Securitization of Mortality Risks in Life Annuities*

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Abstract

The purpose of this paper is to study mortality-based securities, such as mortality bonds and swaps, and to price the proposed mortality securities. We focus on individual annuity data, although some of the modeling techniques could be applied to other lines of annuity or life insurance.

1. Introduction

The purpose of this paper is to study the securitization of mortality risks and to price proposed mortality bonds and swaps with embedded mortality options. Mortality-based securities will expand the array of tools available to insurers and reinsurers to manage mortality risks. The potential for greater underwriting capacity, innovative long-term contracting and lower costs make securitization worth investigating as a supplement to traditional reinsurance. Swiss Re issued the first mortality bond as we were writing this paper (Swiss Re, 2003b; Morgan Stanley, 2003). Although other securities have been based on life insurance and annuity portfolios, this is the first to be traded on pure mortality risk. Betteto (1999) describes the logic behind the deals as price efficiency. Actually, none of these is a pure mortality transaction, like the Swiss Re bond. All of the earlier securitizations were used to effectively sell future cash flows, much like other asset-backed securitizations.

The Swiss Re bond is based on a mortality index of the general population of the United States, United Kingdom, France, Italy and Switzerland. The term of the bond is three years; the price is $400 million; and it pays LIBOR plus 135 basis points. If the mortality index exceeds 130 percent of the 2002 level, the principal is reduced. If it goes above 150 percent, the principal is exhausted. Morgan Stanley's announcement describes this as a one-in-25-year event (Morgan Stanley, 2003). It goes on to say that "the appetite for this security from investors was strong." This is the same reaction investors have had to catastrophe bonds. In this paper, we focus primarily on the other side of the "mortality tail" for which annuity writers (insurers, reinsurers or pension plans) have the greatest concern.

The longevity risk is a dynamic phenomenon. Life expectancy throughout the world in recent decades has improved, but that does not necessarily imply that trend can be projected into the future. Mortality analysis has a long tradition in actuarial science because mortality trends can have a profound influence on a life insurer's financial condition. However, since no one can accurately predict the future, risk management of mortality risk is an indispensable part in the insurer's operation. In addition to uncertainty in mortality forecasts, there are economic and policy changes that make management of longevity risk more important than ever.

Ten years ago, Friedman and Warshawsky (1990) argued that it was puzzling that so few people avail themselves of the private market for annuities. They listed three potential answers to this puzzle. Firstly, people save not for motives related to the usual life-cycle reasoning but, instead, to leave bequests to their heirs. Secondly, most individuals automatically receive life annuities from Social Security and, for a significant fraction of the labor force, employer-sponsored pension plans. Lastly, a more plausible explanation is that people shun individual annuities because they are not priced "fairly" in the actuarial sense (Friedman and Warshawsky, 1990). Although the individual annuity market is currently relatively small, it has attracted substantial interest from researchers and policymakers concerned with the evolving system of retirement income provision. Current discussions of Social Security reform, and the shift from defined-benefit to defined-contribution private pension plans, suggest that there may be increased interest in individual annuity products in the future, according to Mitchell, Poterba, Warshawsky and Brown (2001). They also find evidence that an individual annuity contract appears to be a more attractive product to consumers today than 10 years ago.

As demand for individual annuities increases, insurers' needs for risk management of the potential mortality improvements increase as they write new individual annuity business. As Rappaport, Mercer and Parikh (2002) describe, insurers manage the risk in issuing these new annuity policies. They are keenly interested in understanding the future course of longevity, as well as the protection provided by hedging, asset-allocation strategies, reinsurance and perhaps securitization of mortality risks.

Section 2 covers the potential expansion of the individual annuity market in the United States. In section 3 we discuss the demand for mortality-based securities and describe how insurers can use mortality-based securities and why they may want to sell them. In section 4 we describe the difficulties arising in making mortality projections. We discuss annuity data, including the Individual Annuity Mortality tables and the Group Annuity Experience Mortality (GAEM) reports from reports of the *Transactions of the Society of Actuaries* (*TSA*). We decided to use the GAEM experience for our mortality forecasts with a model by Renshaw, Haberman and Hatzoupoulos (1996). In section 5 we define mortality swaps and show how they can be used to hedge mortality risk. In section 6 we introduce mortality-risk bonds and price them using the Wang transform. Section 7 is for discussion and conclusions.

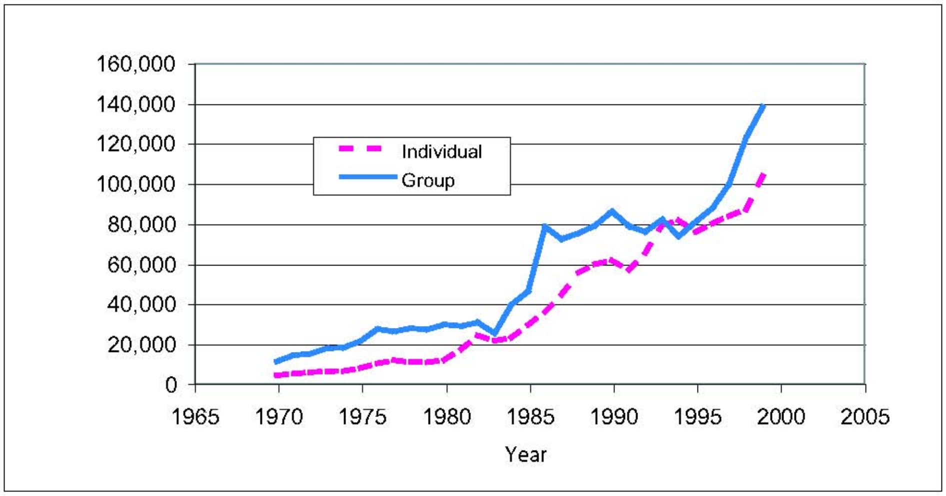
2. Individual Annuity Market in the United States

The annuity market, including fixed annuities, variable annuities, individual annuities and group annuity contracts, has grown sharply in the last decade. With the baby boom cohort approaching retirement, Social Security reform, the decline in the growth of defined-benefit pension plans, and the increase in the growth of defined-contribution plans, we expect that the individual annuity market will expand dramatically.

2.1. Defined Benefit Pension Plans vs. Defined Contribution Plans

Figure 1 shows the individual and group premium income received by insurance companies for annuity policies over the 1970-99 periods, converted to 1994 dollars using the Consumer Price Index. Poterba (2001) notes that although premiums on group policies were three to five times greater than the premiums on individual polices throughout the 1950s and 1960s, individual annuities grew more rapidly from the 1970s until the mid-1990s. In 1994, premium income from individual annuities exceeded that from group annuities. By the 1990s, annuity reserves were more than twice the value of life insurance reserves. The long-term growth of individual relative to group annuity premium reflects both the decline in the growth of defined-benefit pension plans and the rapid expansion of individual annuity products, particularly variable annuities.

Figure 1. Annuity premium annual income of life companies in millions of 1994 dollars from Poterba (2001).



2.2. Baby Boom

According to Mitchell et al. (2001), as the baby boom cohort in the United States nears and moves into retirement, analysts, policymakers and advisors in many nations are devoting increased attention to issues of old-age income security. Increased longevity imposes a greater risk to individuals of outliving their resources who may be forced to substantially reduce their living standards at an advanced age. The demand for individual annuities should increase. This should increase demand for annuity reinsurance. Reinsurers could use mortality bonds to increase their capacity to write annuity reinsurance.

2.3. Social Security Reform

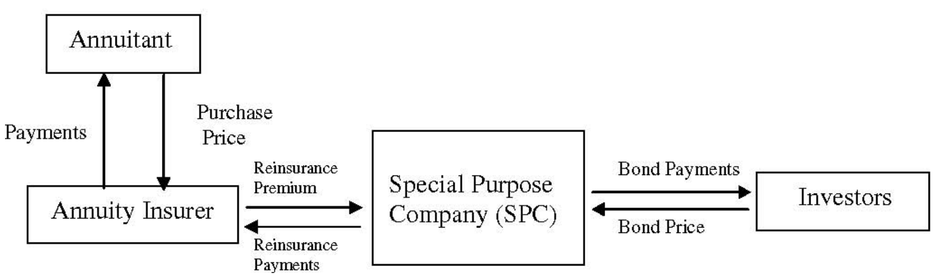
Mitchell et al. (2001) argue that Social Security reform discussions in the United States and other nations have the potential to increase the demand for private annuities. These reforms, if enacted, could also substantially affect the structure of the annuity marketplace. Securitization may be an efficient means for a reinsurer to increase its capacity to write coverage of individual annuities. In this way, reinsurers could facilitate the privatization of public plans.

