlying, unobserved health. Health is a state variable that follows a first-order Markov process. An individual reaches the state "death" either by accident from any health state or by progressively declining health state. Health state is one-dimensional, in the sense that health can either "improve" or "deteriorate" by moving further or closer to the "death" state, respectively. The probability of death in a given year is a function of health state, not of age. Therefore, in this model a person is exactly as old as he feels.

We first demonstrate that a multistate, ageless Markov model can match the mortality patterns in the common annuity mortality tables. The model is extended to consider several types of mortality improvements: permanent through decreasing probability of deteriorating health; temporary through improved distribution of initial health state; and plateau through the effects of past health improvements.

We then construct an economic model of optimal policyholder behavior, assuming that the policyholder either knows his health state or has some limited information. The value of mortality risk transfer through purchasing a life-contingent annuity is estimated for each health state under various risk-aversion parameters. Given the economic model for optimal purchasing of annuities, the value of underwriting (limited information about policyholder health state) is demonstrated.

Keywords: Payout annuity, self-selection, Markov, mortality, heterogeneity.

1. Introduction

One of the controlling realities of life insurance is that the policyholder generally has more accurate information about the state of his or her health than the insurance company. In the case of life insurance, the degree of asymmetry can be reduced through underwriting; almost all life insurance policies written are underwritten. Annuity policies are not generally underwritten. The typical exception is a structured settlement case involving an impaired life.

In the theoretical and academic community, one of the remaining puzzles of the insurance market is the thinness of the lifetime payout annuity market. One possible contributor is this asymmetry of information: the policyholder does not expect to live long enough to recoup the initial premium. Likewise, insurance companies pad annuity mortality tables expecting that only people with optimistic views of their life expectancy will purchase payout annuities.

This antiselection by policyholders, together with the insurance company response, could combine to eliminate the payout annuity market. The process operates as follows. Suppose population mortality suggests a population payout annuity rate that is too expensive to 40 percent of the population due to their private information about mortality. When this 40 percent is eliminated from the mortality pool, the life expectancy of potential policyholders improves, which increases the cost of the payout annuity to the buying public. This price increase, in turn, leads additional potential policyholders (always the least healthy or most likely to die) to avoid the market. The size of the market dwindles with each machination.

This paper develops a model of annuity mortality that facilitates a more thorough evaluation of this issue. The annuitant has a health state. Aging occurs as the health state changes. Death occurs either as a terminal health state or as an accident. By knowing the current health state, the annuitant or the insurance company can improve its knowledge about the life expectancy of the annuitant.

The literature on the importance of adverse selection in the annuity market is in the same stage of development as the market itself. Poterba (2001) gives a good general overview of the issues related to adverse selection in the annuity market. Brunner and Pech (2000, 2002) develop a two-period general equilibrium model in which they can characterize the nature of the equilibria possible within a market with adverse selection. Finkelstein and Poterba (2004) analyze the policies of a large U.K. insurer and find evidence that different types of payout annuities have different mortality characteristics, which is evidence that knowledgeable insurers self-select the annuity that best matches their mortality risk. Finkelstein and McGarry (2003) find direct evidence that purchasers of long-term care (LTC) insurance (a health-contingent form of payout annuity) have private information about their probability of needing nursing home care. Mitchell and McCarthy (2002) use the difference between population mortality tables and the annuity mortality tables as evidence of self-selection in the annuity market.

The paper is organized as follows. First we estimate a Markov model which matches the most common annuitant mortality tables. We compare parameter estimates derived from various mortality tables. The striking result is that annuitant mortality can be modeled using very few parameters within a Markov model.

We then present an economic model of annuitant utility. The optimal consumption decision is derived for the case where no annuity market exists. Using the same utility framework, we derive the threshold price for an immediate annuity. Employing several competing utility models, we show the degree of market thinness created by the asymmetry of information.

2. Markov Model

Rather than focus on chronological age, the annuitant mortality model in this paper is concerned only with health status. As a gross approximation of actual human health, consider a model where an annuitant's mortality is described by a one-dimensional health status, S_t, where S_t is an integer from 0 to J. A person "ages" as health state decreases. $S_t = 0$ represents death. In any period, the health status can either improve or deteriorate.