3. Insurance Securitization

Insurance risk, usually catastrophic property damage risk, has been successfully passed to bondholders. These are the so-called cat bonds. Cox and Pedersen (2000) describe a model for pricing cat bonds and several examples of cat bonds. Cox, Pedersen and Fairchild (2000) discuss the conditions under which a market for cat bonds is viable. They argue that, if cat risks are uncorrelated with the stock and traditional bond markets, then adding cat bonds to the market improves investment opportunities. An investor with traditional high-yield bonds will prefer to hold cat bonds of the same investment quality, because of their lower covariance with the market. This helps explain why there were over 30 cat bond transactions reported in the financial press. Mortality risk bonds are different in several important ways. However, in both cases transaction costs are likely to be high relative to reinsurance on a transaction basis.

A mortality securitization works like a cat risk securitization. For example, the reinsurer (insurer or annuity provider) purchases reinsurance from a special purpose company (SPC). The SPC issues bonds to the investors. The bond contract and reinsurance convey the risk exactly from the annuity provider to the investors. The SPC invests the reinsurance premium and cash from the sale of the bonds in default-free securities, as shown in Figure 2 below. This allows the SPC to pay the benefits under the terms of the reinsurance with certainty.

Figure 2. Mortality Bond Cash Flow Diagram



The rationale for a market for cat bonds was (at least in part) based on the notion that cat bonds returns should be uncorrelated with market returns—that is, cat securities have a zero beta. According to some experts with whom we have discussed this idea, the same argument is not likely to apply to mortality-based securities due to the life-cycle theory of Modigliani (1986). Actually, Modigliani discusses the correlation of wealth and longevity, not market returns and longevity. Certainly one can conjecture that mortality securities have a nonzero beta, but it seems to be an open question. In any case, we are not asserting that mortality securities have a zero beta. Although investors may want to know the beta of a proposed mortality bond, it does not have to be zero in order for mortality securities to be attractive. Indeed, according to the Morgan Stanley announcement of the Swiss Re bonds, "appetite for this security from investors was strong."

The same rationale for hedging longevity risk with reinsurance can be applied to securitization. The advantages of securitization may be lower costs in the long run, more favorable contracts and elimination of default risk. More importantly, securitization can bring additional capital to the life and annuity industry.

3.1. Raising Required Capital

When an insurer sells an immediate annuity, it usually pays a commission (and incurs other costs). Insurance and accounting regulations require that the company hold capital to provide for future annuity benefits. It is possible that the sum of acquisition costs and statutory capital required exceed the premium paid by the annuity owner.

For example, suppose the premium is $5,000,000 for a male annuitant age 65. The commission and issue expense might be 4 percent, or $200,000. The total monthly payout is $39,058 based on the average immediate annuity market quotes in 1995 (about 7.81 dollars per month per 1,000 dollars of premium). The statutory reserve is determined by valuation regulations as

$$39,058 \times 12 a_{65}^{(12)}$$

With level annual interest rates of 6 percent and the 1996 US Annuity 2000 Basic Annuity Table, the liability value for an annuity of one dollar per year is $a_{65}^{(12)}$=10.63566. The problem for the insurers is that \$39,058 $\times$ 12 $a_{65}^{(12)}$=\$4,984,891 which exceeds the net price the company gets after commission (\$4,800,000) by \$184,981. This is about 3.7 percent of the premium. This does not mean the business is not profitable. On a market-valuation basis, the present value of future benefit (PVFB) using realistic mortality and market interest rates is less than the premium. That is,

$$\text{PVFB} + \$200,000 < \$5,000,000,$$

so the company adds to its market value on a present-value basis. In other words, the company must dedicate capital to the annuity business in order to grow. Reinsurance and securitization can be used to provide capital as the annuity business grows.

3.2. Innovative Contracting

Cummins and Lewis (2002) describe securitization as the repackaging and trading of cash flows that traditionally would have been held on balance sheet by financial intermediaries or industries. Securitizations generally involve the agreement between two parties to trade cash-flow streams to manage and diversify risk and/or to take advantage of arbitrage opportunities. Reinsurance is a traditional way for insurers to transfer their risks to reinsurers. Securitization may be a way for reinsurers to increase capacity to write annuity reinsurance. Evidently increasing capacity to write life insurance coverage was Swiss Re's motivation for its mortality bond.

One advantage of securitization is that it provides new capacity. The bond contact can be customized for the borrower and lender and could be very different from traditional reinsurance contracts. For example, the bond contract might provide for 30 years of coverage. The transaction cost of issuing bonds is expensive relative to buying reinsurance. However, billions of dollars of assets (mortgages, auto loans and so on) are securitized each year. If the technology used in these securitizations is brought to annuity securitization, and if large numbers of annuitants are involved, then the price per unit may be competitive with reinsurance.

Renewing reinsurance frequently may pose higher transaction costs on the primary annuity insurer than those of securitization. The term of coverage provided by a mortality bond could be 30 years, or more, although typically corporate bonds are issued for 10 to 20 years and are callable at the issuer's option. As far as we can determine, life and annuities reinsurance contracts normally provide much shorter coverage.

As Edwalds (2003) notes, longevity risk could easily extend over 50 years or more. Most long-term bonds mature within 30 years. Reinsurance contracts of which we are aware have much shorter term coverage. It is conceivable that a reinsurer can issue a very long-term bond (through the SPC), essentially default free except for mortality risk, which would appeal to investors. This would increase the reinsurer's capacity to issue long-term contracts to its client companies.

Mitchell et al. (2001) describe dramatic advances in life expectancy in the United States over the last century. Today's typical 65-year-old man and woman can expect to live to age 81 and 85, respectively. Perhaps even more striking is the fact that almost one-third of 65-year-old women and almost one-fifth of 65-year-old men are likely to live to age 90 or beyond. Thus, long-term hedging is especially important for the annuity insurer. According to Eason, Hirst and Vukelic (1999), "The other issue facing reinsurers is that they also, because it is insurance, have the same kind of target capital needs that the insurance company does, which has typically a higher risk-based capital (requirement) than a pure bank does."

Therefore, reinsurers may find annuity securitization to be an efficient means of increasing capacity despite transaction costs, simply because reinsurers must hold more capital to write the same risk. With greater capacity, better contracting terms (longer terms, for example) and potentially lower cost (more efficient use of capital), securitization may be a feasible tool for reinsurer to hedge its mortality risks.

4. Difficulties in Accurate Mortality Projection

General and insured population mortality has improved remarkably over the last several decades. At old ages, probabilities of death are decreasing, increasing the need for living benefits. The calculation of expected present values (needed in pricing and reserving) requires an appropriate mortality projection in order to avoid underestimation of future costs which will jeopardize an insurer's profit.

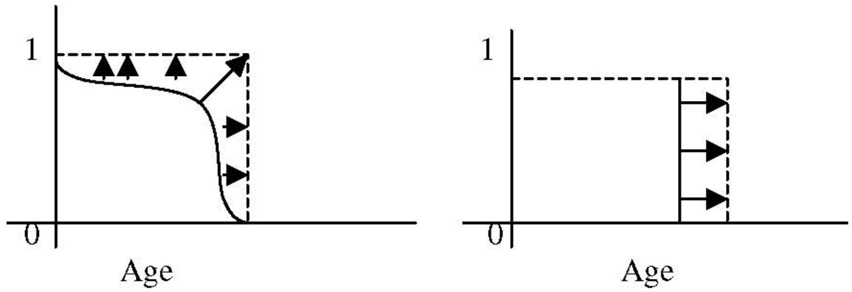
Rogers (2002) shows that mortality operates within a complex framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors. Not all of these factors improve with time. For example, for biological variables, recent declines in mortality rates were not distributed evenly over the disease categories of underlying and multiple causes of death. According to Stallard (2002), successes against the top three killers (heart diseases, cerebrovascular diseases and malignant neoplasms) did not translate into successes against many of the lower ranked diseases.

4.1. Different Opinions in Mortality Trend

4.1.1. Improvement

Buettner (2002) concludes that there are today two alternative views about the future improvement of mortality at older ages: compression vs. expansion (sometimes also called rectangularization vs. steady progress), illustrated in Figure 3. Mortality compression occurs when age-specific mortality declines over a widening range of adult ages, but meets natural limits for very advanced ages. As a result, the survivor curve would approach a rectangle and mortality across countries may indeed converge to similar patterns.

Figure 3. Two views of mortality improvement—rectangularization on the left and steady progress on the right.



In the case of steady progress, there are no natural limits to further reductions in mortality at higher ages. The age at which natural limits set in does not exist. Consequently, all age groups, especially higher age groups, would continue to experience declining mortality. The Human Genome Project is producing a rapidly expanding base of knowledge about life processes at their most fundamental level. Some experts have predicted that the genes for the aging process will be identified and drugs to retard the aging process will be developed in the not-distant future. It is worth noting that genetic technology, including the mapping of the human genome, has developed much faster than forecasts. Anti-aging drugs may be available sooner than anyone forecasts.

4.1.2. Life Table Entropy

Life table entropy refers to a phenomenon that further improvement of already high life expectancies may become increasingly more difficult. The gains in survival a century ago were greater than they have been more recently. For instance, Rogers (2002) shows that the survival gains achieved between 1900 and 1920 are large compared to the modest gains realized between 1980 and 1999. Hayflick (2002) suggests that:

> ...Those who predict enormous gains in life expectation in the future based only on mathematically sound predictions of life table data but ignore the biological facts that underlie longevity determination and aging do so at their own peril and the peril of those who make health policy for the future of this country.

4.1.3. Deterioration

Although general population mortality has improved over time, the improvement may be overstated. Substantial mortality improvements often come after periods of mortality deterioration. For example, between 1970 and 1975, males aged 30-35 saw annual mortality improvement of over 2 percent, but this may be an adjustment to the 1.5 percent annual mortality decline that occurred during the previous five-year period. Moreover, there is still a chance for a resurgence of infectious diseases. Deaths due to influenza could increase with the introduction of new influenza strains or with shortages of the influenza vaccine. Rogers (2002) argues that although HIV is now controlled, it is not eradicated and could expand, or variants of HIV could develop that could increase mortality. Drug-resistant infectious diseases like tuberculosis could increase. Goss, Wade and Bell (1998) find that age-adjusted annual death rates for ages 85 and over in the United States actually deteriorated by 0.72 percent per year for males and by 0.52 percent for females during the observation period 1990-94.

There is no agreement among experts on the future of mortality. Steady improvement is the trend, but changes in either direction are feasible.

4.2. Technical Difficulties in Mortality Projections

4.2.1. Quality of Data

Good quality complete data is a prerequisite for a reliable mortality projection. However, in reality it is not easy to obtain data for research. For example, although detailed data on old-age mortality are collected in most countries of the developed world, they are not so commonly available for developing countries. Buettner (2002) claims that even in developed countries, the quality of age reporting deteriorates among the very old.

The Society of Actuaries' series of studies of life annuity experience is of limited value for several reasons. First, it is not timely. Second, it is appropriate only for the products the policyholders owned (whole life, term life or annuities, for example). So it cannot be used directly to assess mortality for new products or similar products issued on a new basis (e.g., underwriting annuities for select mortality).

Thulin, Caron and Jankunis (2002) note that complexity of annuity products nowadays often makes mortality projection difficult. Sometimes, an insurer has to introduce new entries with different mortality assumptions into the insured pool. For instance, trends in the marketplace are blurring traditional distinctions in the following two key areas:

(1) Worksite products sold on an individual basis increasingly show features traditionally associated with group products.

(2) Group products sold on the basis of individual election in the workplace (voluntary products) with minimal participation requirements compete directly with individual

products.

They severely limit insurers' ability to underwrite to discern mortality differentials. New sources of underwriting information are becoming a way of life for insurers, as pressure on costs and hastened issue pressure create an underwriting environment with less documentation and information. One solution is making more data available to researchers and making it available sooner.

The Society of Actuaries publishes tables and mortality reports from time to time. The individual annuity mortality (IAM) tables are intended for estimation of insurance company liabilities. While these tables are based on actual insurance industry experience, the rates are projected or loaded in order to produce conservative estimates of annuity liabilities. Until 1992 the Society published periodic group annuity mortality reports of actual experience. While the reports do not contain complete mortality tables, they are not adjusted; instead these reports reflect actual industry experience. Moreover, the experience reports were made more frequently than tables were constructed. For these reasons, based on the above information, we decided that the loaded or projected tables are not appropriate for our illustration and prediction.

The *1983 Transactions Reports of the Society of Actuaries* on Annuities (1983) present calendar year experience of retired individuals who are covered under insured pension plans in the United States and Canada. The report includes experience of contracts providing insurer-guaranteed annuity benefits to ongoing pension plans and experience of contracts covering closed groups of lives for which purchases are made by a single payment at issue (single-premium-close-out business); it excludes contracts which do not contain insurer guarantees of future payments (immediate participation guarantee contract direct payment benefits). The reports summarize calendar year exposures and deaths in five-year age groups. Male and female data are displayed by number of lives and amount of annual annuity income.

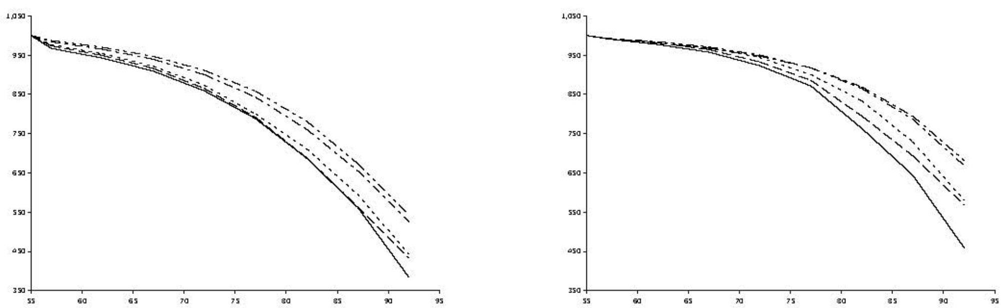
The *1983 TSA Reports* on Annuities (1983) describe problems encountered in collecting data:

> The last published report of insured group annuity mortality experience appeared in the 1975 reports covering calendar years 1969-71. It was hampered by data collection problems during the subsequent ten years: several companies discontinued experience submission, and several others submitted data which were inconsistent or riddled with reporting errors.

The problems remain although the Society is working to revive its experience studies. In the end, we decided to use the *GAM Experience Reports* since they are based on actual mortality improvement.

The *GAM Experience Reports* on Annuities (1952, 1962, 1975, 1983, 1984, 1987, 1990, 1994, 1996) describe the mortality improvement from 1951-1992. The *Reports* give the number of deaths observed among a cohort of annuitants in five-year age groups observed for one year. The observations of deaths and exposures are summarized in the appendix to this paper. The *Reports* provide data, but do not construct mortality tables. We show graphs of this experience in Figure 4. The data comes from experience for retired individuals covered by pension plans in the United States and Canada. For male and female data, the survival curves generally rise with the observation period. The change between 1981 and 1991 for females is an exception since there is some deterioration at the later ages. That is, the lowest rates at each age are for the 1951 observations, the next to lowest are for 1961, and so on. The trend in improvement is increasing on average, with the largest increase occurring between 1971 and 1981 for males and females.

Figure 4. Number of survivors of an initial cohort of 1,000 male (left) and female lives at age 55, based on the Society of Actuaries *TSA Reports* for 1951, 1961, 1971, 1981, and 1991 on group annuity experience, without adjustments.



4.2.2. Projection Models

Recent changes in mortality challenge mortality projection models. The competitive nature of the insurance market means that an insurer cannot raise its price at will. A sound projection model is crucial. However, the revealed weakness and problems of poor fitting may arise because most projection models do not capture the dynamics of mortality that is changing in a dramatic and fundamental way.

Renshaw et al. (1996) suggest a generalized linear model which showed mortality declining over time with the rates of decline not being necessarily uniform across the age range. It incorporates both the age variation in morality and the underlying trends in the mortality rates. The advantage of this model is that the predictions of future forces of mortality come directly from the model formula. We adopt this model for investigating the performance of mortality derivatives based on a portfolio of life annuities.