For the general population of annuitants aged t, let p_t be the J+1 by 1 vector of probabilities of annuitants with health status j, j = 0, … , J. Let T represent the J+1 by J+1 transition matrix, with typical element $T_{ij} = \mathrm{Prob}(S_{t+1}=j|S_t=i)$. Then $p_{t+1} = Tp_t$.

In order to restrict the number of parameters in the model, the transition matrix T is defined as follows.

- For i > 1, $T_{i,i-1} = p_d$, the probability of decreasing health state by one.
- For J > i > 0, $T_{i,i+1} = p_u$, the probability of improving health state by one.
- For i > 1, $T_{i,0} = p_a$, the probability of death by accident.
- $T_{1,0} = p_d + p_a$.
- $T_{i,i} = 1 - p_d - p_a - p_u$(if i < J).

- $T_{i,j}$ = 0, unless defined above.

Estimating a Markov mortality model to calibrate to any particular data set requires estimating the parameters P_u, P_d, and P_a, and the initial state conditions p_0. Define $q_{0|t}$ (probability of death in period t conditional on being alive in period 0) as (p_{t0} – $p_{t-1,0}$), where $p_{t,0}$ is the first element of the state probability vector p_t. For death data $\{d_t\}$, the likelihood of the data conditional on the model is

$$L() = \Pi_t \, q_{0|t}^{dt}, \text{ or } \ln(L) = \sum_t d_t \ln(q_{0|t}).$$

The statistical estimation exercise is to maximize ln(L) with respect to P_u, P_d, P_a and the initial state conditions p_0. The data used in this exercise are mortality tables, which are themselves aggregations of raw mortality data. Since no specific sample or sample size is available, statistical inference or testing is beyond the scope of this paper.

The above model is fit to U.S. Census population mortality tables for the years 1900–1990. Since the focus of this research is retirement, the model was calibrated to the ages 50-90. To obtain snapshots of specific birth cohorts, several mortality tables were interpolated to obtain cohort mortality tables. For example, the 1850 birth cohort uses the age-50 mortality from the 1900 census, the age-60 mortality from the 1910 census and so forth.

Table 1 shows maximum likelihood point estimates for each of the raw mortality tables and for birth cohort tables constructed for decennial birth cohorts from 1850 to 1900. In all cases P_u and P_a are estimated at zero and not reported. Health state initial probabilities are grouped, although health states 7-9 are estimated to be zero for all mortality tables.

Table 1 – Mortality Estimates Based on Common Mortality Tables

Data Set (Mortality Table)	P_d	State 10	States 4-6	States 1-3
1996 US Annuity 2000 Basic, Male	32.4%	85%	11%	4%
1996 US Annuity 2000 Basic, Female	31.1%	90%	8%	2%
RP-2000 Male Combined Healthy	32.6%	87%	10%	3%
RP-2000 Female Combined Healthy	32.0%	89%	9%	2%
RP-2000 Male Healthy Annuitant	32.6%	84%	11%	5%
RP-2000 Female Healthy Annuitant	32.0%	86%	11%	3%
US (SSA AS 107) 1900, Age Nearest, Male	36.8%	57%	26%	17%
US (SSA AS 107) 1910, Age Nearest, Male	36.8%	58%	26%	16%