During a certain period, the force of mortality, $\mu(x,t)$, at age x, in calendar year t, is modeled using the following formula:

$$\mu(x,t) = \exp\left[\beta_0 + \sum_{j=1}^{s}\beta_j L_j(x') + \sum_{i=1}^{r}\alpha_i t'^i + \sum_{i=1}^{r}\sum_{j=1}^{s}\gamma_{i,j}L_j(x')t'^i\right]$$
$$= \exp\left\{\sum_{j=0}^{s}\beta_j L_j(x')\right\}\exp\left\{\sum_{i=1}^{r}\left(\alpha_i + \sum_{j=1}^{s}\gamma_{ij}L_j(x')\right)t'^i\right\} \quad (1)$$

where

$$t' = \frac{t-1971.5}{20.5} \quad \text{and} \quad x' = \frac{x-74.5}{17.5}.$$

Sithole, Haberman and Verrall (2000) use the same model. They note that first factor in (1) is the equivalent of a Gompertz-Makeham graduation term. The second multiplicative term is an adjustment term to predict an age-specific trend. The γ_{ij} terms may be preset to 0. The age and time variables are rescaled to x' and t' so that both are mapped onto the interval $[-1,+1]$after transforming ages and calendar years. $L_j(x)$is the Legendre polynomial defined below:

$$L_0(x) = 1$$
$$L_1(x) = x$$
$$L_2(x) = (3x^2-1)/2$$
$$L_3(x) = (5x^3-3x)/2$$
$$\vdots$$
$$(n+1)L_{n+1}(x) = (2n+1)xL_n(x) - nL_{n-1}(x)$$

where n is a positive integer and $-1 \le x \le 1$.

The data are the actual group annuity mortality experience for calendar years t=1951,1961,1971,1981,...,1992. Since the *GAM Experience Reports* are five-year age group results, we assume that the ratio of the total number of deaths in each group over the total number of exposures in that group (the average death rate in that group) represents the death rate of the middle-point age of that group. We use the middle-point age as our observation in the regression. The experience was analyzed for the middle-point age ranges x=57 to 92 years for both male and female, giving a total of 120 data cells for males and 120 for females.

In fitting the equation (1), we found that when the parameter $\gamma_{1,2}$ is excluded from the formula (for male and female), all of the remaining six parameters in the model are statistically significant. Although the six-parameter model which excludes the quadratic coefficient in age effects from the trend adjustment term was next fitted to the data, the revised models seem to be appropriate for making predictions of future forces of mortality.

$$\mu(x,t) = \exp\left[\beta_0 + \beta_1 L_1(x') + \beta_2 L_2(x') + \beta_3 L_3(x') + \alpha_1 t' + \gamma_{11} L_1(x') t'\right] \quad (2)$$

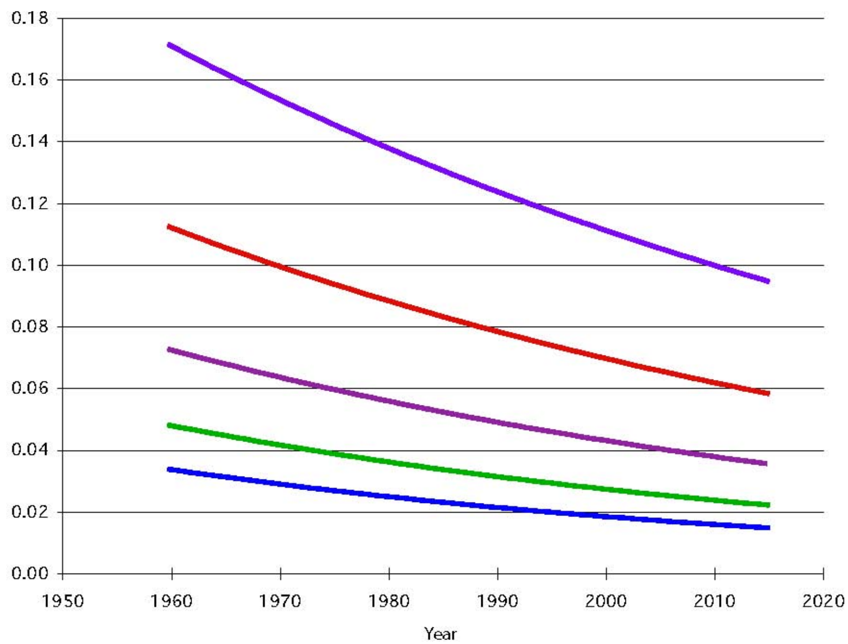
Details of the revised fit are given in Table 1.

Table 1. Group annuities, six-parameter log-link model. All of the coefficients are significant at the 1 percent level.

	Male		Female	
Coefficient	Value	Standard error	Value	Standard error
β_0	2.7744	0.0087	3.3375	0.0111
β_1	1.3991	0.0139	1.7028	0.0179
β_2	0.1053	0.0114	0.1543	0.0146
β_3	0.1073	0.0127	0.0872	0.0163
α_1	0.2719	0.0116	0.2660	0.0149
$\gamma_{1,1}$	0.0839	0.0178	0.1294	0.0228
Adjusted R_2	0.9944		0.9930	
Sum of squared errors	0.0701		0.0899	

Figure 5 shows the male group annuity predicted forces of mortality based on the six-parameter model given by (3). All of the predicted forces of mortality progress smoothly with respect to both age and time, and the model naturally predicts a reduction in the rate of improvement in mortality at the old ages. We will use the values predicted by the six-parameter model to investigate the mortality derivative's performance. We have a model based on experience from 1951 to 1992. There are errors in the estimate which should tell us how confident we can be in projecting mortality into the future, assuming the dynamics of mortality improvement continue as they have in the observation period. This is potentially dangerous. As we have pointed out earlier, there is a good bit of controversy with regard to the dynamics of mortality improvement.

Figure 5. Male Group Annuity Mortality, predicted forces of mortality based on six-parameter log-link model and *TSA Reports* 1951-1992. The top curve is the force of mortality for age 85; the one just below it is for age 80, then 75, 70 and the bottom curve is for age 65. The greatest improvement (steepest slope) is at age 85.



We note also that these results are based on group annuity experience. Individual annuity experience may be very different. For example, antiselection should be a much more important issue. As the market for individual immediate annuities develops, insurers will have to adjust their estimates to reflect the change in the market mortality. They may have to apply underwriting techniques and control for moral hazard and antiselection when they issue annuities, just as they now do for life insurance.

5. Mortality Swaps

Insurers need to manage their risk in issuing annuity policies and are therefore keenly interested in understanding the future course of longevity, as well as the potential uncertainties that they must insure themselves against through hedging, asset-allocation strategies and reinsurance. Recently we have seen that reinsurers use bonds with embedded options (cat bonds, mentioned earlier) and swaps (Swiss Re, 2003a) to manage catastrophic property losses. No one can predict future mortality levels and managing mortality risks is always going to be a problem, so we expect reinsurers will use mortality swaps and mortality bonds. The Swiss Re bond may be the beginning of a mortality security market that is much larger than the cat bond market.

Certainly the dynamics of interest rates play an important role in pricing and hedging annuity liabilities. However, we are going to focus on mortality and take the interest-rate dynamics as given and independent of mortality dynamics.

A swap can be regarded as a series of forward contracts, and hence they can be priced using the concept of forwards. We assume the initial number of the survivors is 1,000,000 at age 55. Our idea of mortality swap is motivated by the insurer's desire to pay fixed-level payments for a series of variable-level payments. The characteristics of the mortality swap we propose are very similar to the plain vanilla interest swap. So we call our proposed swap "the plain vanilla mortality swap."

As an example of a mortality swap, consider an insurer[1] that must pay immediate life annuities to N annuitants[2] now all aged x. Set the notional principal at $1,000 per year per annuitant. The insurer's actual payments could be used, but to keep the concept as simple as possible we fix the principal amount as 1,000 per year per annuitant. Let l_{x+t} denote the number of survivors to year t. The insurer pays (at least) $1{,}000\,l_{x+t}$ to its annuitants. The swap is designed to hedge this portion of the insurer's payments to its annuitants.

The insurer and its swap counterparty agree on a level X_t for each year. In year t the insurer pays a fixed amount $1000\,X_t$ (varying only perhaps by duration but not random) to the counterparty and receives $1000\,l_{x+t}$. The insurer and counterparty agree at the beginning as to the annuitant pool in much the same way that mortgage loans are identified in construction of a mortgage-backed security. The insurer and counterparty payments are made on a net basis, so if there are more survivors to year t than expected (relative to the preset level) the company gets $1000\,(l_{x+t} - X_t)$>0. The insurer's net cash flow to annuitants is offset by positive cash flow from the swap:$1000 l_{x+t} - 1000(l_{x+t} - X_t) = 1000 X_t$.