US (SSA AS 107) 1920, Age Nearest, Male	36.5%	63%	23%	14%
US (SSA AS 107) 1930, Age Nearest, Male	36.4%	59%	26%	15%
US (SSA AS 107) 1940, Age Nearest, Male	36.3%	60%	25%	15%
US (SSA AS 107) 1950, Age Nearest, Male	35.6%	62%	25%	13%
US (SSA AS 107) 1960, Age Nearest, Male	35.4%	63%	25%	12%
US (SSA AS 107) 1970, Age Nearest, Male	35.3%	63%	25%	12%
US (SSA AS 107) 1980, Age Nearest, Male	34.9%	69%	21%	10%
US (SSA AS 107) 1990, Age Nearest, Male	34.4%	73%	19%	8%
US (SSA AS 107) 1900, Age Nearest, Female	36.3%	61%	24%	15%
US (SSA AS 107) 1910, Age Nearest, Female	36.2%	63%	23%	14%
US (SSA AS 107) 1920, Age Nearest, Female	36.1%	65%	22%	13%
US (SSA AS 107) 1930, Age Nearest, Female	35.6%	66%	22%	12%
US (SSA AS 107) 1940, Age Nearest, Female	35.3%	71%	19%	10%
US (SSA AS 107) 1950, Age Nearest, Female	34.2%	76%	16%	8%
US (SSA AS 107) 1960, Age Nearest, Female	33.5%	79%	14%	7%
US (SSA AS 107) 1970, Age Nearest, Female	33.0%	80%	14%	6%
US (SSA AS 107) 1980, Age Nearest, Female	32.5%	80%	14%	6%
US (SSA AS 107) 1990, Age Nearest, Female	32.4%	82%	13%	5%
Male Cohort 1850	36.4%	59%	25%	16%
Male Cohort 1860	36.5%	60%	24%	16%
Male Cohort 1870	36.0%	58%	27%	15%
Male Cohort 1880	35.6%	60%	25%	15%
Male Cohort 1890	35.5%	60%	25%	15%
Male Cohort 1900	35.1%	61%	26%	13%
Female Cohort 1850	35.9%	62%	23%	15%
Female Cohort 1860	35.6%	64%	22%	14%
Female Cohort 1870	34.8%	65%	22%	13%
Female Cohort 1880	34.1%	69%	19%	12%

Female Cohort 1890	33.5%	72%	17%	11%
Female Cohort 1900	32.9%	75%	16%	9%

Notice that the trend in mortality is markedly different between males and females when estimated using this model. Based on 1900 census data, there is little difference between male and female mortality. By 1990, female mortality improvement is dramatic both with respect to the percentage of the age-50 population in the highest health state as well as with respect to the probability of declining health in a year. The common annuity tables show even further pads and improvements, consistent with the findings of Mitchell and McCarthy (2002).

Charts 1 and 2 show the health dynamics based on estimates using the Annuity 2000 Basic Male table. Notice that 85 percent of the male population is estimated to be in the top health state at age 50. By age 65, there is considerable heterogeneity of health status. The population of most unhealthy lives (state = 1) never gets large, but as a percentage of the remaining lives it becomes important starting after age 65. Recall that the population mortality rate in this model is the percentage of lives with health status 1 times the probability of declining health.