[1]The "insurer" could be an annuity writer, an annuity reinsurer or private pension plan. The counter party could be a life insurer or investor.

Of course, if mortality the other way, the insurer still has a net cash flow of 1000 X_t since the insurer will pay the excess$1000(X_t - l_{x+t})$ to the counterparty. In this way a mortality swap can transform a segment of the insurer's annuity payments into a fixed cash flow.

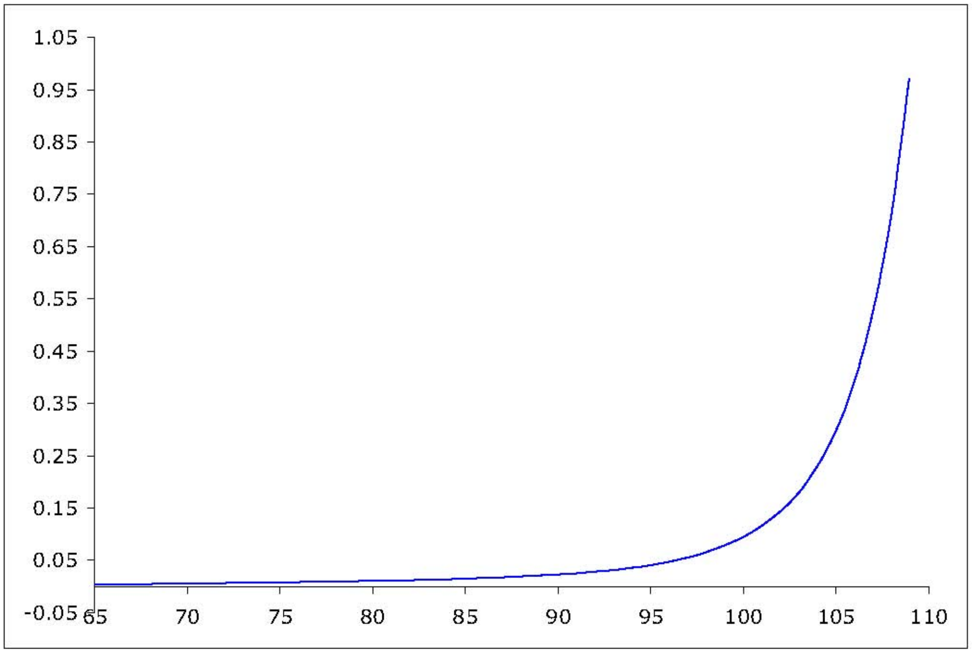
Under the valuation model we are assuming, the value of the cash flow to the insurer (fixed payor) for an n–year swap is:

$$V = 1000\left[\sum_{t=1}^{n} \mathsf{E}(\ell_{x+t})d(0,t) - \sum_{i=t}^{n} X_t d(0,t)\right] \qquad (3)$$

where $\mathrm{E}(l_{x+t})$ denotes the expected number of survivors among the N initial annuitants and $d(0,t)$ is the discount factor based on the current bond market prices. If the counterparties agree to $X_t = \mathrm{E}(l_{x+t})$ then V=0, then V=0 and no initial exchange of cash is required to initiate the swap.

We point out that, given the distribution of survivors, there is very little variance in the cash flows. For example, given the survivor function ${}_tp_x$ we can describe l_{x+t} as a binomial distribution. It is the number of successes in N trials with the probability of a success on a given trial of ${}_tp_x$. The distribution of l_{x+t} is approximately normal with parameters $\mu_t = N{}_tp_x$ and $\sigma_t = \sqrt{N{}_tp_x(1-{}_tp_x)}$. The coefficient of variation is the ratio of σ_t/μ_t. The graph of the coefficient of variation of the number of survivors for an initial group of 10,000 annuitants, based on the 1994 GAM female (65) survival distribution is shown in Figure 6. Note that for a swap of duration 30 years, the coefficient of variation rises to a maximum of about 1 percent , so there is little risk, *given the table*. The risk arises from uncertainty in the table. In calculating the swap value, we have to evaluate the expected value $\mathrm{E}(l_{x+t})$ carefully. It is not enough to estimate a mortality table and then estimate the expected value. That approach would ignore the uncertainty in the table.

Figure 6. The ratio of standard deviation to expected number of survivors of an initial group of 10,000 annuitants, based on the 1994 GAM female (65) mortality distribution



In order to illustrate this further, suppose that the possible tables are labeled with a random variable θ. The conditional distribution $l_{x+t}|\theta$ depends on θ. The unconditional moments are:

$$\mathrm{E}[l_{x+t}] = \mathrm{E}[\mathrm{E}[l_{x+t}|\theta] = N\mathrm{E}[\mathrm{E}[{}_tp_x|\theta]]$$
$$\mathrm{Var}[l_{x+t}] = \mathrm{E}[\mathrm{Var}[l_{x+t}|\theta]] + \mathrm{Var}\ [\mathrm{E}[l_{x+t}|\theta]]$$

Even if, as in Figure 6, there is very little variance in E[$l_{x+t}|\theta$] for all θ and the range of $t \le 30$, there is still variance due to table uncertainty (the first term). We have little experience to guide us in estimating the terms E[E[${}_tp_x \mid \theta$]] and E[Var[${}_tp_x \mid \theta$]]. Of course, this uncertainty occurs in all kinds of mortality derivatives, not just swaps.

6. Mortality Risk Bonds

Wang (1996, 2000, 2001) has developed a method of pricing risks that unifies financial and insurance pricing theories. We are going to apply this method to price mortality risk bonds. Let $\phi(x)$ be the standard normal cumulative distribution function with a probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-X^{2/2}}$$

for all x. Wang defines the distortion operator as

$$g\lambda(u) = \phi\left[\phi^{-1}(u) - \lambda\right]$$

for 0<u<1 and a parameter λ. Now, given a distribution with cumulative density function F(t), a "distorted" distribution $F^*(t)$ is determined by λ according to the equation:

$$F^*(t)=g_\lambda(F)(t)=g_\lambda(F(t))(6).$$

Consider an insurer's liability X over a time horizon [0,T]. The value or fair price of the liability is the discounted expected value under the distribution obtained from the distortion operator. Omitting the discount for now, we have the formula for the price:

$$H(X,\lambda)=E^*(X)=xdF^*(x)(7)$$

where $F^*(x)=g_\lambda(F)(x)=\Phi[\Phi-^{1}(F(x)) - \lambda]$. The parameter λ is called the market price of risk, reflecting the level of systematic risk. Thus, for an insurer's given liability X with cumulative density function F, the Wang transform will produce a "risk-adjusted" density function F^*. The mean value under F^*, denoted by $E^*[X]$, will define a risk-adjusted "fair value" of X at time T, which can be further discounted to time zero, using the risk-free rate. Wang's paper describes the utility of this approach. It turns out to be very general and a generalization of well known techniques in finance and actuarial science. Our idea is to use observed annuity prices to estimate the market price of risk for annuity mortality, then use the same distribution to price mortality bonds.

6.1. Market Price of Risk

First we estimate the market price of risk λ. We defined our transformed distribution F^* as:

$$F^*(t)=g_\lambda(F)(t)=\Phi[\Phi-^{-1}({}_tq_{65}) - \lambda]. \quad (8)$$

For the distribution function $F(t)={}_tq_{65}$ we use the 1996 IAM 2000 Basic Table for a male life age 65 and, separately, for a female life age 65. Then assuming a commission rate equal to 4 percent, we use the 1996 market quotes of nonqualified immediate annuities and the 1996 U.S. Treasury yield curve to get the market price of risk λ by solving the following equations numerically:

(12)

$128.40 = 7.48 a_{65}$ for males,

(12)

$138.39 = 6.94 a_{65}$ for females.

The market price of risk for males and females respectively is shown in Table 2 and Figure 6. The risk loads are 0.2134 for male annuitants and 0.2800 for female annuitants. Figure 6 shows that the market prices of the annuities are higher than the mortality experience of the 1996 IAM 2000 Basic Table, and the market curve lies above the 1996 IAM 2000 Basic mortality experience curve. We think of the 1996 IAM 2000 Basic Table as the actual or physical distribution, which requires a distortion to obtain market prices. That is, a risk premium is required for pricing annuities.

6.2. Mortality Bond Structure

Like the mortality swap, a designed portfolio of annuities underlies the mortality bond. Suppose that N annuitants are specified, all age x=65 at the time the bond is issued. Mortality bond contracts may specify a mortality table on which both the bondholders and the insurer agree (e.g., the 1996 U.S. Annuity 2000 Basic Mortality Table).