Chart 1 – Health Status as Percentage of Surviving Population

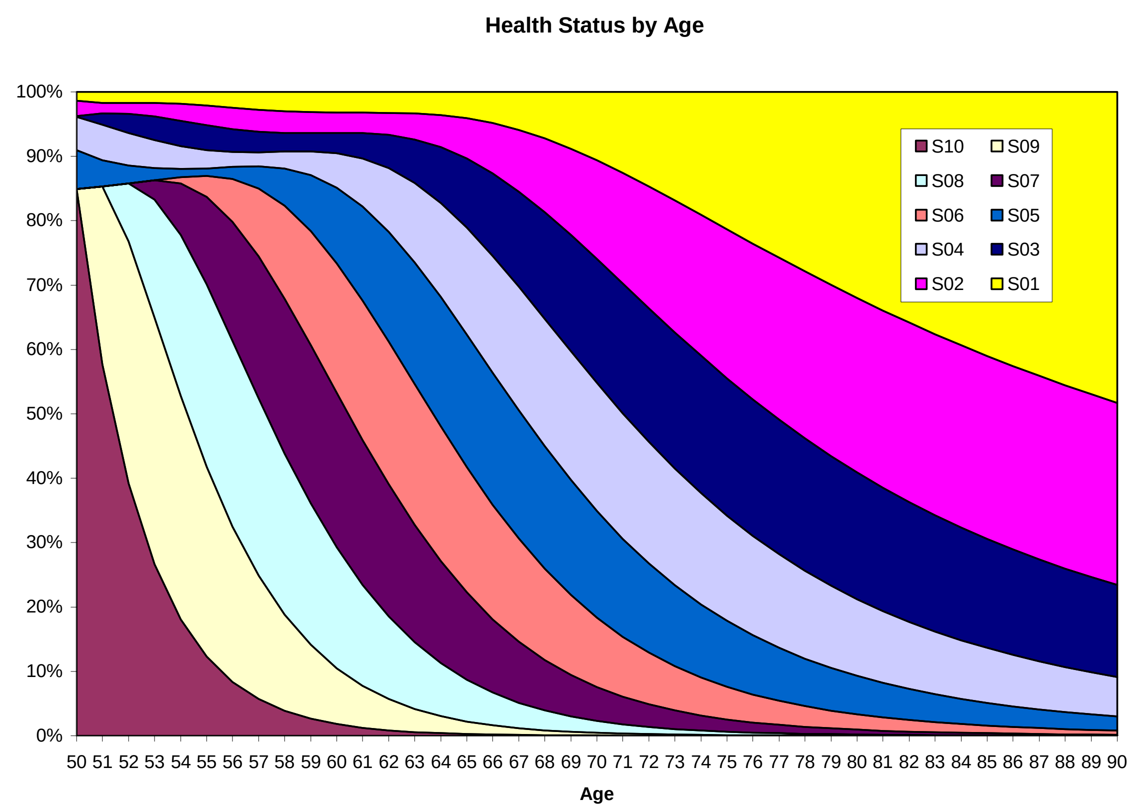
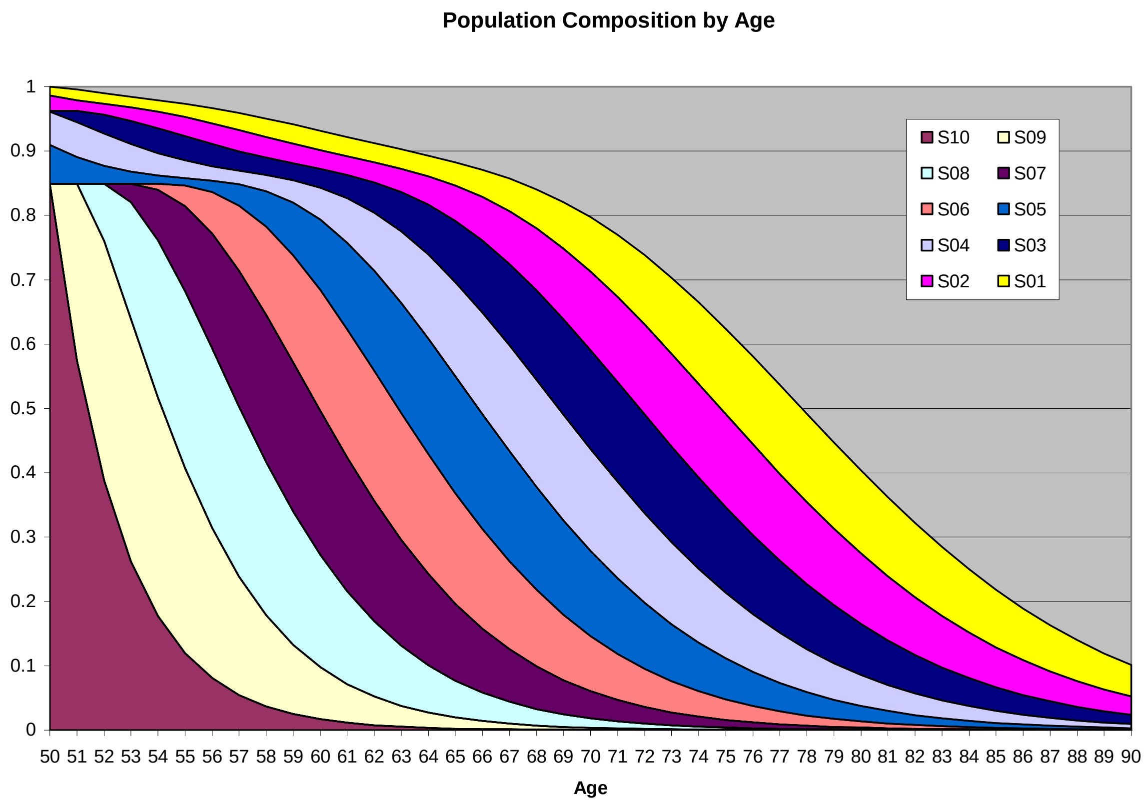