Figure 7. The result of applying the Wang transform to the survival distribution based on 1996 IAM experience for males (65) and prices from Best's Review, 1996.

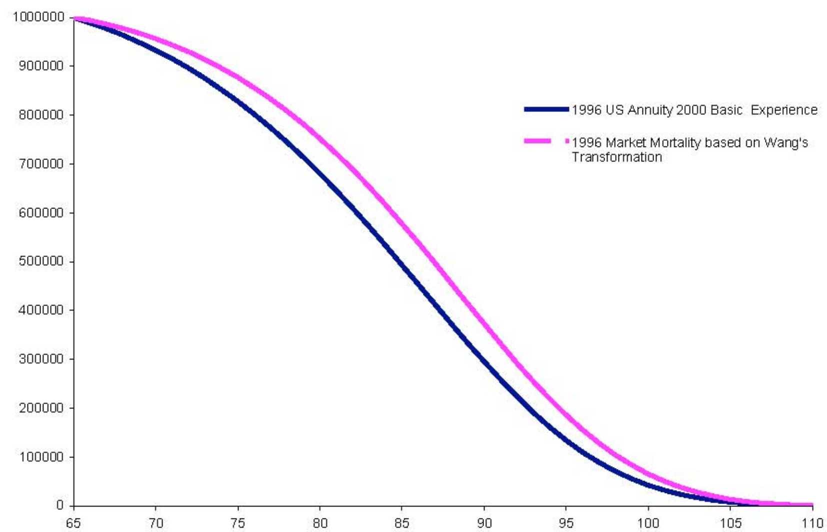


Table 2. The market price of risk, determined by the 1996 IAM 2000 Basic Table, the U.S. Treasury constant maturity interest rate term structure for December 29, 1996 and annuity market prices (without commission) from Best's Review (1996). The payment rate is the dollars per month of life annuity per $1,000 of annuity premium at the issue age. The market value is the price (net of commission) for $1 per month of life annuity.

Payment Rate		Market Value	Market price of risk
Male (65)	7.48	128.40	0.2134
Female (65)	6.94	138.39	0.2800

Moreover, the mortality contract may also set several improvement levels on the forces of mortality of each age to reflect the future mortality improvement. In our example, we set three different improvement levels for male and female (65) immediate annuities:

(i) 0.0070 for age from 65 to 74;
(ii) 0.0105 for age from75 to 84;
(iii) 0.0140 for age from 85 to 94.

Including the above improvement factors, the corresponding strike level for each age will be $\overline{l}_{65+t}$. The number of survivors l_{65+t} is the number of lives attaining age in the survivorship group set in the contract. We define the bond contract so that the coupons are risky, but the principal is always paid at maturity. The bondholders will get the coupon payment C if the actual number of survivors at time t is smaller than the strike level $\overline{l}_{65+t}$. Otherwise, they will get nothing. That is, the bondholder's payment at the end of year t is

$$D_t = \begin{cases} C & \text{if } l_{65+t} \leq \overline{l}_{65+t} \\ 0 & \text{if } l_{65+t} > \overline{l}_{65+t} \end{cases}$$

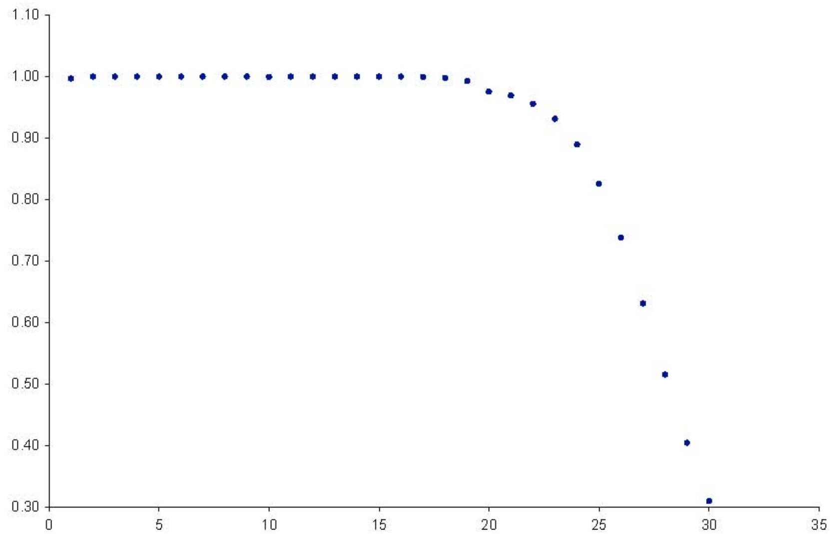
for t=1,2,...,T where T is the term of the mortality bond, 30 years when the bond is issued.

Suppose we know the survival distribution for the pool of N annuitants upon which the bond is based, so we know the survival probability ${}_tp_{65}$. Then the distribution of the number of survivors has a binomial distribution with number of trials N and success probability ${}_tp_{65}$. Since N is rather large, we can use the normal approximation with parameters $m_t=N{}_tp_{65}$ and $s_t=N{}_tp_{65}(1\ {}_tp_{65})$ to get the expected value of the bondholder's coupon:

$$\begin{aligned} \mathrm{E}[D_t] &= \Pr(l_{65+t} \leq \overline{l}_{65+t}) \\ &\approx \phi \frac{\overline{l}_{65+t} - m_t}{s_t} \end{aligned}$$

where Φ(z) denotes the standard normal cumulative density. Figure 8 shows the $E[D_t]$ based on the 1996 US Annuity 2000 Basic mortality experience (female age 65).

Figure 8. The expected values of bondholder's payment E[Dt] for a coupon rate of C=1, based on the 1996 U.S. Annuity 2000 Basic tables for males age 65.



This is a calculation that one might perform when the bond is designed. The strike levels ◎$_{65+t}$ can be specified at this point. Lower levels provide more protection for the issuer and greater risk to the bondholder. With this mortality bond design, the bondholders are more likely to get the coupons in the earlier years than in the later years. If we assume that the bondholder will get the face value when the mortality bond matures, the price of the mortality bond will be

$$P = Fd(0,T) + C\sum_{t=0}^{T} E^{*}[Dt]d(0,t)$$

where d(0,t) is the discount factor based on the risk-free-interest-rate-term structure at the time the bond is issued. The face amount F is not at risk; it is paid at time T regardless of the number of surviving annuitants. E^* [D_t] denotes the expected value based on the market mortality table. The survival distribution in equation (12) is the distribution derived from the annuity market. It is based on the 1996 U.S. Annuity 2000 Basic Mortality Tables and the Wang transform (8) with λ=0.2134 for male annuitants and λ=0.2800 for females. The discount factors are from the U.S. Treasury interest-rate-term structure on December 29, 1996. Table 3 shows prices for a mortality bond for a group of 10,000 male annuitants, with the strike levels defined above and a 7 percent coupon rate. The price of the mortality bond for a bond based on male (65) immediate annuitants is $981.53 per $1000 of face value. Similarly, we can get the bond price for the female (65) immediate annuitants, which is $959.10 per $1000.

Table 3. The survival distribution underlying the 1996 immediate annuity market based on the 1996 U.S. Annuity 2000 Basic Mortality Table, the Wand transform and 1996 US Treasury interest rates on December 29.

	Male (65)	Female (65)
Market price of risk (λ)	0.2134	0.2800
Face value	1,000	1,000
Coupon rate	0.07	0.07
Number of annuitants	10,000	10,000
Improvement level age 65-74	-0.0070	-0.0070
Improvement level age 75 -84	-0.0105	-0.0105
Improvement level age 85 94	-0.0140	-0.0140
Price	981.53	959.10
Straight Bond Price	1013.32	1013.32

6.3. Insurer's mortality bond hedge

The actual annuity payments of an insurer in the future are based on the future actual mortality experience. However, we can study how it might turn out under different scenarios. Assume an insurer has to pay a group of 10,000 annuitants $1,000 per year if the annuitants survive at the end of the year. Suppose also that annuity-based bonds, as described above, are available as a hedge. The insurer sells k-bonds for a total face amount of 1,000 k with each bond based on the same pool of 10,000 annuitants. At the same time the insurer buys k-straight bonds with the same coupon rate as the annuity-based bonds. Assuming the annuitants are females, the net cost of the two bond transactions is 1013.32 k $-$959.10k=54.22k.

The number of bonds can be selected by the insurer and the market. That is, a bond contract can be designed for a given annuitant pool, and then the bond can be marketed in units of $1,000 of face value. The annuitant pool plays the role of an index with each bond providing an embedded option on the index. If the insurer creates a hedge involving k-mortality bonds and k-straight bonds, then the insurer's net cash flow corresponding to $1,000 of initial annuity liability is random each year. It can be written as the payments to annuitants, plus payments to mortality bondholders, less payments from straight bond issuers:

Annuity payments =1,000 l_{x+t}

Plus coupons to mortality bondholders $= \begin{cases} kC & \text{if } l_{x+t} \le \bar{l}_{x+t} \\ 0 & \text{if } l_{x+t} > \bar{l}_{x+t} \end{cases}$

Minus coupons from straight bond issuers $= kC$

Equals net cash outflow per 1,000 $= \begin{cases} 1000 l_{x+t} & \text{if } l_{x+t} \le \bar{l}_{x+t} \\ 1000 l_{x+t} - kC & \text{if } l_{x+t} > \bar{l}_{x+t} \end{cases}$

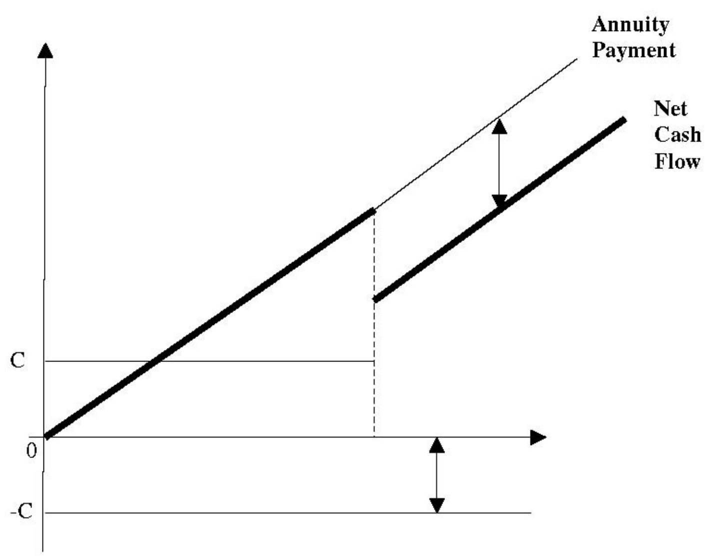
In our example C=70 and l_x=10,000. In this case the insurer might issue k=10,000 bonds with a total face value of $10 million. The cost of the hedge is $542,200 and the hedge provides coverage in each of 30 future years. The hedge pays $700,000 in each year in which the number of annuitants exceeds the strike level. The present value varies with the mortality tables, of course.

One of the most important functions of introducing mortality bonds is to hedge the cash flows of an insurer and reduce the impact of mortality improvement. The following example, illustrated in Figure 9, shows how mortality bonds function as a hedge against improving mortality. Suppose that an insurer sells a $10,000,000 face value of mortality bonds based on a group of 10,000 male (65) annuitants with a 7 percent coupon rate and at the same time buys a $10,000,000 straight bond with a 7 percent coupon rate. The insurer has to pay the surviving annuitants $1,000 per year. If the actual number of survivors is less than the strike level $\bar{l}_{x+t}$ in the contract, the mortality bond coupons are exactly offset by the coupons from the straight bond. If the actual number of survivors is more than the strike level $\bar{l}_{x+t}$, the insurer does not pay the mortality bond coupon so the straight bond coupon reduces the cash outflow. The total cash outflow is shifted down, below the actual annuity payment level. This is how mortality bonds mitigate the impact of excess mortality improvement relative to the insurer's expectation.

7. Discussion and Conclusions

Financial innovation has led to the creation of new classes of securities that provides opportunities for insurers to manage their underwriting and to price risks more efficiently. Cummins and Lewis (2002) establish that risk expansion helps to explain the development of catastrophic risk bonds and options in the 1990s. A similar expansion is needed to manage longevity risk. There is a growing demand for a long-term hedge against improving annuity mortality. We have shown how innovation in swaps, options and bond contracts can provide new securities which can provide the hedge insurers need.

Figure 9. The number of survivors l_{x+t} is on the horizontal axis and the insurer's payment on the vertical axis. If the number of survivors is more than the strike level l_{x+t}, the insurer does not pay the mortality bond coupons so the regular bond coupons reduce the cash outflow. If the number of survivors is below the strike level, the coupons are equal and cancel each other. The total cash outflow drops by the coupon amount when the number of survivors exceeds the strike level.



There is a trend of privatizing social security systems with insurers taking more longevity risk. Moreover, the trend to defined-contribution corporate pension plans is increasing the potential market for immediate annuities. This is an opportunity and also a challenge to insurers. Insurers will need increased capacity to take on longevity risk, and securities markets can provide it. This will allow life insurers to share this "big cake." Compared with the reinsurance market, securitization of mortality risks has longer duration, higher capacity and possibly lower cost. Demand for new securities arises when new risks appear and when existing risks become more significant in magnitude. And we now have the technology to securitize the mortality risks based on modern financial models. Securitization in the annuity and life insurance markets has been relatively rare, but we have argued that this may change. We explored the securitization of mortality risks showing how it can help solve the difficulties in managing annuity mortality risk.

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Appendix

Summary of Data

We collected the data from the Society of Actuaries Transactions Reports for each of the years for which there was data. We used reports for calendar years published for the years 1951, 1961, 1971 and each year from 1981 to 1992. The last report is based on 1992 experience. We understand that the Society of Actuaries is reviving its experience studies.

1951						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	335.70	11.00	1174.25	10.00	1509.95	21.00
60-64	12102.34	308.00	3847.76	57.00	15950.10	365.00
65-69	39871.68	1413.00	4602.89	91.00	44474.57	1504.00
70-74	17218.98	958.00	1737.57	63.00	18956.55	1021.00
75-79	5873.40	484.00	666.00	37.00	6539.40	521.00
80-84	1774.33	226.00	209.00	26.00	1983.33	252.00
85-89	374.08	68.00	51.25	8.00	425.33	76.00
90-94	47.42	15.00	7.00	2.00	54.42	17.00

1961						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	1,371.88	36.00	2,454.63	18.00	3,826.51	54.00
60-64	23,718.46	605.00	9,902.34	116.00	33,620.80	721.00
65-69	96,620.43	3,371.00	19,390.30	333.00	116,010.73	3,704.00
70-74	60,560.45	3,371.00	10,594.01	349.00	71,154.46	3,720.00
75-79	26,772.96	2,275.00	3,901.58	195.00	30,674.54	2,470.00
80-84	7,701.84	1,002.00	1,057.17	109.00	8,759.01	1,111.00
85-89	1,717.08	310.00	275.00	35.00	1,992.08	345.00
90-94	254.42	59.00	39.00	7.00	293.42	66.00

1971						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	3,611.23	85.00	3,574.90	26.00	7,186.13	111.00
60-64	33,806.66	791.00	18,521.74	177.00	52,328.40	968.00
65-69	120,227.85	4,022.00	41,802.04	595.00	162,029.89	4,617.00
70-74	93,795.47	4,955.00	28,542.94	746.00	122,338.41	5,701.00
75-79	63,066.93	5,269.00	16,284.46	747.00	79,351.39	6,016.00
80-84	28,166.41	3,113.00	6,815.79	510.00	34,982.20	3,623.00
85-89	8,022.23	1,315.00	1,699.37	213.00	9,721.60	1,528.00
90-94	1,328.05	338.00	251.95	51.00	1,580.00	389.00

1981						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	26,599.21	440.00	11,124.59	99.00	37,723.80	539.00
60-64	82,756.29	1,568.00	32,978.18	347.00	115,734.47	1,915.00
65-69	185,232.93	4,924.00	73,727.06	1,003.00	258,959.99	5,927.00
70-74	157,276.45	6,571.00	68,210.94	1,397.00	225,487.39	7,968.00
75-79	97,763.34	6,189.00	42,614.73	1,347.00	140,378.07	7,536.00
80-84	48,755.90	4,727.00	20,588.86	1,093.00	69,344.76	5,820.00
85-89	19,601.58	2,719.00	7,936.75	681.00	27,538.33	3,400.00
90-94	4,980.49	990.00	2,087.62	294.00	7,068.11	1,284.00