Chart 2 – Health Status With Death as the Omitted State



3. Economic Model

The policyholder is assumed to make asset and consumption decisions with the intent of maximizing their expected lifetime utility. Utility in each period is possibly influenced by the health state of the policyholder. Expected lifetime utility is the net present value of the utility derived from consumption in each period of the policyholder's life. Assets not consumed are invested, earning a fixed rate of return.

This model assumes a single policyholder. For two policyholders making joint asset and consumption decisions based on their combined mortality, the health state model could be assumed to apply to the pair. Likewise, this model assumes no bequests, or no purpose for assets other than consumption.

Mathematically, assume that per-period utility, conditional on health state j, is defined as

$$U_j(c_t) = K_c + (\alpha+1)^{-1}(\theta_j c_t)^{(\alpha+1)}, \qquad (1)$$

where c_t is consumption in period t, θ_t is a health-state consumption modifier, $\alpha \leq 0$ is the risk-aversion parameter, and K_c is a utility constant whose sole purpose is to make utility positive.

Marginal utility with respect to consumption is $U_j'(c_t) = (\theta_t c_t)^{\alpha}$.

The choice of utility function is based on mathematical simplicity and the fact that this particular class of utility functions is characterized by constant relative risk aversion. Absolute risk aversion is measured by U'(c)/U"(c). For this class of utility functions, absolute risk aversion is linear in consumption. Relative risk aversion (risk aversion divided by consumption or wealth) is constant. For a discussion of alternative utility functions used in retirement wealth analysis, see Gerber and Shiu (2000). Brown (2003) uses linearly separable log utility, which is a special case of the power utility model used here for $\alpha = -1$.

The discounted net present expected value of total lifetime utility is a function of the health state and the current level of assets. Mathematically, this is the value of being in health state j with assets a_t.

$$V_j(a_t) = \max\{c_s,\ s \geq t\} \sum_{s \geq t} \beta^{s-t} E[U_j(c_t)], \tag{2}$$

subject to the budget constraints that $a_s \geq 0$, and

$$a_{t+1} = (a_t - c_t)*(1+r), \tag{3}$$

where r is the investment rate of return. The expectation (2) is with respect to the health state j. Equation (2) can be written recursively, as a maximization with respect to only current consumption. This is the set of Bellman equations.

$$V_j(a_t) = \max\ \{c_t\}\ [\ U_j(c_t) + \beta\ \{\ p_s V_j(a_{t+1}) + p_u V_{j+1}(a_{t+1}) + p_d V_{j-1}(a_{t+1})\ \}], \tag{4}$$

where p_s is the probability of staying in the current health state, p_u is the probability of improving health state, and p_d is the probability of deteriorating health state.

Recall from (3) that a_{t+1} is a function of c_t, which means that the value equation can be solved by differentiating the right-hand side of (4) with respect to c_t. The difficulty is that while we have specified the functional form for the utility function $U_j(c)$, we have not specified $V_j(a)$. Furthermore, the functional form for $V_j(a)$ must be derived from $U_j(c)$. The appendix verifies that the consistent functional form is

$$V_j(a) = K_j + \delta_j(\alpha + 1)^{-1}a^{\alpha+1}. \tag{5}$$

Furthermore, the appendix verifies that the optimal consumption is a set percentage of remaining assets.

$$c_t^* = (1 - \gamma_j)a_t, \text{ and } a_{t+1}^* = \gamma_j a_t(1+r). \qquad (6)$$

The health-state constants δ_j and γ_j must match the utility parameters. Substituting (5) and (6) into (4) and differentiating the right-hand side of (4) with respect to c_t delivers the following level and first order conditions, (7) and (8), respectively.