Group annuity experience 1951, 1961, 1971 and 1981

1982						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	28,631.53	453.00	11,754.62	92.00	40,386.15	545.00
60-64	89,455.43	1,753.00	35,433.49	336.00	124,888.92	2,089.00
65-69	192,308.39	5,097.00	75,640.56	985.00	267,948.95	6,082.00
70-74	162,420.78	6,740.00	72,661.69	1,354.00	235,082.47	8,094.00
75-79	103,419.33	6,465.00	48,058.37	1,540.00	151,477.70	8,005.00
80-84	52,549.11	4,861.00	23,671.10	1,231.00	76,220.21	6,092.00
85-89	21,392.48	2,989.00	9,443.51	832.00	30,835.99	3,821.00
90-94	5,716.77	1,082.00	2,526.42	322.00	8,243.19	1,404.00

1983						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	33,163.65	510.00	13,783.18	117.00	46,946.83	627.00
60-64	98,632.53	1,868.00	41,665.68	435.00	140,298.21	2,303.00
65-69	195,074.64	5,153.00	79,663.64	1,103.00	274,738.28	6,256.00
70-74	170,348.65	6,995.00	72,621.93	1,511.00	242,970.58	8,506.00
75-79	107,213.60	6,964.00	48,482.16	1,613.00	155,695.76	8,577.00
80-84	57,936.04	5,399.00	24,237.52	1,388.00	82,173.56	6,787.00
85-89	22,035.27	3,111.00	9,528.77	895.00	31,564.04	4,006.00
90-94	6,136.86	1,218.00	2,725.40	373.00	8,862.26	1,591.00

1984						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	40,574.69	580.00	16,305.25	132.00	56,879.94	712.00
60-64	119,381.14	2,212.00	48,941.94	448.00	168,323.08	2,660.00
65-69	221,883.84	5,695.00	91,062.97	1,241.00	312,946.81	6,936.00
70-74	200,590.93	8,196.00	86,304.56	1,870.00	286,895.49	10,066.00
75-79	129,357.81	8,141.00	60,361.35	2,106.00	189,719.16	10,247.00
80-84	67,297.97	6,288.00	31,781.28	1,771.00	99,079.25	8,059.00
85-89	26,575.80	3,766.00	12,400.26	1,211.00	38,976.06	4,977.00
90-94	7,743.72	1,574.00	3,681.76	573.00	11,425.48	2,147.00

1985						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	43,299.71	656.00	17,016.15	146.00	60,315.86	802.00
60-64	123,040.09	2,386.00	50,603.92	565.00	173,644.01	2,951.00
65-69	223,999.93	6,226.00	93,571.37	1,368.00	317,571.30	7,594.00
70-74	207,718.42	9,000.00	90,306.94	2,050.00	298,025.36	11,050.00
75-79	137,102.94	9,186.00	65,194.85	2,426.00	202,297.79	11,612.00
80-84	71,953.72	7,141.00	35,412.31	2,137.00	107,366.03	9,278.00
85-89	28,655.87	4,287.00	14,095.45	1,437.00	42,751.32	5,724.00
90-94	8,411.94	1,812.00	4,179.97	671.00	12,591.91	2,483.00

Group annuity experience 1982 - 1985

1986						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	44,010.72	627.00	16,677.86	112.00	60,688.58	739.00
60-64	122,620.42	2,163.00	50,381.10	476.00	173,001.52	2,639.00
65-69	227,995.35	5,699.41	95,512.26	1,261.00	323,507.61	6,960.41
70-74	216,055.50	8,098.29	93,727.78	1,966.00	309,783.28	10,064.29
75-79	146,182.97	8,610.00	68,834.32	2,324.00	215,017.29	10,934.00
80-84	78,070.67	7,153.00	38,836.55	2,108.00	116,907.22	9,261.00
85-89	31,484.42	4,005.00	15,650.49	1,406.00	47,134.91	5,411.00
90-94	9,097.10	1,678.00	4,672.65	690.00	13,769.75	2,368.00
1987						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	47,303.94	598.00	17,781.62	134.00	65,085.56	732.00
60-64	129,028.29	2,138.00	53,226.99	533.00	182,255.28	2,671.00
65-69	238,848.85	5,773.00	101,240.19	1,356.00	340,089.04	7,129.00
70-74	223,665.17	8,714.00	98,442.35	2,054.00	322,107.52	10,768.00
75-79	157,461.29	9,443.00	74,752.64	2,525.00	232,213.93	11,968.00
80-84	83,820.45	7,671.00	43,600.05	2,452.00	127,420.50	10,123.00
85-89	34,094.97	4,590.00	18,036.28	1,677.00	52,131.25	6,267.00
90-94	9,836.78	1,921.00	5,395.54	825.00	15,232.32	2,746.00
1988						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	49,424.32	683.00	18,162.87	141.00	67,587.19	824.00
60-64	132,778.58	2,252.00	53,788.54	513.00	186,567.12	2,765.00
65-69	235,874.82	5,587.00	102,022.53	1,295.00	337,897.35	6,882.00
70-74	221,164.05	8,388.00	99,853.21	2,116.00	321,017.26	10,504.00
75-79	162,202.31	9,530.00	78,542.78	2,630.00	240,745.09	12,160.00
80-84	88,225.65	8,012.00	47,418.51	2,583.00	135,644.16	10,595.00
85-89	35,929.54	4,707.00	20,142.57	1,879.00	56,072.11	6,586.00
90-94	10,484.98	2,002.00	5,926.74	845.00	16,411.72	2,847.00
1989						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	45,167.60	580.00	19,788.90	138.00	64,956.50	718.00
60-64	120,348.84	2,008.00	53,312.98	488.00	173,661.82	2,496.00
65-69	201,223.57	4,827.00	94,345.49	1,235.00	295,569.06	6,062.00
70-74	180,723.00	6,748.00	88,016.87	1,829.00	268,739.87	8,577.00
75-79	134,297.88	7,852.00	70,107.48	2,357.00	204,405.36	10,209.00
80-84	72,524.22	6,606.00	41,921.07	2,353.00	114,445.29	8,959.00
85-89	29,672.14	3,992.00	18,031.93	1,628.00	47,704.07	5,620.00
90-94	8,245.34	1,704.00	5,114.09	820.00	13,359.43	2,524.00

Group annuity experience 1986 - 1989

1990						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	53,375.95	686.00	24,851.00	174.00	78,226.95	860.00
60-64	146,190.29	2,333.00	67,235.53	596.00	213,425.82	2,929.00
65-69	258,735.98	5,949.00	122,669.86	1,562.00	381,405.84	7,511.00
70-74	238,694.07	8,911.00	116,031.28	2,327.00	354,725.35	11,238.00
75-79	189,088.76	11,105.00	95,064.28	3,186.00	284,153.04	14,291.00
80-84	109,583.14	9,912.00	62,967.19	3,520.00	172,550.33	13,432.00
85-89	48,022.47	6,572.00	30,700.37	2,778.00	78,722.84	9,350.00
90-94	14,672.14	2,842.00	10,005.89	1,445.00	24,678.03	4,287.00

1991						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	50,731.54	661.00	22,245.01	158.00	72,976.55	819.00
60-64	137,582.08	2,383.00	60,722.23	543.00	198,304.31	2,926.00
65-69	240,820.91	5,774.00	114,994.74	1,557.00	355,815.65	7,331.00
70-74	230,909.08	8,685.00	115,825.34	2,433.00	346,734.42	11,118.00
75-79	188,317.23	10,961.00	96,727.27	3,360.00	285,044.50	14,321.00
80-84	112,587.59	10,048.00	66,245.62	3,791.00	178,833.21	13,839.00
85-89	48,883.89	6,713.00	33,022.70	2,996.00	81,906.59	9,709.00
90-94	15,033.98	2,901.00	10,909.55	1,624.00	25,943.53	4,525.00

1992						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	47,790.52	689.00	20,925.44	156.00	68,715.96	845.00
60-64	122,033.83	2,143.00	55,616.52	466.00	177,650.35	2,609.00
65-69	216,153.60	5,124.00	107,068.38	1,429.00	323,221.98	6,553.00
70-74	212,415.17	7,526.00	111,099.67	2,260.00	323,514.84	9,786.00
75-79	173,061.53	9,440.00	91,863.84	3,044.00	264,925.37	12,484.00
80-84	106,152.91	9,177.00	63,719.81	3,349.00	169,872.72	12,526.00
85-89	47,214.93	6,190.00	33,278.32	2,984.00	80,493.25	9,174.00
90-94	15,059.41	2,859.00	11,268.86	1,634.00	26,328.27	4,493.00

Group annuity experience 1990 - 1992