$$K_j + \delta_j(\alpha + 1)^{-1}a^{\alpha+1}$$

$$= K_c + (\alpha + 1)^{-1}(\theta_j(1-\gamma_j)a)^{\alpha+1} + \beta\{ K_{jp} + (\alpha+1)^{-1}[(1+r)\gamma_j a]^{\alpha+1}(p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}) \}. \quad (7)$$

$$0 = [\theta_j(1-\gamma_j)]^{\alpha} - \beta[(1+r)\gamma_j]^{\alpha}(p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}), \text{ where} \quad (8)$$

$$K_{jp} = p_sK_j + p_uK_{j+1} + p_dK_{j-1}.$$

Setting first K_c and K_j to zero, (7) and (8) can solve for the optimal asset allocations, γ_j, and the value function multipliers, δ_j. Setting assets to zero, (7) can separately solve for utility and value function constants K_c and K_j, respectively.

Table 2 shows the solutions to this dynamic programming problem for varying values of the fundamental parameters of risk aversion (α). Notice that in this self-funding mode, the optimal behavior is to consume a very small fraction of assets in the healthy states.

Table 2

◎ = -0.5, ◎ = 98.00%, r = 5.00%

State	◎	◎	ä	◎	◎◎◎	◎/ä
10	5.65	96.56%	6.25%	6.12%	3.44%	97.89%
9	5.30	96.10%	6.57%	6.30%	3.90%	95.92%
8	4.95	95.51%	6.98%	6.55%	4.49%	93.93%
7	4.57	94.74%	7.52%	6.90%	5.26%	91.86%
6	4.17	93.70%	8.25%	7.40%	6.30%	89.62%
5	3.74	92.19%	9.30%	8.10%	7.81%	87.08%
4	3.28	89.84%	10.91%	9.16%	10.16%	83.96%
3	2.76	85.69%	13.62%	10.86%	14.31%	79.73%
2	2.15	76.59%	19.10%	13.95%	23.41%	73.05%
1	1.36	43.61%	35.64%	21.12%	56.39%	59.26%

◎ = -1.5, ◎ = 98.00%, r = 5.00%

State	◎	◎	ä	◎	◎◎◎	◎/ä
10	95.66	95.07%	6.25%	5.70%	4.93%	91.20%
9	87.26	94.76%	6.57%	5.87%	5.24%	89.30%
8	78.35	94.37%	6.98%	6.08%	5.63%	87.13%
7	68.95	93.87%	7.52%	6.36%	6.13%	84.59%
6	59.10	93.21%	8.25%	6.73%	6.79%	81.55%
5	48.89	92.29%	9.30%	7.24%	7.71%	77.78%
4	38.41	90.95%	10.91%	7.95%	9.05%	72.92%
3	27.85	88.80%	13.62%	9.03%	11.20%	66.28%
2	17.51	84.75%	19.10%	10.77%	15.25%	56.41%
1	7.88	74.14%	35.64%	14.10%	25.86%	39.57%

◎ = -2.5, ◎ = 98.00%, r = 5.00%

State	◎	◎	ä	◎	◎◎◎	◎/ä
10	1,756.91	94.87%	6.25%	5.53%	5.13%	88.44%
9	1,566.19	94.63%	6.57%	5.67%	5.37%	86.28%
8	1,367.15	94.33%	6.98%	5.84%	5.67%	83.77%
7	1,162.04	93.95%	7.52%	6.07%	6.05%	80.78%
6	954.03	93.45%	8.25%	6.37%	6.55%	77.17%
5	747.46	92.78%	9.30%	6.76%	7.22%	72.71%
4	548.05	91.83%	10.91%	7.31%	8.17%	67.01%
3	363.20	90.37%	13.62%	8.10%	9.63%	59.44%
2	202.22	87.83%	19.10%	9.31%	12.17%	48.76%
1	76.51	82.07%	35.64%	11.44%	17.93%	32.10%

Note that the policyholder in this model cannot experience financial ruin. Unlike the self-funded retirement model of Milevsky and Robinson (2000), consumption is re-optimized each period as a function of current wealth. For each health state, the policyholder consumes a constant proportion of his wealth in each time period. While continuing good health may result in continually decreasing consumption, assets are never exhausted.

The above analysis assumes that the policyholder has no access to payout annuities, and must bear all the longevity risk associated with retirement. Suppose instead that the policyholder can purchase an annuity that pays amounts that result in level utility for all remaining periods of the policyholder's life. The value of wealth is simply the utility value of the annuity purchased with the assets.

$$W_j(\pi a_t) = U_j(\pi a_t) + \beta\{p_s W_j(\pi a_t) + p_u W_{j+1}(\pi a_t) + p_d W_{j-1}(\pi a_t)\}, \quad (9)$$

where π is the payout annuity factor (the fraction of annuity premium paid in each period). In the estimated case where $p_u = 0$, $p_s + p_d = 1$, and $\beta = 1$, the value of an annuity is linear in life expectancy. In general, one can show that W_j is of the form

$$W_j(c) = \varepsilon_j(\alpha+1)^{-1}c^{\alpha+1}. \quad (10)$$

Knowing both V_j and W_j, one can solve for the value of π_j which makes the policyholder indifferent between purchasing an annuity and self-insuring retirement. $\pi_j = (\delta_j/\varepsilon_j)^{1/(\alpha+1)}$. Table 2 also shows this as a percentage of the naïve price of the annuity assuming the same interest rates and mortality. Not surprisingly, the higher the risk-aversion parameter, the greater the policyholder's premium for retirement risk protection.

Notice also that the least healthy seem also to be the most risk-averse. This is precisely because they face the greater mortality risk. While the healthy may face more aggregate uncertainty as to the age of time to death, that uncertainty is discounted in the future. A person with a health status of 10 and 32 percent probability of declining health in any one period is certain to survive for 10 years, and has less than an 8 percent probability of death in 20 years.

4. Competitive Model

This section focuses its attention on the age-65 male. Mortality is given by parameter estimates from the Annuity 2000 Basic mortality table. Assume that the base policyholder utility parameters are $\alpha = -1.5$, $\beta = .98$, and r = 5 percent. A summary of all relevant parameters and coefficients is given below.

Assume that the insurance company has no information about policyholder health and that the policyholder has perfect information of health state. Further, assume that the insurance company can competitively price an annuity using the same interest rate facing the policyholder. Using the above population initial conditions, the insurance company can derive an annuity mortality table, which not coincidentally matches the Annuity 2000 Basic mortality table. Using an interest rate of 5 percent, the payout annuity factor is 9.62 percent.

This payout factor is sufficiently low to make purchasing an annuity unprofitable for anyone with health states 1 or 2. Eliminating this portion of the population delivers a payout annuity factor of 9.01 percent. This in turn eliminates health state 3 from the buying population. The annuity factor for mortality conditional on initial health state >3 at age 65 is 8.61 percent, which is optimal to purchase for any policyholder with health state > 3.

In this example, the effect of self-selection is to decrease the payout factor from 9.62 percent to 8.61 percent and shrink the market by making the annuity unattractive to the least healthy 21 percent of the age-65 population. However, the market still exists for the remaining healthy population.

Suppose instead that the insurance company bases its prices on an interest rate which is lower than that used by the policyholder. Even if the interest rate earned by the insurance company was 4 percent (a full 1 percent lower), the annuity payout factor conditional on health state > 3 is 8.00 percent, which is still attractive to the same population as before.

Alternatively, suppose that the risk-aversion parameter is -0.5. To give some context, with this risk-aversion parameter the policyholder in health state 1 with no access to the annuity market optimally consumes over 55 percent of his assets in any period even though the probability of death is only 32.5 percent. This gamble causes over a 50 percent decrease in wealth in each remaining time period, with utility declining by 30 percent each year. Brown (2001) uses answers to simple risk-aversion questions to estimate that only about 10 percent of the population has risk-aversion parameters greater than -1. In this environment, with the insurance company facing the same interest rate as the policyholder, annuities are not supported for the bottom four health states, but the equilibrium holds for 62 percent of the population.

Only if you combine a risk-aversion parameter of -0.5 with an interest rate differential of greater than 0.50 percent does the market collapse due to adverse selection. With a company interest rate of 4.5 percent, the mortality table conditional on initial health state 10 delivers an annuity payout factor of 5.93 percent. Health state 10 policyholders with risk-aversion parameter $\alpha = -.5$ prefer annuities with payout factors greater than 6.12 percent.

5. Policy Considerations

It has been suggested that capital market considerations make it optimal for policyholders to buy a "term annuity" and invest the difference, at least at the retirement age of 65. The key assumption in this conclusion is that direct access to capital markets provides a superior return to the typical fixed return earned on a payout annuity. Individual knowledge of mortality increases the richness of this decision.

The major finding of this paper is that the annuity puzzle is still a puzzle. In a model with the starkest of adverse selection, the consumption-smoothing advantages of annuities dominate the adverse selection issues. Based on the utility functions used in this paper, selection issues restrict only a small portion of the market.

The newly retired policyholder finds for the first time in his or her life that good health is expensive, at least in the sense that it lengthens the expected cost of providing for retirement. This effect exacerbates the negative correlation between health status and retirement age (see, for example, Dwyer (2001)). Healthy workers may optimally delay retirement to avoid drawing down assets which they perceive they will need to fund a long retirement period.

Based on population estimates, the annuitant population at the typical retirement age of 65 is much more homogeneous than the population at 75 or 85. If the annuity market can generate a product mix rich enough to differentiate (separate) between healthy and unhealthy annuitants, the problem of self-selection is ameliorated.

The brute-force solution to mortality differentiation would be to introduce underwriting. Alternatively, any annuity feature which can distinguish between health states might be useful. For example, a payout annuity could be bundled with LTC insurance. A payout annuity with direct inflation protection or increasing future payments would be viewed as more attractive to those with longer life expectancies.

Finally, proper accounting for the adverse selection in payout annuities suggests using a select-ultimate table for payout annuity pricing. Select-ultimate tables more accurately correct for selection effects than do loading factors.

Appendix

We wish to show that the value function (5) and optimal consumption function (6) solve the dynamic programming problem faced by the policyholder.

LEMMA (C): The consumption function (6) is optimal given value function (5)

The key result in (6) is not that a proportion of current assets are consumed each period, but that the proportion is independent of the level of assets a_t. Substituting (1) and (5) into (4) yields the optimization problem

$$V_j(a_t) = \max_{\{ct\}}[(\alpha+1)^{-1}(\theta_j c_t)^{(\alpha+1)}+\beta(\alpha+1)^{-1}[(a_t-c_t)(1+r)]^{\alpha+1}\{p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}\}] + K_j^*, \quad \text{(A1)}$$

Differentiating the right-hand side of (A1) gives

$$0 = \theta_j^{\alpha+1}c_t^{\alpha} - \beta(1+r)^{\alpha+1}\{p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}\}(a_t-c_t)^{\alpha}, \quad \text{(A2)}$$

or equivalently

$$c_t = [1 + (\theta_j^{\alpha+1}/(\beta(1+r)^{\alpha+1}\{p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}\}))^{1/\alpha}]^{-1}a_t = (1-\gamma_j)a_t. \text{ QED} \quad \text{(A3)}$$

LEMMA (V): The value function (5) is consistent with the utility function (1) and the optimization problem (4).

Substituting (5) and (6) into (4) yields

$$V_j(a_t) = (\alpha+1)^{-1}(\theta_j(1-\gamma_j)a_t)^{(\alpha+1)}+\beta(\alpha+1)^{-1}[\gamma_j a_t(1+r)]^{\alpha+1}\{p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}\} + K_j^*$$

$$= (\alpha+1)^{-1}[(\theta_j(1-\gamma_j))^{(\alpha+1)}+\beta[\gamma_j(1+r)]^{\alpha+1}\{p_s\delta_j + p_u\delta_{j+1} + p_d\delta_{j-1}\}]a_t^{\alpha+1} + K_j^*. \text{ QED} \quad \text{(A4)}$$

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